Solving the real business cycles model of small-open economies by a sample-independent approach

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Abstract

One hallmark of small-open economy models with a time-separable preference assumption is the non-uniqueness of their steady states. Following King et al. (1988), most studies compute a log-linear approximation solution to their small-open economies around the sample means of the corresponding variables. The resulting reliance of the outcome on a particular sample may lead to different implications about the business cycles properties of a small-open economy. This paper proposes a sample-independent approach to solving this kind of model and shows its superiority over a sample-dependent method through some simulation results.

Keywords: Real business cycles; Small-open economy; Log-linear approximation; Steady state

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1. Introduction

Studies on real business cycles in a closed economy endeavor to replicate the main features of the economy's macro time series. Based on the existence of a unique steady state, most studies adopt a log-linear approximation method proposed in King et al. (1988) to characterize a stochastic competitive equilibrium. However, works on real business cycles in small-open economies are relatively scarce, with one fundamental reason being attributable to the non-uniqueness feature of the steady state implied by most small-open economy models. Facing an exogenous world interest rate, consumption of a small-open economy will be permanently changed by a temporary shock, and hence its steady state is consistent with any level of net foreign asset holdings [cf. Mendoza (1991), Correia et al. (1995), and Turnovsky (1997, p. 55)]. This unpleasant feature of small-open economy models makes applying the approximation method of King et al. (1988) more difficult when one aims to analyze the business cycles of a small-open economy. This paper intends to develop a new approach which does not rely on a unique steady state when applying the log-linear approximation method and which can generate business cycles statistics at the same standpoint as those obtained in closed economies.

Efforts made along this line to resolve the non-stationary implication of small-open economy models include imposing an assumption of non-separable preferences or a finite horizon. For example, Mendoza (1991) endogenizes the subjective time preference rate, while Cardia (1991) and Feve and Langot (1996) consider a probability of death. On the other hand, some studies simply take the non-unique steady-state balance of trade or net holdings of foreign assets as a deep parameter to conduct a log-linear approximation. Correia et al. (1995) and Harjes (1997) further assume such a deep "parameter" to coincide with its sample mean in performing a numerical simulation. However, such an additional assumption is not only inconsistent with the non-stationary implication of the models, but also in contrast with the time-series characteristics. Furthermore, to assume that a time-varying steady-state variable is constant and is equal to its sample mean makes the simulation results of a small-open economy sample-dependent and is likely to result in incorrect evaluations on the business cycles properties of the economy.

This paper proposes a sample-independent approach to solving the real business cycles model of small-open economies. We log-linearize the model around its conditional expectations of a steady state. The conditional expected steady state is consistent with the model's long-run properties, and can be solved recursively under the rational expectations assumption with the help of the model's intertemporal budget constraint. Aside from the certainty equivalence that many studies in this field assume to hold, our method does not make any additional assumption. Employing our method to analyze real business cycles of a small-open economy makes the application of King et al. (1988)'s method straightforward and not sample-dependent. Thus, our method can generate the economy's genuine data. Moreover, the calibration results of a small-open economy model by using our method are expected to gain a higher degree of accuracy than those obtained by other methods that are sample-dependence in nature.

The remainder of the paper is organized as follows. Section 2 studies a simple small-open endowment economy and its steady state, whereby we describe a non-uniqueness feature of the steady state. We then present some empirical evidence to confirm the non-stationary feature of the model. Section 3 introduces a sample-independent approach to our simple model and describes its solving procedure. Section 4 extends the simple model to including capital in production and conducts simulation experiments to support our method's superiority over a sample-dependent method by comparing impulse response results of these two methods. A sensitivity analysis with different sample means also shows the weakness of the statistics obtained from the sample-dependent method. Section 5 concludes.

2. A simple small-open endowment economy and its "steady state"

Consider a representative agent who owns a flow of stochastic endowment in a simple small-open economy without production. The agent can consume, borrow and lend in the world credit market, and at any time *s*, seeks to maximize the following expected lifetime utility:

$$E_s \sum_{t=s}^{\infty} \beta^{t-s} \frac{C_t^{1-\gamma}-1}{1-\gamma}, \;\; \gamma > 0, \;\; 0 < \beta < 1,$$

where E_s is an expectation operator conditioned on the information available at time *s*, C_t denotes consumption at time *t*, β is the constant discount factor, and γ is the coefficient of relative risk aversion. For $\gamma = 1$, the instantaneous utility function $(C_t^{1-\gamma}-1)/(1-\gamma)$ is replaced by its limit, $\ln(C_t)$.

