Nonlinear Exchange Rate Dynamics under Stochastic Official Intervention

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Abstract

Many studies employ non-linear models to explain or forecast the exchange rate and find their superiority. This article builds an exchange rate model of managed float under conditional official intervention. In the model, the government minimizes social loss through a trade-off between targeting the exchange rate and lowering intervention costs. We obtain an endogenous threshold model and derive an analytical solution of the exchange rate stochastic interventions. The implication of a managed float causing a lower volatility of the exchange rate has been found by past empirical studies. Our model provides not only a justification for the central banks’ conditional interventions but also a rationale for the use of regime-switching models of two states (intervention vs. non-intervention) in the empirical studies of exchange rates.

Keywords: Sterilized intervention; Exchange rate dynamics; Managed float; Regime-switching

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1. Introduction

The dynamics of the exchange rate are a continuous and contentious issue in open macroeconomics. Empirically, many studies find that multiple regime models explain or predict the exchange rate better than single regime ones, i.e., a floating exchange rate regime.\footnote{Since the pioneering work of Engel and Hamilton (1990), many have followed in estimating regime-switching models. Recent examples include Bollen, Gray and Whaley (2000), Dewachter (2001), Kirikos (2002), Beine, Laurent and Lecourt (2003), Clarida, et al (2003), Klaassen (2005), Bergman and Hansson (2005), Frömmel, MacDonald and Menkhoff (2005), Chen (2006), Lee and Chang (2007), Beine, de Grauwe, and Grimaldi (2009), and Altavilla and de Grauwe (2010).} To understand the practical evolution of the exchange rate, one needs new models that describe the multiple regime characteristics of the foreign exchange market. The IMF (2009) recommends, “A member should intervene in the exchange market if necessary to counter disorderly conditions, which may be characterized \textit{inter alia} by disruptive short-term movements in the exchange rate of its currency.” In reality, even in the most developed countries (the U.S., Japan, Germany, etc.) an intervention in foreign exchange markets was made after the breakdown of the Bretton Woods System. Switching between intervention and non-intervention can be a natural rationale for a nonlinear exchange rates process.

This article tries to provide a rational expectation model of managed float to explain the empirical success of multiple regime models. Managed float is an intermediate exchange rate regime which allows governments to intervene when they think it’s better to do so. Previous multiple regime models, though not purely empirical ones, either assume that exchange rates are driven by multiple regime state variables or solve the exchange rate under an individual permanent regime to obtain a regime-switching process of the exchange rate. The former appears to be superficial in explaining the nonlinearity of exchange rate dynamics, while the latter is...
inconsistent with the forward-looking characteristic of the exchange rate in a rational market that takes the possibility of future policy change into account. To fix these flaws, we propose to build an endogenous threshold model and provide an analytical solution of the forward-looking exchange rate switching between intervention and non-intervention under a managed float of an optimizing central bank.

Among the literature addressing the exchange rate process under official intervention, some researchers extend the work of Krugman (1991) in applying target zone type models to explain exchange rate dynamics. Despite the target zone model being nearly perfect in its theoretical framework, it has a key weakness in its treatment of monetary policy. It assumes that the money supply is used only to maintain the target zone. Therefore, except for exchange rates, monetary policy plays no role in stabilizing the macro-economy in the target zone model, which is obviously contrary to the main purpose of adopting that macro policy. In addition, although it seems an obvious way to model governments’ management in the foreign exchange market, such kind of set-ups concurrently change the fundamentals in other markets and are an exchange rate policy with a non-sterilized intervention operation, which is far less popular in practice.²

One important feature of the Krugman (1991) model which this article tries to duplicate is the absence of official intervention when only small disturbances happen. Dornbusch (1986) points out that “The case for intervention is usually made as one of countering disorderly market conditions. But there is no very good case why small

² The same problem applies to other studies of managed float using a macroeconomic model with a monetary policy rule directly relating money supply to the exchange rate (Turnovsky, 1984; Djajic and Bazzoni, 1992; Hsieh, 1992), or adding the exchange rate to a Taylor rule (Leitemo and Söderström, 2005; de Andrade and Divino, 2005; Lubik and Schorfheide, 2007).
noise in the market should be smoothed...” Chiu (2003)’s survey shows that many central banks intervene only when the exchange rate is thought to be clearly out of line with economic fundamentals. The fact that central banks intervene only for a big departure of the exchange rate from its target has been found by past empirical studies. For instance, Baillie and Osterberg (1997) found the probability of official intervention to be determined by the magnitude of the deviation of the nominal exchange rate from its target.

Our model has the novel implication that the decision of intervention/nonintervention is endogenously determined by the magnitude of the deviation of the market equilibrium exchange from the exchange rate target. Specifically, intervention is present (absent) if the magnitude of the departure is large (small). Sometimes governments intensively intervene in the foreign exchange market while in other times they totally disappear from the market. Because intervention operations are often carried out with variable frequency, Ito and Yabu (2007) employ ordered probit models for the investigation of official interventions.

This article is a complementary study to the work of Ito and Yabu (2007). In Ito and Yabu (2007), interventions are modeled to be auto-correlated by assuming an intervention cost depending on the interventions of the current and last periods. In this article optimal interventions are series-correlated due to the persistency of their determinants (i.e., market fundamentals). Moreover, the endogenous exchange rate and intervention amounts in our model are conditioned on the possibility of future policy changes in the rational market, while the equation of exchange rate determination in Ito and Yabu (2007) is assumed to be backward-looking and depends on the concurrent intervention as well as the exchange rate of the last period with a coefficient one. In addition, Ito and Yabu (2007) estimate an ordered probit model
with an intervention indicator being the left-hand side variable. Since very few
countries reveal their intervention operations, this article suggests a multiple regime
model with the easily available data (the exchange rate) as the left-hand side variable.

The fact that interventions are not conducted in every period indicates the
coeexistence of both benefits and costs of intervention under a managed float. This
substantiates the assumption of an optimization problem for official foreign exchange
market interventions. That the government makes a tradeoff between the benefit of
intervention and the cost of it and chooses to adopt a managed floating exchange rate
regime is an appealing start, as explained in this article.

