Optimal Trade Policies and Production Technology in Vertically Related Markets

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This paper shows that optimal trade policies for vertically related markets depend crucially on the production technology. When the technology is of Cobb-Douglas type, exhibiting a strongly diminishing return to scale, the optimal trade policy is one of subsidizing either the import of the intermediate good, or the export of the final good. This result runs contrary to the findings of Ishikawa and Spencer (1999) in which fixed-coefficient technology with constant returns to scale was assumed. Moreover, the welfare level under the optimal import policy is higher than (equivalent to) that under the optimal export policy when the technology in question exhibits increasing (constant) returns to scale, whilst the ranking is, in general, reversed where the returns to scale become diminishing.

I. Introduction

Up to the 1990s, the strategic trade policies in imperfect competition focused on the horizontal aspects of market structures, notably demonstrated in the 1980s by Brander and Krugman (1983), Dixit (1984), Brander and Spencer (1984, 1985) and Eaton and Grossman (1986). This approach has been refined recently by Spencer and Jones (1992), Bernhofen (1997) and Ishikawa and Spencer (1999), among others, to examine the impacts of vertically related markets on strategic trade policies.

Spencer and Jones (1992) investigated whether an import tariff on an intermediate input may reduce the input price under a vertically integrated structure model. Bernhofen (1997) examined whether the optimal government intervention in the final-good market depended on the type of pricing schemes, uniform or discriminatory pricing, employed by the intermediate-good producer. Ishikawa and Spencer (1999) suggested that export subsidies that were intended to shift rents from foreign producers to domestic producers of a final good may also serve to shift rents to foreign firms supplying an intermediate input, thereby weakening the incentive for the subsidy. They argued that the optimal export subsidy could therefore be reversed if intermediate-good firms are in foreign countries. Apparently, the structure of the intermediate-good market plays an important role in the determination of trade policies on final goods.

When modeling vertically related markets, most of the literature on trade assumes a fixed-coefficient technology, even though variable-coefficient ones have been well developed in industrial organization literature.¹ This is evident in the papers by, for example, Katsoulacos (1989), Spencer and Jones (1991), Chang and Chen (1994), Spencer and Raubitschek (1996), Bernhofen (1997), Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Hwang et al. (2003). Whilst our understanding of the issue of optimal policies has been much improved by these papers, their assumption of a fixed-coefficient technology remains rather restricted. The purpose of this paper, therefore, is to re-examine the issue under the assumption of a variable-coefficient production function, and to show that production technology plays a significant role in

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¹ See, for example, Warren-Boulton (1974), Maliea and Nahata (1980), Waterson (1982), and Quirmbach (1986).
determining trade policies and also in ranking welfare levels of different trade policies. Note that this extension also bears empirical relevance, as empirical literature on trade policies has often used variable-coefficient estimates to parameterize supply functions. See for instance, Dixit (1988) and Baldwin and Krugman (1988).

In addition to investigating optimal trade policies regarding the crucial changes in the firm’s production function, we will also consider implications of alternative trade policies (import tariffs and export subsidies). The model we shall use for the analysis is a three country model with the home country importing an intermediate good from a foreign monopolistic supplier, manufacturing the entire final product at home, and then exporting it to another foreign market in which it has monopoly power. Obviously, the only distortion in this model, from the home government’s point of view, arises from the intermediate-good market. To correct this distortion, the home government can adopt either an import policy (such as import tariffs) or an export policy (such as export subsidies). As Bhagwati (1971) pointed out, the optimal policy for combating a distortion is one that affects the distortion at its source. Accordingly, an import policy should be employed to remove the monopoly distortion in the input market. Nevertheless, we shall show that this principle is not applicable to our model when the production exhibits considerable decreasing returns to scale.

This paper is organized as follows. Section 2 outlines the model. Section 3 analyzes the optimal import policy with the optimal export policy then being examined in section 4. In section 5, we compare welfare levels under the two policies. Section 6 extends the model by allowing a general demand and two inputs in the production function. Section 7 concludes the paper.

II. The Model

Let there be three countries, a home country, a foreign country and a third country. The home country has only one firm, $H$, operating in a certain industry. This firm imports intermediate input $x$ from the foreign country, manufacturing the entire final product $Q$ at home, and then exporting it to the third country. To ignore strategic distortion in the final good market and to concentrate on the important interdependence between upstream and downstream production, it is assumed that the foreign firm holds monopoly power in the $x$ market while the home firm holds monopoly power in the final good market. Since the home firm is a monopolist in the final good market, from the perspective of the home government, there is no distortion in the export market and the only distortion in this model comes from the monopoly of intermediate input by the foreign firm. To correct this distortion, the home government is allowed to set a specific tariff $t$ on its imports of the intermediate input or to impose a specific subsidy $s$ on its exports of the final product.

