

A Revisit to the Non-linear Mean Reversion of Real Exchange Rates:  
Evidence from a Series-specific Non-linear Panel Unit-root Test

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Abstract

The purpose of this paper is to construct a series-specific non-linear panel unit-root test and then apply it to examine the non-linear mean reversion of real exchange rates for two different panels of industrial countries. We find that the non-linear series-specific panel unit root test achieves higher power and more reasonable size than the linear one suggested by Breuer et al. (2002, Oxford Bulletin of Economics and Statistics 64, 527-546) when the data generating process is calibrated to reflect significant non-linear behaviors. Applying the test to examine the stationarity of real exchange rates with two different panels of countries, we find that about half of the real exchange rates are non-linear stationary in each panel. Moreover, we find that our bootstrap tests achieve a reasonable size based upon a bootstrap-after-bootstrap method. Our findings point out significant non-linearity in the dynamics of real exchange rates.

Keywords: panel unit-root tests, real exchange rates, non-linearity, bootstrap-after-bootstrap.

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## 1. Introduction

Purchasing power parity (PPP), a major cornerstone of many theoretical models in international finance, has been a hot research topic during the last two decades. The long-run PPP requires that real exchange rates must be stationary, which implies that shocks may have long-lasting but ultimately transitory effects on deviations from PPP, thus making the real rate a mean-reverting process. Empirically, conventional literature examines the stationarity of real exchange rates based on a single equation based unit-root test. No consensus exists yet as to what the answer should be. Different findings on the validity of long-run PPP depend on the numeraire currency, the length of data span, and econometric methods.

Ever since the seminal paper by Abuaf and Jorion (1990) and Levin and Lin (1992), the literature on panel unit-root tests and their applications to the stationarity of real exchange rates has grown tremendously (Wu, 1996; Papell, 1997; Taylor and Sarno, 1998; Maddala and Wu, 1999; Chou and Chao, 2001; Kuo and Mikkola, 2001; Chang, 2002; Levin et al., 2002; Im et al., 2003; Smith et al., 2004; Choi and Chue, 2007; Pesaran, 2007).<sup>1</sup> The major advantage for adopting panel unit-root tests is their high power by exploiting cross-section information. Among these existing methods, all studies in the literature adopt a linear structure to test the joint unit-root hypothesis under a well known restriction that rejecting the null hypothesis provides little information for the stationarity of individual series. To overcome this restriction, there have been several approaches in the literature. Taylor and Sarno (1998) provide the JLR test to examine the hypothesis of at least one unit-root series in the panel and conclude that all series in the panel are stationary if the null

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<sup>1</sup> A detailed survey on the development of panel unit-roots can be found in Breitung and Pesaran (2007) and Choi (2006).

hypothesis is rejected. However, the power of the JLR test is poor based on their simulation results. Hadri (2000) tests the hypothesis that all the individual series are stationary against the alternative of at least a single unit root in the panel.<sup>2</sup> An alternative interesting approach is to test the unit-root hypothesis for each specific series in the panel. Breuer et al. (2002, hereafter BMW) propose a series-specific unit-root test that allows researchers to distinguish I(1) and I(0) series in the panel.

A number of articles have pointed out that a non-linear exponential smooth transition autoregressive (hereafter ESTAR) model, provided by Granger and Teräsvirta (1993), is useful in modeling real exchange rates (see, for example, Michael et al., 1997; Taylor et al., 2001). In addition, the issue of exchange rate predictability has also been linked to real exchange rate non-linearity by Kilian and Taylor (2003). Theoretically, there are at least two reasons supporting the above findings. First, the presence of market frictions and the allowance of transportation to take time impeding the smooth operation of arbitrage (Dumas, 1992; Sercu et al., 1995; and Coleman, 1995).<sup>3</sup> Second, models of pricing to market and exchange rate pass-through give rise to impediments to a commodity's arbitrage (Krugman, 1987; Dixit, 1989; and Froot and Klemperer, 1989). The impact of a transitory shock may last for a long period of time when the non-linearity of the series is significant. This indicates that a non-linear stationary series may be highly persistent which causing it to be indistinguishable from a unit-root series.<sup>4</sup> Taylor (2001) points out that the

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<sup>2</sup> If the null hypothesis is not rejected by Hadri's (2000) test, then there is evidence that all series in the panel are stationary.

<sup>3</sup> If transportation takes time, then arbitrage occurs when the expected profit from arbitrage is positive.

<sup>4</sup> We simulate series from a stationary ESTAR model given as follows, with different parameters:

$$\Delta y_{k,t} = \gamma_k y_{k,t-1} \left\{ 1 - \exp(-\theta_k y_{k,t-1}^2) \right\} + \varepsilon_{k,t}.$$

power of the conventional ADF test is poor if the series under investigation follow a non-linear threshold process, which calls for a non-linear unit-root test in the literature. To fill the gap in the literature, Kapetanios et al. (2003) propose a non-linear unit-root test based on an ESTAR(1) model and show that the power of their test is higher than that of the ADF test.

It is therefore interesting to propose a series-specific non-linear panel unit-root test that allows researchers to exploit the cross-section information and to test the unit-root hypothesis for each series in the panel. This is the purpose of the paper. Our procedure applies the seemingly unrelated regression (SUR) method and handles the issues of contemporaneous correlation and heterogeneous serial correlation. We also perform the power analysis in which we compare the power of our non-linear panel unit-root test to that of the BMW test. Finally, we apply our proposed test to re-examine the unit-root hypothesis of real exchange rates in two different panels: the panel of group 10 (G10) and group 20 (G20).<sup>5</sup>

Based on our simulation results, we find that the power of our non-linear panel unit-root test is higher than that of the BMW test when the data generating process is significantly non-linear; otherwise, the power dominance of the BMW test is observed. In addition, we find that, based on our non-linear panel unit-root test, real exchange rates in the panel of G10 and G20 are indeed mixed with I(0) and I(1) in roughly

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With a small value of  $\theta$  ( $\theta = 0.01$ ), we find that simulated series reveal no significant mean-reverting behavior and look similar to that from a driftless random walk. In addition, the ADF  $\tau_{\mu}$  statistic does not reject the unit root hypothesis for those series. We do not report our simulation results in the paper but they are available upon request from authors.

<sup>5</sup> The panel of G10 includes the United Kingdom, Belgium, France, Germany, Italy, the Netherlands, Sweden, Switzerland, Canada, and Japan. The panel of G20 includes those in G10 as well as Australia, New Zealand, Austria, Norway, Finland, Greece, Ireland, Portugal, Spain, and Denmark.

equal proportions, which is in sharp contrast to that of Breuer et al. (2002). Furthermore, we find that our non-linear panel unit-root test has reasonable size in general based on the procedure of bootstrap-after-bootstrap. The effective size of the BMW test is far less than the nominal size in several countries when real exchange rate dynamics are non-linear. This finding indicates that the BMW test is too conservative in testing unit-root hypothesis when variables under investigation are non-linear.

The additional part of this paper is organized as follows: The model and testing procedures are given in Section 2. Power analysis of the linear and non-linear series-specific unit-root tests is given in section 3. In section 4 we apply both tests to re-examine the unit-root hypothesis of real exchange rates in the panels of G10 and G20, respectively. Finally, we summarize our conclusions in the last section.