Except for the interaction between the private and public sectors, recently
another stream of theoretical rationale for generating a nonlinear process of the
exchange rate has revived and caught the academia’s attention, which is the
coexistence of “heterogeneous agents” in the foreign exchange market. For example,
de Grauwe and Grimaldi (2006), following the asset pricing models of Brock and
Hommes (1997, 1998), argue that fundamentalists and chartists have different beliefs
about the future exchange rate. Together with a time-varying proportion of specific
types of agents, the heterogeneous agent model leads to non-linear features in the
dynamics of the exchange rate. This approach is commonly applied in the financial
literature of nonlinear dynamics of the exchange rates (e.g. Reitz and Taylor, 2008;
Beine, de Grauwe and Grimaldi, 2009; Bauer, de Grauwe and Reitz, 2009; Wan and
Kao, 2009; Dieci and Westerhoff, 2010, among others). Nonetheless, imposing
heterogeneous beliefs into a rational expectation model has intractable mathematical
problems. Models of behavioral finance are usually solved as each agent ignores the
existence of other agents in influencing the exchange rate and forms exchange rate
expectations in a backward-looking/adaptive way. Therefore, the exchange rate
expectations of the market participators may be inconsistent with the one derived from the referenced economic model.

The organization of this paper is as follows: Section 2 provides a model of exchange rate determination under a managed floating regime. It depicts official intervention as an endogenously chosen variable through optimizing a government’s objective function. Section 3 gives the solutions of models with assumptions of fundamentals being integrated of order one and two kinds of exchange rate target settings: one is consistent with the fundamentals, while the other is not. The final section offers our conclusions and remarks.

2. An exchange rate model under a managed float

To focus on the effects of an intervention operation on the exchange rate, this article provides an exchange rate model consistent with perfect sterilized intervention under a managed float. A model of perfectly sterilized intervention means that the money supply is an autonomous policy variable and, along with other economic fundamentals, is exogenously determined outside the foreign exchange market. The popularity of sterilized interventions has been found in Chiu (2003) and other empirical studies.

To minimize ad hoc assumptions on model specification, we use the definition equation of the balance of payments which must hold for any open economy model whether with or without micro-foundation. Let’s begin with the following definition equation of the foreign exchange market:

$$CA_t + FA_t = BOP_t,$$

where $CA_t$ denotes the balance of the current account, $FA_t$ denotes the balance of capital and the financial account, and $BOP_t$ denotes the balance of payments, which equals the excess supply of the foreign exchange market. If the foreign exchange
authority lets the exchange rate freely adjust in the foreign exchange market, then the market forces drive the balance of payments to its equilibrium and thus \( \text{BOP}_t = 0 \) is applied. Otherwise, if the bank intervenes, then the balance of payments equation becomes \( \text{BOP}_t = \text{INT}_t \), where \( \text{INT}_t \) is the foreign exchange authority’s net buying of the foreign currencies at time \( t \).

Equation (1) is an essential market constraint for any open macroeconomic model with two behavior equations – one to describe the current account and the other to describe the capital and financial accounts – to define the foreign exchange market constraint. Let \( e_t \) be the logarithm of the exchange rate (defined as the domestic currency price of foreign currency). Regarding the current account in the balance of payments, the exchange rate has a predominant influence on it through a relative price effect, while to the financial account the expected devaluation rate \( (E_t e_{t+1} - e_t) \) has a decisive effect on the flow of international capital. Term \( E_t \) denotes the market expectations based on the information set available at time \( t \), and \( E_t x_{t+i} \equiv E[x_{t+i} | \Omega_t] \) is the mathematical expectation of any variable \( x_{t+i} \) for time \( t + i \), based on the information set available at time \( t \) (\( \Omega_t \)).

The analytical framework for the exchange rate behavior can simply be described by the following constraint of the foreign exchange market as in the Fleming (1962) model:

\[
f_t + \delta e_t + \gamma(e_t - E_t e_{t+1}) = \text{INT}_t,
\]

(2)

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3 As for those models built with a micro-foundation, the market constraint is presented as a budget constraint which relates the country’s foreign reserves changes to its net exports plus factor income inflows (the current account), as well as the accumulation of foreign assets owned by the private sector (the financial account).
where $f_i$ is a linear combination of fundamental factors (in the balance of the current account, the balance of the capital account, and the balance of the financial account) influencing the exchange rate, while $\gamma$ is the positive interest semi-elasticity of capital and financial account, and $\delta$ captures the price elasticity of the current balance. Note that even though Equation (2) looks like a reduced-form equation, $\gamma$ and $\delta$ are structural parameters due to their economic interpretation and are irrelevant to an absence or presence of a central bank’s intervention.\(^5\)

The components of fundamentals $f_i$ may contain technology shocks, physical capital stock for production, financial assets for earning revenue, the price differential between a home country and its foreign counterpart, as well as the interest differential between a home country and its foreign counterpart, which enters $f_i$ with a coefficient $\gamma$ as the expected devaluation rate does. Monetary models make an extreme assumption of perfect capital mobility (i.e., $\gamma \to \infty$). We consider that capital mobility practically can be very high but still imperfect even in industrialized countries so as to keep $\gamma$ as a finite number.

An additional advantage of Equation (2) is its simplicity in capturing the forward-looking feature of the exchange rate, which is one of the most important characteristic of asset prices.\(^6\) If we set $\text{INT}_i$ to be zero, then an exchange rate

\(^4\) Regarding factors influencing the exchange rate, this article separates market fundamentals from official interventions, while some authors (e.g., Miller and Zhang, 1996) consider interventions to be part of market fundamentals.

\(^5\) For example, $\delta$ can be a function of deep parameters in the preference function, production function, or parameters of policy other than intervention (e.g., the tariff on imported goods). Therefore, the neat specification of the foreign exchange market is free from Lucas’ critique.

