The Fisher Hypothesis: A Revisit with Covariate Tests

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Suggested Running Head: The Fisher Hypothesis

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Abstract
This paper reexamines the validity of the Fisher hypothesis by employing a battery of covariate tests to the \textit{ex post} real interest rates in 13 developing economies. We first present simulation evidence using the method proposed by Rudebusch (1993), showing that for over half of the countries, the univariate tests of Ng and Perron (2001) suffer from low power for determining their time-series properties. However, when covariate tests are used, strong evidence favoring mean reversion in real interest rates is uncovered due to power improvement. In sharp contrast, the nonlinear unit root test of Kapetanios et al. (2003) only makes two rejections of unit roots for the same countries. In sum, our findings support the Fisher hypothesis in almost all developing countries with the exception of Indonesia.

\textit{JEL} classifications: C32; E44; G12

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1 Introduction

The Fisher hypothesis is a main tenet of economic and financial theory. According to the hypothesis, the nominal interest rate and expected inflation move together one-for-one in the long run, implying that changes in inflation expectations are fully reflected in nominal interest rate adjustment and therefore conforms to the theory of long-run neutrality. For the validity of the Fisher hypothesis, the real interest rate—the difference between the nominal interest rate and expected inflation rate—should not contain a unit root.

An empirical investigation of the time-series properties of the real interest rate is important. As mentioned above, it can confirm the validity of the Fisher effect and the theory of long-run neutrality. More importantly, consumption-based asset pricing models, for example, Lucas (1978), Breeden (1979), and Hansen and Singleton (1983), imply that the real interest rate and growth rate of consumption should share similar time-series properties. Hence, if the real interest rate is found to contain a unit root, as in Rose (1988), this will be puzzling since the empirical evidence unanimously shows that shocks to the consumption rate tend to be short-lived.

Since Rose (1988), mixed results have been obtained by a number of studies using different econometric approaches (e.g., Mishkin, 1995; Evans and Lewis, 1995; Crowder and Hoffman, 1996; Lai, 1997). Noticeably, even with the state-of-the-art univariate unit root tests of Ng and Perron (2001) tests (NP, henceforth), Rapach and Weber (2004), for example, failed to find strong evidence supporting mean reversion of the real interest rates for most developed countries. Similar results can be found in Koustas and Serletis (1999) and Lanne (2001).

Two possible explanations for this perplexing finding are frequently documented
in the literature. First, the univariate unit-root tests usually lack adequate power against highly persistent series, such as the real interest rate. This argument will be verified in the present empirical study. Second, the commonly used tests often take the unit root as the null hypothesis, which cannot be rejected unless there is very strong evidence against it.

To deal with these two deficiencies inherent to the conventional unit root tests, we will employ a battery of covariate tests to re-examine the time series properties of the real interest rates in 13 developing countries. Intuitively, macroeconomic variables are correlated and rarely observed in isolation. For example, based on the Fisher hypothesis, the real interest rate should have correlations with the nominal interest rate and the inflation rate. As noted by Hansen (1995), the related variables contain useful information that can be exploited to boost the power of the univariate unit root tests. Moreover, it is easy to find correlated covariates for the covariate tests in our empirical study.

The covariate tests we will use include the covariate ADF (CADF) test of Hansen (1995), the covariate feasible point optimal (CPT) test of Elliott and Jansson (2003), and the stationarity test proposed by Jansson (2004). The first two tests take nonstationarity as the null hypothesis, whereas the last one differs in that the series is stationary under the null, avoiding biasing test results in favor of the unit root. These covariate tests have been applied to various empirical studies and have yielded results that are in sharp contrast to those from their univariate counterparts (e.g., Amara and Papell, 2006; Amara and Murphy, 2005; Elliott and Pesavento, 2006).

The contribution of this paper to the empirical literature on the Fisher hypothesis is threefold. First, to the best of our knowledge, this is the first paper to employ the extant covariate tests to examine the time-series behavior of real interest rates in
developing countries. In this respect, previous studies mainly focused on the U.S. and other developed countries rather than on developing countries. Whether the results for developed countries can be extended to the case of developing countries is, however, an open question. Arguably, researchers seem to be curious about the applicability of the Fisher effect to not only developed but also developing countries.

Second, we provide simulation evidence on the performance of the NP tests using Rudebusch’s (1993) method. Specifically, we simulate the finite sample distributions of the NP tests under the null and alternative, respectively. Aided by the simulated distributions, the sizes and powers of the statistics can easily be evaluated for each real interest rate under investigation. The compelling evidence shows that the NP tests often have low power to reject the unit-root hypothesis and spuriously invalidate the Fisher effect for over half of the 13 developing countries investigated.

Third, for comparison, the nonlinear unit root test developed by Kapetanios et al. (2003) (hereafter, the KSS test) is also applied to assess whether the possible presence of nonlinearity in the real interest rate can explain the non-rejection of the null hypothesis for the NP tests. The literature has documented a few potential factors for the presence of nonlinear behavior in the real interest rate, such as the conduction of monetary policies, and the transaction cost. In this case, the KSS test will therefore make more rejections than the NP tests.

The empirical results of our study reveal that using covariate tests yields strong evidence against the unit-root dynamics in the real interest rates for countries where the NP tests are unable to reach a decisive conclusion based on simulated distributions. In contrast, the KSS test performed poorly in finding evidence in favor of mean reversion in that it only made two rejections of the unit root for the same countries. Combining the results of the NP and covariate tests, our empirical study provide
extensive evidence in favor of no unit root in the real interest rates and supports the validity of the Fisher effect for 12 developing countries except for the case of Indonesia.