## 2. Econometric Methodology

Consider the following specification:

$$\Delta y_{k,t} = \alpha_k + \beta_k y_{k,t-1} + \sum_{j=1}^{p_k} \phi_{k,j} \Delta y_{k,t-j} + \varepsilon_{k,t}, \quad t = 1, 2, \dots, T, \quad k = 1, 2, \dots, N, \quad (1)$$

where  $\varepsilon_{k,t}$  is a zero-mean stationary process. Conventional panel unit-root tests examine the following joint unit-root hypothesis:  $H_0 : \beta_1 = \dots = \beta_N = 0$  (Levin et al., 2002; Im et al., 2003; and Maddala and Wu, 1999). However, there are several pitfalls in conventional panel unit-root tests. First, the rejection of the joint null hypothesis does not imply that all series in the panel are stationary. Second, panel unit-root tests may lead to a very high probability of rejecting the joint unit-root hypothesis when there exists at least a single stationary series in the panel (Taylor and Sarno, 1998). Third, failure to consider contemporaneous correlation among data

will bias the panel unit-root test toward rejecting the joint unit-root hypothesis (O'Connell, 1998). To take into account these pitfalls, Breuer et al. (2002) provides a series-specific panel unit-root test within a SUR framework, which allows us to test the unit-root hypothesis for each series in the panel.<sup>6</sup> Referring to Equation (1), Breuer et al. (2002) provides the SURADF<sup>k</sup> statistic to test the following null and alternative hypotheses:

$$H_0^k : \beta_k = 0; \quad H_1^k : \beta_k < 0, \quad \forall k = 1, 2, \dots, N.$$

These hypotheses are tested using t-statistics, constructed from SUR estimates, with simulated critical values.<sup>7</sup>

There is ample literature pointing out that it may not be appropriate to test the stationarity of financial data based on a linear framework (Chortareas et al., 2002; Chortareas and Kapetanios, 2004; Liew et al., 2004). Kapetanios et al. (2003) provide non-linear unit-root tests to examine the hypothesis of unit-root against the alternative of a globally stationary ESTAR process. The t statistic ( $t_{NL}$ ) of their test is constructed by regressing the following auxiliary equation using ordinary least squares:<sup>8</sup>

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{j=1}^p \phi_j \Delta y_{t-j} + \varepsilon_t. \quad (2)$$

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<sup>6</sup> The SUR method is applied since it exploits the information of contemporaneous correlation in residuals.

<sup>7</sup> The critical values of the t-statistics are simulated from Monte-Carlo simulations since the finite sample distribution of the t-statistics from the SUR system of (1) is not known.

<sup>8</sup> Applying the Taylor expansion to an ESTAR (1) model, one obtains the following auxiliary regression:  $\Delta y_t = \delta y_{t-1}^3 + \varepsilon_t$ . To take into account the likely serial correlation in residuals,  $\varepsilon_t$ , the lagged variables of  $\Delta y_t$  are included in equation (2) as suggested by Kapetanios et al. (2003).

If the hypothesis of  $\hat{\delta} = 0$  against  $\hat{\delta} < 0$  is rejected, then we claim that  $y$  is a non-linear stationary series.<sup>9</sup> To accommodate a stochastic process with non-zero means, Kapetanios et al. (2003) suggest using de-meaned data in estimation. Using de-meaned data in a non-linear model implies a specific view of the way that an intercept enters the model under the alternative.<sup>10</sup>

To generalize the non-linear unit-root test of Kapetanios et al. (2003) to a panel framework and to allow for testing stationarity for each series in a panel, we consider the following system equations:

$$\Delta y_{k,t} = \delta_k y_{k,t-1}^3 + \sum_{j=1}^{p_k} \eta_{k,j} \Delta y_{k,t-j} + \varepsilon_{k,t}, \quad k = 1, 2, \dots, N. \quad (3)$$

After estimating the system in (3) with SUR, the t-statistic ( $\text{SUR}t_{\text{NL}}^k$  statistic) for the hypothesis of  $\hat{\delta}_k = 0$  is then constructed to test for the stationarity of the specific series,  $y_{k,t}$ . Since the test statistics have a non-standard distribution, their critical values are obtained through bootstrap. The computed critical values will be specific to the estimated covariance matrix for the series tested, the sample size (T), and the panel size (N).

### 3. Power Analysis

It is interesting to investigate the finite sample performance of the  $\text{SUR}t_{\text{NL}}^k$  statistic and to compare its power with that of the  $\text{SURADF}^k$  statistic. To this end, we construct finite sample critical values through Monte Carlo simulations and then

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<sup>9</sup> The asymptotic distribution of the t-statistic for the hypothesis of  $\hat{\delta} = 0$  is non-standard as pointed out by Kapetanios et al. (2003).

<sup>10</sup> If the model includes a constant and a trend, then the  $t_{\text{NL}}$  statistic is constructed by estimating equation (3) with de-meaned and de-trended data.

compare their powers. We first simulate artificial data based on a system of driftless random walks with high cross-correlation in residuals together with varying panel sizes and numbers of observations.<sup>11</sup> The range for high residual cross-correlations is 0.7-0.8.<sup>12</sup> After having simulated data, we then estimate the following system equations with de-meaned data using SUR:

$$\Delta \tilde{y}_{k,t} = \delta_k \tilde{y}_{k,t-1}^3 + \varepsilon_{k,t}, \quad k = 1, 2, \dots, N. \quad (4)$$

Here,  $\tilde{y}_{k,t}$  denotes for the de-meaned simulated variable. The  $\text{SURt}_{\text{NL}}^k = \hat{\delta}_k / \sigma_{\delta}^k$  statistics, in which  $\sigma_{\delta}^k$  being the standard deviation of  $\delta_k$ , is then constructed. In each experiment, the 5% critical values are simulated based on 10,000 replications.<sup>13</sup>

Table 1 reports the averages of the 5% absolute critical values across the panel for the  $\text{SURt}_{\text{NL}}^k$  and  $\text{SURADF}^k$  statistics, respectively.<sup>14</sup> We find that the averages of the critical values for these two statistics respectively decrease as the number of observations (T) increases. Moreover, for given T, the averages of the

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<sup>11</sup> Following Breuer et al. (2002), the panel size (N) is set to 5, 10, and 20, respectively. For each panel size the number of observation for each series is set to 50, 100, and 250, respectively.

<sup>12</sup> The reason for us to focus on experiments with high cross correlation is that the cross correlation figure among real exchange rates is high. Simulation results based on low or middle cross correlation among residuals are also constructed. They are not reported here, but are available upon request.

<sup>13</sup> After generating data in each experiment, we regress the system equations (4) with de-meaned data and the linear system with intercepts to obtain their SUR estimates. We then construct the finite sample distribution of  $\text{SURt}_{\text{NL}}^k$  and  $\text{SURADF}^k$  statistics, respectively, based on 10,000 replications. We focus on the system with intercepts rather than the one with trends being consistent with the existing empirical models for real exchange rates.

<sup>14</sup> Because the critical values for each series in a given experiment are similar, only the averages of critical values across the panel are reported. Series specific results are available on request from authors.



critical values increase as the panel size,  $N$ , increases. In addition, the averages of the critical values for the  $SURADF^k$  statistic are generally higher than those of the  $SURt_{NL}^k$  statistic.<sup>15</sup>

We then document the power of  $SURt_{NL}^k$  and  $SURADF^k$  tests in various environments. We begin with considering a linear stationary alternative. The data generating process (DGP) in this case is a system of stationary AR(1) processes with the autoregressive (AR) coefficients set to either 0.95 or 0.9 and high contemporaneous correlations in residuals. The power results for  $SURt_{NL}^k$  and  $SURADF^k$  tests are obtained in experiments through 5,000 replications.

Table 2 presents results on the power performance of the tests under a linear AR(1) alternative hypothesis, and the number in the table is the average of the power for the  $SURt_{NL}^k$  and  $SURADF^k$  statistics across the panel. Results from Table 2 indicate that the average of the power for these statistics increases with the number of observations for a given panel size, but they are not significantly affected by the increase of the panel size for a given number of observations. This finding is similar to that of Breuer et al. (2002). The average of the power of the  $SURADF^k$  statistic is higher than that of the  $SURt_{NL}^k$  statistic, which is expected under the assumption of a linear AR(1) alternative hypothesis.

