Re-examining the Sources of Real Exchange Rate Fluctuations: A Rational Expectations Structural VAR Approach

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To explicitly take into account market expectations for real exchange rate fluctuations, this paper proposes a rational expectations structural VAR (RE-SVAR) method to decompose structural shocks. An exogenous shock not only has effects on real exchange rates directly, but also has indirect effects through the change in predictions for other fundamental variables. The RE-SVAR method imposes all these direct and indirect channels to decompose the sources of real exchange rate fluctuations. We find that the over-identifying restrictions implied by our model cannot be rejected for Canada, France, Italy, and Japan. Our results also indicate that private expenditure shocks are the most important source of real exchange rate variations for France and Italy, monetary shocks are most important for Canada, and foreign price shocks are most important in the long run for Japan. Moreover, supply shocks have a small but significant explanatory power fot real exchange rate variations only in France.

Keywords: Rational expectations, Structural VAR, Real exchange rates
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1 Introduction

The driving force of volatile real exchange rate movements has yet to be fully documented. What is the source of real exchange rate variations? Due to the fact that

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the theoretical dynamic effects on real exchange rates are quite diversified, little empirical work has been done on developing structural models to answer this question. However, given the theoretically and empirically undoubted importance of expectations on the future environment to exchange rate movements, it should be a good idea to answer this question based on a model that can account for market expectations explicitly.

This paper interprets dynamic links between the real exchange rate and several structural shocks through a small-open economy model. We do this in three steps. First, we present a rational expectations model and demonstrate that forward-looking expectations are important to characterize the dynamic behavior of real exchange rates. Second, to do our empirical study we adopt the rational expectations-structural vector autoregression (RE-SVAR) approach, which is proposed by Lee and Lin (1998), to account for the expectations in deriving the cross-equation restrictions in the parameters of the autoregression and of the structural innovations. Third, we use variance decompositions to determine the relative importance of each shock in explaining the fluctuations of the real exchange rate empirically. The impulse response functions generated by the restricted vector autoregression representation (VAR) also help us to characterize the channels of structural shocks affecting real exchange rates.

Concerning relative importance of the sources in real exchange rate fluctuations, previous empirical studies use very distinct econometric methodology to achieve the identification. Huizinga (1987) and Cumby and Huizinga (1990) used Beveridge and Nelson's decomposition without much theoretical implication. Apergis and Karfakis (1996) used Bernanke's (1986) methodology by imposing an economic structure on the variance-covariance matrix. Lastrapes (1992), Evans and Lothian (1993), Clarida and Gali (1994), and Chen and Wu (1997) imposed just-identified long-run restrictions in the coefficients' matrix of structure as in Blanchard and Quah (1989). Zhou (1995) used the approach of King, *et al.* (1991) and focused on the relevant variables' common trend. Lee and Lin (1998) adopted the RE-SVAR approach which can disentangle the different dynamic effects of several shocks in a theoretical model,[1] but they did not consider productivity shocks. Since the volatility of the real exchange rate is also one of the stylized facts concerned by real business cycle theorists who believe technical change is the all-important source of economic shocks (e.g., Ahmed, *et al.* 1993), it is interesting to include productivity shocks in our structural model.

To take into account more structural shocks, this paper interprets the dy-

[1] This paper extends Keating's (1990) identification methodology for a structural VAR.

namics of real exchange rates in floating-exchange-rate economies using a rational expectations model, in which purchasing power parity and uncovered interest parity can be easily imposed as extreme cases. This model can generate fluctuations and persistence in real exchange rates in response to a variety of domestic and foreign shocks. A merit of the RE-SVAR approach is that it allows us to investigate the full effects raised by a specific shock. For example, an increase in government spending directly results in real appreciation through the excess demand for domestic output that it creates. Furthermore, a government spending shock (and other structural shocks) can have indirect effects on the real exchange rate through its predictability of future fundamentals.

There are three total distinctions when applying this paper's RE-SVAR approach here. First, knowing that asset prices are vulnerable to market expectations, this method helps us explain the dynamics of real exchange rates in accordance with a rational expectations model. Second, the estimation with the RE-SVAR method provides an evaluation of the model's performance. Third, this paper considers a shift-segmented trend model in which relatively infrequent occurring events either shift the constant term or change the slope. Since whether the real exchange rate is empirically integrated of order one or not remains controversial, unlike the long-run restriction approach that usually uses a stochastic trend assumption, we adopt an alternative trend assumption. The first two characters are due to the unique distinction of the RE-SVAR method while the last is purely an assumption made in this paper.

Our empirical results indicate that private expenditure shocks are the most important source of real exchange rate variations for France and Italy, monetary shocks are most important for Canada, and foreign price shocks are most important in the long run for Japan. A government spending shock is not significant for the four countries' real exchange rate fluctuations. Lastly, supply shocks have a small but significant explaining power in the real rate variation only for France.

Apart from the introduction, the remainder of this paper is organized as follows. Section 2 solves a stochastic discrete-time rational expectations version of Frenkel and Rodriguez's (1982) model. In Section 3 we use the RE-SVAR approach to derive a set of identifying restriction. Under an alternative segmented trend assumption, Section 4 explains the data and reports empirical results. Section 5 concludes.

2 A stochastic discrete-time Frenkel and Rodriguez (1982) model

The rational expectations model of Frenkel and Rodriguez (1982) that we specify in discrete time is as follows:

$$d_t \equiv \delta_0 + \delta_1 y_t - \delta_2 r_t + \delta_3 g_t + \delta_4 \left(p_t^* + e_t - p_t\right) + n_t, \tag{1}$$

$$p_t - p_{t-1} = \pi(d_t - y_t), \tag{2}$$

$$m_t - p_t = l_0 + l_1 y_t - l_2 r_t, \tag{3}$$

$$f_0 + f_1\left(p_t^* + e_t - p_t\right) - f_2 y_t + f_3\left(r_t - r_t^* - E_t e_{t+1} + e_t\right) = 0, \tag{4}$$

where the "structural" parameters $\delta_i (i = 1, \cdots 4)$, $l_i (i = 0, \cdots 2)$, $f_i (i = 0, \cdots 3)$ and π are all positive. In our model all variables except the domestic nominal interest rate r_t and the foreign nominal interest rate r_t^* are in logarithms. Term d_t is the aggregate demand for domestic output at time t, δ_0 is the autonomous demand for domestic output, y_t is the domestic output at time t, g_t is the domestic real government spending at time t, p_t^* is the foreign price level at time t, e_t is the nominal exchange rate: one unit of the foreign currency is e_t units of the domestic currency, p_t is the domestic price level at time t, n_t is the serially-uncorrelated autonomous private demand shock at time t, and m_t is the nominal money stock at time t. Finally, $E_t X \equiv E[X|\Omega_t]$ for any random variable X in which E is the expectation operator and Ω_t is the agent's information set at time t.