The remainder of this study is set out as follows. Section 2 provides an overview of the covariate tests used in this paper. Section 3 describes the data and reports the preliminary results of the empirical analysis by applying Rudebusch’s (1993) method to the NP tests. Section 4 reports results of the covariate tests. For comparison, the results of the KSS test are presented in Section 5. Some concluding remarks are provided in Section 6.

2 Covariate tests

2.1 Covariate augmented Dickey-Fuller (CADF) test

As noted in Hansen (1995), the ADF test, which ignores the information contained in other macroeconomic variables related to the time series to be tested, generally suffers from low statistical power. By incorporating a number of stationary covariates into the regression equation, Hansen then put forward the CADF test claiming that the test can deliver substantial power gains over the ADF test due to a more precise estimate of the AR coefficient. For easy exposition, let $y_t$ and $x_t$ denote the real interest rate and an $m$-vector of stationary covariate, respectively. The CADF test is the conventional $t$-statistic for testing the unit-root null hypothesis that $H_0 : \phi = 0$ against the stationary alternative hypothesis that $H_1 : \phi < 0$ in the following regression:

$$\Delta y_t = \mu + \phi y_{t-1} + \sum_{i=1}^{\rho} \alpha_i \Delta y_{t-i} + \sum_{j=-q_2}^{q_1} \beta_j x_{t-j} + \epsilon_t,$$  

which is the conventional ADF regression augmented with the leads ($q_2$) and lags ($q_1$)
of } x_j \text{ provided that } \beta_j \neq 0. \text{ Hansen also derives the asymptotic distribution of the CADF test, which is a convex combination of the ADF and the standard normal distributions; the combination depends on the nuisance parameter of the squared correlation } (r^2) \text{ between the equation disturbance } (\varepsilon_t) \text{ and the ADF regression error } (v_t = \sum_{j=-q_1}^{q_1} \beta_j x_{t-j} + \varepsilon_t). \text{ The smaller the value of } r^2, \text{ the larger are the power gains the CADF test can achieve. }

Because of the dependence of the distribution on the nuisance parameter, when implementing the CADF test, one needs to consistently estimate it and then finds suitable critical values from Table 1 in Hansen (1995). As recommended by Hansen (1995), the parameter of } r^2 \text{ is estimated using the nonparametric method given by }

\[
\hat{r}^2 = \frac{\hat{\sigma}_\varepsilon^2}{\hat{\sigma}_\varepsilon^2 + \hat{\sigma}_\varepsilon^2},
\]

where

\[
\begin{bmatrix}
\hat{\sigma}_\varepsilon^2 & \hat{\sigma}_{ve} \\
\hat{\sigma}_{ve} & \hat{\sigma}_\varepsilon^2
\end{bmatrix} = \sum_{k=-M}^{M} \omega(k/M) \frac{1}{T} \sum_t \hat{\eta}_{t-k} \hat{\eta}_t,
\]

with \( \hat{\eta}_t = (\hat{\varepsilon}_t, \hat{\varepsilon}_t) \); \( \hat{v}_t = \sum_{j=-q_1}^{q_1} \hat{\beta}_j (x_{t-j} - \bar{x}) + \hat{\varepsilon}_t \), where \( \hat{\varepsilon}_t \) and \( \hat{\beta}_j \)'s are the ordinary least squares (OLS) residual and estimates for \( \beta_j \)'s in Eq. (1), respectively. The functions \( \omega \) and \( M \) in Eq. (3) are the kernel function and bandwidth, respectively. We used the Bartlett kernel and the method in Andrew (1991) to choose the bandwidth in the following empirical study. In addition, the optimal values of \( p, q_l \) and \( q_z \) in Eq. (1) were determined by minimizing the Bayesian information criterion (BIC) with the maximum lag set at 5.

### 2.2 Covariate point optimal test (CPT)
The second covariate test employed in our empirical study is the CPT test put forward by Elliott and Jansson (2003). Motivated by the sequence of Neyman-Pearson tests as found in Elliott et al. (1996), Elliott and Jansson (2003) derive the power envelope for the CPT test and show that the test can achieve higher power than the CADF test.

To construct the CPT test for our empirical purpose, suppose $y_t$ co-varies with $x_t$ in the following way:

$$
\begin{bmatrix}
y_t \\
x_t
\end{bmatrix}
= \begin{bmatrix}
\beta_{y0} \\
\beta_{x0}
\end{bmatrix} + \begin{bmatrix}
u_{y,t} \\
u_{x,t}
\end{bmatrix}, \quad t = 1, \ldots, T
$$

(4)

and

$$
A(L) \begin{bmatrix}
(1-\rho L)u_{y,t} \\
u_{x,t}
\end{bmatrix} = \epsilon_t,
$$

(5)

where $A(L)$ is a $k$-order polynomial in the lag operator $L$. The CPT test examines the unit root null of $\rho = 1$ against the alternative of $\rho < 1$, and is calculated by running a VAR($k$) on the model of the generalized least squares (GLS) detrended data under the null and alternative and is given by

$$
\text{CPT}(1, \overline{\rho}) = T \left( tr \left[ \tilde{\Sigma}(1)^{-1}\tilde{\Sigma}(\overline{\rho}) \right] - (m + \overline{\rho}) \right),
$$

(6)

where $m$ is the dimension of $x_t$, $\overline{\rho} = 1 - 7/T$ in our case, and $\tilde{\Sigma}$’s are the estimated variance-covariance matrices of residuals obtained by running VAR’s under $\rho = 1$ and $\overline{\rho}$. Again, the criterion of BIC was used for the selection of the VAR lag, as suggested by Elliott and Jansson (2003), and the maximum lag is set at 5. Like the CADF test, the asymptotic distribution of the CPT test is also dependent on a nuisance parameter ($R^2$), which represents the contribution of covariates to the power of the test. The larger the value of $R^2$ is, the more powerful the CPT test will be.
Similarly, the parameter of $R^2$ must be consistently estimated, and the corresponding estimate ($\hat{R}^2$) is used to determine the appropriate critical value from Table 1 in Elliott and Jansson (2003).  