### **3.1. Power analysis under the hypothesis that all series in the panel follows a stationary ESTAR process.**

In this subsection we investigate the power of the  $SURt_{NL}^k$  statistic in various environments under the alternative hypothesis that all series are generated from a

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<sup>15</sup> Holding  $T$  and  $N$  constant, the absolute critical values also increase with the value of the cross-correlation in residuals. These simulation results are available upon request.

system of stationary ESTAR model, and then compare it with that of the SURADF<sup>k</sup> statistic. Following Kapetanios et al. (2003), the DGP is given as follows for a general power comparison:

$$\Delta y_{k,t} = 0.1y_{k,t-1} + \gamma_k y_{k,t-1} \left\{ 1 - \exp(-\theta_k y_{k,t-1}^2) \right\} + \varepsilon_{k,t}, \quad (5)$$

where  $\varepsilon_{k,t}$  is assumed to follow a standard normal distribution,  $\gamma_k = \{-1.5, -1, -0.5\}$ , and  $\theta_k = \{0.01, 0.05, 0.1, 1\}$ . Term  $\theta_k$  denotes the speed of transition between regimes. The transition function in equation (5) is indicated by:  $F(\theta_k, y_{k,t-1}) = 1 - \exp(-\theta_k y_{k,t-1}^2)$ , which captures the non-linearity of the model. The value of the transition function is bounded between zero and one.

Another representation of equation (5) is given as follows:  $y_{k,t} = (1.1 + \gamma_k * F(.))y_{k,t-1} + \varepsilon_{k,t}$ . The value of the transition function,  $F(.)$ , varies over time, and hence the slope of the AR(1) process varies over time. If  $\theta_k$  approaches zero, then the value of transition function is close to zero, and the AR(1) coefficient of  $y_{k,t}$  will be larger than one. The non-linear process of  $y_{k,t}$  in equation (5) will therefore degenerate to a linear explosive process in this case, otherwise, it follows a non-linear stationary process. The magnitude of  $\gamma_k$  controls the significance of the non-linear process to be locally non-stationary. If  $\gamma_k$  approaches zero, then equation (5) degenerates to a linear explosive process. Therefore equation (5) means that the non-linear process is locally explosive while globally remaining geometrically ergodic.<sup>16</sup> Testing the significance of the

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<sup>16</sup> A zero coefficient for the first regressor,  $y_{k,t-1}$ , in equation (5) means that the stationary nonlinear process behaves as a random walk in the middle range. There is no significant difference in

transition parameter is equivalent to testing the hypothesis that the real exchange rate follows a linear explosive process against the alternative of a mean-reverting ESTAR process. The asymptotic distribution for the t-statistic of the transition parameter ( $\gamma=0$ ) is not conventional but its empirical marginal significance levels can be constructed through a bootstrap method (Taylor and Peel, 2000).

We first generate artificial data of  $y_{k,t}$  based on equation (5) with the high residual cross-correlations as mentioned above. We then construct the  $SURt_{NL}^k$  and  $SURADF^k$  statistics by estimating the system equations (3) and (1), respectively, using SUR, and compare these statistics with their respective critical values. The powers of the  $SURADF^k$  and  $SURt_{NL}^k$  statistics under different panel sizes and sample sizes are obtained by repeating the previous procedures 5,000 times. Numbers in Table 3 are the averages of the powers across the panel for both statistics with high cross-correlation in residuals, and the number in a parenthesis is the average of the power with low cross-correlation in residuals.<sup>17</sup>

Several interesting findings are observed from Table 3. First, the power of the  $SURt_{NL}^k$  and  $SURADF^k$  statistics increases with sample size (T) and the magnitude of cross-correlation in residuals, holding other parameters constant. Second, the power of the  $SURt_{NL}^k$  statistic is higher than that of the  $SURADF^k$  statistic in general, which is expected given the non-linear data generating process. Third, the powers of both statistics are not systematically affected by panel sizes. Fourth, the

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simulation results between these two cases and hence we do not report results from the case of a zero coefficient for  $y_{k,t-1}$ . These results are available upon request from the authors.

<sup>17</sup> The low cross-correlation coefficients refer to the case where they are restricted to vary in the range of (0.2, 0.3).

powers of both statistics generally increase with the value of  $\theta$ , and they are close to one when  $\theta$  equals one unless the absolute value of  $\gamma$  is small. The superiority of the  $SURt_{NL}^k$  test relative to the  $SURADF^k$  test is observed in general when  $\theta$  is equal to 0.05 and 0.1. In the case where  $\theta$  is large (e.g.  $\theta=1$ ), the power of the  $SURADF^k$  test exceeds that of the  $SURt_{NL}^k$  test. The reason could be that the model becomes approximately linear as  $\theta$  grows large, which leads to the power dominance of the  $SURADF$  statistic (Kapetanios et al., 2003). On the other hand, the model becomes approximately linear and explosive as  $\theta$  is small (e.g.  $\theta=0.01$ ), which is the reason for both statistics to have low power except  $T$  being sufficiently large.

Fifth, there is no significant differences in the power of both statistics when the sample size is large ( $T=250$ ) unless the value of  $\theta$  is small ( $\theta=0.01$ ). Finally, and perhaps most importantly, the power of the  $SURt_{NL}^k$  test is low when both  $T$  and  $\gamma$  are small (e.g.  $T=50$ ,  $\gamma=-0.5$ ). This is not surprising, because the model in (5) degenerates to a linear explosive process if  $\gamma=0$ , and the power of both statistics tend to be lower when the absolute value of  $\gamma$  is small. Given the sample size and  $\theta$ , the power of the  $SURt_{NL}$  statistic increases with the absolute value of  $\gamma$ , and hence the non-linearity of data. In summary, we find that the power of the  $SURt_{NL}^k$  statistic is higher than that of the  $SURADF^k$  statistic when the data generating process is calibrated to reflect significant non-linear behaviors.

### **3.2. Power analysis under the hypothesis that the panel mixes with I(0) and I(1) series**

It is likely that the series under investigation in a panel are mixed with I(0) or I(1) series. In this case the conventional panel unit-root tests tend to over-reject the

joint unit-root hypothesis. We thus explore the power of the  $SURt_{NL}^k$  test in a panel with the mixed  $I(0)$  and  $I(1)$  series, and then summarize in Table 4 the power performance, with high cross-correlation in residuals and  $T=100$ . This is because the number of quarterly real exchange rates used in our empirical analysis is about 100 and the cross correlations among real exchange rates are high in data. The DGP for the stationary series is described by the non-linear  $ESTAR(1)$  process given in equation (5) and the DGP for the  $I(1)$  series is the driftless random walk. The true DGP for both the  $I(1)$  and  $I(0)$  series do not include intercepts, but all regressions are estimated with intercepts.