Equation (1) is the aggregate demand function for the domestic output at time t. Equation (2) describes how the domestic price level adjusts to excess demand in the goods market. In the limit as π tends to infinity, p_t adjusts instantaneously, thereby continuously maintaining market-clearing in the goods market. If δ_4 is infinite, then purchasing power parity holds continuously. Equation (3) is a representation of money market clearing and equation (4) represents the foreign exchange markets' equilibrium condition. The magnitude of f_3, which is the speed of adjustment in net capital inflow, is a key factor determining the dynamics of exchange rates. When f_3 is infinite, the foreign exchange market equilibrium condition is replaced by the interest parity condition as in the Dornbusch (1976) model.[2]

The solution of the model for a real exchange rate can be obtained by first deriving the solutions for e_t and p_t. From equations (1)–(4), we have the expectational difference equation for the nominal exchange rate:

[2]Stockman (1987) pointed out that the behavior of real exchange rates could reflect not the importance of a sluggish price level adjustment, but rather the influence of real shocks with large permanent components. However, a sticky-price model does not imply real shocks are unimportant; it is the uncovered interest parity commonly shared in both the sticky-price model and the equilibrium model that is vulnerable in empirical studies (see Hodrick (1987) for a survey).

$$E_t e_{t+1} - \mu E_{t-1} e_t - \left[\pi \delta_4 \left(\frac{1}{l_2} - \frac{f_1}{f_3}\right)\mu + \frac{f_1}{f_3} + 1\right] e_t + \left(\frac{f_1}{f_3} + 1\right)\mu e_{t-1} = k_e + x_t,$$

where $\mu = (1 + \pi(\delta_2/l_2) + \pi\delta_4)^{-1}$ and $k_e = (f_0/f_3 + l_0/l_2)(1 - \mu) + \pi(\delta_0 - (l_0\delta_2/l_2))(1/l_2 - f_1/f_3)\mu$. Define $\delta_1' = 1 - \delta_1$, $\gamma \equiv \pi(\delta_1' + \delta_2 l_1/l_2)\mu$, $\phi_1 \equiv 1/l_2 - f_1/f_3$, $\phi_2 \equiv l_1/l_2 - (f_1/f_3)/(f_1/f_2)$, and $\phi_3 \equiv 1 + f_1/f_3$. The driving variable, x_t, is then given by

$$x_t \equiv -(\gamma\phi_1 - \phi_2)\, y_t - \mu\phi_2 y_{t-1} + \left(\frac{\pi\delta_2}{l_2}\phi_1\mu - \frac{1}{l_2}\right) m_t + \frac{\mu}{l_2} m_{t-1} + \pi\delta_3\phi_1\mu g_t - r_t^* + \mu r_{t-1}^* + (\phi_3 - 1 + \pi\delta_4\phi_1\mu)\, p_t^* - (\phi_3 - 1)\,\mu p_{t-1}^* + \pi\phi_1\mu n_t.$$

From the associated characteristic equation, two characteristic roots exist. Let β_1 and β_2 denote the two roots with $\beta_1 + \beta_2 = \mu + \pi\delta_4\phi_1\mu + \phi_3$, $\beta_1\beta_2 = \mu\phi_3$, and $\beta_1 < \beta_2$. It is easy to show that β_1 and β_2 satisfy $0 < \beta_1 < 1$ and $\beta_2 > 1$ by the medium-value theorem. Clearly, the above equation has a saddle point configuration with one root (β_1) stable and the other (β_2) unstable, and the unique convergent solution of e_t is:

$$e_t = \beta_1 e_{t-1} + \frac{k_e}{1 - \beta_2} - \frac{\mu}{\beta_2(\mu - \beta_2)} \sum_{j=0}^{\infty} \beta_2^{-j} E_{t-1} x_{t+j} + \frac{1}{\mu - \beta_2} \sum_{j=0}^{\infty} \beta_2^{-j} E_t x_{t+j}.$$

In this paper q_t denotes the real exchange rate at time t and

$$q_t = e_t - p_t + p_t^*,$$

where an increase in q_t means a depreciation in the real exchange rate of the domestic currency. To derive the real exchange rate, we first need to obtain the solution of p_t. After some calculations, equations (1)–(4) yield the following result:

$$p_t = \beta_1 p_{t-1} + \frac{k_p}{1 - \beta_2} - \frac{1}{\beta_2} \sum_{j=0}^{\infty} \beta_2^{-j} E_t w_{t+j},$$

in which $k_p = \pi[\delta_4((f_0/f_3) + (l_0/l_2)) - (\delta_0 - (\delta_2 l_0/l_2))(f_1/f_3)]\mu$ and the driving variable, w_t, is given by

$$w_t \equiv -\gamma y_{t+1} + (\gamma\phi_3 + \pi\delta_4\mu\phi_2)\, y_t + \frac{\pi\delta_2\mu}{l_2} m_{t+1} - \frac{\pi\,(\delta_2\phi_3 + \delta_4)\,\mu}{l_2} m_t$$
$$+\pi\delta_3\mu g_{t+1} - \pi\delta_3\mu\phi_3 g_t - \pi\delta_4\mu r_t^* + \pi\delta_4\mu p_{t+1}^* - \pi\delta_4\mu p_t^* - \phi_3\pi\mu n_t.$$

Collecting the expressions for e_t and p_t, we then see that the dynamic behavior of q_t can be characterized by the following equation:[3]

$$q_t = p_t^* + \beta_1\left(q_{t-1} - p_{t-1}^*\right) - \frac{1}{\beta_2 - \mu}\sum_{j=0}^{\infty}\beta_2^{-j}\left(E_t - \frac{\mu}{\beta_2}E_{t-1}\right)x_{t+j}$$
$$+\frac{1}{\beta_2}\sum_{j=0}^{\infty}\beta_2^{-j}E_t w_{t+j}. \tag{5}$$

For our model, an unanticipated once-and-for-all increase in money supply is neutral in the long run. An unanticipated permanent increase in government spending leads to a permanent real appreciation and an increase in the interest rate, both of which are required to restore equilibrium in the goods market and foreign exchange market. A permanently higher productivity and foreign interest rate also raise the real exchange rate. However, given the insulation properties of a flexible exchange rate, changes in the foreign price level have no real effects on the home country.

3 Econometric methodology — RE-SVAR approach

In the characterization of economic dynamics, effects from structural shocks on real exchange rates can be ascertained only when the structural VAR implied by equation (5) is identified. This section shows how the theoretical model characterizes the dynamic structure in a VAR, and how to recover the structural parameters from the VAR model.

Let $Z_t \equiv [q_t\ y_t\ m_t\ g_t\ p_t^*\ r_t^*]'$. For simplicity of presentation, we assume that the dynamic behavior of Z_t is governed by the structural first-order VAR denoted VAR (1):[4]

$$Z_t = A_0 Z_t + A_1 Z_{t-1} + U_t, \tag{6}$$

[3] For simplicity of presentation we have omitted the constant term $(1 - \beta_2)^{-1}(k_e - K_p)$.