\(^6\) This has been neglected in many empirical studies.
under a clean floating regime ($e_i^F$) can be solved forward as:

$$e_i^F = \frac{\gamma E_i e_{i+1} - f_i}{\delta + \gamma} = E_i \sum_{i=0}^{\infty} \rho^i \left( - \frac{f_{i+1}}{\delta + \gamma} \right),$$

where the discount factor $\rho \equiv \gamma / (\delta + \gamma)$, which measures the relative importance of the capital and financial accounts in the balance of payments. In general, $0 < \delta, \gamma < \infty$ and $0 < \rho < 1$.

The intervention decision of the central bank can be explained through consideration of a social objective function. IMF (2009, pp. 37) recommends: “A member should intervene in the exchange market if necessary to counter disorderly conditions, which may be characterized inter alia by disruptive short-term movements in the exchange rate of its currency.” To decrease the departure of an exchange rate from its target should be the first consideration of a central bank’s intervention as pointed out by previous literature (Almekinders and Eijffinger, 1996; Miller and Zhang, 1996; Bhattacharya and Weller, 1997; Vitale, 1999; and Ito and Yabu, 2007).

However, there will be some costs to intervene. As argued by Miller and Zhang (1996) and Ghosh (2002), a central bank that intervenes must use foreign reserves for exchanging domestic currency and vice versa, and must buy or sell treasury bills for sterilization. Moreover, resistance to the adjustment of the exchange rate leads to deterioration in the balance of payments and affects the domestic economy through spillover effects. Almekinders and Eijffinger (1996) and Ito and Yabu (2007) also claim that official intervention causes “political costs” which reflect costs of discussion with the Ministries of Finance, both their own and of other major countries regarding interventions. These costs are not independent of the intervention size. A larger amount of intervention causes the central bank to be more concerned.

In addition, the loss arising from interventions may include not only a variable
cost of participation in the foreign exchange market, but also a fixed intervention cost of exchanging domestic currencies with a foreign counterpart. Cadenillas and Zapatero (1999) and Ito and Yabu (2007) have emphasized the implication of the existence of a fixed cost in the central banks’ intervention operations is compatible with the empirical evidence. The fixed cost may not be a monetary cost but rather a psychological or political cost. It can be attributed to a central bank’s inherent reluctance to intervene in the foreign exchange market as mentioned by Ghosh (2002, fn.7).

This article therefore considers two sources of social loss: the departure of the exchange rate from its target and the intervention cost. Assume that a central bank minimizes the following expected social loss function:

$$
\text{LOSS} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \phi(e_t - e_t^*)^2 + (\text{INT}_t)^2 + cI_t \right],
$$

(4)

where $\beta$ is the central bank’s discount factor, $\phi$ is the relative weight on the exchange rate’s departure from its target, $e_t^*$ is a pre-announced target of the exchange rate set by the government, $I_t$ is an indicator function (which equals one if the government intervenes in the foreign exchange market, or zero if it does not), and the constant term $c$ is the fixed cost associated with any foreign exchange market intervention. It is obvious that if the bank is concerned with targeting the foreign exchange rate only ($\phi \to \infty$) and the intervention cost is finite, it will choose a fixed exchange rate regime. On the other hand, if the bank is concerned about intervention costs only ($\phi \to 0$), it will choose a clean floating exchange rate regime.

At the beginning of the period, the market fundamentals are realized and observed by the participants of the foreign exchange market and the central bank. The latter thus announces its target of the exchange rate. Then the rational private
sector forms expectations of the exchange rate while the government decides whether or not to intervene. Given the market’s expected exchange rate, the central bank achieves optimization by setting its intervention strategy to minimize the expectations of the loss function, (4).\footnote{This article assumes that the foreign exchange authority always has enough foreign exchange assets for the intervention - that is, we assume either the authority has sufficient foreign reserves to operate or it can borrow enough to operate.} For a positive fixed cost \((c > 0)\), this strategy leads to a trigger rule, conditional on the central bank’s exchange rate stabilization firmness \((\phi):\)

\[
\text{INT}_t = \frac{\phi}{(\delta + \gamma)^2 + \phi} \left[ f_t + (\delta + \gamma)e_t^* - \gamma E_t e_{t+1} \right],
\]

when

\[
\gamma E_t e_{t+1} - f_t - (\delta + \gamma)e_t^* > (\delta + \gamma)\sqrt{[(\delta + \gamma)^2 + \phi]c / \phi}, \quad (6a)
\]

or

\[
\gamma E_t e_{t+1} - f_t - (\delta + \gamma)e_t^* < -(\delta + \gamma)\sqrt{[(\delta + \gamma)^2 + \phi]c / \phi}; \quad (6b)
\]

otherwise, \(\text{INT}_t = 0\). The trigger rule indicates that the government compares the market conditions with its thresholds to decide whether or not to intervene before choosing the optimal amount of intervention. Therefore, the first question facing the foreign exchange authority is: “To intervene or not to intervene?” If intervention is the answer, then the following question arises: “How much to intervene?”

The exchange rates obey the following formula under managed floating. When the government decides not to step into the foreign exchange market:

\[
e_t = e_t^n = \frac{\gamma E_t e_{t+1} - f_t}{\delta + \gamma},
\]

where \(e_t^n\) denotes the exchange rate in the absence of intervention under the
managed float. As well, the exchange rate obeys the following formula under managed floating when there is government intervention:

$$e_t = e'_t = \frac{(\delta + \gamma)^2}{(\delta + \gamma)^2 + \phi \left[ \frac{\gamma E_t e_{t+1} - f_t}{\delta + \gamma} \right]} + \frac{\phi}{(\delta + \gamma)^2 + \phi} e^*_t$$

$$= \alpha e^*_t + (1 - \alpha) e'_t,$$  \hspace{1cm} (8)

where \( \alpha = (\delta + \gamma)^2 / [(\delta + \gamma)^2 + \phi] \) and \( e'_t \) denotes the exchange rate in the presence of intervention under the managed float. Equation (8) indicates that \( e'_t \) is a weighted average of the rate in the absence of intervention and the target of the exchange rate. When the government is concerned more for the departure of the exchange rate and its target (\( \phi \) getting higher), the weight for the rate in the absence of intervention (\( \alpha \)) decreases while the weight for the exchange rate target rises.