Paying greater attention to the production function, we assume that the demand function for the final good is linear $P = a - bQ$, where $P$ and $Q$ represent the price and quantity, respectively, for the final good market. Production is subject to the

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2 Assuming the monopoly of the intermediate input by the foreign firm allows a better focus on the trade policy issue and is also empirically relevant in NICs. For example, according to a report published in the Asian Wall Street Journal on January 23, 1995, Indonesia has been forcing foreign buyers of its plywood to purchase solely through an agent designated by the Indonesian Wood Panel Association (known as Apkindo) since 1980 and “has blossomed into a mighty cartel that claims to control about 80% of world exports of tropical plywood.” Another approach is to assume that the input price is determined through a bargaining process between the foreign and domestic firms. This can be extended from this model.
following technology: \( Q = x^r \), where \( x \) is the only input to produce \( Q \); the parameter \( r \) is defined as the degree of returns to scale. The value of \( r \) is less than (equal to, greater than) 1 for decreasing (constant, increasing) returns to scale technology. Given these assumptions, the firm \( H \)'s profit function is given by:

\[
\max_{Q} \pi^H(Q, w, s) = (a - bQ)Q - wx + sQ
\]

where \((a - bQ)Q\) is the total revenue, \( w \) is the price of input \( x \) and \( s \) is the export subsidy rate.

The foreign firm \( F \)'s profit function can be expressed as

\[
\max_{w} \pi^F = x(w - t - c)
\]

where \( c \) is the constant marginal cost to produce the intermediate input \( x \) and \( t \) is the tariff rate of the intermediate input.

The welfare function of the home country is the firm \( H \)'s profit \((\pi^H)\) plus tariff revenue \((tx)\) minus export subsidy \((sQ)\):

\[
\max_{t, s} W^H = \pi^H(Q, w, s) - sQ + tx
\]

Under these conditions, the game consists of three stages in the decision-making process. In stage one, the home government can choose to implement an export subsidy rate, \( s \), or an import tariff rate, \( t \), to maximize its welfare. In stage two, the foreign monopolistic supplier of the intermediate input takes \( t \) as given while determining the price of \( x \). In the third stage, the home firm determines the exports of the final product, taking \( w, t \) and/or \( s \) as given.

We are now ready to examine how the returns to scale affect the optimal export and import policies. In the next section, we begin with our analysis of the import trade policy by assuming that the subsidy rate is zero, and then, in section 4, investigate the optimal export trade policy by assuming that the import tariff is zero.

**III. Optimal Import Policy**

As is normally the case with a multi-stage sub-game perfection model, the equilibrium is solved in a backward fashion by considering the third stage (the home firm’s decision) first, followed by the second stage (the foreign firm’s decision) and finally the first stage (the home government’s decisions).

**A. The home firm’s decision**
Let \( s = 0 \) in (1), the profit function for the domestic firm becomes
\[
\max_Q \pi^H = (a - bQ)Q - wQ^{1/r}
\]
(4)
The first-order condition of profit maximization is:
\[
\frac{d\pi^H}{dQ} = a - 2bQ - w \frac{1}{r} Q^{\frac{1}{r} - 1} = 0
\]
(5)
Assume that the second-order condition is satisfied. That is: \( \pi_{QQ}^H = -2b - w \frac{1}{r} (1/r - 1)Q^{(1/r) - 2} < 0 \).\(^5\) Equation (5) defines \( Q^* = Q(w) \).  Since \( x = Q^{1/r} \), the derived demand for input \( x \) is therefore \( x^* = Q^{(1/r)} \). A change in input price affects the output and the demand for input as follows:
\[
Q^*_x = \frac{dQ^*}{dw} = \frac{1}{r} Q^{\frac{1}{r} - 1} - 2b - \frac{1}{r} (\frac{1}{r} - 1)w Q^{\frac{1}{r} - 2} < 0
\]
(6)
\[
x^*_w = \frac{dx^*}{dw} = \frac{dQ^*}{dQ^*} \frac{dQ^*}{dw} = \frac{1}{r} Q^{\frac{1}{r} - 1} Q^*_w < 0
\]
(7)
As expected, output and the demand for input fall in response to an increase in the input price.