[4] The VAR (1) system is not quite restrictive as it might first appear since any finite-order VARMA model can be transformed into a VAR (1) system by appropriately redefining the vector Z_t.

in which A_0 and A_1 are 6×6 matrix coefficients with $A_{k,ij}$ being the (i, j) element of $A_k (k = 0, 1)$ and $U_t = [u_{1t} \cdots u_{6t}]'$ is a 6×1 vector of contemporaneously and temporally-uncorrelated structural shocks with the diagonal covariance matrix: $\sum_U = E[U_t U_t']$. The implied restricted reduced-form VAR (1) of Z_t is:

$$Z_t = A^* Z_{t-1} + U_t^*,$$

where $A^* = [I - A_0]^{-1} A_1$ and $U_t^* = [I - A_0]^{-1} U_t$ with the variance-covariance matrix $\sum_{U^*} = [I - A_0]^{-1} \sum_U [I - A_0]^{-1'}$. Here, I is a 6×6 identity matrix.

The data can recover only the unrestricted reduced-form VAR (1) model of Z_t:

$$Z_t = A Z_{t-1} + V_t, \tag{7}$$

where A is a 6×6 matrix of coefficients with a_{ij} being the element coefficient, and $V_t = [v_{1t} \cdots v_{6t}]'$ is a 6×1 vector of temporally-uncorrelated, reduced-form shocks with the covariance matrix: $\sum_V = E[V_t V_t']$. The identification problem is simply this: can we uniquely recover A_0, A_1, and $\sum_U$ in (6) from A and $\sum_V$ in (7)? Evidently, there are more unknown parameters in A_0, A_1, and $\sum_U$ than known estimates in A and $\sum_V$. The structural VAR cannot be identified without additional restrictions on A_0, A_1, and $\sum_U$.

This paper proposes the RE-SVAR approach and shows that the forward-looking expectations in (5) imply a set of identifying restrictions on A_0, A_1, and $\sum_U$. To see this, note that the driving variables, x_t and w_t, in equation (5) can be written as

$$x_t = B_1 Z_t + B_2 Z_{t-1} + \pi \phi_1 \mu n_t,$$
$$w_t = B_3 Z_{t+1} + B_4 Z_t - \pi \phi_3 \mu n_t,$$

in which the 1×6 vectors $B_i (i = 1, \cdots, 4)$ are defined as

$$\begin{aligned}
B_1 &\equiv \begin{bmatrix} 0 & -(\gamma\phi_1 - \phi_2) & \frac{(\pi\delta_2\phi_1\mu - 1)}{l_2} & \pi\delta_3\phi_1\mu & \phi_3 - 1 + \mu\delta_4\phi_1\mu & -1 \end{bmatrix}, \\
B_2 &\equiv \begin{bmatrix} 0 & -\phi\mu & \mu/l_2 & 0 & (1-\phi_3)\,\mu & \mu \end{bmatrix}, \\
B_3 &\equiv \begin{bmatrix} 0 & -\gamma & \pi\delta_2\mu/l_2 & \pi\delta_3\mu & \pi\delta_4\mu & 0 \end{bmatrix}, \\
B_4 &\equiv \begin{bmatrix} 0 & \gamma\phi_3 + \pi\delta_4\phi_2\mu & \frac{-\pi(\delta_2\phi_3 + \delta_4)\mu}{l_2} & -\pi\delta_3\phi_3\mu & \pi\delta_4\mu & -\pi\delta_4\mu \end{bmatrix}.
\end{aligned}$$

Since Z_t has a VAR (1) representation, the least-square projection of Z_{t+j} on $\{Z_t, Z_{t-1}, \cdots\}$ is given by $A^{*j} Z_t$, for $j \geq 0$. Therefore, the forward-looking expectations in equation (5) can be transformed as follows:

$$\begin{aligned}
\sum_{j=0}^{\infty} \beta_2^{-j} E_{t-1} x_{t+j} &= \sum_{j=0}^{\infty} \beta_2^{-j} \left[B_1 A^{*j+1} + B_2 A^{*j} \right] Z_{t-1} \\
&= \left[B_1 A^* + B_2 \right] \left[I - \beta_2^{-1} A^* \right]^{-1} Z_{t-1}, \\
\sum_{j=0}^{\infty} \beta_2^{-j} E_t x_{t+j} &= \sum_{j=0}^{\infty} \beta_2^{-j} \left[B_1 A^{*j} + \beta_2^{-1} B_2 A^{*j} \right] Z_t + B_2 Z_{t-1} + \pi \phi_1 \mu n_t \\
&= \left[B_1 + \beta_2^{-1} B_2 \right] \left[I - \beta_2^{-1} A^* \right]^{-1} Z_t + B_2 Z_{t-1} + \pi \phi_1 \mu n_t, \\
\sum_{j=0}^{\infty} \beta_2^{-j} E_t w_{t+j} &= \sum_{j=0}^{\infty} \beta_2^{-j} \left[B_3 A^{*j+1} + B_4 A^{*j} \right] Z_t + \pi \mu \phi_3 n_t \\
&= \left[B_3 A^* + B_4 \right] \left[I - \beta_2^{-1} A^* \right]^{-1} Z_t + \pi \mu \phi_3 n_t.
\end{aligned}$$

Let h_i' denote the column vector with six elements, all of which are zero except for the ith unit element. Substituting the formula for the expected present value of x_t and w_t into equation (5) yields the following real exchange rate equation:

$$\begin{aligned}
q_t = &\left[\left[\frac{B_3 A^* + B_4}{\beta_2} - \frac{B_1 + \beta_2^{-1} B_2}{\beta_2 - \mu} \right] \left[I - \beta_2^{-1} A^* \right]^{-1} + h_5 \right] Z_t \\
&+ \left[\beta_1 (h_1 - h_5) - \frac{B_2}{\beta_2 - \mu} + \frac{\mu (B_1 A^* + B_2)}{\beta_2 (\beta_2 - \mu)} \left[I - \beta_2^{-1} A^* \right]^{-1} \right] Z_{t-1} \\
&- \pi \mu \left[\frac{\phi_1}{\beta_2 - \mu} + \frac{\phi_3}{\beta_2} \right] n_t. \qquad (8)
\end{aligned}$$

By substituting the restricted reduced-form VAR into (8) for Z_t, the resulting equation implies that the first rows of both A^* and $\sum_{U^*}$ are highly non-linear functions of structural parameters, $\delta_i (i = 1, \cdots 4)$, $l_i (i = 1, 2)$, $f_i (i = 1, \cdots 3)$, and π:

$$\begin{aligned}
h_1 A^* = &\left\{ \left[\frac{B_3 A^* + B_4}{\beta_2} - \frac{B_1 + \beta_2^{-1} B_2}{\beta_2 - \mu} \right] \left[I - \beta_2^{-1} A^* \right]^{-1} + h_5 \right\} A^* \\
&+ \beta_1 (h_1 - h_5) + \frac{\mu (B_1 A^* + B_2)}{\beta_2 (\beta_2 - \mu)} \left[I - \beta_2^{-1} A^* \right]^{-1} - \frac{B_2}{\beta_2 - \mu}. \qquad (9)
\end{aligned}$$

$$D \sum\nolimits_{U^*} D' = \left[\pi \mu \left[\frac{\phi_1}{\beta_2 - \mu} + \frac{\phi_3}{\beta_2} \right] \right]^2 \sigma_n^2 = h_1 \sum\nolimits_U h_1',$$

where D is a 1×6 vector:

$$D = \left[\frac{B_3A^* + B_4}{\beta_2} - \frac{B_1 + \beta_2^{-1}B_2}{\beta_2 - \mu}\right]\left[I - \beta_2^{-1}A^*\right]^{-1} + (h_5 - h_1).$$

It is clear from (9) that once $\delta_i(i = 1, \cdots 4), l_1, l_2, f_i(i = 1, \cdots 3), \pi$, and σ_n^2 are recovered from A and $\sum_V$, the first rows of A_0, A_1, and $\sum_U$ can be determined. On the other hand, given $A^* = [I - A_0]^{-1}A_1$, the elements left in A_1 can be recovered from A once A_0 is determined. Therefore, what remains to be identified are the other parameters in A_0 and $\sum_U$.