Several interesting findings are observed from Table 4, in which  $AVEI(0)$  is the average power result for the  $I(0)$  series in the panel while  $AVEI(1)$  is the average rejection rate for the  $I(1)$  series in the same panel. First, the average power of the  $SURt_{NL}^k$  statistic is high in general under the case of a mixed panel unless the absolute values of  $\gamma$  and  $\theta$  are low (e.g.  $\gamma = -0.5$  and  $\theta = 0.01$ ). For example, the power of the  $SURt_{NL}^k$  statistic is 0.405 for the case of  $\gamma = -0.5$ ,  $\theta = 0.01$ ,  $N = 5$  and the number of  $I(0)$  variables in the panel is one, but it is 0.998 when  $\theta = 1.0$  and other parameters remain the same. Holding other parameters constant, the average power of the  $SURt_{NL}^k$  statistic increases with the non-linearity of the model (that is, a high  $\theta$  or a high absolute value of  $\gamma$ ). Second, the average power of the  $SURt_{NL}^k$  statistic tends to increase if the proportion of stationary series in the panel decreases, which is consistent with the finding of Breuer et al. (2002) for the  $SURADF^k$  statistic.<sup>18</sup> However, the effect of the number of stationary variables in a panel on the average power of the  $SURt_{NL}^k$  statistic is limited especially when the

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<sup>18</sup> The likely reason for this finding is provided in Breuer et al. (2002).

value of  $\theta$  is larger than 0.01. Third, the average rejection rates of the  $\text{SURt}_{\text{NL}}^k$  statistic for both I(0) and I(1) series are not significantly affected by panel sizes. Finally, the average rejection rate of the I(1) series, AVEI(1), is small regardless of other parameter values, and it decreases as the number of I(0) variables in the panel increases.<sup>19</sup>

To summarize, results from Table 4 point out that the average power of the  $\text{SURt}_{\text{NL}}^k$  statistic is high as long as the speed of adjustment between regimes is greater than 0.01 ( $\theta > 0.01$ ). In addition, the average power tends to decrease as the number of stationary variables in the panel increases, but its effect on the average power is limited especially when  $\theta > 0.01$ .

#### **4. Empirical Investigation**

Data for quarterly nominal exchange rates and consumer price indices are taken from International Monetary Fund's *International Financial Statistics*. The empirical period runs from the second quarter of 1973 (1973Q2) to the last quarter of 1998 (1998Q4) with 102 quarterly observations. Twenty industrial countries are included in our sample: United Kingdom (UK), Belgium (BEL), France (FRA), Germany (GER), Italy (ITA), Netherlands (NET), Sweden (SWE), Switzerland (SWI), Canada (CAN), Japan (JAP), Australia (AUS), New Zealand (NEZ), Austria, (AUS), Norway (NOR), Finland (FIN), Greece (GRE), Ireland (IRE), Portugal (POR), Spain (SPA), and Denmark (DEN). For those countries without a consumer price index, their real exchange rates are then constructed based on a wholesale price index. The reason we end our sample period at 1998Q4 is due to the introduction of the Euro in

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<sup>19</sup> In addition, controlling for other parameters, the power of  $\text{SURt}_{\text{NL}}^k$  statistic increases as the cross-correlation in residuals increases (results are not reported here but are available upon request from the authors).

1999. Two different panels are discussed. They are group 10 (G10) and group 20 (G20), respectively.<sup>20</sup>

We first apply the ADF statistic to examine the stationarity of real exchange rates among countries in our sample. The model without a linear trend is applied and the lag order is selected based on the recursive t-test procedure based on Campbell and Perron (1991). Results from the second column of Table 5 point out that the unit-root hypothesis is rejected for three out of twenty real exchange rates at the 5% level of significance. This result is consistent with that of existing literature and is due to the low power of the ADF test when the real exchange rates are highly persistent.

The alternative reason for the failure of rejecting the unit-root hypothesis could be due to the recent argument that real exchange-rate processes are likely to be non-linear due to the existence of transaction costs, and hence the power of the ADF test is poor in this situation. To justify this conjecture, we therefore apply a non-linear unit-root test as provided by Kapetanios et al. (2003) to re-investigate the unit-root hypothesis of real exchange rates. The lag order is the same as those in the ADF test. Since a non-trend model is applied, we first de-mean the data and then construct the  $t_{NL}$  statistic as suggested by Kapetanios et al. (2003). However, results from the third column of Table 5 indicate that the unit-root hypothesis is not rejected for all countries in either panel.

The failure of rejecting the unit-root hypothesis given linear and non-linear unit-root tests leads us to conjecture that the power of a single equation test is low (Levin et al., 2002). We therefore apply panel data unit-root tests to re-examine the unit-root hypothesis of real exchange rates. We first apply the  $SURADF^k$  statistic

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<sup>20</sup> See footnote 4 for the definitions of G10 and G20.

provided by Breuer et al. (2002) to examine the stationarity of real exchange rates based on the panel of G10 and G20, respectively. The  $SURADF^k$  statistics are reported in the fourth column of Table 5, and their 5% and 10% critical values are reported in the fifth and sixth columns, respectively. These critical values are simulated through a non-parametric bootstrap with 10,000 replications, and hence we take into account the effect of contemporaneous correlation across residuals on critical values. Findings from columns four to six indicate that real exchange rates are stationary only in two out of ten countries for the G10 panel and in one out of twenty countries for the G20 panel. These results are similar to those of Breuer et al. (2002).

It is worth noting that the failure of rejecting the unit-root hypothesis of real exchange rates based on a series-specific linear panel unit-root test may be due to the fact that real exchange rates are indeed non-linear. To justify this conjecture, we then apply the series-specific non-linear panel unit-root test to re-investigate the unit-root hypothesis of real exchange rates. Our empirical results are reported in columns seven to nine of Table 5, in which column seven reports the  $SURt_{NL}^k$  statistics and its 5% and 10% critical values are reported in columns eight and nine. We find strong evidence for non-linear stationarity of real exchange rates since the number of countries that rejects the unit-root hypothesis of real exchange rates is six out of ten in the G10 panel and ten out of twenty in the G20 panel. In brief, half the countries in the panel reject the unit-root hypothesis of real exchange rates based on the  $SURt_{NL}^k$  statistics.

Given the finding of evidence to reject the unit-root hypothesis for real exchange rates, we then ask whether our rejections resulted from the size distortion of our bootstrap test. To address this issue, we apply a bootstrap-after-bootstrap



method, suggested by Berkowitz and Kilian (2000), to investigate the effective size of our bootstrap tests given the nominal size of 5%.<sup>21</sup> Results from Table 6 indicate that the size of  $SURt_{NL}^k$  ranges from 0.040 to 0.054 for the G10 panel and from 0.033 to 0.066 for the G20 panel. Among those 10 countries in the G20 panel that reject the unit-root hypothesis for real exchange rates, their size ranges from 0.041 to 0.066. In general, the bootstrapped  $SURt_{NL}^k$  statistics achieve reasonable size. As for the  $SURADF^k$  statistic, we find that its size varies from 0.025 to 0.047 for the G10 panel and from 0.016 to 0.055 for the G20 panel, with these being lower than those of  $SURt_{NL}^k$  statistic. Results from Table 6 conclude that failing to control the non-linearity of data leads the  $SURADF^k$  test to be a conservative test relative to the  $SURt_{NL}^k$  test. This could be the reason for failing to reject the unit-root hypothesis when the  $SURADF^k$  statistic is applied. Briefly, our evidence points out that about half the real exchange rates in G10 and G20 panels are non-linear stationary, which in turn sheds light on the significance of non-linear dynamics of real exchange rates.

## 5. Conclusions

Conventional literature examines the unit-root hypothesis of real exchange rates based on a linear framework, but findings for long-run PPP are mixed at best. There are several studies in the literature that consider an alternative possibility whereby real exchange rates follow a smooth transition autoregressive (STAR) model, which is consistent with the theoretical development emphasizing transaction costs or the interaction of heterogeneous agents in the foreign exchange market. However, most existing studies in the literature assume non-linear dynamics of real exchange rates without rigorous tests.

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<sup>21</sup> A detailed description of the procedure of bootstrap-after-bootstrap is given in the Appendix.

Kapetanios et al. (2003) provide a non-linear unit-root test to examine the unit-root hypothesis of a variable. The purpose of this paper is to provide a series-specific non-linear panel unit-root test that allows us to test the stationarity of individual variables in the panel. The reason for focusing on a series-specific test is to avoid the well known pitfall in testing the joint unit-root hypothesis. We first examine the power of the series-specific non-linear panel unit-root test and find that the test has higher power and a more reasonable size than the linear one suggested by Breuer et al. (2002) when the data generating process is calibrated to reflect significant non-linear behaviors. Its power remains high if the panel mixes with stationary and unit-root variables.