### 2.3 Covariate stationarity test

The covariate stationarity test we will use, denoted by $Q_T$, is proposed by Jansson (2004), and is briefly described as follows. Consider the following model given by

$$
\begin{pmatrix}
y_t \\
x_t
\end{pmatrix} = \begin{pmatrix} \mu_y \\
\mu_x
\end{pmatrix} + \begin{pmatrix} v_t^y \\
v_t^x
\end{pmatrix}, \quad t = 1, 2, \ldots, T, 
$$

(7)

where $v_t^y$ and $v_t^x$ are error processes and are respectively generated by

$$
\Delta v_t^y = (1 - \theta L) u_t^y, \quad \text{and}
$$

$$
v_t^x = u_t^x
$$

(8)

with initial condition $v_1^y = u_1^y$ and stationary processes of $u_t^y$ and $u_t^x$ having the long-run and semi-long-run covariance matrices as

$$
\Omega = \begin{pmatrix} \sigma_{yy} & \sigma_{yx} \\
\sigma_{xy} & \Sigma_{xx}
\end{pmatrix} \quad \text{and} \quad \Gamma = \begin{pmatrix} \gamma_{yy} & \gamma_{xy} \\
\gamma_{xy} & \Gamma_{xx}
\end{pmatrix}, \text{respectively.}
$$

(10)

As indicated in Eq. (8), the stationary null hypothesis that $H_0: \theta = 1$ is tested against the unit-root alternative hypothesis that $H_1: \theta < 1$. Following Elliott et al. (1996), Jansson uses the local GLS detrended data to construct the $Q_T$ test. More specifically, OLS is first used to estimate the following model under the null ($\theta = 1$) and alternative ($\theta = \bar{\theta}$), respectively:

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1 For more information on the estimation of $R^2$, refer to p.81 in Elliott and Jansson (2003).

2 Although two covariate stationarity tests, the $Q_T$ and $L_T$, are proposed by Jansson (2004), the $Q_T$ test generally has higher power than the $L_T$ test as shown in his simulation results; therefore, the former is adopted in the present empirical study.
\[ z_i(\theta) = d_i(\theta)' \beta_0 + \nu_i, \]  

(11)

where \( z_i(\theta) = (y_i(\theta), x_i')' = (\sum_{s=0}^{\tau-1} \theta^s \Delta y_{t-s}, x_i')' \) with initial condition \( \Delta y_1 = y_1 \),

\[ d_i(\theta) = \left( \begin{array}{cc} \theta^{-1} & 0 \\ 0 & I_m \end{array} \right), \]

and the OLS residuals are obtained:

\[ \hat{\nu}_i(\theta, \hat{\Omega}) = z_i(\theta) - d_i(\theta)' \left( \sum_{s=1}^{T} d_s(\theta) \hat{\Omega}^{-1} d_s(\theta)' \right)^{-1} \times \left( \sum_{s=1}^{T} d_s(\theta) \hat{\Omega}^{-1} z_s(\theta) \right), \]

(12)

where \( \bar{\theta} = 1 - 7/T \) for our case, and \( \hat{\Omega} \) is a consistent estimator of \( \Omega \), obtained by a nonparametric method.

Then the \( Q_T \) test takes the following form:

\[ Q_T(\bar{\theta}, \hat{\Omega}, \hat{\Gamma}) = P_T - 2T(1-\bar{\theta})\hat{\sigma}_{yy,x}^{-1} \hat{\gamma}_{yy,x} \]

(13)

with

\[ P_T = \sum_{t=1}^{T} \left[ \hat{\nu}_t(1, \hat{\Omega})' \hat{\Omega}^{-1} \hat{\nu}_t(1, \hat{\Omega}) \right] - \sum_{t=1}^{T} \left[ \hat{\nu}_t(\bar{\theta}, \hat{\Omega})' \hat{\Omega}^{-1} \hat{\nu}_t(\bar{\theta}, \hat{\Omega}) \right], \]

(14)

where \( \hat{\Gamma} \) is also a nonparametric estimator of \( \Gamma \), \( \hat{\gamma}_{yy,x} = \hat{\gamma}_{yy} - \hat{\sigma}_{yy}^{-1} \hat{\Sigma}_{xx} \hat{\sigma}_{xy} \), and

\[ \hat{\sigma}_{yy,x} = \hat{\sigma}_{yy} - \hat{\sigma}_{y}^{-1} \hat{\Sigma}_{xx} \hat{\sigma}_{xy}. \]

As shown in Jansson (2004), the asymptotic distribution of the \( Q_T \) test depends on the nuisance parameter of \( R^2 = \sigma_{yy}^{-1} \sigma_{xy} \Sigma_{xx} \sigma_{xy} \), which measures the contribution of covariates to the power of the test. The larger the \( R^2 \), the more powerful is the \( Q_T \) test. Note that without covariates, or in the case wherein \( R^2 = 0 \), the univariate counterpart of \( Q_T \) pertains to the Saikkonen and Luukkonen (1993) test. Likewise, the estimated value of \( R^2 \) is used for acquiring the appropriate critical value. Note that the \( Q_T \) statistic is a one-sided upper tail test, that is, the stationary null will be rejected if the value of the test statistic is large enough.
2.4 Covariate selection

The implementation of three covariate tests involves the choice of stationary covariates, which has a great influence on the test power and then affects the empirical results. Unfortunately, no econometric theory is available for the selection of a particular set of covariates that can produce the highest power among a number of covariates. In the related literature, the commonly used approach is to use a single covariate in their studies according to economic theory (e.g., Amara and Papell, 2006; Elliott and Pesavento, 2006).