From $\sum_V = [I - A_0]^{-1}\sum_U[I - A_0]^{-1'}$, we can see that to recover the remaining 30 parameters in A_0 and $\sum_U$ from $\sum_V$ requires at least nine additional restrictions. At first, this paper assumes that all foreign variables are predetermined to the small-open economy under consideration. Furthermore, we assume that the full-employment output is predetermined to all kinds of demand shocks in the home country. Thus

$$\begin{aligned} &A_{0,51} = A_{0,52} = A_{0,53} = A_{0,54} = A_{0,61} = A_{0,62} = A_{0,63} = A_{0,64} = 0, \\ &A_{0,21} = A_{0,23} = A_{0,24} = 0. \end{aligned} \tag{10}$$

Once A_0 is determined, the remaining elements in A_1 can be recovered using $A = [I - A_0]^{-1}A_1$. With 18 restrictions in equations (9) and (10), the 71 structural parameters, $\delta_i(i = 1, \cdots 4), l_1, l_2, f_i(i = 1, \cdots 3), \pi, A_{0,21}, \cdots A_{0,66}$, $A_{1,21} \cdots A_{1,66}$, and $\sum_U$ can be obtained from 57 estimates in A and $\sum_V$. It is immediately apparent that the rational-expectations VAR model is overidentified. Note that the precise restrictions on A_0 and A_1 vary with the lag structure in the structural VAR model. Introducing a longer lag length in the VAR model induces more complicated identifying restrictions.

4 Interpreting the evidence from RE-SVAR

In this section we present the empirical results. Using the RE-SVAR approach to achieve the results through our stochastic version of Frenkel and Rodriguez's (1982) model, we seek to answer two questions: what are the main sources of real exchange rate fluctuations since the collapse of Bretton Woods? How important is each shock? The countries under study are Canada, France, Italy, and Japan, and each country is taken to be the home country.

4.1 An alternative shifting-segmented trend model

The theoretical model (1)–(4) is not specified to consider steady-state growth for the postwar experience in above countries. To avoid such counter-fact features, the variables in equations (1)–(4) should be interpreted as deviations form their secular trends.

This paper considers an alternative trend model in which relative infrequently-occurring economic events either shift the deterministic trend or change its slope. Wu (1997) and Papell (2002) have identified that there is a breaking point near 1985 in the real exchange rate series for most industrialized countries, while Ben-David, Lumsdaine and Papell (2003, Table 2) find that the second oil price shock (1979–1980) is a breaking point for many OECD countries' output. For simplicity, we therefore specify two breaking points in the sample period of 1975:1–1997:1: The second oil price shock of 1979 and the G-5 Plaza accord of 1985. One shifting-segmented trend model for Z_t is thus

$$Z_t = \varphi_0 + \varphi_1 t + \varphi_2 DT1979 + \varphi_3 DT1985 + \varphi_4 D1979 + \varphi_5 D1985 + \tilde{z}_t. \quad (11)$$

The dummy variables in the equation are defined as follows: $DT1979 = t - 1979{:}4$ for $t > 1979{:}4$; 0, otherwise. $DT1985 = t - 1985{:}3$ for $t > 1985{:}3$; 0, otherwise. $D1979 = 1$ for $t > 1979{:}4$; 0, otherwise. $D1985 = 1$ for $t > 1985{:}3$; 0, otherwise. We then use the residuals obtained from regression (11) as the data series in the estimation.

4.2 The choice of data and estimation

Data series on exchange rates, price levels, money supply, real output, and interest rates are obtained from the IMF *International Financial Statistics (IFS)*. The domestic price index (p_t) for each of the G4 is represented by the country's implicit GDP deflator, while the nominal exchange rate (e_t) is the nominal effective exchange rate (*IFS*, line l00neu). The foreign price index p_t^* can be constructed using the equivalence:

$$p_t^* = q_t - e_t + p_t,$$

where q_t is the real effective exchange rate defined on the value-added deflator (*IFS*, line l99byl10). The reported international interest rate in the *IFS* World Table – London interbank offer rate on 3-month U.S. dollar deposits (*IFS*, line 111l60ldd) – is chosen to represent the foreign interest rate (r_t^*), while M1 and real GDP are chosen as the measures of money stock and output, respectively.

Government spending in Canada, France, and Japan are measured as the sum of real government consumption and gross domestic fixed capital formation by the public sector. Canada's and France's data series are taken from *OECD Quarterly National Accounts*, while Japan's data series is obtained from *Annual Report of National Account* published by its Economic Planning Agency. Only government expenditure series were available for Italy, which are taken from *IFS* and then deflated by Italy's price index.[5]

We take a logarithm on all the data series in Z_t except for r_t^*, and then remove the shifting and segmented trend for each variable. All data are quarterly series. The sample period begins in 1975 and ends in the first quarter of 1997.[6]

To justify the specification of the shifting-segmented trend model, we use the dummy-variables' technique and conduct the likelihood ratio test for the null hypothesis of no structural break. As displayed in the upper panel of Table 1, the likelihood ratio statistics firmly reject the null hypothesis of $\varphi_{2i} = \varphi_{3i} = \varphi_{4i} = \varphi_{5i} = 0$, with i being each variable of Z_t in (11) for Canada, France, Italy, and Japan. After removing the constant term and time trend, we use the Ljung-Box Q test to diagnose our VAR (1) assumption for the de-trended variables. According to the Q statistics in Table 1, of twenty-four regressive equations, only Japan's money supply and France's real exchange rate and foreign price reject the serially-uncorrelation hypothesis at a 5% level of significance, thus it may not be improper to set the lag length as one.[7] We then estimate the reduced-form VAR for Z_t to obtain A and $\sum_V$, and recover the parameters in A_0, A_1 and $\sum_U$ in (6) with the cross-equation restrictions and pre-determinedness assumptions imposed.