Based on two different panels of real exchange rates among industrial countries, we apply the test to examine the unit-root hypothesis for each real exchange rate in the panel. Our results point out that the unit-root hypothesis is rejected for about half the real exchange rates in each panel, which significantly differs from that of Breuer et al. (2002). Our findings therefore support significant non-linear dynamics of real exchange rates among industrial countries.

## Appendix

The purpose of this appendix is to describe the procedures for constructing the size of the panel data bootstrap test. The idea is to nest two non-parametric bootstrap procedures together.

1. Obtain the bootstrap sample of the error term  $\varepsilon_t^0 = [\varepsilon_{1t}^0, \varepsilon_{2t}^0, \dots, \varepsilon_{Nt}^0]$  by estimating the following system equations with the iterative, seemingly unrelated regression (SUR) method:

$$\Delta x_{k,t} = \hat{\lambda}_k^0 + \hat{\delta}_k^0 x_{k,t-1}^3 + \sum_{j=1}^{h_i} \hat{\phi}_{k,j}^0 \Delta x_{k,t-j} + \varepsilon_{k,t}^0, \quad k = 1, \dots, N,$$

where  $\Delta x_{k,t} = x_{k,t} - x_{k,t-1}$ .

2. Generate  $x_{k,t}^{*,0}$  as follows:

$$\Delta x_{k,t}^{*,0} = \hat{\lambda}_k^0 + \sum_{j=1}^{h_i} \hat{\phi}_{k,j}^0 \Delta x_{k,t-j}^{*,0} + \varepsilon_{k,t}^{*,0}.$$

We randomly draw  $T+100$  disturbances, with replacement, from estimated residuals,  $\varepsilon_t^0 = (\varepsilon_{1t}^0, \dots, \varepsilon_{Nt}^0)'$ , to generate a series of residuals,  $\varepsilon_t^{*,0} = (\varepsilon_{1t}^{*,0}, \dots, \varepsilon_{Nt}^{*,0})'$ , for our pseudo sample. Since we draw the residuals,  $\varepsilon_{1t}^0, \dots$  and  $\varepsilon_{Nt}^0$ , in tandem, the simulated sample preserves the contemporaneous correlation in the disturbances presented in the original data. After generating  $x_{k,t}^{*,0}$  we drop the first 100 observations and then construct the  $\text{SURt}_{NL}^{k,0}$  statistic,  $\text{SURt}_{NL}^{k,0}$ , by regressing the following equation:

$$\Delta \tilde{x}_{k,t}^{*,0} = \delta_k^{*,0} (\tilde{x}_{k,t-1}^{*,0})^3 + \sum_{j=1}^{h_i} \eta_{k,j}^0 \Delta \tilde{x}_{k,t-j}^{*,0} + \varepsilon_{k,t}^{a,0}, \quad k = 1, \dots, N,$$

where  $\tilde{x}_{k,t}^{*,0}$  is the de-meaned series of  $x_{k,t}^{*,0}$ .

3. Obtain the estimated residuals,  $\varepsilon_t^1 = [\varepsilon_{1t}^1, \varepsilon_{2t}^1, \dots, \varepsilon_{Nt}^1]$ , by estimating the following system equations with the SUR method:

$$\Delta x_{k,t}^{*,0} = \hat{\lambda}_k^1 + \hat{\delta}_k^1 (x_{k,t-1}^{*,0})^3 + \sum_{j=1}^{h_i} \hat{\phi}_{k,j}^1 \Delta x_{k,t-j}^{*,0} + \varepsilon_{k,t}^1, \quad k = 1, \dots, N.$$

Randomly draw  $T+100$  disturbances, with replacement, from estimated residuals,

$\varepsilon_t^1 = [\varepsilon_{1,t}^1, \dots, \varepsilon_{N,t}^1]$ , to generate a series of residuals,  $\varepsilon_t^{*1} = [\varepsilon_{1,t}^{*1}, \dots, \varepsilon_{N,t}^{*1}]$ , Generate

$x_{i,t}^{*,1}$  as follows:

$$\Delta x_{k,t}^{*,1} = \hat{\lambda}_k^1 + \sum_{j=1}^{h_i} \hat{\phi}_{k,j}^1 \Delta x_{k,t-j}^{*,1} + \varepsilon_{k,t}^{*1}.$$

We then we drop the first 100 observations and then construct the  $\text{SURt}_{\text{NL}}^k$  statistic,

by regressing the following equation:

$$\Delta \tilde{x}_{k,t}^{*,1} = \delta_k^{*1} (\tilde{x}_{k,t-1}^{*,1})^3 + \sum_{j=1}^{h_i} \eta_{k,j}^1 \Delta \tilde{x}_{k,t-j}^{*,1} + \varepsilon_{k,t}^{a1}, \quad k = 1, \dots, N.$$

4. Repeat step 3 for 500 times and the collection of realized  $\text{SURt}_{\text{NL}}^k$  statistics forms

the bootstrap distribution of  $\text{SURt}_{\text{NL}}^k$  under the null hypothesis.

5. Check if the unit-root hypothesis is rejected by comparing  $\text{SURt}_{\text{NL}}^{k,0}$  with the 5% critical value from its bootstrap distribution.

6. Repeat steps 2-5 1000 times and the probability of rejection in step 5 reveals the size of the panel data bootstrap test.

## Reference

- Abuaf, N. and Jorion, P. (1990), "Purchasing Power Parity in the Long Run," *Journal of Finance* 45, 157-174.
- Berkowitz, J. and Kilian, L. (2000), "Recent Developments in Bootstrapping Time Series," *Econometric Reviews* 19, 1-48.
- Breitung, J. and Pesaran, M. H. (2007), "Unit Roots and Cointegration in Panels," in Matyas, L. and Sevestre, P. (eds), *The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice*, Kluwer: Dordrecht; forthcoming.
- Breuer, J. B., McNown, R. and Wallace, M. (2002), "Series-specific Unit Root Tests with Panel Data," *Oxford Bulletin of Economics and Statistics* 64, 527-546.
- Campbell, J. Y. and Perron, P. (1991), "Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots?" in Blanchard, O. J. and Fischer, S. (eds), *NBER Macroeconomics Annual 1991*, pp. 141-201, Stanley Cambridge and London: MIT Press.
- Chang, Y. (2002), "Nonlinear IV Unit Root Tests in Panels with Cross-sectional Dependency," *Journal of Econometrics* 110, 261-292.
- Choi, I. (2006), "Nonstationary Panels," in Mills, T.C., Patterson, K. (eds), *Handbooks of Econometrics*, Vol. I, pp. 511-539, Palgrave Macmillan: Basingstoke.
- Choi, I. and Chue, T. K. (2007), "Subsampling Hypothesis Tests for Nonstationary Panels with Applications to Exchange Rates and Stock Prices," *Journal of Applied Econometrics* 22, 233-264.
- Chortareas, G., Kapetanios, G. and Shin, Y. (2002), "Nonlinear Mean reversion in Real Exchange Rates," *Economics Letters* 77, 411-417.