Employing Equation (7), Equations (5), (6a), and (6b) can be re-written as follows:

$$\text{INT}_t = \begin{cases} (1 - \alpha) \left[ f_t + (\delta + \gamma) e'_t - \gamma E_t e_{t+1} \right] & \text{if } e''_t \geq \bar{e}_t \text{ or } e''_t \leq \underline{e}_t, \\ 0, & \text{if } \bar{e}_t \geq e''_t \geq \underline{e}_t, \end{cases}$$  \hspace{1cm} (5')

where \( \bar{e}_t \equiv e'_t + \sqrt{[(\delta + \gamma)^2 + \phi] c / \phi} \) and \( \underline{e}_t \equiv e'_t - \sqrt{[(\delta + \gamma)^2 + \phi] c / \phi} \). Unlike previous studies such as Lewis (1995), Baillie and Osterberg (1997), and Mundaca (2001) that assumed the determinants of intervention decision, from our model we derive the endogenous contingent intervention rule that the central bank participates in the foreign exchange market only when the exchange rate goes beyond the non-intervention band.

When there is no fixed cost from intervening, a variable intervention cost still exists. If \( c = 0 \), it can be shown that the central bank always intervenes to minimize social losses. The only problem remaining is then “How much to intervene?” However, that the fixed cost \( c \) should be positive is argued by Cadenillas and Zapatero.
In addition, when the government is concerned for the intervention cost, and thereby $\phi$ is a finite number and $\alpha > 0$, the optimal policy is partial intervention (and a toleration for the departure of the exchange rate from its target) rather than full intervention (pegged exchange rate to its target) since the intervention cost is increasing in the bank’s intervention amount.

The participants of the foreign exchange market embody the government’s intervention rule to their expectation formation. To get more insight from the model, we re-write the rational expectation of the exchange rate under the managed float as:

$$E_t e_{t+1} = \int_{-\infty}^{\infty} e_{t+1} h(e_{t+1} | \Omega_t) de_{t+1}$$

$$= \int_{-\infty}^{e_t} e'_{t+1} h(e_{t+1} | \Omega_t) de_{t+1} + \int_{e_t}^{e_t'} e'_{t+1} h(e_{t+1} | \Omega_t) de_{t+1} + \int_{e_t'}^{\infty} e''_{t+1} h(e_{t+1} | \Omega_t) de_{t+1},$$

where $h(e_{t+1} | \Omega_t)$ is the probability density function of the exchange rate. Because the probability of non-intervention in period $t + 1$, $\int_{-\infty}^{e_t} h(e_{t+1} | \Omega_t) de_{t+1}$, depends on variables in the information set of period $t$ and is time-varying, there is no easy way to solve the highly non-linear model without further assumptions on the processes of $f_t$.

Even when there is no intervention in period $t$, it is obvious that a future intervention possibility obviously changes the market expectations of rational agents and thus the exchange rate at time $t$, as will be discussed later. The expectations of the exchange rate in turn influence the intervention amount of the foreign exchange authority.

Our model, in describing the exchange rate process under managed floating, is established as an endogenous threshold model with two states (intervention and

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8 This is an important feature of the multi-regime model proposed by Lee and Chen (2006): a higher probability of a central bank’s future interventions raises the rational-expectations discrepancy between the exchange rate and its fundamentals, even though the bank does not step in the foreign exchange market during that period.
non-intervention) and with endogenous transition probabilities. To some degree, our model is somewhat like a target zone model with an endogenous non-intervention area. Here the margins are not constant according to the implication of Equation (5*). Once the exchange rate target changes, the target zone is re-aligned. On the one hand, the government only intervenes when the market-equilibrium exchange rate is going to float past the thresholds \( e^*_t \pm \sqrt{[(\delta + \gamma)^2 + \phi]c / \phi} \), and in that case the operation moves the exchange rate back to a weighted average of its non-intervention counterpart \( (e^n_t) \) and its target \( (e^*_t) \). On the other hand, it seems that, when the government does not intervene, the exchange rate \( (e^n_t) \) equals the rate under a clean float \( (e^F_t) \). However, this is not true since the formation of the expected exchange rate under managed float is different from that under a clean float.

3. **A rational expectations solution of the exchange rate under a managed float**

Attention should be paid to the endogeneity of exchange rate expectations. Exchange rate expectations depend on the exchange rate regime a government adopts. If it adopts a managed floating regime and intervenes stochastically, the expectations depend on whether the government intervenes or not. However, it is not easy to solve the expectation of the exchange rate due to the persistent features of the economic fundamentals. The persistency will cause the transition probability of intervention/nonintervention to be serially correlated. Therefore, given the same \( \Omega_{t-1} \), the expectation for tomorrow’s exchange rate depends on today’s states and thus the values of \( E\text{te}_{t+1} | s \) in Equations (3), (7), and (8) are different.

To make the model tractable, we assume that the fundamentals follow an autoregressive process and are integrated of order one, and consider a special disturbance distribution from which we could derive an analytical solution of the
exchange rate. First, we assume
\[
\Delta f_t = \lambda + \lambda_t \Delta f_{t-1} + \varepsilon_t, \tag{9}
\]
in which \( \varepsilon_t \) is a white noise.\(^9\)

The market expectation of the present value of the fundamental can be represented as:
\[
E_t \sum_{j=0}^{\infty} \rho^j f_{t+j} = \frac{\delta + \gamma}{\delta} f_t + \frac{\delta + \gamma}{\delta} E_t \sum_{j=1}^{\infty} \rho^j \Delta f_{t+j}
= \frac{\delta + \gamma}{\delta} \kappa + \frac{\delta + \gamma}{\delta} f_t + \frac{\delta + \gamma}{\delta} \frac{\rho \lambda_i}{1 - \rho \lambda_i} \Delta f_t,
\]
where \( \kappa \equiv \lambda \rho / [(1 - \rho)(1 - \rho \lambda_i)] \). Therefore, the exchange rate under a clean float regime can be solved forward as:
\[
e_t^f = -\frac{1}{\delta} \kappa - \frac{1}{\delta} f_t - \frac{\rho \lambda_i}{\delta(1 - \rho \lambda_i)} \Delta f_t. \tag{10}
\]
This equation clearly implies a co-integration relationship between the equilibrium exchange rate and its market fundamentals through a co-integration vector \([1, 1/\delta]\).