Before estimating the structural parameters with the identified restriction, notice in equations (1) and (2) that δ_1', δ_2, δ_3, and δ_4 enter the cross-equation restrictions as scalar multiples of π, and f_2 and f_3 enter as scalar multiples of $1/f_1$. These cause difficulty in the separable identification of δ_1', δ_2, δ_3, and δ_4 from π and f_2 and f_3 from f_1^{-1}. The result with $\pi = 1$ and $f_1 = 1$ is reported in the next subsection. It is obvious that if $\pi(f_1^{-1})$ was set to be 0.5, the optimal estimates of δ_1', δ_2, δ_3, and δ_4, (f_2 and f_3) would be double at the same time.

The RE-SVAR approach generates over-identifying restrictions, and thus opens the possibility for statistical evaluation of the theory. The likelihood ratio statistics in the lower panel of Table 1 indicate that the rational expectations

[5] Data series in 1991:4–1994:4 are interpolated with annual data due to a lack of data.

[6] From 1998 onwards, the IMF did not report valued-added real exchange rate statistics.

[7] Moreover, a χ^2 test for the null hypothesis that Z_t follows VAR (1) against VAR (2) cannot be rejected at a 10% level of significance in France and in Japan.

Table 1: Empirical Results (1975:1–1997:1)

	Canada	France	Italy	Japan
I. Likelihood Ratio Tests for Structural Breaks				
$\chi^2(24)$	934	1,198	1,032	1,040
	(0.00)	(0.00)	(0.00)	(0.00)
II. Q tests for the Selection of Order One				
q_t	15.63	36.28	25.76	11.80
	(0.83)	(0.03)	(0.26)	(0.96)
y_t	24.58	16.39	11.78	13.69
	(0.32)	(0.80)	(0.96)	(0.91)
m_t	27.41	23.14	15.55	39.33
	(0.20)	(0.39)	(0.84)	(0.01)
g_t	12.57	30.14	17.93	17.24
	(0.94)	(0.12)	(0.71)	(0.75)
p_t^*	22.84	73.79	25.14	17.66
	(0.41)	(0.00)	(0.29)	(0.73)
r_t^*	25.42	20.20	26.54	21.50
	(0.28)	(0.57)	(0.23)	(0.49)
III. Likelihood Ratio Tests for Over-identifying Restrictions				
χ^2 (6)	3.284	0.499	0.582	4.954
	(0.772)	(0.998)	(0.991)	(0.550)
IV. Implied Auto-regressive Coefficient (β_1) in Real Exchange Rates				
β_1	0.796	0.503	0.947	0.646

Note: The number in parentheses is the significance level of the statistics.

cross-equation restrictions and pre-determinedness assumptions imposed by the theoretical model cannot be rejected by the data even at a 10% significance level.[8] The restrictions fair quite well in Canada, France, Italy, and Japan.

Note that the real exchange rate has been found to be well characterized by an AR (1) process in recent PPP literature. The auto-regressive coefficient estimated usually implies a very long effect for transitory shocks on real exchange

[8]With the value π and f_1 being specified a priori, there are sixty-nine structural parameters to be recovered from 57 free parameters in A and $\sum_V$. However, given the additional eighteen restrictions in equations (9) and (10), there are 51 free structural parameters left to be estimated. The problem is thus over-identified and the degrees of freedom of the χ^2 distribution are six.

rates of developed countries. However, Papell (2002) and Wu (1997) investigated the time series properties of real exchange rates and confirmed that real exchange rates are stationary with possible structural breaks in regression. To justify our segmented- trend assumption, we also report the implied auto-regressive coefficient (β_1) in the reduced form of real exchange rates (equation (5)). As displayed in the last panel of Table 1, the relatively low estimates of β_1 in the four countries are consistent with a stationary real exchange rate with structural breaks.

4.3 Variance decomposition

In this subsection, we provide evidence on the relative importance of the various structural shocks in the theoretical model by performing a variance decomposition. Define the k-quarter-ahead forecast error in the real exchange rate as the difference between the actual realization of q_t and its forecast from (6) as of k quarters earlier. Table 2 presents the percentage of the variance of the real exchange rate's k-quarter-ahead forecast error that is due to each of the structural shocks, for $k = 1, 2, 4, 8, 16$, and 32 in Canada, France, Italy, and Japan.[9]

Five main results emerge from Table 2. First, the private demand shock (n_t) dominates all other structural shocks in explaining real exchange rate variations at all horizons for France and Italy. For example, at the 4-quarter horizon, the contributions of n_t are 59.4 percent and 94.1 percent of the variance of the real exchange rate in France and Italy, respectively. Even though the relative importance of the real private demand shock declines slowly over time, it accounts for more than 56 percent of the variance of the real exchange rate for the two countries in the long run. On the other hand, a private demand shock is the most import shock at the 1-quarter horizon for Japan, contributing 64.6 percent of variance of real exchange rates in Japan. Our results provide support that demand shocks are very important in explaining real exchange rate behavior, and are consistent with the findings for France and Italy in Lastrapes (1992).

The second result is that a supply shock (u_{2t}) is not important in explaining the short-run and long-run behavior of real exchange rates for Canada, Italy, and Japan. Clarida and Gali (1994) also found that none of the real exchange volatility in different forecasting horizons could be attributed to supply shocks in Canada and Japan, and Zhou (1995) verified that the effect of the productivity differential is relatively minor in the long run. Nonetheless, for France, we find a small but significant part of the variance in the medium and long run is attributed to supply shocks.

[9] The statistics in Table 2 are the results of 500 replications of the Monte Carlo simulation.

Table 2: Variance Decomposition of Real Exchange Rate

	n_t	u_{2t}	u_{3t}	u_{4t}	u_{5t}	u_{6t}
			Canada			
1quarter ahead	1.25	0.84	89.79	0.90	6.99	0.23
	(9.51)	(5.23)	(11.98)	(2.20)	(1.73)	(3.70)
2quarter ahead	2.42	0.54	89.10	0.52	7.33	0.09
	(11.93)	(3.67)	(12.65)	(0.52)	(1.87)	(1.49)
4quarter ahead	4.67	0.31	86.13	0.35	8.49	0.05
	(12.77)	(1.97)	(12.66)	(0.21)	(2.50)	(0.73)
8quarter ahead	11.04	0.45	76.54	0.94	10.96	0.07
	(12.61)	(0.92)	(12.48)	(0.73)	(3.66)	(0.79)
16quarter ahead	21.09	0.95	62.97	3.17	11.74	0.09
	(11.44)	(1.11)	(11.07)	(1.72)	(3.70)	(1.10)
32quarter ahead	21.42	1.01	62.49	3.67	11.33	0.09
	(11.31)	(1.16)	(10.71)	(1.81)	(3.44)	(1.02)
			France			
1quarter ahead	65.82	1.93	3.24	0.91	28.04	0.05
	(12.23)	(1.99)	(5.51)	(1.24)	(10.79)	(0.38)
2quarter ahead	62.72	2.83	4.84	0.89	28.64	0.07
	(10.77)	(2.39)	(4.20)	(1.21)	(9.91)	(0.43)
4quarter ahead	59.41	4.72	7.50	0.80	27.48	0.08
	(9.73)	(2.83)	(3.79)	(1.07)	(8.98)	(0.47)
8quarter ahead	57.47	6.99	7.72	1.21	26.53	0.08
	(9.47)	(2.98)	(3.71)	(1.04)	(8.57)	(0.47)
16quarter ahead	56.43	7.52	8.15	1.73	26.10	0.08
	(9.32)	(2.89)	(3.60)	(1.17)	(8.36)	(0.46)
32quarter ahead	56.38	7.51	8.20	1.75	26.08	0.08
	(9.30)	(2.88)	(3.59)	(1.18)	(8.34)	(0.46)
			Italy			
1quarter ahead	99.10	0.01	0.54	0.00	0.35	0.00
	(0.99)	(0.05)	(0.88)	(0.02)	(0.26)	(0.00)
2quarter ahead	96.03	0.01	3.70	0.00	0.26	0.00
	(1.25)	(0.11)	(1.17)	(0.01)	(0.25)	(0.00)
4quarter ahead	94.10	0.03	4.68	0.00	1.19	0.00
	(1.52)	(0.27)	(1.28)	(0.02)	(0.47)	(0.00)
8quarter ahead	88.25	0.04	4.83	0.00	6.88	0.00
	(2.58)	(0.52)	(1.14)	(0.02)	(1.74)	(0.00)
16quarter ahead	83.26	0.05	4.56	0.00	12.13	0.00
	(3.33)	(0.56)	(0.98)	(0.02)	(2.63)	(0.00)
32quarter ahead	83.16	0.05	4.55	0.00	12.24	0.00
	(3.34)	(0.55)	(0.98)	(0.02)	(2.63)	(0.00)