In this paper, a similar approach is utilized for the choice of covariates. As stated by the Fisher equation, the real interest rate is equivalent to the nominal interest rate minus actual inflation rate plus a stationary inflation forecast error under rational expectations. Obviously, the real interest rate is, to a certain extent, related to the nominal interest rate and actual inflation rate. For this reason, while testing for the unit-root dynamics of the real interest rate in a particular country, its own nominal interest rate and inflation rate are regarded as the sources of potential covariates. Following the suggestions of Elliott and Pesavento (2006), the two potential covariates are taken first differences to rule out their possible non-stationary or near unit-root properties that can cause these tests to suffer from inflated size. Then, the desired covariate is chosen between the two stationary covariates according to the criterion that the estimated value of $\hat{R}^2$ ($\hat{r}^2$) is the largest (smallest) for the CPT and $Q_T$ (CADF) tests.

3 Data and preliminary results

3.1 The data
This study examines the empirical validity of the Fisher equation, or equivalently, the behavior of real interest rates of 13 developing countries, including Argentina, Brazil, Chile, China, Indonesia, Korea, Malaysia, Mexico, the Philippines, Singapore, Taiwan, Thailand, and Turkey. The data for all countries are taken from the International Monetary Fund’s IFS database except Taiwan, which is taken from the Central Bank of China. Quarterly data are used; however, the sample periods are different for the countries considered due to limited data availability. The ex post real interest rates are calculated as short-run nominal interest rates minus actual inflation rates. As usual, the growth rate of the consumer price index is used as a proxy for the inflation rate for all countries, while various nominal interest rates are used due to the data availability. Table 1 provides details regarding sample periods as well as different nominal interest rates for each country.

3.2 Some preliminary results

Since the work of Ng and Perron (2001), the state-of-the-art univariate unit root tests such as DF-GLS and $MZ_{\alpha}^{GLS}$ (the NP tests) have been widely used in testing the stationarity properties of time series. Following Elliott et al. (1996) with the use of local GLS detrended data, they show that these two tests have higher power than the counterparts with OLS detrended data. Moreover, a brand new criterion entitled the modified Akaike information criterion (MAIC) is proposed to determine the lag order for constructing the tests. Unlike the traditional information criteria such as the BIC or AIC, the criterion of MAIC can effectively alleviate serious size distortions of the NP tests, especially when the series to be tested have a negative moving average (MA) root. Therefore, the NP tests along with MAIC were applied to the real interest rates
in the present study.

Instead of blindly trusting the asymptotic distributions of the NP tests, we estimated the finite sample distributions under the null and alternative hypotheses with the method proposed by Rudebusch (1993). Strictly speaking, the finite sample critical values of the NP tests can be different from their asymptotic counterparts and will depend on the sample size and the characteristics of each series under investigation. This problem can be tackled by the simulated distribution under the null. In addition, with the simulated distribution under the alternative hypothesis, the probability of obtaining the sample statistic from the estimated alternative model becomes available. This additional information is also helpful for hypothesis testing but is often ignored. Further, with the simulated distributions, we can analyze asymptotic sizes and size-adjusted powers of the NP tests, as well as draw inferences on which one of the two hypotheses is more appropriate for the real interest rate.3

[Table 2 should be here]

Table 2 reports the finite sample properties of the NP tests, including sample statistics, \( p \)-values, size-adjusted powers, and asymptotic sizes at the 5% significance level for the real interest rates. In general, the DF-GLS test has stable asymptotic sizes across all countries except for Chile and Singapore where inflated sizes occur. For the \( MZ_{\alpha}^{GLS} \) test, in contrast, the asymptotic sizes exhibit more variation, where inflated size for Chile, but under size for Korea, Malaysia, Mexico, Singapore, Taiwan, Thailand, and Turkey. This finding indicates that the DF-GLS test is more reliable than the \( MZ_{\alpha}^{GLS} \) test if one depends on the asymptotic tests for examining the stationarity properties of real interest rates.

On the basis of \( p \)-values, the evidence shows that the real interest rates of

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3 To avoid distraction, the Rudebusch (1993) method has been placed in the appendix.
Argentina, China, Malaysia, the Philippines and Thailand exhibit long-run mean reversion, lending strong support to the Fisher relationship since the $pv - LS$ is greater than 0.05 and the $pv - DS$ is smaller than 0.05 for the two tests. In contrast, for Indonesia, the real interest rate contains a unit root, violating the Fisher hypothesis. For the rest of the countries, the two $p$-values are larger than 0.05, showing that we cannot discriminate which distribution generates the sample statistics; this therefore requires further investigation. It is worth noting that when the asymptotic critical values are used for the indecisive countries, the two tests would falsely conclude that their real interest rates behave like a unit root process. Failing to take into consideration the distribution under the alternative gives rise to the unit-root results. This also indicates that the distribution under the alternative hypothesis contains valuable information for statistical inferences and should not be ignored.

To further clarify this approach, the small sample distributions of the DF-GLS test were plotted in Figure 1 for Taiwan. Note that the dotted lines represent the sample statistic (-1.55) of the DF-GLS test, and the empirical distribution of the sample statistic conditional on the LS model (LS-distribution) is always to the left of the distribution conditional on the DS model (DS-distribution). The area under the LS-distribution and to the right of the sample statistic is 0.3; whereas the area under the DS-distribution and to the left of the sample statistic is 0.13, implying that we cannot discern which distribution the sample statistic is more likely to come from.

For countries where the time-series properties of real interest rates are unable to be determined by the $p$-values, we find that their size-adjusted powers are also low. The average values of these size-adjusted powers for the DF-GLS and $M_{GLS}^{\alpha}$ tests are 0.38 and 0.35, respectively. For this reason, we will use the covariate tests for
these countries as well as provide clearer inferences about the empirical validity of the Fisher effect.