B. The foreign firm’s decision

Substituting \( x^* = Q^{(1/r)} \) into (2), we can rewrite the profit function of the foreign upstream firm as follows:
\[
\pi^F = x^* (w) \cdot (w - t - c) = Q^* (w)^{\frac{1}{r}} (w - t - c)
\]
(8)
The first-order condition for profit maximization is derived as follows:
\[
\frac{d\pi^F}{dw} = Q^* (w)^{\frac{1}{r}} + (w - t - c) \frac{1}{r} Q^* (w)^{\frac{1}{r} - 1} Q^*_w = 0
\]
(9)
The second-order condition for profit maximization requires \( \pi_{ww}^F < 0 \) (see Appendix 1 for derivation).

Totally differentiate (9) with respect to \( w \) and \( t \) to yield the change of input price

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\(^5\) If the production function exhibits very high increasing returns to scale (i.e., \( r \) is much larger than 1), the second-order condition may be violated, and therefore we assume that \( r \) cannot be too high.

\(^6\) All equilibrium variables are denoted by an asterisk superscript.
with respect to tariff:

\[ w_t^* = \frac{dw}{dt} = -\frac{\pi^*_w}{\pi^*_w} = \frac{1}{r} \left[ (3 - \frac{1}{r}) - \frac{1}{(r-1)^2} \right] \frac{w}{Q^*_w} > 0 \]  

(10)

This implies that an increase in tariff rate increases the input price since the tariff can be seen as part of the cost.

**C. Optimal import policy**

In stage one of the game, the government decides on its import policy based on the input price decision (i.e., Eqs. (9) and (10)) in the second stage and on the input usage and output decisions in the third stage (i.e., Eqs (6) and (7)). By substituting \( s = 0 \), \( w^* = w(t) \) and \( x^* = Q^{(t/r)} \) into (3), we can have the social welfare function of the home country as follows:

\[ W^H = \pi^H(Q(w(t)), w(t)) + tQ(w(t))^{\frac{1}{r}} \]

\[ = (a - bQ(w(t)))Q(w(t)) - (w(t) - t)Q(w(t))^{\frac{1}{r}} \]  

(11)

The home government sets its import tariff to maximize its national welfare; hence, the first-order condition for the welfare maximization is:

\[ \frac{dW^H}{dt} = \frac{d\pi^H}{dQ} Q^*_w w_t^* + \frac{d\pi^H}{dw} w_t^* + Q^r + t \left( \frac{1}{Q^r} Q^*_{w} w_t^* \right) = 0 \]  

(12)

Assume that the second-order condition is satisfied. Substituting \( d\pi^H / dQ = 0 \) and \( d\pi^H / dw = -Q^{(t/r)} \) into (12), we can rewrite (12) as follows:

\[ \frac{dW^H}{dt} = -Q^r w_t^* + \frac{1}{r} \frac{1}{Q^r} Q^*_{w} w_t^* = 0 \]  

(13)

The first term on the right-hand side, which captures the term of trade effect, is negative. The second term that captures the direct tariff effect is positive, implying that a higher tariff rate yields a higher welfare even if the domestic output level is unchanged. The last term that represents the output effect is negative, as a higher tariff rate tends to push up the input price that lowers the domestic output level.

Solving (13) yields the optimal import tariff (see Appendix 2 for proof):

\[ t^* = -\frac{rQ(1-w_t^*)}{Q^*_w w_t^*} \]

\[ = r^3 Q^{\frac{1}{r}} \left[ \frac{2}{r} - 3 + \frac{1}{r} (\frac{1}{r-1} + 2) Q^*_w w_t^* \right] \left[ -2b + \frac{1}{r} (\frac{1}{r-1} wQ^r - 2) \right] \]  

(14')
As $Q_w^* < 0$ and $w_t^* > 0$, the sign of (14) is determined by the sign of $(1 - w_t^*)$. If $w_t^* > 1$, namely, the input price rises by more than the tariff, an import subsidy (i.e., $t < 0$) is needed to improve the term of trade of the intermediate input. On the other hand, an import tariff (free trade) will be optimal if $w_t^* < (=) 1$.

Now, we turn to investigate the relationship between optimal tariff $t^*$ and the scale of the production function $r$. From (14'), When $r = 1$ ($r = 0.5$), then $t^*$ is positive (negative). This leads us to the following proposition:

**Proposition 1:** If the production function of the home firm exhibits constant returns to scale or increasing returns to scale ($r \geq 1$), the optimal import tariff is positive. In contrast, if the production function exhibits considerable decreasing returns to scale, the optimal import tariff becomes negative.