(Continued)

	n_t	u_{2t}	u_{3t}	u_{4t}	u_{5t}	u_{6t}
			Japan			
1quarter ahead	64.58	0.04	1.55	0.32	33.51	0.00
	(15.20)	(0.19)	(7.03)	(0.19)	(12.00)	(0.00)
2quarter ahead	25.10	0.04	0.89	6.19	67.74	0.04
	(14.40)	(0.20)	(4.19)	(2.66)	(16.21)	(0.54)
4quarter ahead	14.56	0.02	0.45	6.94	77.80	0.22
	(12.95)	(0.11)	(2.34)	(5.87)	(18.50)	(2.58)
8quarter ahead	14.01	0.02	0.29	5.07	80.22	0.39
	(13.77)	(0.08)	(1.61)	(5.10)	(18.95)	(3.93)
16quarter ahead	14.14	0.04	0.28	5.01	80.14	0.40
	(13.78)	(0.22)	(1.52)	(4.95)	(18.88)	(4.01)
32quarter ahead	14.29	0.04	0.27	4.96	80.03	0.40
	(13.81)	(0.24)	(1.50)	(4.89)	(18.86)	(4.01)

Note: n_t: private expenditure shock; u_{2t}: domestic supply shock; u_{3t}: monetary shock; u_{4t}: government spending shock; u_{5t}: foreign price shock; u_{6t}: foreign interest rate shock. The numbers in parentheses are the standard deviations.

The third result is that a monetary shock (u_{3t}) is a major source of real exchange rate variations for Canada. The finding of a significant but less than 10 percent level of the variance of real exchange rates in the long run for France and Italy is consistent with the result that a transitory shock has a small but important explanatory power in Evans and Lothian (1993). However, for Japan our finding that a monetary shock is unimportant is inconsistent with Clarida and Gali (1994).

The fourth result is that the real government spending shock (u_{4t}) is not important in explaining the short-run and long-run behavior of real exchange rates for Canada, France, Japan, and Italy. Our findings are in contrast to those in Froot and Rogoff (1991). Froot and Rogoff verified by regression results that the cross-country difference in government consumption accounts for long-run real exchange rate movements.

The fifth and last result is that in the two foreign shocks coming from outside (u_{5t} and u_{6t}), the price shock (u_{5t}) plays a larger role in explaining real exchange rate variations for Canada, France, Italy, and Japan, while the interest rate shock does not. The foreign price shock explains about 25% of the variations in the real exchange rate for France and 80% of the variations for Japan in the long run. Our empirical results are consistent with Zhou (1995) who found foreign and domestic oil prices to be most important for the variance in Japanese real exchange rates. For Canada and Italy, the foreign price shock also plays a relatively small but non-negligible role in the long run (about 10% of the variance of the real exchange rates).

In summary, the empirical results herein about the low explanatory power of supply shocks on real exchange rates confirm the previous findings from a long-run restriction approach. Our findings about the non-importance of government spending shocks on real exchange rates are in contrast to Froot and Rogoff (1991)'s regression results that the cross-country difference in government consumption accounts for long-run real exchange rate movements. However, since this paper is based on a structural model of rational expectations, we can identify the relative importance of private spending shocks, government spending shocks, monetary shocks, and foreign price shocks on the variance decomposition of real exchange rates. Of them, foreign price shocks are non-negligible for the G4 that we consider.

4.4 Impulse response functions

Having assessed the relative contribution of structural shocks to fluctuations in the real exchange rate, the next step is to show the dynamic effects of structural shocks for the two countries. The focus is on interpreting the dynamic effects of major shocks on real exchange rate fluctuations in Canada, France, Italy, and Japan. Namely, they include the private demand shocks, monetary shocks, and foreign price shocks.

According to the impulse responses with respect to n_t for France and Italy in Figures 2 and 3, we find that a private demand shock has very important explanatory power and it causes an immediate real appreciation. Theoretically, a positive demand shock induces a higher domestic price level and the real exchange rate appreciates. The more flexible the price is, the bigger the real exchange rate changes. As an increase in the domestic price level creates an excess demand for money, the money market is cleared by a higher domestic interest rate. When the domestic interest rate is higher than the world interest rate, then capital flows in. The initial real appreciation together with the improvement in the capital account clears the foreign exchange market. Ceteris paribus, after the immediate appreciation the real exchange rate begins to depreciate along with the declining domestic price level.

As displayed in Figure 1, a positive domestic monetary shock (u_{3t}) causes a real appreciation in Canada. Theoretically, when there is an excess supply of