Given that the exchange rate’s fundamental is an I(1) process, if the target of the exchange rate shares the same trend as its fundamentals, then both the realized exchange rate and its target share the common trend. Therefore, both the deviation between the exchange rate and its target and the amount of the official intervention as well are stationary. In this case, the managed floating exchange rate regime is sustainable and the social loss function is well defined. However, the government can set its exchange rate target to a level not co-integrated with the fundamentals at least for some period. The deviation between the exchange rate and its target and the

\(^9\) An extension for \( \Delta f_t \) to be a VAR(\( P \)) process, in which \( P \geq 1 \), is simple and will not change our conclusions.
amount of the official intervention are then non-stationary, and the possible economic results are speculative attacks and a collapse of the exchange rate regime.

We hereafter specify two types of exchange rate targets to show the exchange rate dynamics. The first type assumes that the government targets its exchange rate to the trend portion of the exchange rate in the foreign exchange market which results in making the exchange target fundamental-consistent. The second type assumes that the government sets a constant target of the exchange rate being inconsistent with non-stationary economic fundamentals.

3.1 Targeting the exchange rate at its fundamental trend

Usually a government prefers to choose a relatively stable target level for its exchange rate. Chiu (2003)’s survey shows that Japan and Korea intervene “to smooth excess exchange rate fluctuations that are judged to be clearly out of line with economic fundamentals.” Switzerland previously intervened “when there were sharp fluctuations in the exchange rate that were judged to be inconsistent with market fundamentals;” while the U.S. intervenes when “the exchange rate is thought to be clearly out of line with the economic fundamentals.” Neely (2008) also points out that successful foreign exchange authorities act consistently with fundamentals. In practice, the fundamental rate of the foreign exchange appears to be a prevailing exchange rate target of the central banks.

For analytical simplification, suppose that the government sets the trend part of the exchange rate as the exchange rate target:

\[ e_t^* = -\frac{\kappa}{\delta} f_t. \]  \hspace{1cm} (11)

Equation (11) can represent a fundamental-consistent target level.

Note that the endogenous probability of intervention is time varying in general. Thus, a simplified distribution assumption can help us remove the non-linear problem.
in solving the exchange rate in the absence of intervention. Let us guess that the form of the exchange rate in the absence of government intervention under the managed float is as follows:

\[ e^n_t = -\frac{1}{\delta}f_t + a + b\Delta f_t, \]  

(12)

where \( a \) and \( b \) are undetermined coefficients to be solved.

By subtracting Equation (11) from Equation (12) and substituting Equation (9) into the result, we obtain:

\[ e^n_t - e^*_t = \frac{\kappa}{\delta} + a + b\Delta f_t, \]

\[ = \frac{\kappa}{\delta} + a + b(\lambda + \lambda_i\Delta f_{t-1}) + \omega_t, \]  

(13)

where \( \omega_t \equiv b\varepsilon_t \). Due to the persistence of \( \Delta f_t \) in Equation (13), it is obvious that the probability of intervention at time \( t + 1 \) depends on the state at time \( t \).

Assume that \( \omega_t \) follows a uniform distribution between a sufficiently large range \([-H, H]\], then we would have the following expectations of the exchange rate (refer to Appendix A for derivation):

\[ E_t e^*_{t+1} = \int_{-\infty}^{\infty} e^*_{t+1} f(e_{t+1} | \Omega_t) d(e_{t+1}) \]

\[ = -\frac{\lambda}{\delta} - (1-\alpha)\frac{\kappa}{\delta} + \alpha(a + b\lambda) - \frac{f_t}{\delta} + \left(\alpha b - \frac{1}{\delta}\right)\lambda_i\Delta f_t. \]  

(14)

Substituting Equation (14) into (7), the exchange rate in the absence of government intervention is therefore:

\[ e^n_t = -\frac{f_t}{\delta + \gamma} + \rho\left(-\frac{1}{\delta}f_t - \frac{\lambda}{\delta} - (1-\alpha)\frac{\kappa}{\delta} + \alpha(a + b\lambda) + \left(\alpha b - \frac{1}{\delta}\right)\lambda_i\Delta f_t\right) \]

\[ = \rho\left(-\frac{\lambda}{\delta} - (1-\alpha)\frac{\kappa}{\delta} + \alpha(a + b\lambda) - \frac{f_t}{\delta} + \rho\left(\alpha b - \frac{1}{\delta}\right)\lambda_i\Delta f_t\right). \]  

(15)
By comparing Equations (12) and (15) we can solve the undetermined coefficients, as the coefficients for each variable must be equal. Starting with $\Delta f_i$, we have:

$$b = \rho \left( ab - \frac{1}{\delta} \right) \lambda_i.$$

Therefore, coefficient $b$ can be solved as:

$$b = -\frac{\rho \lambda_i}{\delta(1 - \alpha \rho \lambda_i)}.$$

When $\alpha = 1$, $b$ degenerates to $-\rho \lambda_i / [\delta(1 - \rho \lambda_i)]$ as the coefficient of $\Delta f_i$ in Equation (10).

The following equation must then hold for the constant term:

$$a = \rho \left( -\frac{\lambda}{\delta} (1 - \alpha) \kappa + \alpha (a + b \lambda) \right).$$

This equation implies:

$$a = \rho \left( \frac{1}{1 - \alpha \rho} \left( -\frac{\lambda}{\delta} (1 - \alpha) \kappa + ab \lambda \right) \right)$$

$$= \frac{\rho}{1 - \alpha \rho} \left( \frac{\lambda}{\delta(1 - \alpha \rho \lambda_i)} - (1 - \alpha) \frac{\kappa}{\delta} \right).$$

When $\alpha = 1$, the coefficient $a$ degenerates to $-\kappa / \delta$ as the constant part in Equation (10).