If the production function exhibits sufficiently decreasing returns to scale, the derived demand for the intermediate input will become very convex to the origin (see Appendix 3 for proof). Under such a circumstance, a tariff on the intermediate input will raise the price of the intermediate input by more than the tariff rate, thus lowering the profit of the domestic firm and the welfare of the domestic country. Therefore, the best import policy is to subsidize the import.

This outcome is similar to the findings made by Brander and Spencer (1984). They found that optimal tariff is negative if the demand for a final good is sufficiently convex to the origin. In the current case, we have demonstrated that the optimal tariff for the intermediate input is also negative if the derived demand curve is sufficiently convex to the origin.

**IV. Optimal Export Policy**

Notably, there is only one distortion (imperfection in the input market) in the model, which requires only one policy instrument to correct it. In the previous section, we assumed that the government implemented an import policy to correct the distortion. Here, we will allow the government to choose an alternative policy, an export subsidy, as a means of removing the distortion.

**A. The home firm’s decision**

In the case of an export subsidy, the foreign firm’s profit function in (2) is now assumed as $t = 0$. Following a similar procedure used in the previous section, we can derive the first-order condition for profit maximization of the home firm as follows:

$$\frac{d\pi^H}{dQ} = a - 2bQ - w - \frac{1}{r} Q^{\frac{1}{r-1}} + s = 0$$

(15)

Assuming that the second-order condition $\pi_\theta^H = -2b - \frac{1}{r} (-1)Q^{(1/r)-2} < 0$ is met, we can obtain from (15) the optimal output levels $Q^* = Q(w, s)$ and the derived demand $x^* = Q(w, s)^{1/r}$. The comparative static results for the effects of the input price on $Q$ and $x$ can be obtained as follows:
Equations (16) and (17) show that an increase in input price pushes up the home firm’s production cost, reduces its output level and leads to a lower input demand. Similarly, we can also derive from (15) the comparative static effects of \( s \) on \( x \) and \( Q \) as well:

\[
Q_*^s = \frac{dQ_*}{ds} = \frac{1}{r} \frac{1}{Q_*^{r-1}} - \frac{1}{r} \frac{1}{Q_*^{r}} > 0
\]  

\[
x_*^s = \frac{dx_*}{ds} = \frac{dx_*}{dQ_*} \frac{dQ_*}{ds} = \frac{1}{r} \frac{1}{Q_*^{r-1}} \frac{dQ_*}{ds} > 0
\]  

An increase in export subsidy on the final good increases the output and hence the demand for the intermediate input.

**B. The foreign firm’s decision**

In the second stage, the foreign supplier determines the optimal input price based on the derived demand. Substitute \( t = 0 \) and \( x^* = Q^* (w, s)^{1/r} \) into (2) to yield the profit function of the foreign input supplier as follows:

\[
\pi^F = x^* (w, s) \cdot (w - c) = Q^* (w, s)^{1/r} (w - c)
\]  

Take the first derivative of (20) with respect to \( w \) to yield the following first-order condition for profit maximization:

\[
\frac{d\pi^F}{dw} = Q^* (w, s)^{\frac{1}{r}} + (w - c) \frac{1}{r} Q^* (w, s)^{\frac{1}{r} - 1} Q_*^w = 0
\]  

Assuming the second-order condition for profit maximization \( \pi^F_{ww} = [(3 - 1/r) - (1/r - 1) (1/r - 2) w Q_*^r / Q] Q^{1/r-1} Q_*^w < 0 \) to be satisfied, we can then derive from (21) the following comparative static result (see Appendix 4 for derivation):
Since \( \pi_{ww}^F < 0 \) and \( Q_w^* < 0 \) (by (16)), the denominator of (22) is positive. The term in the numerator \( 1 + (1/r - 1)wQ_w^*/Q = -2b/(b - (1/r - 1)wQ_w^*Q'(r)^{-2}/r) \) is also positive. Hence, we can get:

\[
W_s^* > 0, \quad \text{if} \quad r > \frac{1}{2} \tag{23}
\]

A rise in subsidy tends to increase \( Q^* \), leading to a higher derived demand and therefore a higher input price. One would normally expect \( W_s^* \) to be positive. Nevertheless, our analysis has revealed that \( W_s^* \) can be either positive or negative. It is negative when \( r < 1/2 \). This finding is illustrated in Figure 1 which shows that the derived demand curve, \( x(w, s) \), becomes very convex to the origin when \( r < 1/2 \). The corresponding marginal revenue curve is \( mr \) and the optimal input price is \( w^* \). If \( s \) increases to the level of \( s' \), the derived demand \( x(w, s) \) shall shift to \( x(w, s') \). The corresponding marginal revenue curve becomes \( mr' \), and the input price reduces from \( w^* \) to \( w' \).