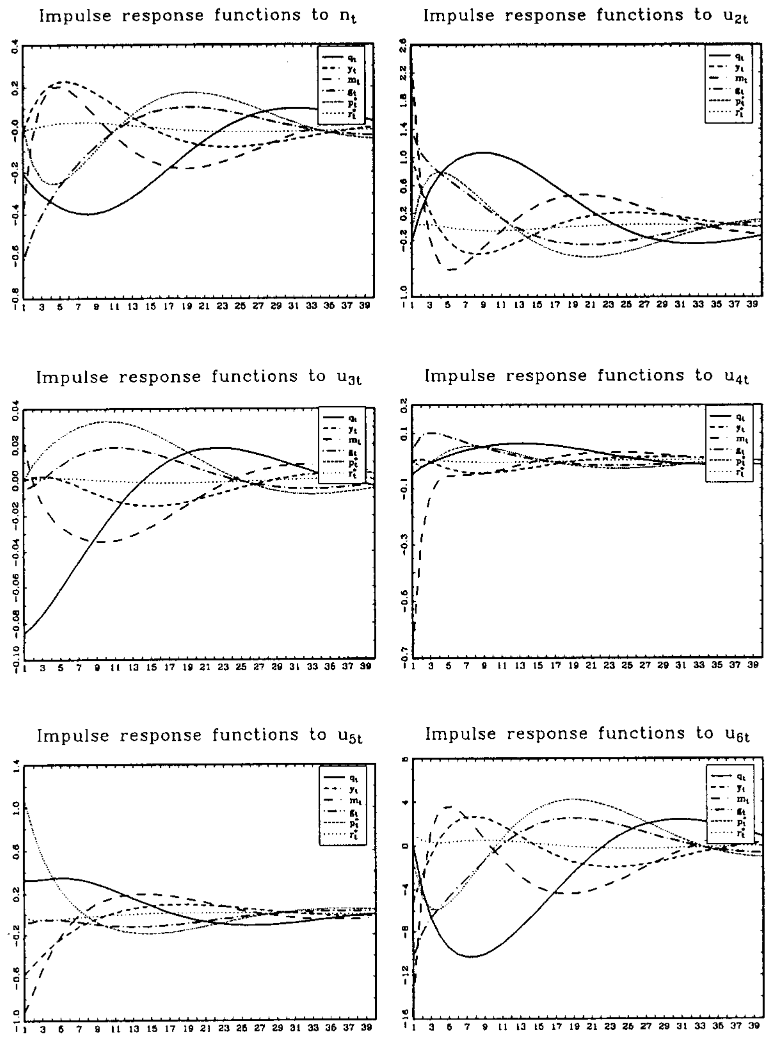

Figure 1: Impulse Responses in Canada

money induced by the money supply shock, a decrease in the domestic interest rate and an increase in the price level are needed to clear the money market. Since a higher domestic price worsens the current account and a lower domestic interest rate worsens the capital account, the current nominal exchange rate must depreciate to balance the foreign exchange market. That is, the effect of a higher domestic price must be dominated by the offsetting effect of a higher nominal exchange rate. Thus, the real exchange rate should depreciate after the money supply shock happens.

Canada's real appreciation seems to counter the above implication at first glance as in Figure 1. However, monetary shocks cause Canada's money supply to increase only one quarter, and then the supply shrinks and stays below its steady state value for almost 6 years. The monetary shock here causes the market to expect the Canadian dollar to appreciate. An appreciation expectation incurs capital inflow and improves the capital account. A real appreciation and a worsening current account balance is needed to clear the foreign exchange market.

According to the impulse responses with respect to a foreign price shock (u_{5t}) in Figures 1, 2, and 4, an unanticipated increase in the foreign price causes an immediate real depreciation for Canada, France, and Japan. After that, the real exchange rate keeps stable in Canada, while it continues to increase in France and Japan. The reaction of the home government - monetary policy mainly - determines the pattern of impulse response of real exchange rates. As described previously, an expansionary monetary policy lowers the domestic interest rate and a real depreciation is thus needed to clear the foreign exchange market. That is, ceteris paribus, a temporary increase in money supply causes the real exchange rate to depreciate immediately and to appreciate to its long-run level then. French and Japanese authorities seem to have accommodated the higher foreign price by increasing their own money supplies. The real exchange rates therefore depreciated further for France and Japan. On the contrary, Canada's monetary authority appears to have countered the inflation pressure and to have contracted its money supply. A tight monetary policy causes the domestic interest rate to increase and postpones the timing for the real exchange rate to decline to its long-run level. There is a different story for Italy, as displayed in Figure 3. Italy's government reacted to the foreign shock immediately by an expansionary fiscal policy and then a tight monetary policy. The policy mix delayed the timing for the real depreciation.