We therefore solve the exchange rates by the method of undetermined coefficients as:

$$e_i^n = -\frac{\rho(1 - \alpha) \kappa}{\delta(1 - \alpha \rho)} - \frac{\rho \lambda}{\delta(1 - \alpha \rho)(1 - \alpha \rho \lambda_i)} - \frac{1}{\delta} f_i - \frac{\rho \lambda_i}{\delta(1 - \alpha \rho \lambda_i)} \Delta f_i,$$  

(16)

and

$$e_i^i = -\frac{(1 - \alpha) \kappa}{\delta(1 - \alpha \rho)} - \frac{\alpha \rho \lambda}{\delta(1 - \alpha \rho)(1 - \alpha \rho \lambda_i)} - \frac{1}{\delta} f_i - \frac{\alpha \rho \lambda_i}{\delta(1 - \alpha \rho \lambda_i)} \Delta f_i.$$  

(17)
In this example, the exchange rate target is fundamental-consistent, so that the exchange rate under managed float and the fundamental are co-integrated with the same co-integration coefficient as that under a clean float. In addition, from Equation (5) or (5'), it can be seen that the optimal intervention is stationary in the case of a fundamental-consistent target. This guarantees the sustainability of the exchange rate regime of a managed float in the long run.

With the targeting of the exchange rate being a concern of the foreign exchange authority \( \phi > 0 \), please be reminded from Equation (8) that in determining \( e'_i \), the weight of the exchange rate target lies between one and zero \( (0 < \alpha < 1) \). By comparing Equations (16) and (10), one finds that even in the absence of intervention the exchange rate under managed floating \( (e''_i) \) does not equal the exchange rate under clean floating \( (e'^C_i) \). Only when the foreign exchange authority disregards its exchange rate target \( (\phi = 0 \text{ and thereby } \alpha = 1) \), \( e''_i = e'^C_i \).

Parameter \( \alpha \) is the weight of the market equilibrium rate on the exchange rate in the presence of intervention under the managed float. When the weight of the market equilibrium rate is closer to unity, it reflects that the government is less firm in committing itself to its exchange rate target. No wonder in this case that there is a smaller deviation between the rate in the absence of intervention under a managed float and the rate under a clean float. Note also that the effects of the short-run fundamental fluctuation on the exchange rates in both states under a managed float are decreasing functions of the firmness of the exchange rate target. It is therefore easy to prove that the effects of the short-run fundamental fluctuations on the exchange rates under a managed float can be much lower than their counterpart under a clean
float when the government is acutely concerned about its exchange rate target.\textsuperscript{10}

3.2 Targeting the exchange rate at a fundamental-inconsistent value

In previous subsections we discussed exchange rate targets consistent with the fundamentals. The implied intervention amounts and the exchange rate deviation (between the realized rate and its target) are stationary, and the government loss function is well defined. However, some countries set their exchange rate targets at a constant level while their economic fundamentals are not compatible with that target.\textsuperscript{11} In this situation ($e^* = \bar{c}$), the realized exchange rate diverges from its target. Although maintaining a constant exchange rate target when facing a non-stationary fundamental makes no sense in theory, constant targets are indeed set by some countries trying to rebuild the confidence in their currency while lacking of sufficient consideration for their fundamentals.

Because the exchange is diverging from its target, the probability of intervention keeps increasing, finally approaching 100%. In this case:

$$E_i e_{t+1} = E_i \left[ e_{t+1} | I_{t+1} = 1 \right]$$

$$= E_i \left[ \alpha e_{t+1} + (1 - \alpha)\bar{c} | I_{t+1} = 1 \right]$$

$$= \alpha E_i e_{t+1} + (1 - \alpha)\bar{c}.$$

By substituting the above equation into Equation (7), we get:

$$e_i^n = \frac{\gamma \alpha E_i e_{t+1} + \gamma (1 - \alpha)\bar{c} - f_i}{\delta + \gamma}.$$

\textsuperscript{10} A simple simulation and its resulting graphs for this point are presented in Appendix B.

\textsuperscript{11} For example, Nicaragua adopted a fixed exchange rate during 1991-1992, with a serious inflation relative to its trade counterparts, and it soon found the fixed exchange rate regime to be unsustainable and turned instead to adopt a crawling peg.
\begin{equation}
= -\frac{1}{\delta + \gamma} E_t \sum_{j=0}^{\infty} \rho^{*j} \left[ -\gamma(1-\alpha)\tilde{e} + f_{t+j} \right], \tag{18}
\end{equation}

where the discount factor \( \rho^* \equiv \alpha \gamma / (\delta + \gamma) \) is less than \( \rho \) in previous subsections.

The constant intervention of the foreign exchange authority lowers the importance of future fundamentals on the equilibrium exchange rate. Furthermore, the firmer the government commits to its exchange rate target (i.e., a lower \( \rho^* \)), the lower the influence of the economic fundamentals.

By substituting Equation (9) into Equation (18), we have:

\begin{equation}
e^*_t = -\frac{1}{\delta + (1-\alpha)\gamma} \kappa' + \frac{\gamma(1-\alpha)}{\delta + (1-\alpha)\gamma} \tilde{e} - \frac{1}{\delta + (1-\alpha)\gamma} f_t, \\
+ \frac{\rho^* \lambda_i}{[\delta + (1-\alpha)\gamma](1-\rho^* \lambda_i)} \Delta f_t, \tag{19}
\end{equation}

where \( \kappa' \equiv \rho^* \lambda' / [(1-\rho^*)(1-\rho^* \lambda_i)] \). While the exchange rate with official intervention under managed float, which is always equal to the realized exchange rate, is then solved as:

\begin{equation}
e'_t = -\frac{\alpha}{\delta + (1-\alpha)\gamma} \kappa' + \frac{(1-\alpha)(\delta + \gamma)}{\delta + (1-\alpha)\gamma} \tilde{e} - \frac{\alpha}{\delta + (1-\alpha)\gamma} f_t, \\
+ \frac{\alpha \rho^* \lambda_i}{[\delta + (1-\alpha)\gamma](1-\rho^* \lambda_i)} \Delta f_t. \tag{20}
\end{equation}

Several points are found from Equations (19) and (20). First, through the rational expectation of the government intervention, the coefficient of the exchange rate target in Equation (20) is higher than the weight of the target in Equation (8), i.e., \( 1-\alpha \). Second, in this case there is not only a level effect, but also a change in the co-integrating relationship between the exchange rates and the fundamentals under managed float compared with that under a clean float. As the government is concerned for targeting the exchange rate (\( \phi > 0 \) and \( \alpha < 1 \)), it can be seen that the
co-integration coefficient of economic fundamentals \( f_t \) on the exchange rate under a managed float is less than that \( 1/\delta \) under a clean float. Third, as there is more governmental commitment to the exchange rate target (thereby a lower \( \alpha \)), the effect of the short-run fundamental fluctuation on the exchange rate under a managed float is much lower than that under a clean float.