- Chortareas, G. and Kapetanios, G. (2004), "The Yen Real Exchange Rate May Be Stationary after All: Evidence from Nonlinear Unit-root Tests," *Oxford Bulletin of Economics and Statistics* 66, 113-131.
- Chou, W. L. and Chao, C.-C. (2001), "Are Currency Devaluations Effective? A Panel Unit Root Test," *Economics Letters* 72, 19-25.
- Coleman, A. M. G. (1995), "Arbitrage, Storage and the Law of One Price: New Theory for the Time Series Analysis of an Old Problem," Discussion Paper, Department of Economics, Princeton University.
- Dixit, A. K. (1989), "Hysteresis, Import Penetration and Exchange Rate Pass Through," *Quarterly Journal of Economics* 104, 205-228.
- Dumas, B. (1992), "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World," *Review of Financial Studies* 5, 153-180.
- Froot, K. and Klemperer, P. (1989), "Exchange Rate Pass-Through When Market Share Matters," *American Economic Review* 79, 637-654.
- Granger, C. W. J. and Teräsvirta, T. (1993), *Modelling Non-linear Economic Relationships*, Oxford: Oxford University Press.
- Hadri, K. (2000), "Testing for Stationarity in Heterogeneous Panel Data," *The Econometrics Journal* 3, 148-161.
- Im, K. S., Pesaran, M. H. and Shin, Y. (2003), "Testing for Unit Roots in Heterogeneous Panels," *Journal of Econometrics* 115, 53-74.
- Kapetanios, G., Shin, Y. and Snell, A. (2003), "Testing for a Unit Root in the Nonlinear STAR Framework," *Journal of Econometrics* 112, 359-379.
- Kilian, L. and Taylor, M. P. (2003), "Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?" *Journal of International Economics* 60, 85-107.
- Krugman, P. (1987), "Pricing to Market When the Exchange Rate Changes," in Arndt,

- S., and Richardson, J. D. (Eds.), *Real Financial Linkages Among Open Economies*, MIT Press, Cambridge, MA.
- Kuo, B.-S. and Mikkola, A. (2001), "How Sure Are We About Purchasing Power Parity? Panel Evidence with the Null of Stationary Real Exchange Rates," *Journal of Money, Credit and Banking* 33, 767-789.
- Levin, A. and Lin, C.-F. (1992), "Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties," Department of Economics, UC San Diego, University of California at San Diego, Economics Working Paper Series.
- Levin, A., Lin, C.-F. and Chu, C.-S. J. (2002), "Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties," *Journal of Econometrics* 108, 1-24.
- Liew, V. K.-S., Baharumshah, A. Z. and Chong, T. T.-L. (2004), "Are Asian Real Exchange Rates Stationary?" *Economics Letters* 83, 313-316.
- Maddala, G. S. and Wu, S. (1999), "A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test," *Oxford Bulletin of Economics and Statistics*, Special Issue, 631-652.
- Michael, P., Nobay, A. R. and Peel, D. A. (1997), "Transaction Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation," *Journal of Political Economy* 105, 862-879.
- O'Connell, P. G. J. (1998), "The Overvaluation of Purchasing Power Parity," *Journal of International Economics* 44, 1-19.
- Papell, D. H. (1997), "Searching for Stationarity: Purchasing Power Parity under the Current Float," *Journal of International Economics* 43, 313-332.
- Pesaran, M. H. (2007), "A Simple Panel Unit Root Test in the Presence of Cross-Section Dependence," *Journal of Applied Econometrics* 22, 265-312.
- Sercu, P., Uppal, R. and van Hulle, C. (1995), "The Exchange Rate in the Presence of

- Transactions Costs: Implications for Tests of Purchasing Power Parity,” *Journal of Finance* 50, 1309-1319.
- Smith, L. V., Leybourne, S., Kim, T.-H. and Newbold, P. (2004), “More Powerful Panel Data Unit Root Tests with an Application to Mean Reversion in Real Exchange Rates,” *Journal of Applied Econometrics* 19, 147-170.
- Taylor, A. (2001), “Potential Pitfalls for the Purchasing Power Parity Puzzle? Sampling and Specification Biases in Mean-Reversion Tests of the Law of One Price,” *Econometrica* 69, 473-498.
- Taylor, M. P. and Peel, D. A. (2000), “Nonlinear Adjustment, Long-Run Equilibrium and Exchange Rate Fundamentals,” *Journal of International Money and Finance* 19, 33-53.
- Taylor, M. P., Peel, D. A. and Sarno, L. (2001), “Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution to the Purchasing Power Parity Puzzles,” *International Economic Review* 42, 1015-1042.
- Taylor, M. P. and Sarno, L. (1998), “The Behavior of Real Exchange Rates during the Post Bretton Woods Period,” *Journal of International Economics* 46, 281-312.
- Wu, Y. (1996), “Are Real Exchange Rates Nonstationary? Evidence from a Panel-Data Test,” *Journal of Money, Credit, and Banking* 28, 54-63.



Table 1. The averages of the 5% critical values across the panel

Panel size (N)	Observations (T)	$SURt_{NL}^k$	$SURADF^k$
N = 5	50	-3.956	-4.125
	100	-3.866	-3.987
	250	-3.791	-3.924
N = 10	50	-4.701	-5.095
	100	-4.427	-4.746
	250	-4.242	-4.539
N = 20	50	-6.213	-7.146
	100	-5.128	-5.864
	250	-4.716	-5.414

Notes: Numbers in the table are the averages of the 5% critical value of the  $SURt_{NL}^k$  and  $SURADF^k$  statistics, respectively, under a specific sample and panel size. These critical values are simulated based on 10,000 replications.

Table 2. The averages of the power under the AR(1) alternative hypothesis

Panel size	T	$\beta = 0.95$		$\beta = 0.90$	
		$SURt_{NL}^k$	$SURADF^k$	$SURt_{NL}^k$	$SURADF^k$
N = 5	50	0.076	0.101	0.110	0.206
	100	0.111	0.217	0.218	0.591
	250	0.308	0.762	0.664	0.996
N = 10	50	0.065	0.098	0.088	0.191
	100	0.089	0.199	0.165	0.543
	250	0.235	0.736	0.572	0.996
N = 20	50	0.055	0.099	0.066	0.179
	100	0.075	0.183	0.134	0.512
	250	0.192	0.698	0.499	0.995

Notes: Numbers in the table are the averages of the power for the  $SURt_{NL}^k$  and  $SURADF^k$  statistics, respectively, under a specific sample and panel size. These powers are simulated based on 5,000 replications. All series in a given panel are constructed based on a stationary AR(1) process with the AR coefficient set at 0.95 or 0.90.

Table 3. The averages of the power under the nonlinear alternative hypothesis,  $N = 5$

	$\theta = 0.01$		$\theta = 0.05$		$\theta = 0.1$		$\theta = 1$	
	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$
$\gamma = -1.5$								
T = 50	0.318 (0.210)	0.240 (0.180)	0.852 (0.706)	0.670 (0.490)	0.969 (0.916)	0.924 (0.850)	1.000 (1.000)	1.000 (1.000)
T = 100	0.616 (0.479)	0.378 (0.252)	0.997 (0.990)	0.984 (0.982)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
T = 250	0.981 (0.971)	0.844 (0.907)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -1$								
T = 50	0.225 (0.164)	0.225 (0.179)	0.658 (0.470)	0.456 (0.284)	0.870 (0.725)	0.746 (0.562)	0.995 (0.976)	1.000 (0.999)
T = 100	0.414 (0.304)	0.286 (0.227)	0.966 (0.929)	0.878 (0.820)	0.999 (0.993)	0.995 (0.992)	1.000 (1.000)	1.000 (1.000)
T = 250	0.878 (0.848)	0.574 (0.577)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -0.5$								
T = 50	0.150 (0.134)	0.242 (0.183)	0.308 (0.192)	0.241 (0.168)	0.460 (0.295)	0.357 (0.217)	0.655 (0.535)	0.926 (0.744)
T = 100	0.212 (0.223)	0.312 (0.312)	0.683 (0.534)	0.483 (0.316)	0.889 (0.785)	0.783 (0.641)	0.974 (0.945)	1.000 (0.999)
T = 250	0.429 (0.423)	0.304 (0.295)	0.995 (0.990)	0.958 (0.984)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)

Table 3. (continued, N = 10)