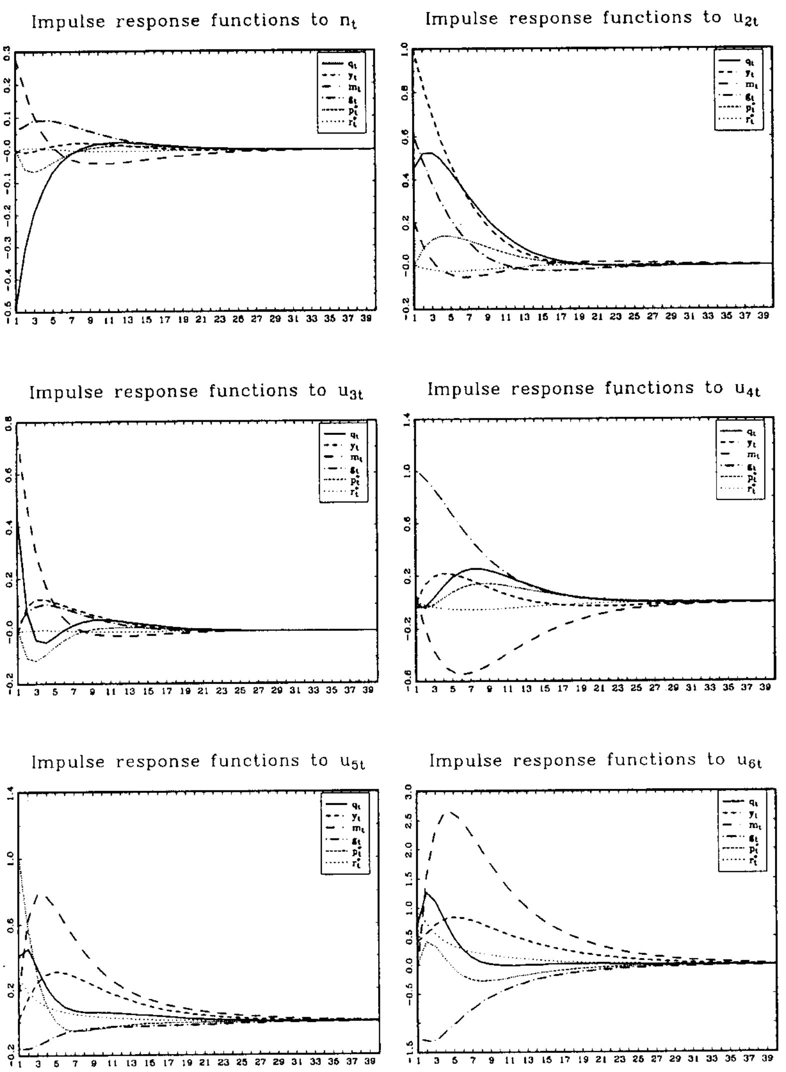

Figure 2: Impulse Responses in France

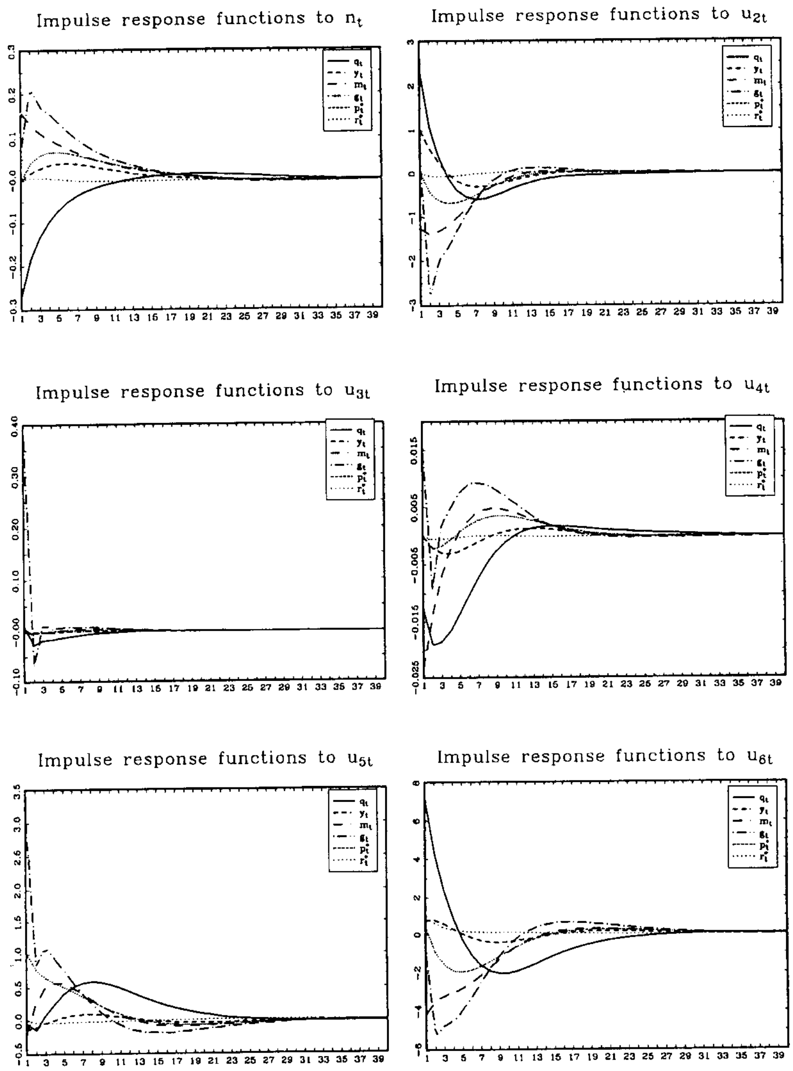

Figure 3: Impulse Responses in Italy

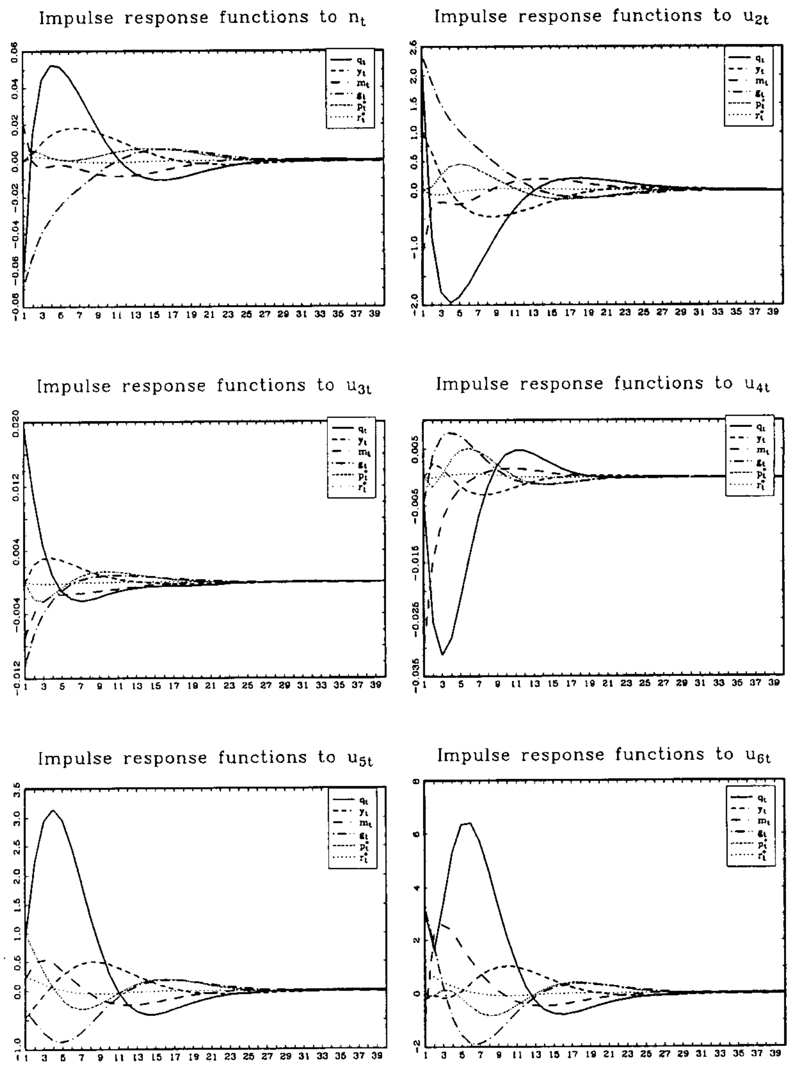

Figure 4: Impulse Responses in Japan

5 Conclusions

This paper proposes a rational expectations structural VAR (RE-SVAR) method to decompose the variance of real exchange rates by a variety of structural shocks in a rational expectations model. We identify the structural parameters with the rational-expectations restrictions implied by the theoretical dynamics of real exchange rates. When applying our method to Canada, France, Italy, and Japan, we find that the statistical performance of the theoretical model fairs quite well and the impulse responses of the real exchange rate with respect to structural shocks are consistent with the prediction of the model.

Our empirical results also indicate that private expenditure shocks are the most important source of real exchange rate variations for France and Italy, monetary shocks are most important for Canada, and foreign price shocks are most important in the long run for Japan. A government spending shock is not significant for the four countries' real exchange rate fluctuations. Finally, supply shocks have a small but significant explaining power in the real rate variation only for France.

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從理性預期結構性 VAR 模型分析實質匯率波動之來源

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為了探討理性預期下小型開放經濟實質匯率波動的來源，本文提出「理性預期結構性 VAR 模型」的分析方式。一個外生干擾除了對實質匯率有著理論上的直接影響，它也可能會對預測自己和其他的外生變數有幫助，因此產生各種間接的影響，而「理性預期結構性 VAR 模型」的分析方式可以將這些直接與間接效果一起限制於 VAR 模型的係數之上。經過最大概似法的估計與檢定，加拿大、法國、義大利與日本均無法拒絕本文之理論模型。實證結果顯示：國內私人部門的自發性需求是法國與義大利最重要的實質匯率波動來源，貨幣衝擊是加拿大實質匯率波動的主因，而來自國外的衝擊則是日本長期實質匯率波動的最重要原因。同時政府支出衝擊並不具有顯著之說明能力；生產力的衝擊只有在法國存在微小但顯著的解釋力。

關鍵詞: 實質匯率, 理性預期, 結構性 VAR 模型
JEL 分類代號: C51, C52, F3