4 Results of covariate tests

[Table 3 should be here]

Based on the results in Table 2, seven countries require to be further investigated, including Brazil, Chile, Korea, Mexico, Singapore, Taiwan, and Turkey. Table 3 collects the results of the CADF, CPT, and $Q_{t}$ tests. When the CADF test statistics are compared with the 5% critical values, the unit root null is rejected for Brazil, Chile, Mexico, Singapore, Taiwan and Turkey. The CADF test still shows little evidence in favor of mean reversion in the real interest rate for Korea, however, despite the inclusion of the covariate. Rejecting the null hypothesis of the CADF test seems to be sensible since these rejections are supported by smaller $\hat{R}^2$ values ranging from 0.004 (Singapore) to 0.701 (Chile). The chosen covariate is the differenced inflation rate in each country except for Chile where the differenced nominal interest rate is chosen as covariate.

Turning to the results of the CPT test, we also find that it provides strong evidence supporting the Fisher relationship for six out of the seven countries with the exception of Chile. The chosen covariate across countries is the same as that for the CADF test. Generally speaking, these rejections of the unit-root dynamics can be explained by the improved powers of the CPT test, which are reflected by the values of $\hat{R}^2$. For example, except for Chile and Korea, all the values of $\hat{R}^2$ are more than 0.4, ranging from 0.446 (Mexico) to 0.945 (Singapore). As mentioned earlier, the higher the value of $\hat{R}^2$, the larger the power improvement obtained for the CPT test.

As a supplement to the CADF and CPT tests, the $Q_{t}$ test was applied to the data
where the null hypothesis is stationarity against the alternative of non-stationarity. The results in Table 3 indicate that for five of the seven countries, strong evidence supporting the stationary null hypothesis is provided by the $Q_T$ test at the 5% significance level, with two exceptions, namely Brazil and Taiwan, where the stationary null is rejected. The estimated values of $R^2$ show different variations across countries.

When the results of the three covariate tests are combined, overall, strong evidence can be unveiled to support the Fisher effect in all cases, since at least two of the three tests favor mean reversion in these real interest rates. Among these seven countries, all the three tests uniformly support the Fisher effect for Mexico, Singapore and Turkey, while, for the remaining countries, exactly two covariate tests convey adequate evidence against non-stationary behavior in real interest rates.

Finally, when these results are combined with those of the NP tests shown in Table 2, the conclusion can be drawn that the real interest rates in the 12 developing countries exhibit mean-reverting dynamics, empirically validating the Fisher relationship. Indonesia is the only one exception where shocks to the real interest rate appear to be infinitely persistent.

5 Unit-root test with nonlinear alternatives

Kapetanios et al. (2003) have recently proposed the KSS unit root test; a test which expands the conventional ADF test by retaining the null hypothesis of nonstationarity against the alternative of nonlinear but globally stationary exponential smooth transition autoregressive (ESTAR) process. They show, by simulation, that the KSS test is more powerful than the ADF test if the tested series is stationary and behaves like an ESTAR process. As noted in Kapetanios et al. (2003), due to the
transaction cost and other frictions, it is quite sensible that the more the real interest rate is away from its equilibrium, the larger the force induced by arbitrage or investment will serve to drive it back. By using the KSS test, Kapetanios et al. (2003) found strong evidence supporting the Fisher hypothesis in seven out of eleven industrialized countries. In addition, as suggested in Kousta and Lamarche (2006), the real interest rate may exhibit nonlinear dynamics due to the implementation of monetary policy that depends on the inflation rate. Accordingly, the KSS test was applied to those countries where the NP tests fail to convey clear evidence upon the dynamics of real interest rates.

The KSS test is the $t$-statistic for testing $\delta = 0$ in the following regression estimated by OLS:

$$
\Delta \hat{y}_t = \delta \hat{y}_{t-1}^3 + \sum_{j=1}^{p} \rho_j \Delta \hat{y}_{t-j} + \epsilon_t, \quad (15)
$$

where $\hat{y}_t = y_t - T^{-1} \sum_{t=1}^{T} y_t$ denotes the demeaned real interest rate. Following the suggestion of Kapetanios et al. (2003), the lag length of $p$ in Eq. (15) was selected with the top-down testing procedure.

[Table 4 should be here]

Table 4 reports empirical evidence on the real interest rates with the KSS test. Longer lags were chosen, varying from 6 to 8, for all countries with the exception of Brazil, implying that the degree of serial correlations may be high in the case of real interest rates. Given the 5% significance level, the KSS test made rejections of the non-stationary hypothesis only for Brazil and Turkey, indicating that the real interest rates in the two countries behave like a stationary ESTAR process and in favor of the Fisher relationship. For the remaining countries, there is no strong evidence against the unit-root dynamics. Coincidentally, the covariate unit root tests of CADF and CPT
also lend support to the Fisher effect in Brazil and Turkey, even if their rates may exhibit nonlinear dynamics. Overall, even after accommodating possible non-linearity in the real interest rates, it is still hard for the KSS test to establish the validity of the Fisher effect. These results are inconsistent with those of Kapetanios et al. (2003) that more rejections of unit root were obtained with the KSS test for developed countries. We think that this inconsistency can result from the fact that real interest rates in the developing countries may behave rather differently to those in developed countries. In other words, the behavior of interest rates in developing countries may or not be linear, and even if they are non-linear, the ESTAR process may be not a good approximation of them, thus leading to the unit root findings.