	$\theta = 0.01$		$\theta = 0.05$		$\theta = 0.1$		$\theta = 1$	
	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$
$\gamma = -1.5$								
T = 50	0.306 (0.186)	0.224 (0.161)	0.838 (0.627)	0.624 (0.416)	0.962 (0.864)	0.879 (0.747)	1.000 (0.999)	1.000 (1.000)
T = 100	0.604 (0.445)	0.340 (0.246)	0.997 (0.986)	0.968 (0.958)	1.000 (1.000)	0.999 (1.000)	1.000 (1.000)	1.000 (1.000)
T = 250	0.979 (0.968)	0.783 (0.869)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -1$								
T = 50	0.209 (0.145)	0.198 (0.158)	0.645 (0.404)	0.439 (0.258)	0.850 (0.636)	0.695 (0.473)	0.990 (0.940)	1.000 (0.996)
T = 100	0.390 (0.282)	0.257 (0.216)	0.965 (0.906)	0.842 (0.758)	0.998 (0.990)	0.986 (0.980)	1.000 (1.000)	1.000 (1.000)
T = 250	0.868 (0.835)	0.494 (0.540)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -0.5$								
T = 50	0.123 (0.115)	0.192 (0.160)	0.292 (0.169)	0.229 (0.152)	0.433 (0.247)	0.341 (0.200)	0.582 (0.427)	0.885 (0.639)
T = 100	0.180 (0.198)	0.269 (0.284)	0.671 (0.488)	0.433 (0.298)	0.880 (0.740)	0.735 (0.583)	0.968 (0.919)	1.000 (0.997)
T = 250	0.397 (0.404)	0.265 (0.292)	0.995 (0.988)	0.932 (0.969)	1.000 (1.000)	0.999 (1.000)	1.000 (1.000)	1.000 (1.000)

Table 3. (continued, N = 20)

	$\theta = 0.01$		$\theta = 0.05$		$\theta = 0.1$		$\theta = 1$	
	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$	$\text{SURt}_{\text{NL}}^k$	$\text{SURADF}^k$
$\gamma = -1.5$								
T = 50	0.300 (0.140)	0.217 (0.131)	0.838 (0.627)	0.624 (0.416)	0.962 (0.864)	0.879 (0.747)	1.000 (0.999)	1.000 (1.000)
T = 100	0.645 (0.398)	0.325 (0.234)	0.997 (0.986)	0.968 (0.958)	1.000 (1.000)	0.999 (1.000)	1.000 (1.000)	1.000 (1.000)
T = 250	0.979 (0.960)	0.692 (0.814)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -1$								
T = 50	0.198 (0.111)	0.180 (0.124)	0.645 (0.404)	0.439 (0.258)	0.850 (0.636)	0.695 (0.473)	0.990 (0.940)	1.000 (0.996)
T = 100	0.429 (0.250)	0.248 (0.201)	0.965 (0.906)	0.842 (0.758)	0.998 (0.990)	0.986 (0.980)	1.000 (1.000)	1.000 (1.000)
T = 250	0.870 (0.814)	0.405 (0.498)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
$\gamma = -0.5$								
T = 50	0.106 (0.086)	0.153 (0.121)	0.292 (0.169)	0.229 (0.152)	0.433 (0.247)	0.341 (0.200)	0.582 (0.427)	0.885 (0.639)
T = 100	0.183 (0.166)	0.243 (0.246)	0.671 (0.488)	0.433 (0.298)	0.880 (0.740)	0.735 (0.583)	0.968 (0.919)	1.000 (0.997)
T = 250	0.411 (0.379)	0.226 (0.284)	0.995 (0.988)	0.932 (0.969)	1.000 (1.000)	0.999 (1.000)	1.000 (1.000)	1.000 (1.000)

Notes: Numbers in the columns are the averages of the powers with high correlation in the panel and the number in a parenthesis is with a low one. To compute the rejection probabilities, the data under the alternative are generated by  $y_t = 1.1y_{t-1} + \gamma y_{t-1}[1 - \exp(-\theta y_{t-1}^2)] + \varepsilon_t$ .

Table 4. The average power result under mixed panels (T=100)

	$\theta = 0.01$		$\theta = 0.05$		$\theta = 0.1$		$\theta = 1$	
	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)
N = 5								
$\gamma = -1.5$								
No. of I(0)=1	0.716	0.029	0.997	0.016	1.000	0.012	1.000	0.012
No. of I(0)=2	0.703	0.019	0.997	0.008	1.000	0.005	1.000	0.004
No. of I(0)=3	0.656	0.017	0.997	0.006	1.000	0.004	1.000	0.002
No. of I(0)=4	0.648	0.014	0.997	0.005	1.000	0.002	1.000	0.001
No. of I(0)=5	0.616	NA	0.997	NA	1.000	NA	1.000	NA
$\gamma = -1$								
No. of I(0)=1	0.555	0.029	0.976	0.018	0.998	0.013	1.000	0.009
No. of I(0)=2	0.532	0.021	0.975	0.009	0.999	0.007	1.000	0.003
No. of I(0)=3	0.476	0.020	0.968	0.007	0.999	0.005	1.000	0.001
No. of I(0)=4	0.456	0.017	0.969	0.007	0.999	0.003	1.000	0.001
No. of I(0)=5	0.414	NA	0.966	NA	0.999	NA	1.000	NA
$\gamma = -0.5$								
No. of I(0)=1	0.405	0.029	0.784	0.023	0.938	0.016	0.998	0.009
No. of I(0)=2	0.363	0.018	0.771	0.012	0.932	0.008	0.998	0.003
No. of I(0)=3	0.301	0.014	0.727	0.011	0.908	0.005	0.993	0.002
No. of I(0)=4	0.263	0.012	0.716	0.007	0.905	0.005	0.991	0.000
No. of I(0)=5	0.212	NA	0.683	NA	0.889	NA	0.974	NA
N = 10								
$\gamma = -1.5$								
No. of I(0)=1	0.679	0.025	0.996	0.017	1.000	0.015	1.000	0.018
No. of I(0)=3	0.664	0.014	0.996	0.006	1.000	0.004	1.000	0.005
No. of I(0)=5	0.645	0.011	0.997	0.004	1.000	0.002	1.000	0.002
No. of I(0)=7	0.627	0.007	0.997	0.002	1.000	0.001	1.000	0.001
No. of I(0)=9	0.616	0.005	0.997	0.003	1.000	0.001	1.000	0.000
No. of I(0)=10	0.604	NA	0.997	NA	1.000	NA	1.000	NA
$\gamma = -1$								
No. of I(0)=1	0.521	0.028	0.967	0.019	0.998	0.016	1.000	0.014
No. of I(0)=3	0.496	0.016	0.967	0.006	0.997	0.004	1.000	0.003
No. of I(0)=5	0.459	0.013	0.967	0.004	0.998	0.003	1.000	0.001
No. of I(0)=7	0.431	0.009	0.966	0.003	0.998	0.002	1.000	0.001
No. of I(0)=9	0.410	0.006	0.966	0.003	0.998	0.002	1.000	0.000
No. of I(0)=10	0.390	NA	0.965	NA	0.998	NA	1.000	NA
$\gamma = -0.5$								
No. of I(0)=1	0.372	0.030	0.753	0.021	0.919	0.017	0.997	0.012
No. of I(0)=3	0.336	0.016	0.735	0.008	0.911	0.005	0.997	0.002
No. of I(0)=5	0.270	0.011	0.715	0.006	0.901	0.003	0.993	0.000
No. of I(0)=7	0.234	0.009	0.698	0.004	0.894	0.002	0.990	0.000
No. of I(0)=9	0.201	0.006	0.684	0.003	0.886	0.001	0.982	0.000
No. of I(0)=10	0.180	NA	0.671	NA	0.880	NA	0.968	NA

Table 4. (continued)