**Figure 1.** The optimal price of the input when \( r < 1/2 \)

When modeling vertically related markets, most of the literature on trade and industrial organization assumes a fixed-coefficient technology. Under this assumption, an export subsidy necessarily leads to a higher input price (i.e., \( W_s^* > 0 \)). Nevertheless, the input price may go down if the production function exhibits sufficiently decreasing returns to scale.

In the next section, we shall show that the degree of return to scale is a crucial determinant of the optimal export policy, and therefore a more general type of production function should be considered in determining trade policies.
C. Optimal export policy

Substituting \( w^* = w(s) \) into \( x^* = Q(w(s), s)^{1/r} \) and letting \( t = 0 \) in (3), the national welfare for the home country is given by:

\[
W^H = \pi^H(Q(w(s), s), w(s), s) - sQ(w(s), s) = (a - bQ)Q - w^{1/r}
\]  

(24)

The first-order condition for welfare maximization is:

\[
\frac{dW^H}{ds} = \frac{d\pi^H}{dQ}(Q^*_w w^*_s + Q^*_s) + \frac{d\pi^H}{dw} w^*_s - Q - s(Q^*_w w^*_s + Q^*_s) = 0
\]  

(25)

Assume that the second-order condition \( d^2W^H/ds^2 < 0 \) is satisfied. Substituting \( d\pi^H/dQ = 0, \ d\pi^H/ds = Q, \) and \( d\pi^H/dw = -Q^{1/r} \) into (25) yields:

\[
\frac{dW^H}{ds} = -Q^{1/r} w^*_s - s(Q^*_w w^*_s + Q^*_s) = 0
\]  

(26)

The first term in (26) \(-Q^{1/r} w^*_s\) captures the terms of trade effect whose sign is determined by \( w^*_s \), which in turn depends on the value of \( r \) (i.e., \( r > \frac{1}{2} \)); the term in the parenthesis \((Q^*_w w^*_s + Q^*_s)\) measures the output effect whose sign is positive (see Appendix 5 for derivation). Solving (26) yields

\[
s^* = \frac{-Q^{1/r} w^*_s}{Q^*_w w^*_s + Q^*_s} > 0 \quad \text{if} \quad r \geq \frac{1}{2}
\]  

(27)

Based on (27), we can establish the following proposition.

Proposition 2: When the degree of returns to scale of the production function of the home firm is greater than (equal to, less than) \( 1/2 \), the optimal export subsidy is negative (zero, positive).

The intuition behind this proposition is as follows. In this model, the sole distortion arises from the foreign monopolistic input market. An export policy, if properly designed, can improve the home country’s terms of trade in the input market; and thereby reduces the foreign monopoly distortion. From (23), if \( r > (<) 1/2 \), an export subsidy increases (decreases) the input price, which worsens (improves) the terms of trade. Therefore, the optimal export policy is to tax (subsidize) the export.

The above results run in contrast to the findings of Ishikawa and Spencer (1999). In a similar framework, Ishikawa and Spencer (1999) assumed a fixed-coefficient production technology and concluded that: (i) the optimal import tariff is positive and the optimal export subsidy is negative; and (ii) both policies generate the same welfare level. It is straightforward to prove that their first result holds true by substituting \( r = \)}
1 into our model. Nevertheless, they failed to show that optimal trade policy, either import or export, may be reversed when the production function exhibits decreasing returns to scale. In the next section, we shall prove that their second result is no longer valid under a certain range of returns to scale.

V. Comparison of Import and Export Policies

Assuming a fixed-coefficient production function, Ishikawa and Spencer (1999) found that the welfare level under the optimal import policy is equivalent to that under the optimal export policy. Moreover, as Bhagwati (1971) pointed out, the optimal policy for combating a distortion is one that affects the distortion at its source. Accordingly, the optimal import policy is superior to the optimal export policy in the current model since the import policy is aimed more directly to the source of distortion. However, we shall demonstrate in what follows that the principle set out by Bhagwati (1971) does not hold when \( r \neq 1 \). It implies that welfare under the export policy can be superior, equal or inferior to that under the import policy, depending on the value of \( r \).

Three cases for different values of \( r \) will be examined in order.