The agent's endowment at time *t*, Y_t, can be decomposed by two parts and written as $Y_t = A_t W_t$, where A_t denotes a stationary endowment shock and W_t represents the growing component of endowment at time *t*. Assume that W_t grows at a constant rate λ as:

$$W_t = W_0 e^{\lambda t},$$

and that the logarithm of A_t follows a stationary AR(1) process as:

$$\ln A_t - a = \rho_a (\ln A_{t-1} - a) + \varepsilon_t^a, \quad |\rho_a| < 1,$$

where $a = \ln A$, and $\{\varepsilon_t^a\}$ is an i.i.d. $N(0, \sigma_a^2)$ sequence.

Furthermore, the agent faces an exogenous real rate of interest r_t between *t* and $t+1$ in the world credit market. Let B_t denote the net foreign assets at the beginning of period *t*. The evolution of foreign assets at time *t*, and thus the agent's time *t* temporal budget constraint can be expressed as:

$$B_{t+1} = e^{r_{t-1}} B_t + A_t W_t - C_t.$$

In addition, the world interest rate follows a stationary AR(1) process:

$$r_{t-1} - r = \rho_r (r_{t-2} - r) + \varepsilon_t^r, \quad |\rho_r| < 1,$$

where $\{\varepsilon_t^r\}$ is an i.i.d. $N(0, \sigma_r^2)$ sequence.

The decision problem of the representative agent in such a simple economy is to choose a time path of consumption and foreign assets holding that maximizes the expected lifetime utility, subject to the whole sequence of temporal budget constraints from time s to infinity.

According to the growing aspect of endowment level W_t, the model's other variables such as consumption and foreign assets will grow around a time trend. A standard method to analyze closed-economy models with steady-state growth divides all growing variables in such models by their growth component, and hence transforms the economy into a stationary one. The same transformation method can be applied to small-open economies like ours to remove the time trend of the growing variables. However, as we will point out later, this method cannot make these economies stationary.

For notational convenience, we define $\widetilde{V}_t = V_t / W_t$ for any growing variable V_t in our economy. The Euler equation for the agent's optimization problem after transformation is simply the following:

$$E_s\left[\left(\frac{\widetilde{C}_t}{\widetilde{C}_{t+1}} e^{-\lambda}\right)^{\gamma} \beta e^{r_t}\right] = 1 . \tag{1}$$

And the transformed time t temporal budget constraint now becomes:

$$\widetilde{B}_{t+1} = [e^{r_{t-1}} \widetilde{B}_t + \widetilde{Y}_t - \widetilde{C}_t] e^{-\lambda} . \tag{2}$$

Combining the sequence of temporal budget constraints with the no-Ponzi-game condition leads to an intertemporal budget constraint as:

$$\widetilde{B}_s = -e^{-r_{s-1}}(\widetilde{Y}_s - \widetilde{C}_s) - e^{-r_{s-1}} E_s \sum_{t=s+1}^{\infty} e^{\Sigma_{j=0}^{t-s-1}(\lambda - r_{s+j})} (\widetilde{Y}_t - \widetilde{C}_t)\,.$$

Here, the initial value of the foreign assets, $\widetilde{B}_s$, determines the sum of the current trade balance ($\widetilde{Y}_s - \widetilde{C}_s$) and all its discounted future values for this economy.

The first step in studying this model's properties is to abstract from the presence of stochastic shocks and describe its deterministic steady state. Along with characterizing the economy's long-run features, a non-uniqueness feature of the steady state will be derived and this gives us sufficient motive to take the model's non-stationarity property more seriously.