6 Concluding remarks

In this paper, we re-examined the time-series properties of the ex post real interest rate of 13 developing economies. Initially, the NP tests were applied to the real interest rates. In contrast to studies in the relevant literature, we did not blindly rely on the asymptotic distributions to make inferences, but employed Rudebusch’s (1993) method to simulate the finite-sample distributions for the tests under the null and alternative hypotheses. The characteristics of each series that may give rise to the asymptotic distributions different from their finite-sample counterparts are thereby able to be taken into consideration. Moreover, equipped with the two simulated distributions, we can easily discern which hypothesis the observed series is more likely to obey. The asymptotic size and power is also able to be quantified. We find that for the countries where the time-series properties of real interest rates cannot be determined by the simulated distributions, the powers of the NP tests are quite low. These countries, therefore, require further investigation.
To this end, a battery of covariate tests is then employed to deal with the low-power problem. By exploiting useful information contained in the covariates, these tests enjoy a great power boost in contrast with their univariate counterparts. Thanks to the improvement in power, strong evidence can be found that, for these countries, the real interest rates are mean-reverting and in favor of the Fisher equation.

We also evaluated the possibility of non-linearity in the data by conducting the KSS test that allows for a stationary ESTAR process under the alternative. Surprisingly, even though allowing for non-linearity in each country, the KSS test still lacks great ability to reject the null hypothesis of a unit root, and therefore provides only minimum support to the Fisher hypothesis. Our results are apparently different from those of Kapetanios et al. (2003) which states that more rejections of unit root were obtained with the KSS test for developed countries. We think that this inconsistency can be reconciled by noting that real interest rates in developing countries may or not exhibit linear stationary behavior, and even if they behave non-linearly, the ESTAR process may not be a good approximation of interest rates.

In sum, this study shows that the nominal interest rate and the inflation rate move together in the long run for the developing countries considered in this analysis thus supporting the empirical validity of the Fisher effect, and which also accords with the classical theory of long-run neutrality. In addition, the findings of mean reversion in real interest rates in the present study resolve the puzzling inconsistency concerning intertemporal consumption behavior. For policy implications, our stationary evidence suggests that market interest rates are good indicators of inflationary expectations, which provides policy makers with a credible signal to take adequate policies to counter inflationary pressures and achieve price stability.
References


Appendix

This appendix briefly describes Rudebusch’s (1993) method for obtaining the simulated distributions of NP tests under the null and alternative hypotheses. As in Rudebusch (1993) and Kuo and Mikkola (1999), only AR processes are, for the sake of simplicity, taken into consideration but without loss of reasonable flexibility. Under the alternative hypothesis, a level stationary (LS) $\text{AR}(p)$ model is fitted to the real interest rate ($y_t$)

$$y_t = \hat{\beta}_0 + \sum_{i=1}^p \hat{\beta}_i y_{t-i} + \hat{\epsilon}_t. \quad \text{(A1)}$$

Under the unit root null, a difference stationary (DS) model of $\text{AR}(1)$ for the series is estimated as

$$\Delta y_t = \hat{\phi} \Delta y_{t-1} + \hat{\nu}_t, \quad \text{(A2)}$$

where the constant term in Eq. (A2) was not empirically significant and was excluded, conforming to the stationary specification with a constant term in Eq. (A1). As recommended by Rudebusch (1993), the optimal lag $p$ in Eqs. (A1) and (A2) is determined by the top-down testing procedure of Ng and Perron (1995) with the maximum lag set at 16 in the following empirical study.

Based on Eqs. (A1) and (A2), the finite sample distributions of the NP tests can be easily obtained. First, artificial data series of $y_t$ were generated from the estimated LS and DS models with normal independently and identically distributed errors and the same sample size as in the original. To avoid the effect of initial values, 100 more observations were generated and an equivalent number was dropped from the beginning of the generated series. Next, the NP tests were calculated for each of the 5000 generated series to establish the simulated distributions under the two estimated
models. Given the 5% significance level in the subsequent empirical study, the asymptotic sizes and size-adjusted powers can easily be obtained on the basis of the simulated distributions.

Finally, by comparing the sample test statistics ($\hat{\tau}_{\text{sample}}$) with their simulated distributions, we can find which one of the two estimated models will generate $\hat{\tau}_{\text{sample}}$ with a higher probability. This is facilitated by calculating the $p$-values of the test statistic under the two estimated processes corresponding to the alternative and the null hypotheses, and are, respectively, denoted by

$$p_\text{V-LS} \equiv P\left( \hat{\tau} \geq \hat{\tau}_{\text{sample}} \mid f_{\text{LS}}(\hat{\tau}) \right), \quad \text{and} \quad (A3)$$

$$p_\text{V-DS} \equiv P\left( \hat{\tau} \leq \hat{\tau}_{\text{sample}} \mid f_{\text{DS}}(\hat{\tau}) \right), \quad (A4)$$

where $\hat{\tau}$ is the test statistic calculated from the artificial data; $f_{\text{LS}}(\hat{\tau})$ and $f_{\text{DS}}(\hat{\tau})$ are the simulated distributions of $\hat{\tau}$, conditional on the LS and DS models, respectively. Given the 5% significance level, if one $p$-value is greater than 0.05, and the other is less than 0.05, one can infer that the $\hat{\tau}_{\text{sample}}$ is more likely to come from the estimated model corresponding to the larger $p$-value. If both of the $p$-values are greater than 0.05, we are unable to determine which estimated model generates the $\hat{\tau}_{\text{sample}}$, and therefore, the test result is indecisive and needs to be further investigated.
Table 1 Data description