**Case 1:** \( r = 1 \)

Given \( r = 1 \), it is straightforward to show that the optimal export subsidy is \( s^* = -\frac{(a-c)}{3} \), and the social welfare is \( \left( a-c \right)^2 / 12b \) under an export policy; the optimal import tariff is \( t^* = \frac{(a-c)}{3} \) and the welfare level is \( \left( a-c \right)^2 / 12b \) under an import policy. Obviously, both policies generate the same welfare level, thus confirming the finding of Ishikawa and Spencer (1999).\(^7\)

**Case 2:** \( r = 1/2 \)

Under an export policy, the optimal export subsidy is \( s^* = 0 \), namely free trade, when \( r = 1/2 \) (by (27)) and the corresponding welfare level is \( \frac{a^2}{8(b+c)} \). On the other hand, the optimal import policy requires \( t^* = \frac{-(b+c)}{3} < 0 \) (by (14')) and the resulting welfare level is \( \frac{9a^2}{64(b+c)} \). Obviously, the welfare under the optimal import policy is necessarily greater than that under the optimal export policy. It implies that the second result in Ishikawa and Spencer (1999) does not hold true when the production technology does not exhibit constant returns to scale.

**Case 3:** \( r = r^C \)

(\( \text{Let } r = r^C \text{ satisfies } t^* = 0. \))

From proposition 1, there exists an optimal import policy which is free trade at \( r = r^C \) \((0.5 < r^C < 1)\), and the corresponding welfare level is \( \frac{a^2}{8(b+c)} \). On the other hand, since \( r^C > 1/2 \) the optimal export subsidy is negative at \( r = r^C \) by Proposition 2 and its welfare is necessarily greater than that under free trade, which is equivalent to the welfare level under the optimal import policy. Thus, the optimal

\(^7\) We obtain \( t - s = (a-c)/3 \) if the home government adopts both policies simultaneously, and if \( r = 1 \). It shows that the same welfare level can be achieved by many combinations of the two policies.
import tariff results in a lower social welfare level, as compared with the optimal export subsidy when \( r = r^c \).

In general, the welfare ranking of an export and an import policy is ambiguous, as it depends on the value of \( r \). A series of computations have therefore been undertaken for different values of \( r \) ranging from 0.4 to 1.2 with the following parameter settings: \( a = 10 \), \( b = 1 \), and \( c = 0.2 \). The results, as shown in Figure 2, reveal that welfare under an import policy is greater than that under an export policy for lower levels of \( r \). However, when the values of \( r \) are between approximately 0.709 and 1, welfare under an export policy exceeds that under an import policy. Finally, when the value of \( r \) is greater than one, welfare under an import policy becomes higher again. Based on the findings, we can establish the following proposition.

**Figure 2.**  Welfare comparison

\[
\begin{align*}
W^H &
\end{align*}
\]

**Proposition 3:** The welfare ranking of an export and an import policy is ambiguous, as it depends on the value of \( r \). For lower values and higher values (greater than 1) of \( r \), welfare under an import policy is greater than that under an export policy; otherwise, welfare under an export policy exceeds that under an import policy.

According to Figure 2, welfare under an export policy is higher than that under an import policy when the value of \( r \) ranged between (0.709, 1). Thus, the principle set out by Bhagwati (1971) is no longer applicable when production technologies do not exhibit constant returns to scale. Similarly, the welfare-equivalence finding in Ishikawa and Spencer (1999) also crucially depends on the assumption of fixed-coefficient technology. In general, the welfare levels under the two policies are not equivalent if production functions are not of constant returns to scale.

**VI. Extensions**

For analytical and expository convenience, we have assumed the demand to be linear and the production function uses only one input. Nevertheless, the model can be easily extended in the following two directions: Allow a general demand function \( P = P(Q) \) and a two-input production function such as \( Q = x^r k^B \) with \( k \) being a fixed factor. Under this more generalized setting, equation (14') that was used to characterize the optimal import policy in section 3 becomes:
\[ t^* = \frac{r^3}{A} Q^{2-\frac{1}{r}} \left[ \frac{(2 - 3) + \frac{1}{r} (1 - 1) Q^r_w}{Q} - rA^{-1} (3P'' + P'''Q) Q^r_s Q^{2-\frac{1}{r}} \right] \]

\[ [2P'' + P'''Q - \frac{1}{r} (-1) wA Q^{\frac{1}{r}}] \]

where \( A = k^{-\beta/r} \). If \( k = 1 \) (the production function restores to the one-input case) and \( P''' = P'' = 0 \) (the linear demand case), (14") reduces to (14'). Moreover, from (14"), it is straightforward to show the following two results. First, if the demand remains linear but the production function uses one variable and one fixed factors, Proposition 1 still holds true. Second, if we assume \( P''' = 0 \), the optimal import tariff is likely to be positive (negative) when the demand curve is concave (convex).