	$\theta = 0.01$		$\theta = 0.05$		$\theta = 0.1$		$\theta = 1$	
	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)	AVEI(0)	AVEI(1)
N = 20								
$\gamma = -1.5$								
No. of I(0)=1	0.742	0.029	0.997	0.021	1.000	0.019	1.000	0.025
No. of I(0)=4	0.698	0.015	0.997	0.007	1.000	0.005	1.000	0.009
No. of I(0)=7	0.688	0.011	0.997	0.004	1.000	0.003	1.000	0.005
No. of I(0)=11	0.678	0.009	0.997	0.003	1.000	0.002	1.000	0.001
No. of I(0)=15	0.665	0.008	0.997	0.002	1.000	0.001	1.000	0.001
No. of I(0)=20	0.645	NA	0.997	NA	1.000	NA	1.000	NA
$\gamma = -1$								
No. of I(0)=1	0.592	0.030	0.983	0.022	0.998	0.020	1.000	0.020
No. of I(0)=4	0.527	0.016	0.976	0.008	0.999	0.006	1.000	0.005
No. of I(0)=7	0.504	0.013	0.974	0.005	0.999	0.003	1.000	0.002
No. of I(0)=11	0.484	0.011	0.974	0.003	0.999	0.002	1.000	0.001
No. of I(0)=15	0.465	0.008	0.972	0.002	0.998	0.001	1.000	0.000
No. of I(0)=20	0.429	NA	0.970	NA	0.998	NA	1.000	NA
$\gamma = -0.5$								
No. of I(0)=1	0.430	0.031	0.816	0.025	0.952	0.021	1.000	0.017
No. of I(0)=4	0.352	0.018	0.772	0.010	0.930	0.007	0.998	0.003
No. of I(0)=7	0.304	0.013	0.757	0.007	0.922	0.004	0.995	0.001
No. of I(0)=11	0.267	0.010	0.738	0.005	0.914	0.002	0.992	0.001
No. of I(0)=15	0.235	0.007	0.721	0.003	0.903	0.001	0.984	0.000
No. of I(0)=20	0.183	NA	0.698	NA	0.890	NA	0.958	NA

Notes: Each panel consists of 100 observations for each panel member. The I(0) series in a given panel is generated by  $y_t = 1.1y_{t-1} + \gamma y_{t-1}[1 - \exp(-\theta y_{t-1}^2)] + \varepsilon_t$ . AVEI(0) is the average of the power of the I(0) series in the panel while AVEI(1) is the average of the rejection rate of the I(1) series in the same panel.

Table 5. Estimation Results (1973Q2-1998Q4)

Country	ADF	$t_{NL}$	SURADF <sup>k</sup>	CV5%	CV10%	SUR $t_{NL}^k$	CV5%	CV10%
G10								
UK	-2.790	-2.848	-2.091	-4.121	-3.685	<b>-3.912**</b>	-3.776	-3.375
BEL	-2.455	-1.726	<b>-5.031*</b>	-5.136	-4.758	<b>-5.051**</b>	-4.927	-4.490
FRA	-2.614	-1.734	-4.767	-5.214	-4.817	<b>-5.021**</b>	-5.011	-4.574
GER	-2.584	-1.709	-4.810	-5.323	-4.956	-4.086	-5.020	-4.628
ITA	-2.584	-2.457	-3.935	-4.456	4.042	<b>-4.062*</b>	-4.149	-3.662
NET	-2.685	-1.765	-4.939	-5.322	-4.941	-4.297	-5.060	-4.665
SWE	<b>-2.937**</b>	-2.670	<b>-3.732*</b>	-4.112	-3.677	<b>-4.632**</b>	-3.908	-3.489
SWI	-2.823	-2.321	-4.274	-4.813	-4.435	-3.712	-4.486	-4.044
CAN	-1.042	-0.270	-0.966	-3.321	-2.960	0.079	-3.283	-2.950
JAP	-1.615	-2.022	-3.471	-3.929	-3.547	<b>-3.406*</b>	-3.636	-3.275
G20								
UK	-2.790	-2.848	-2.831	-5.133	-4.657	<b>-4.436*</b>	-4.586	-4.097
BEL	-2.455	-1.726	-5.077	-7.269	-6.814	<b>-6.308**</b>	-6.306	-5.758
FRA	-2.614	-1.734	-5.069	-7.248	-6.734	<b>-6.222**</b>	-6.186	-5.668
GER	-2.584	-1.709	-4.843	-7.566	-7.103	-5.441	-6.419	-5.893
ITA	-2.584	-2.457	-4.513	-5.772	-5.275	-4.120	-4.989	-4.404
NET	-2.685	-1.765	-5.078	-7.530	-7.074	-5.516	-6.466	-5.929
SWE	<b>-2.937**</b>	-2.670	-4.041	-5.579	-5.018	<b>-4.789*</b>	-4.928	-4.390
SWI	-2.823	-2.321	-4.066	-6.203	-5.704	-4.144	-5.186	-4.644
CAN	-1.042	-0.270	-1.083	-4.322	-3.830	0.292	-4.019	-3.572
JAP	-1.615	-2.022	-4.130	-4.720	-4.264	<b>-4.452**</b>	-4.174	-3.713
AUS	-1.843	-2.270	-1.489	-4.315	-3.868	-1.039	-4.286	-3.791
NEZ	<b>-2.969**</b>	-2.884	-3.707	-4.433	-3.986	<b>-4.621**</b>	-4.298	-3.791
AUS	-2.475	-1.909	-5.351	-7.594	-7.139	<b>-6.091*</b>	-6.497	-5.966
NOR	-2.367	-1.607	-4.439	-6.225	-5.684	<b>-5.349**</b>	-5.349	-4.765
FIN	<b>-2.926**</b>	-2.528	<b>-5.742**</b>	-5.612	-5.019	<b>-5.424**</b>	-4.937	-4.444
GRE	-2.250	-1.566	-2.693	-6.054	-5.534	-3.192	-4.986	-4.484
IRE	-2.191	-2.326	-2.853	-5.149	-4.663	-3.771	-4.433	-3.981
POR	-1.827	-1.655	-2.583	-5.808	-5.295	-4.230	-4.921	-4.384
SPA	-2.592	-2.287	-3.901	-5.664	-5.124	-3.981	-4.950	-4.409
DEN	-2.175	-1.403	-4.820	-7.391	-6.913	<b>-5.893*</b>	-6.210	-5.664

Notes: CV denotes the critical value of the corresponding statistics. \*\* and \* indicate significance at the 5% and 10% level, respectively. The numbers in boldface indicate significance at the 10% level.

Table 6. Size of Tests (1973Q2-1998Q4)

	SURADF <sup>k</sup>	SURt <sub>NL</sub> <sup>k</sup>
G10		
UK	0.032	0.041
Belgium	0.039	0.054
France	0.035	0.040
Germany	0.044	0.052
Italy	0.036	0.047
Netherlands	0.035	0.048
Sweden	0.029	0.045
Switzerland	0.047	0.044
Canada	0.036	0.052
Japan	0.025	0.051
G20		
UK	0.031	0.050
Belgium	0.043	0.053
France	0.036	0.044
Germany	0.055	0.049
Italy	0.024	0.050
Netherlands	0.044	0.052
Sweden	0.017	0.050
Switzerland	0.033	0.033
Canada	0.016	0.047
Japan	0.019	0.043
Australia	0.030	0.038
New Zealand	0.027	0.059
Austria	0.041	0.049
Norway	0.025	0.041
Finland	0.027	0.066
Greece	0.024	0.038
Ireland	0.043	0.046
Portugal	0.031	0.053
Spain	0.023	0.035
Denmark	0.050	0.051

Note: The numbers in the table are empirical sizes of the SURt<sub>NL</sub><sup>k</sup> and SURADF<sup>k</sup> statistics given the 5% nominal size, which are obtained by applying the method of bootstrap-after-bootstrap provided by Berkowitz and Kilian (2000).