The summary of this section is as follows. Even in the absence of intervention, a possibility of future intervention makes the exchange rate under managed float differ from the rate under a clean float. In the case where the government sets an exchange rate target consistent with its I(1) fundamentals, if the government is concerned more about the intervention costs, then the more similar the effect of the short-run fundamental fluctuation on the exchange rate under a managed float regime is to that under a clean float one. In addition, whether the government sets its exchange rate target to be consistent or inconsistent with its I(1) fundamentals, the firmer the government’s application is toward targeting the exchange rate, the less the effect is of the short-run fundamental fluctuation on the exchange rate under a managed float than under a clean float. The theoretical implication of our model resembles Krugman’s (1991) “honeymoon effect” in that the change of foreign exchange fundamentals has a milder effect on the exchange rate under managed float in the absence of intervention than its clean float counterpart.\(^{12}\)

4. Concluding remarks

This article provides a theoretical model of rational expectations dealing with official interventions of variable frequency and obtains an implied regression for the

\(^{12}\) After specifying the bank’s target rate, our model can be estimated by an endogenous threshold model restricted closely by the theory or a simple nonlinear time series model, depending on the researcher’s purpose.
exchange rate. The model implies that the evolution of the exchange rate has two endogenously determined thresholds, unlike Krugman (1991) and subsequent researchers who do not explain the width of their exchange rate target zone. Between the thresholds, the exchange rate is floating without government intervention. When crossing them, official intervention and market fundamentals together determine the process of the equilibrium exchange rate. The concept where central banks intervene only for big departures of the exchange rate from its target is consistent with empirical evidence. Moreover, the chance of realigning the exchange rate target is embodied in the expectations of the future exchange rate and influences the current exchange rate.

The effects of the short-run fundamental fluctuations on the exchange rates under a managed float can be much lower than its counterpart under a clean float. It is shown that the more the government cares about targeting its exchange rate or the smaller the fixed intervention costs are, then the narrower the optimal non-intervention regime is and the more likely for the government to intervene. The theory-based validity of official intervention justifies the intervention operations of a central bank’s pursuit for the stability of the exchange rate.

Another significant feature of our model is that it implies a regime-switching model of the exchange rate, which outperforms a variety of linear models as found by previous research. Moreover, the model can be easily estimated for any country reporting its exchange rate data. That is, we can investigate the effects of official interventions only through observable exchange rate data, even for those countries who fail to reveal their intervention information.

Several remarks need mentioning. First, this article derives an analytical solution of the exchange rate at the cost of neglecting to explicitly employ a model of
micro-foundation. However, the utility function of a representative agent can be approximated to the government’s loss function, if a proper economic structure is specified (see, for example, Wickens, 2008, Ch13; Woodford, 2003, Ch6). Second, this article does not consider asymmetric intervention policies. An easy way to model asymmetric intervention is to assume different fixed costs of intervention. When the cost is considered to be higher in the case of devaluation, a central bank prefers to let its currency devaluate over the other option of appreciating. Finally, after specifying the bank’s target rate, our model can be estimated by an endogenous threshold model which is closely restricted by theory. The concentrated least squares estimation of Kourtellos, Stengos and Tan (2007) which extends the threshold regression framework of Caner and Hansen (2004) to allow for endogeneity of the threshold variable, can be an empirical application.
Appendix A

This appendix is a supplementary mathematical note to solving the expectations for the exchange rate. From the definition of the expectation of the exchange rate in the managed float model, we have:

\[
E_t e_{t+1} = \int_{-\infty}^\infty e_{t+1} f(e_{t+1} \mid \Omega_t) d(e_{t+1})
\]

\[
= \int_{-\infty}^{1-t-C} e_{t+1} f(e_{t+1} \mid \Omega_t) d(e_{t+1}) + \int_{1-t+C}^{C} e_{t+1} f(e_{t+1} \mid \Omega_t) d(e_{t+1})
\]

\[
+ \int_{C-t-C}^\infty e_{t+1} f(e_{t+1} \mid \Omega_t) d(e_{t+1}) ,
\]

where \( C \equiv \sqrt{((\delta + \gamma)^2 + \phi) c / \phi} \). We re-write the expectation equation as:

\[
E_t e_{t+1} = E_t e'^{a}_{t+1} + \int_{-\infty}^{1-t-C} (e_{t+1}' - e'^{a}_{t+1}) f(e_{t+1} \mid \Omega_t) d(e_{t+1})
\]

\[
+ \int_{1-t+C}^{C} (e_{t+1}' - e'^{a}_{t+1}) f(e_{t+1} \mid \Omega_t) d(e_{t+1}) .
\]

From Equation (8) we have \( e_{t}' = e_{t}'' - (1 - \alpha)(e_{t}' - e_{t}^*) \). Let \( \Delta_{t} \equiv e_{t}'' - e_{t}' \) and therefore:

\[
E_t e_{t+1} = E_t e'^{a}_{t+1} - (1 - \alpha) \int_{-\infty}^{C} (\Delta_{t+1}) f(\Delta_{t+1} \mid \Omega_t) d(\Delta_{t+1})
\]

\[
- (1 - \alpha) \int_{C}^{\infty} (\Delta_{t+1}) f(\Delta_{t+1} \mid \Omega_t) d(\Delta_{t+1})
\]

\[
= E_t e'^{a}_{t+1} - (1 - \alpha) \int_{-\infty}^{C} (\Delta_{t+1}) f(\Delta_{t+1} \mid \Omega_t) d(\Delta_{t+1})
\]

\[
+ (1 - \alpha) \int_{-\infty}^{C} (\Delta_{t+1}) f(\Delta_{t+1} \mid \Omega_t) d(\Delta_{t+1}) .
\]