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample period</th>
<th>Nominal interest rate</th>
<th>Series code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1977Q1—2008Q1</td>
<td>Deposit rate</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Brazil</td>
<td>1982Q2—2008Q1</td>
<td>Time deposits</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Chile</td>
<td>1977Q1—2008Q1</td>
<td>Deposit rate</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>China</td>
<td>1980Q1—2008Q1</td>
<td>Deposit rate</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1974Q2—2008Q1</td>
<td>Three-month deposit NC</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Korea</td>
<td>1981Q2—2008Q1</td>
<td>Time deposit</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1966Q4—2008Q1</td>
<td>Fixed deposit 3-month</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Mexico</td>
<td>1978Q1—2008Q1</td>
<td>Treasury bill rate</td>
<td>60C..ZF…</td>
</tr>
<tr>
<td>Philippines</td>
<td>1976Q1—2007Q4</td>
<td>Treasury bill rate</td>
<td>60C..ZF…</td>
</tr>
<tr>
<td>Singapore</td>
<td>1973Q2—2008Q1</td>
<td>Treasury bill rate</td>
<td>60C..ZF…</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1975Q3—2008Q1</td>
<td>Deposit rate</td>
<td>n.a.</td>
</tr>
<tr>
<td>Thailand</td>
<td>1977Q1—2008Q1</td>
<td>Deposit rate</td>
<td>60L..ZF…</td>
</tr>
<tr>
<td>Turkey</td>
<td>1978Q4—2008Q1</td>
<td>Time deposits</td>
<td>60L..ZF…</td>
</tr>
</tbody>
</table>

Note: Data for all countries except Taiwan are retrieved from International Monetary Fund’s *IFS* database. For Taiwan, they are collected from the Central Bank of China.
| Table 2 Finite sample performance of the Ng and Perron unit root tests |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|
| Country       | Sample statistic | $p_v - LS$ | $p_v - DS$ | Size-adjusted power (5%) | Asym. size (5%) |
| Argentina     | DF-GLS           | -5.96      | 0.71        | 0.00               | 0.66            | 0.055             |
|               | $MZ_{a}^{GLS}$           | -48.63     | 0.75        | 0.00               | 0.56            | 0.051             |
| Brazil        | DF-GLS           | -1.29      | 0.13        | 0.24               | 0.72            | 0.058             |
|               | $MZ_{a}^{GLS}$           | -1.38      | 0.11        | 0.52               | 0.68            | 0.053             |
| Chile         | DF-GLS           | 1.02       | 0.00        | 0.96               | 0.13            | 0.176             |
|               | $MZ_{a}^{GLS}$           | 0.56       | 0.07        | 0.86               | 0.12            | 0.149             |
| China         | DF-GLS           | -3.85      | 0.94        | 0.00               | 0.62            | 0.036             |
|               | $MZ_{a}^{GLS}$           | -30.30     | 0.93        | 0.00               | 0.55            | 0.045             |
| Indonesia     | DF-GLS           | -0.48      | 0.02        | 0.59               | 0.69            | 0.054             |
|               | $MZ_{a}^{GLS}$           | -0.53      | 0.04        | 0.64               | 0.64            | 0.044             |
| Korea         | DF-GLS           | -1.67      | 0.83        | 0.07               | 0.12            | 0.039             |
|               | $MZ_{a}^{GLS}$           | -0.08      | 0.24        | 0.61               | 0.09            | 0.015             |
| Malaysia      | DF-GLS           | -3.48      | 0.88        | 0.00               | 0.64            | 0.044             |
|               | $MZ_{a}^{GLS}$           | -20.79     | 0.87        | 0.00               | 0.61            | 0.030             |
| Mexico        | DF-GLS           | -1.13      | 0.24        | 0.28               | 0.31            | 0.057             |
|               | $MZ_{a}^{GLS}$           | -2.65      | 0.36        | 0.23               | 0.32            | 0.032             |

Notes: $p_v - LS$ and $p_v - DS$ are defined in Eqs. (A3) and (A4), respectively. The size-adjusted power is calculated based on the 5% critical value of the estimated distribution of $\hat{\tau}$ conditional on the DS model in Eq. (A2), where $\hat{\tau}$ is the test statistic calculated from the artificial data. The 5% asymptotic size is based on the 5% asymptotic critical values of the tests, which are -1.98 and -8.1 for the DF-GLS and $MZ_{a}^{GLS}$ tests, respectively. The countries in boldface represent decisive testing results, as the two tests unanimously cannot reject the unit root null or are in favor of the alternative hypothesis.
Table 2 Small sample performance of the Ng and Perron unit root tests (Continued)

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample statistic</th>
<th>$pv - LS$</th>
<th>$pv - DS$</th>
<th>Size-adjusted power (5%)</th>
<th>Asym. size (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Philippines</td>
<td>DF-GLS</td>
<td>-3.14</td>
<td>0.48</td>
<td>0.00</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>$M_z^{GLS}$</td>
<td>-16.57</td>
<td>0.49</td>
<td>0.00</td>
<td>0.81</td>
</tr>
<tr>
<td>Singapore</td>
<td>DF-GLS</td>
<td>-0.36</td>
<td>0.04</td>
<td>0.65</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>$M_z^{GLS}$</td>
<td>0.05</td>
<td>0.06</td>
<td>0.73</td>
<td>0.38</td>
</tr>
<tr>
<td>Taiwan</td>
<td>DF-GLS</td>
<td>-1.55</td>
<td>0.30</td>
<td>0.13</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>$M_z^{GLS}$</td>
<td>-5.32</td>
<td>0.48</td>
<td>0.06</td>
<td>0.47</td>
</tr>
<tr>
<td>Thailand</td>
<td>DF-GLS</td>
<td>-2.09</td>
<td>0.72</td>
<td>0.03</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>$M_z^{GLS}$</td>
<td>-7.67</td>
<td>0.74</td>
<td>0.03</td>
<td>0.32</td>
</tr>
<tr>
<td>Turkey</td>
<td>DF-GLS</td>
<td>-0.26</td>
<td>0.03</td>
<td>0.68</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>$M_z^{GLS}$</td>
<td>-0.15</td>
<td>0.07</td>
<td>0.70</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Notes: $pv - LS$ and $pv - DS$ are defined in Eqs. (A3) and (A4), respectively. The size-adjusted power is calculated based on the 5% critical value of the estimated distribution of $\hat{\tau}$ conditional on the DS model in Eq. (A2), where $\hat{\tau}$ is the test statistic calculated from the artificial data. The 5% asymptotic size is based on the 5% asymptotic critical values of the tests, which are -1.98 and -8.1 for the DF-GLS and $M_z^{GLS}$ tests, respectively. The countries in boldface represent decisive testing results, as the two tests unanimously cannot reject the unit root null or are in favor of the alternative hypothesis.
Table 3: Results of the covariate tests