In the case of optimal export policy, equation (27) that was used to characterize the optimal export policy in section 4 becomes:

\[ s^* = -\frac{Q^r w^*_s}{Q^r_w s^*_t + Q^r_s} = \frac{Q^r \pi^F_{ws}}{(Q^r_w s^*_t + Q^r_s) \pi^H_{ws}} > 0 \quad \text{if} \quad \pi^F_{ws} < 0 \quad (27') \]

where \( \pi^F_{ws} = Q^{(1/r)-1} \left[ -(1/r - 2)(1 + (1/r - 1)wQ^r_w/Q) + Q(3P'' + P'''Q) / \pi^H_{QQ} \right] Q^r_s \) and \( \pi^H_{QQ} < 0 \). Obviously, the optimal export subsidy in (27') depends not only on the production technology, but also on the demand concavity. Note that (27') involves \( P''' \) whose sign is ambiguous in nature. Nevertheless, if we assume \( P''' = 0 \) and \( r = 1/2 \), we can derive the following relationship from (27'):

\[ \pi^F_{ws} < 0, \quad \text{if} \quad P'' > 0 \]

This implies that the optimal subsidy schedule in Figure 2 moves downward (upward) and the optimal export subsidy on the final good is likely to become negative (positive), when the demand curve is concave (convex). This leads to the following proposition.

**Proposition 4:** If the production function is extended to the one with one variable and one fixed factors, all the results embedded in propositions 1 and 2 hold. On the other hand, if the demand become non-linear, the optimal export subsidy on the final export good is likely to become negative (positive) and the optimal tariff on the intermediate good likely to be positive (negative) when the demand is concave (convex).

**VII. Conclusions**

This paper has demonstrated the importance of production scale in determining the direction of a country’s import and export policies, and in investigating welfare implications of the two policies. When modeling vertically related markets, most of the literature on trade and industrial organization assumed a fixed-coefficient technology. This paper has set up a three-country model with the home country importing an intermediate good from a foreign monopolistic supplier, manufacturing the entire final product at home, and then exporting it to another foreign market in
which it has monopoly power. It has shown that the optimal trade policies may be reversed if the fixed-coefficient technology assumption is relaxed. Therefore this assumption should be used very carefully when attempting to address or compare alternative optimal trade policies.

This paper has reached three main conclusions. First of all, the optimal import tariff is positive (can be negative) if the production function of the home firm exhibits constant or increasing returns to scale (decreasing returns to scale). Secondly, the optimal export subsidy is negative (zero, positive) if the degree of return to scale is greater than (equal to, less than) 1/2. Thirdly, the welfare ranking of the two policies depends crucially on the degree of returns to scale. If the production function exhibits sufficiently decreasing return to scale or increasing return to scale, welfare under an import policy is greater than that under an export policy; otherwise, the welfare ranking is reversed.

From the first and second conclusions, the optimal trade policy is to subsidize either the foreign intermediate input or the home firm’s final good export if the production function exhibits sufficiently decreasing return to scale. We extended the results in Ishikawa and Spencer (1999) to the cases of non-constant returns to scale. Our third conclusion is in sharp contrast to the principle pointed out by Bhagwati (1971). Bhagwati (1971) argued that the optimal policy for removing a distortion was one that affects the distortion at its source. According to the principle, welfare should be higher under an import policy than under an export policy in the setting. On the contrary, we have shown that it is preferable to use an export policy rather than an import policy when the production function in question exhibits mild decreasing return to scale (i.e., 0.709 < r < 1). Therefore, the principle established by Bhagwati (1971) is not applicable to cases in which production functions exhibit non-constant returns to scale.

To simplify the analysis and focus on the role of returns to scale, we begin with the model assuming a linear demand and simplified production function using only one input. This model has been extended in two directions. First, we have introduced a fixed factor into the production function and concluded that this extension does not change our qualitative results. Second, we allow the demand to be non-linear. It is found that if the demand curve is concave (convex), the optimal import tariff on intermediate good is positive (negative) in the case of optimal import policy. So is the optimal export subsidy policy on the final good in the case of optimal export policy.

