Pitfalls in using Granger causality tests to find an engine of growth

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Abstract

This paper discusses the reliability of using a Granger causality test to find an engine of growth. We first focus growth models' cointegration implications since causality must exist in an error-correction model. As a complementary, Monte Carlo experiments with independently generated I(1) variables also indicate a significant probability for rejecting the Granger non-causality null. Given the persistency and cointegration of variables in growth models, rejecting the non-causality null may reflect a spurious causal relationship, rather than confirm a theoretical causality.

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1. Introduction

Granger causality tests nowadays are widely used to find the engine of an economy's growth. However, empirical studies using the Granger causality test seldom associate with a complete and convincing theory explaining how the cause produces the effect.[1] Although many problems about this test have been recognized early in the literature, this paper proposes more evidence directly related to the time-series implications of growth models. In particular, we focus on two time-series properties that make the Granger non-causality null be easily rejected: the variables' integration and cointegration.

Engle and Granger (1987) and Granger (1988) have demonstrated that a Granger causality must exist in an error-correction model (ECM). As well shown in growth literature, every growth model, endogenous or exogenous, implies some kind of balanced growth conditions between output and state variables of interest. Because these growth conditions mean that the logarithms of output-to-state variable ratios are stationary, they thus lead to cointegration implications between the output and the state variables. Therefore, Granger

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[1] It is well known that the concept of causality tested by Granger and Sims tests is not closely related to the theoretical causality [cf. Feige and Pearce (1979), Engle et al. (1983), and Cooley and LeRoy (1985)]. In addition, Newbold (1978) and Sargent (1989) point out that measurement errors can distort the causal relationship between the pairs of series and make every variable Granger cause every other variable in the system.

causality must be found in a growth model. The key issue is that a variety of competing models implying equivalent balanced growth conditions may share the same Granger causality. If Granger causal relationships are not uniquely implied by a specific model, then it would be too early to make a conclusion by the results of a Granger causality test.

Many macroeconomic variables, on the other hand, are integrated of order one, I(1), data. When estimating the nonstationary time series models, Cheung and Lai (1993) point out that Johansens's test is biased toward finding cointegration between variables if the test's finite sample critical values are not applied. Since cointegration is easy to hold for I(1) variables and Granger causality must exist in an ECM, there is thus a reasonable doubt that the Granger causality test might be biased toward rejecting the non-causality null for even arbitrary integrated data. A naive Monte Carlo simulation with a group of independently generated integrated series is conducted to compute the probability in rejecting the Granger non-causality null.

The remainder of this paper is as follows. Section 2 gives an example to illustrate that Granger causality is merely a reflection of a balanced growth condition between state variables and output, and that the causality test cannot tell whether the state variable is a driving force for output or itself is derived from other forces. Section 3 counts the number of times of rejecting the Granger non-causality null for a group of independently generated integrated series and finds a very significant probability of rejection. Section 4 concludes.

2. Spurious causal relationship I: An exogenous growth example

Consider an exogenous growth model in which the driving force of growth is the ever increasing level of technology, *X*. Assume that $X_t = X_0 e^{gt}$. However, per capita human capital (*H*) is one production factor affecting per capita output (*Y*). Assume the following production function,

$$Y_t = A_t H_t^{\alpha} X_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where A_t denotes a stationary productivity shock. The evolution of human capital is determined by

$$\tilde{H}_{t+1} - \tilde{H}_t = s\tilde{Y}_t - (\delta + g)\tilde{H}_t, \quad 0 < s < 1, 0 < \delta < 1,$$

where the transformed human capital $\tilde{H}_t$ is defined to be $\ln(H_t / X_t)$, the transformed output $\tilde{Y}_t$ is $\ln(Y_t / X_t)$, s is the fraction of income invested in human capital, and δ is the rate of depreciation.

The specification of the production function is a simplified version of Mankiw et al. (1992)'s augmented Solow model by assuming no physical capital in the production process. This is an exogenous growth model, because its growth rate would be zero if X_t cannot grow. Since the ratio of output-to-human capital will be constant in steady state, there exists a balanced growth condition between output and human capital.