From the Euler equation (1) and the budget constraint (2), the long-run restriction on consumption and foreign assets of the economy implies the following equation is satisfied at the steady state:[1]

$$\widetilde{B}(e^{\lambda} - e^{r}) = A - \widetilde{C}\,. \tag{3}$$

Given the long-run values of endowment shocks and the world interest rate, A and r, the economy's steady state is denoted by a couple of ($\widetilde{C}$, $\widetilde{B}$). Note that there are two endogenous variables, $\widetilde{C}$ and $\widetilde{B}$, constrained by one equation (3). Clearly,

[1] We assume the magnitudes of the parameters guarantee holding of the long-run restriction on equation (1), $\lambda\gamma + 1/\beta = 1 + r$. Otherwise, if the differential of the steady-state world interest rate and the rate of time preferences is greater (less) than $\lambda\gamma$, then the foreign assets of the small-open economy will grow (decline) faster than the endowment growth forever, and thus the economy is no longer small.

employing the single long-run restriction is not sufficient to obtain the values of both the steady-state variables. From equation (3), for any value of $\widetilde{B}$ there is a steady-state consumption $\widetilde{C}$ consistent with it: economies with higher net foreign assets will have higher steady-state trade deficits and will sustain higher steady-state levels of consumption. This non-uniqueness feature of the steady state inherited from our model is well documented in the literature of the real business cycles model of small-open economies, although it may be interpreted somewhat differently. For instance, Eaton (1989, p. 1339) agrees that there is a continuum of steady states with the choice among them determined by initial conditions. Correia et al. (1995) claim that this is a steady state to which the economy does not return. Sen (1994, p. 510) argues that there is no steady state at all in this class of models.

The non-uniqueness feature of the steady state implies that the steady-state values of the transformed variables around which we will compute an approximate solution to the stochastic competitive equilibrium are not well defined. However, this non-stationarity property of the theoretical model, in particular, the non-stationarity of transformed consumption, trade balance, and foreign assets, would not cause much trouble in analyzing business cycles if these variables indeed move sufficiently stable. In this case, their sample means can be taken as parameters as a first approximation. Nevertheless, the non-stationary hypothesis of such variables is hardly rejected by evidence collected from a sample of 16 economies shown in Figure 1 and Figure 2.

Figure 1 shows the trade balance-to-GDP ratio and Figure 2 shows the consumption-to-GDP ratio for 16 countries (Australia, Canada, France, Germany, Hong

Kong, Indonesia, Italy, Japan, South Korea, Malaysia, the Netherlands, Singapore, Taiwan, Thailand, the U.K. and the U.S.) with nearly 50 annual observations.[2] From Figure 1, the series of trade balance-to-GDP ratio over time are divergent for all 16 countries. Even so, if each series is stationary with little variance, its sample mean can be approximated to a "parameter". However, based on results of the conventional ADF test with an optimal lag order, the unit-root hypothesis is rejected at a 5% level of significance for only three of the 16 countries (Canada, France and the U.K.).[3] Figure 2 reveals the non-stationary feature of the consumption-to-GDP ratio. Again, only three of the 16 countries (Germany, Hong Kong and the Netherlands) have the unit-root hypothesis rejected by the ADF test.

Evidence laid out in Figure 1 and Figure 2 confirms the conjecture that taking the long-run (steady-state) transformed level of either consumption or trade balance (and thus foreign assets) as a parameter is counterfactual. An approximate solution to the stochastic competitive equilibrium based on using such a "parameter" naturally expects a lower degree of accuracy.[4] To overcome the dilemma of using such a "parameter" around which we compute a stochastic competitive equilibrium and thus improve accuracy of numerical simulations, this paper develops a new approach for solving the

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[2] Except for Taiwan, the data are obtained from the *International Financial Statistic* (IFS) tape. Data of Taiwan are obtained from *Financial Statistics, Taiwan District, R.O.C.* The different duration of time periods chosen reflects availability of different data sets.

[3] For the optimal choice of lag order, refer to Campbell and Perrson (1988).

[4] Correia et al. (1995) and Rebelo and Végh (1995) recognize this problem, but they do not provide a clue for it.

real business cycles model of small-open economies. We note that, given the information set at time *s*, both the expected steady-state consumption and foreign assets are conditioned on the initial value of $\widetilde{B}_s$. In fact, under the assumption of rational expectations, the *conditional expected steady state* (CESS) of a small-open economy is unique at any point of time. The important departure of our method from those of others is that we can compute an approximate solution to the stochastic competitive equilibrium by log-linearizing the Euler equation around the CESS. We then solve the temporal endogenous variables and their CESS sequentially with the help of the intertemporal budget constraint. A description of this method, the CESS method, will be discussed in the next section.