<table>
<thead>
<tr>
<th>Country</th>
<th>CADF Sample statistic</th>
<th>Sample statistic</th>
<th>CPT Covariate ( \hat{R}^2 )</th>
<th>Covariate Sample statistic</th>
<th>Sample statistic</th>
<th>( Q_T ) Covariate ( \hat{R}^2 )</th>
<th>Covariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>-2.40'</td>
<td>0.198</td>
<td>( \Delta \pi_r )</td>
<td>3.88'</td>
<td>0.384</td>
<td>( \Delta \pi_r )</td>
<td>42.84'</td>
</tr>
<tr>
<td></td>
<td>[-2.27]</td>
<td></td>
<td></td>
<td>[3.91]</td>
<td></td>
<td></td>
<td>[-0.77]</td>
</tr>
<tr>
<td>Chile</td>
<td>-3.57'</td>
<td>0.701</td>
<td>( \Delta R_r )</td>
<td>27.52</td>
<td>0.242</td>
<td>( \Delta R_r )</td>
<td>-1.03</td>
</tr>
<tr>
<td></td>
<td>[-2.71]</td>
<td></td>
<td></td>
<td>[3.60]</td>
<td></td>
<td></td>
<td>[-0.81]</td>
</tr>
<tr>
<td>Korea</td>
<td>-2.06</td>
<td>0.240</td>
<td>( \Delta \pi_r )</td>
<td>2.42'</td>
<td>0.227</td>
<td>( \Delta \pi_r )</td>
<td>-4.31</td>
</tr>
<tr>
<td></td>
<td>[-2.32]</td>
<td></td>
<td></td>
<td>[3.58]</td>
<td></td>
<td></td>
<td>[-0.72]</td>
</tr>
<tr>
<td>Mexico</td>
<td>-2.94'</td>
<td>0.568</td>
<td>( \Delta \pi_r )</td>
<td>1.84'</td>
<td>0.446</td>
<td>( \Delta \pi_r )</td>
<td>-4.23</td>
</tr>
<tr>
<td></td>
<td>[-2.62]</td>
<td></td>
<td></td>
<td>[4.16]</td>
<td></td>
<td></td>
<td>[-0.74]</td>
</tr>
<tr>
<td>Singapore</td>
<td>-2.31'</td>
<td>0.004</td>
<td>( \Delta \pi_r )</td>
<td>19.95'</td>
<td>0.945</td>
<td>( \Delta \pi_r )</td>
<td>-4.37</td>
</tr>
<tr>
<td></td>
<td>[-2.11]</td>
<td></td>
<td></td>
<td>[21.94]</td>
<td></td>
<td></td>
<td>[1.28]</td>
</tr>
<tr>
<td>Taiwan</td>
<td>-4.68'</td>
<td>0.017</td>
<td>( \Delta \pi_r )</td>
<td>3.38'</td>
<td>0.786</td>
<td>( \Delta \pi_r )</td>
<td>7.76'</td>
</tr>
<tr>
<td></td>
<td>[-2.11]</td>
<td></td>
<td></td>
<td>[8.77]</td>
<td></td>
<td></td>
<td>[-0.74]</td>
</tr>
<tr>
<td>Turkey</td>
<td>-5.39'</td>
<td>0.387</td>
<td>( \Delta \pi_r )</td>
<td>4.49'</td>
<td>0.595</td>
<td>( \Delta \pi_r )</td>
<td>3.84</td>
</tr>
<tr>
<td></td>
<td>[-2.49]</td>
<td></td>
<td></td>
<td>[5.08]</td>
<td></td>
<td></td>
<td>[0.63]</td>
</tr>
</tbody>
</table>

Notes: \( \Delta R_r \) and \( \Delta \pi_r \) denote the first differences of nominal interest rates and inflation rates, respectively. Numbers in the square brackets are the 5% critical values. Statistical significance is indicated by an asterisk (*) for the 5% level. \( \hat{R}^2 \) and \( \hat{R}^2 \) measure the contribution of covariates to test power.
<table>
<thead>
<tr>
<th>Country</th>
<th>$p$</th>
<th>KSS statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>1</td>
<td>-11.91$^*$</td>
</tr>
<tr>
<td>Chile</td>
<td>8</td>
<td>-1.68</td>
</tr>
<tr>
<td>Korea</td>
<td>6</td>
<td>-1.87</td>
</tr>
<tr>
<td>Mexico</td>
<td>7</td>
<td>-2.26</td>
</tr>
<tr>
<td>Singapore</td>
<td>8</td>
<td>-1.08</td>
</tr>
<tr>
<td>Taiwan</td>
<td>7</td>
<td>-2.28</td>
</tr>
<tr>
<td>Turkey</td>
<td>8</td>
<td>-3.21$^*$</td>
</tr>
</tbody>
</table>

*Notes:* The 5% asymptotic critical value for the KSS test is -2.93. Statistical significance is indicated by an asterisk (*) for the 5% level. $p$ represents the optimal lag for the KSS test selected by the top-down procedure.
Figure 1. Distributions of the DF-GLS test under the LS (Left) and DS (Right) models for Taiwan. The sample statistic is denoted by the dotted lines.