In sum, this paper has employed a production function with variable-coefficient technology to show that return to scale is crucial in determining the direction of government intervention. Therefore, the assumption of fixed-coefficient production technologies, which has been popular in industrial organization and trade literature when modeling vertically related markets, should be used with caution.

REFERENCES


Appendix

A. Differentiating (9) with respect to \( w \) yields:

\[
\pi^*_{ww} = \frac{1}{r} Q^r_w Q^* w^* + \frac{1}{r} Q^r_w Q^* w^* + (w - t - c) - \frac{1}{r} (w - c) Q^r_w Q^* (Q^*)^2 + w - c - \frac{1}{r} Q^r_w \frac{d(Q^* w)}{dw}.
\]

Substituting \( \frac{d(Q^* w)}{dw} = 2 \frac{(1 - 1)}{Q} Q^* (Q^*)^2 + \frac{1}{r} (1 - 2) \frac{1}{Q^2} (Q^*)^3 \) and (9) into the
above equation, we have \( \pi_{ww}^* = \left[ (3 - \frac{1}{r}) - \frac{1}{r} \right] \frac{1}{w} Q_w^* = \frac{1}{r} Q^* \).

B. Substitute (10) \( w_i^* = \frac{1}{r} \) into (13) to yield

\[
t^* = \frac{r Q [1 - w^*_w]}{Q_w w_t} = r^2 Q \left[ \frac{2 - 1}{r} \right] \left[ \frac{2}{r} - 3 + \frac{1}{r} \frac{1}{w} Q_w^* \right] \left[ -2b - \frac{1}{r} (\frac{1}{r} - 1) w Q^* \right].
\]

C. From (7), we have the slope of the derived demand:

\[
w_x = \frac{d w}{d x} = \frac{d w}{d Q} \frac{d Q}{d x} = \frac{1}{Q_w} r x^{r-1} = -2b r^2 x^{2r-2} - (1 - r) w x^{-1} < 0,
\]

and

\[
w_{xx} = \frac{d^2 w}{d x^2} = (1 - r) (4b r^2 x^{2r-3} - w x^{-1} + w x^{-2}),
\]

where \( (4b r^2 x^{2r-3} - w x^{-1} + w x^{-2}) \) is positive. Thus, \( w_{xx} < (\geq, >) 0 \) if and only if \( r > (\leq, <) 1 \), that means the derived demand is concave (linear, convex) to the origin.

D. From (22) \( \frac{d w^*}{d s} = -\frac{\pi_{ww}^* F}{\pi_{ww}^*} \)

where \( \pi_{ww}^* = \frac{1}{r} Q^{1-r} \) \( Q^*_w + (w - c) \frac{1}{r} \frac{1}{r} - 1) Q^{1-r} \) \( Q^*_w Q^*_w^* + (w - c) \frac{1}{r} Q^{1-r} \frac{d (Q^*_w)}{d s} \).

Moreover, substitute \( \frac{d (Q^*_w)}{d s} = (1 - r) \frac{1}{Q_w} Q^*_w Q^*_s + (1 - r) \frac{1}{r} \frac{1}{r} - 2) \frac{w}{Q^2} (Q^*_w)^2 Q^*_s \) and (21)

into \( \pi_{ww}^* \) to get \( \pi_{ww}^* = -\frac{1}{r} \frac{w - c}{Q_w} \) \( 1 + (\frac{1}{r} - 1) \)

E. Substitute \( Q_w^* = \frac{1}{r} Q^{1-r} < 0, Q^*_w = \frac{1}{r} Q^{1-r} > 0, \)

\[
-2b - \frac{1}{r} (\frac{1}{r} - 1) w Q^{r-2} - 2b - \frac{1}{r} (\frac{1}{r} - 1) w Q^{r-2}
\]

and \( w_i^* = -\frac{\pi_{ww}^*}{\pi_{ww}^*} \)

\[
\pi_{ww}^* = \frac{1}{r} \frac{1}{r} - (\frac{1}{r} - 1) \frac{1}{r} - 2) \frac{w}{Q^*_w} \frac{1}{r} - 1) Q^*_w \) \( 1 - Q^*_w \)

into \( Q^*_w w_i^* + Q^*_s \) yields
\[ Q^*_w w^*_s + Q^*_r = \frac{1}{[2b + \frac{1}{r} \left( \frac{1}{r} - 1 \right) w Q^*_w^{\frac{1}{2}}]} \frac{1}{\left[ (3 - \frac{1}{r}) - \left( \frac{1}{r} - 1 \right) \frac{1}{w} Q^*_w \right]} > 0. \]