Previous researchers, such as In and Doucouliagos (1997), claim human capital is the engine of the US's growth and use the result that output is Granger caused by human capital as evidence to support their hypothesis. However, the question is will human capital fail to Granger cause output if the real world is instead guided by the exogenous growth model we have discussed above? The answer is no. Engle and Granger (1987) and Granger (1988) show that either a unidirectional or bi-directional Granger causality must exist in at least the stationary error terms from the cointegration regressions. Our negative answer is simply due to the cointegration relationship between output and human capital, which is a natural implication of the growth model's balanced growth condition.

In general every growth model implies some balanced or unbalanced growth conditions between output and state variables of interest. These state variables include physical capital, human capital, technology level positively affected by trade, efficiency of the financial sector, research and development, etc. The growth conditions in turn lead to cointegration restrictions when forming a time series model to estimate, and we are not surprised to find that the non-causality null is rejected. Moreover, it could be observationally equivalent for the cointegration implied by an endogenous growth model and a competing exogenous one (see, for example, Lau and Sin, 1997). A researcher who believes in some specific growth model will be happy to find that the results of a Granger causality test for his VAR model indeed support his belief. Nevertheless, the existence of a Granger causal relationship cannot be taken as evidence to discriminate different growth models.

3. Spurious causal relationship II: Independent I(1) series

Most macroeconomic variables are integrated and with persistent growth rates. Cheung and Lai (1993) point out that Johansen's tests are biased toward finding cointegration between I(1) variables. And we know Granger causality must exist in an ECM. Therefore, is the Granger causality test bias toward finding a causal relationship when applying to I(1) series?

According to the Granger representation theorem, a cointegration model of two variables needs to be modified with an ECM by augmenting an error-correction term, as follows

$$\Delta x_t = a_{1,0} + \sum_{i=1}^{p-1} a_{1,1i}\Delta x_{t-i} + \sum_{i=1}^{p-1} a_{1,2i}\Delta y_{t-i} + a_{1,3}\text{ecm1}_{t-p} + \varepsilon_{1t},$$
$$\Delta y_t = a_{2,0} + \sum_{i=1}^{p-1} a_{2,1i}\Delta x_{t-i} + \sum_{i=1}^{p-1} a_{2,2i}\Delta y_{t-i} + a_{2,3}\text{ecm1}_{t-p} + \varepsilon_{2t},$$

where $\{x_t\}$ and $\{y_t\}$ are I(1) series generated by the *p*-th order VAR model, $\{\text{ecm1}_t\}$ denotes the stationary error-correction term, and $\{\varepsilon_{it}\}$, $i = 1, 2$, are serially-uncorrelated sequences.

In testing a hypothesis, consider "x_t is not Granger caused by y_t" as an example, then the null can be formulated as

$$H_0:\ a_{1,21} = a_{1,22} = \cdots = a_{1,2p-1} = 0 \ \text{ and } \ a_{1,3} = 0.$$

It might be convenient to refer to the first half of the null as "short-run non-causality" and the second half as "long-run non-causality".

To compute the probability of rejecting a non-causality null, a Monte Carlo experiment based on the following data generation process is conducted:

$$\Delta x_t = 0.9\Delta x_{t-1} + 0.006\varepsilon_t^x,$$

$$\Delta y_t = 0.8\Delta y_{t-1} + 0.004\varepsilon_t^y,$$

where ε_t^x and ε_t^y are contemporary and intertemporal independent standard normal distributed innovations.[2]

We simulate the pair of data series 1000 times. Each replication contains 50 observations, which is a little longer than the annual period available for most newly-industrialized countries, but shorter than for major industrialized countries. Assume that a researcher does not know a priori whether these variables are cointegrated or not and he uses Johansen's cointegration tests to decide the

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[2] Unlike many variables in growth theory, there is no constant term in our simulation. If a constant is added to the data generation process, then the simulated variables grow as time passes by and the probability of rejecting the non-causality null rises sharply.

number of cointegration.[3] Based on Johansen's trace statistics, there may be zero or one or two cointegrating relationship between x and y. If there is no cointegrating relationship between them, then a VAR(1) with first-order difference data is used to conduct a Granger causality test. If there are two cointegrating relationships between them, then the research will conclude the data's stationarity and conduct a non-causality test in a VAR(2) with levels. Otherwise, an ECM will be used to conduct a non-causality test. As for rejecting the Granger non-causality null, we say that there exists a causal relationship between x and y at least once in either the short run or the long run.