3. The CESS method

The basic idea of the CESS method is as follows. Given the fact that the expected steady state of a small-open economy depends on its initial conditions, we can thus obtain a unique solution of the CESS in every period by using the long-run restriction and the intertemporal budget constraint of the model. For our economy, this method implies that for any time *s*, given the initial value of foreign assets $\widetilde{B}_s$, we can solve the CESS expressed by consumption $\widetilde{C}(\widetilde{B}_s)$ and $\widetilde{B}(\widetilde{B}_s)$, together with the contemporary endogenous variables $\widetilde{C}_s$ and $\widetilde{B}_{s+1}$ simultaneously.

The procedure of the CESS method is described in the following two stages.[5] The first stage derives a contingency rule that specifies how consumption evolves for every

[5] The detailed mathematical derivation of the CESS method is available upon request.

possible configuration of the world interest rate and, in particular, for the corresponding CESS. Under the assumption of certainty equivalence, we log-linearize the Euler equation (1) around the CESS to get:

$$E_s c_{t+1} - c = (E_s c_t - c) + \frac{E_s r_t - r}{\gamma}, \quad t \geq s, \tag{4}$$

where $c_t = \ln \widetilde{C}_t$ and $c = \ln \widetilde{C}$.

The decision rule of consumption at time *t* can be obtained through summing up equation (4) from time *t* to infinity, incorporating with the least-square projection of r_t, as well as the definition of the CESS consumption, $\lim_{\tau \to \infty} E_s(c_\tau) = c$:

$$E_s c_t - c = -\frac{\rho_r}{\gamma(1-\rho_r)} E_s(r_{t-1} - r). \tag{5}$$

Note that given the information available at time *s*, the value of the CESS consumption *c* is uniquely determined.

The second stage incorporates the derived contingent consumption from equation (5) into the (log-) linearized intertemporal budget constraint. Together with the steady-state restriction, we solve the endogenous CESS variables (*c* and $\widetilde{B}$) at time *s* as functions of the lagged state variables and driving variables which are all realized in the beginning of period *s*. That is, the two endogenous variables are solved with two equality constraints, equation (3) and

$$\widetilde{B}_s = \widetilde{B} - \left[\frac{e^{c-r}\rho_r}{\gamma(1-\rho_r)(1-\rho_r e^{\lambda-r})} + \frac{\widetilde{B}}{1-\rho_r e^{\lambda-r}}\right](r_{s-1} - r) - \frac{Ae^{-r}}{1-\rho_a e^{\lambda-r}}(a_s - a),$$

where $a_s = \ln A_s$.

When the values of c and $\widetilde{B}$ at time s are obtained, substituting them back into equation (5) and the budget constraint (2) gives the equilibrium consumption at time s, c_s, and the equilibrium foreign assets at the end of period s, $\widetilde{B}_{s+1}$. They are both non-linear functions of the lagged state variables and driving variables. Finally, we repeat the procedure in the second stage to solve the sequences of consumption and foreign assets recursively, and complete the CESS method.

Although the CESS method may seem to be more complicated at first sight than the sample-dependent method (by which we mean some sort of sample means is added to resolve the problem of non-uniqueness of the steady state), most of the recursive work involves numerical computation and is left to a computer. One main advantage of the CESS method over the sample-dependent method is that results obtained by using the CESS method are sample-independent while those using the sample-dependent method are sensitive to the sample chosen. This advantage is notable since, in general, the population means of consumption, trade balance and foreign assets are not well-defined due to the non-stationarity character of the models of small-open economies. Studies based on the sample-dependent method again naturally expect a lower degree of accuracy.

In order to demonstrate the superiority of the CESS method over the sample-dependent method, we perform simulation experiments for an extension of our simple model in the following section. The extension includes capital stock as input in production and therefore transforms our endowment economy into a production economy.