Define \( d_{t} = \kappa / \delta + a + b\lambda + b\lambda_{f} \Delta_{t} \), so that Equation (13) can be re-written as \( \Delta_{t} = d_{t+1} + \omega_{t} \). Substituting Equation (13) into the expectation equation, along with the assumption that \( \omega_{t} \) follows a uniform distribution between a sufficiently large range \([-H, H]\) can further derive the expectation of the exchange rate as:
\[ E, e_{t+1} = E, e_t - (1 - \alpha) \left( \frac{\kappa}{\delta} + a + b(\lambda + \lambda_1 \Delta f_i) \right) \]
\[ + (1 - \alpha) \frac{C}{H} \left( \frac{\kappa}{\delta} + a + b(\lambda + \lambda_1 \Delta f_i) \right) \]
\[ + (1 - \alpha) \int_{c-d_i}^{c-d_i} \omega_{t+1} f(\omega_{t+1} | \Omega, \mathcal{C}) d(\omega_{t+1}) \]
\[ = E, e_t - (1 - \alpha) \frac{H - C}{H} \left( \frac{\kappa}{\delta} + a + b(\lambda + \lambda_1 \Delta f_i) \right) \]
\[ - (1 - \alpha) \frac{C}{H} \left( \frac{\kappa}{\delta} + a + b(\lambda + \lambda_1 \Delta f_i) \right) \]
\[ = -\frac{1}{\delta} f_i - \frac{1}{\delta} (\lambda + \lambda_1 \Delta f_i) + a + b(\lambda + \lambda_1 \Delta f_i) \]
\[ - (1 - \alpha) \left( \frac{\kappa}{\delta} + a + b(\lambda + \lambda_1 \Delta f_i) \right) \]
\[ = -\lambda \frac{1}{\delta} - (1 - \alpha) \frac{\kappa}{\delta} + \alpha(a + b(\lambda + \lambda_1 \Delta f_i)) \]
\[ = -\frac{f_i}{\delta} - \frac{\lambda}{\delta} - (1 - \alpha) \frac{\kappa}{\delta} + \alpha(a + b\lambda) \left( a \delta - \frac{1}{\delta} \right) \lambda_1 \Delta f_i. \]
Appendix B

In the simulation experiment of the exchange rate processes under different regimes, we set $\delta = 1$, $\gamma = 50$, $\phi = 2040$, $\lambda = 0$, $\lambda_1 = 0.8$, and $c = 100$ for a benchmark model. The serial-uncorrelated disturbances of fundamentals are generated from a uniform distribution, U[0, 1]. Figure 1 plots the simulated exchange rate under a clean float ($e_t^F$), the simulated exchange rate under a managed float in the absence of intervention ($e_t^n$), and the simulated realized exchange rate under a managed float ($e_t^M$). Since the fundamental is an I(1) process, we also plot three stationary series: the deviations of three exchange rates respectively from the target of the exchange rate, which is assumed to be the permanent part of the clean float rate. That is, $e_t^F - e_t^*$, $e_t^n - e_t^*$, and $e_t^M - e_t^*$. The feature of a smoother exchange rate process under managed than under clean float is easy to verify by observing Figures 1 and 2.

![Figure 1 Simulated exchange rates](image-url)
We repeat the simulation experiment 5,000 times to obtain the mean statistics for standard deviations of $e_t^F - e_t^*$ and $e_t^M - e_t^*$, $\sigma(e_t^F - e_t^*)$ and $\sigma(e_t^M - e_t^*)$, respectively. The simulation results for the benchmark model listed below show the deviation of the exchange rate from its (trend) target under a managed float is significantly less volatile than that under a clean float. Sensitivity analyses related to various elasticity of terms of trade ($\delta$), degree of capital mobility ($\gamma$), weight about instability of the exchange rate ($\phi$), fixed intervention costs ($c$), and the persistence of fundamentals ($\lambda_1$) are also conducted and the robustness of the result is found.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>$\sigma(e_t^F - e_t^*)$</th>
<th>$\sigma(e_t^M - e_t^*)$</th>
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<tbody>
<tr>
<td>$\delta = 0.1$</td>
<td>17.917 [2.437]</td>
<td>3.561 [0.484]</td>
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<tr>
<td>= 10</td>
<td>0.091 [0.012]</td>
<td>0.053 [0.007]</td>
</tr>
<tr>
<td>$\gamma = 5$</td>
<td>0.908 [0.122]</td>
<td>0.088 [0.009]</td>
</tr>
<tr>
<td>= 500</td>
<td>1.797 [0.244]</td>
<td>1.733 [0.234]</td>
</tr>
<tr>
<td>$\phi = 204$</td>
<td>1.657 [0.227]</td>
<td>1.286 [0.167]</td>
</tr>
<tr>
<td>= 20400</td>
<td>1.652 [0.226]</td>
<td>0.047 [0.005]</td>
</tr>
<tr>
<td>$c = 10$</td>
<td>1.654 [0.229]</td>
<td>0.358 [0.049]</td>
</tr>
<tr>
<td>= 1000</td>
<td>1.654 [0.224]</td>
<td>0.531 [0.047]</td>
</tr>
<tr>
<td>$\lambda_1 = 0.4$</td>
<td>0.201 [0.013]</td>
<td>0.154 [0.009]</td>
</tr>
<tr>
<td>= 0.9</td>
<td>4.388 [0.873]</td>
<td>0.577 [0.112]</td>
</tr>
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</table>

Note: The numbers in the parenthesis are standard errors.
References


Ito, Takatoshi and Tomoyoshi Yabu, 2007. What Prompts Japan to Intervene in the