Table 1 reports the results of the cointegration test and causality test. According to the results of the Johansen cointegration test and correcting for a small sample bias, 83.8% of the data are identified as having no cointegrating relationship. For the 162 data sets identified as having cointegration relationships, the Granger non-causality tests are all rejected in the short run and/or the long run except in five cases. Of those with one cointegration and estimating using an ECM, the non-causality is rejected for all of the cases. For the data correctly identified as not being cointegrated, 16.9% of them still reject the Granger non-causality null.

[3] Assume that the lag length for the VAR model is correctly chosen to be 2.

It is well known that Granger causality is sensitive to the number of variables in the VAR system. One may question that the results in Table 1 are due to the small number in the VAR model.[4] To take into account this problem, we further consider the third variable *z*. Assume that

$$\Delta z_t = 0.7\Delta z_{t-1} + 0.002\varepsilon_t^z,$$

and ε_t^x, ε_t^y and ε_t^z are contemporarily and intertemporally independent N(0, 1) sequences. We repeat the three variables' simulation with the same process as above. As for rejecting the Granger non-causality, we now mean that there exists a causal relationship between *x* and *y*, or *y* and *z*, or *z* and *x* at least once in either the short run or the long run.

Table 2 reports the results. According to the results of the Johansen cointegration test, 73.9% of the data are correctly identified as being not cointegrated. For the 261 data sets that are identified as being cointegrated with at least one cointegration relationship, the Granger non-causality tests are rejected in the short run and/or the long run in all cases. For the data correctly identified as being not cointegrated, 43.4% of those still fail to reject the Grange non-causality null. In sum, nearly 60% of the simulations reject the

[4] Empirically, for example, Esfahani (1991) and Riezman et al. (1996) test the export-led growth hypothesis. They emphasize that if imports are not included as a third variable in the VAR model (or ECM), then the results of the Granger causality test are not correct.

non-causality null for the three-variable case.[5]

Although in the simulation we do not specify which variable is output and which is a state variable, the high probability to reject the non-causality null does demonstrate how easily a spurious causality is derived by applying a Granger causality test to irrelevant ARIMA(1,1,0) series. The naive simulation results also cast a shadow on the Granger causality test in finding the engine of growth and/or feedback effect from output.

4. Conclusion

Many competing growth models with an equivalent cointegration implication can own the same Granger causal relationship, thus the Granger causality test itself alone is not enough to identify an engine of growth. In addition, many macro variables are integrated and with persistent growth rates. Monte Carlo experiments with independently generated ARIMA(1,1,0) variables show that there is a significant probability of rejecting the non-causal null. We conclude that rejecting the Granger non-causality null is easy and it doesn't mean we find the true engine of an economy's growth.

[5] This appears to arise partly from the persistence of variables in first-order difference.

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Table 1. Cases failing to reject the null hypothesis: Two variables (x, y)

Cointegration test		Non-causality test
Rank	# of data sets	H_0
$r = 0$	838	696
$r = 1$	133	0
$r = 2$	29	5
Total	1000	701

Note: 1. The 5% critical values of the cointegration rank test statistics are from Table 1* of Osterwald-Lenum (1992) and are corrected for a small sample bias as suggested by Chenug and Lai (1993).

2. H_0: no causality null (either in the long run or the short run)

3. The Granger non-causality tests are estimated by simultaneous equation models, with and without non-causality restrictions, to improve the efficiency of estimates. Likelihood-ratio statistics are compared with the 5% critical values of the χ^2 statistics to compute the number of cases failing to reject the null.

Table 2. Cases failing to reject the null hypothesis: Three variables (x, y, z)

Cointegration test		Non-causality test
Rank	# of data sets	H_0
$r = 0$	739	415
$r = 1$	199	0
$r = 2$	47	0
$r = 3$	15	0
Total	1000	415

Note: Same as Table 1.