4. A production economy with capital and its simulation experiments

Our simple endowment model in Section 2 is extended to have capital stock in production and the production technology is assumed to exhibit a constant returns to scale:

$$Y_t = A_t K_t^{\alpha} X_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where Y_t now is the final product at time *t*, A_t is interpreted as a stationary productivity shock, K_t is the physical capital at the beginning of period *t*, and X_t represents the technology level at time *t*. For simplicity, the agent's labor service is assumed to be inelastically supplied and is normalized to unity. Assume that X_t grows at the same constant rate λ as W_t in the endowment economy:

$$X_t = X_0 e^{\lambda t}.$$

To mimic the gradual installing process of capital accumulation in the real world and to ensure non-trivial dynamics, we assume capital accumulation involves adjustment costs. Thus, the investment expenditure (Z_t) becomes:

$$Z_t = [K_{t+1} - (1-\delta)K_t]\left[1 + \frac{\phi}{2}\frac{K_{t+1} - (1-\delta)K_t}{K_t}\right], \ 0 < \delta < 1, \ \phi > 0,$$

where ϕ represents the costs of adjusting capital stock, and δ is the rate of depreciation.

After taking into account production technology constraints, the evolution of foreign assets at time *t* that constitutes the agent's time *t* temporal budget constraint now becomes:

$$B_{t+1} = e^{r_{t-1}} B_t + A_t K_t^{\alpha} X_t^{1-\alpha} - [K_{t+1} - (1-\delta)K_t]\left[1 + \frac{\phi}{2}\frac{K_{t+1} - (1-\delta)K_t}{K_t}\right] - C_t .$$

Aside from choosing a time path of consumption and foreign assets, the representative agent in this production economy chooses a time path of capital stock that maximizes the expected lifetime utility, subject to the whole sequence of temporal budget constraints from time *s* to infinity.

The Euler equations for the agent's optimization problem after dividing all growing variables by the growth component X_t are as follows:[6]

$$E_s\left[\left(\frac{\widetilde{C}_t}{\widetilde{C}_{t+1}} e^{-\lambda}\right)^{\gamma} \beta e^{r_t}\right] = 1, \tag{6}$$

$$E_s\left\{e^{r_t} - \frac{\alpha A_{t+1}\widetilde{K}_{t+1}^{\alpha-1} + (1-\delta)[1 + \phi\frac{\widetilde{K}_{t+2}e^{\lambda} - (1-\delta)\widetilde{K}_{t+1}}{\widetilde{K}_{t+1}}] + \frac{\phi}{2}[\frac{\widetilde{K}_{t+2}e^{\lambda} - (1-\delta)\widetilde{K}_{t+1}}{\widetilde{K}_{t+1}}]^2}{1 + \phi\frac{\widetilde{K}_{t+1}e^{\lambda} - (1-\delta)\widetilde{K}_t}{\widetilde{K}_t}}\right\} = 0. \tag{7}$$

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[6] To save the use of notations, the same notation $\widetilde{V}_t$ is redefined as $\widetilde{V}_t = V_t / X_t$, for any growing variable V_t in this section.

Not surprisingly, equation (6) shares the same form as equation (1) in Section 2, although the definitions of transformed consumption in these two equations are somewhat different. The transformed time t temporal budget constraint is revised accordingly:

$$\widetilde{B}_{t+1} = [e^{r_{t-1}}\widetilde{B}_t + A_t\widetilde{K}_t^{\alpha} - \widetilde{Z}_t - \widetilde{C}_t]e^{-\lambda}, \tag{8}$$

where $\widetilde{Z}_t = \left[\widetilde{K}_{t+1}e^{\lambda} - (1-\delta)\widetilde{K}_t\right]\left[1 + \frac{\phi}{2}\left(\widetilde{K}_{t+1}e^{\lambda} / \widetilde{K}_t - (1-\delta)\right)\right]$. Similarly, the intertemporal budget constraint is:

$$\widetilde{B}_s = -e^{-r_{s-1}}(\widetilde{Y}_s - \widetilde{Z}_s - \widetilde{C}_s) - e^{-r_{s-1}}E_s\sum_{t=s+1}^{\infty} e^{\sum_{j=0}^{t-s-1}(\lambda - r_{s+j})}(\widetilde{Y}_t - \widetilde{Z}_t - \widetilde{C}_t).$$

From the Euler equations (6) and (7) and the budget constraint (8), the long-run restrictions on consumption, capital stock, and foreign assets of the economy imply the following two equations are satisfied at the steady state:

$$e^r = \frac{\alpha A\widetilde{K}^{\alpha-1} + \frac{\phi}{2}(e^{\lambda} - 1 + \delta)^2}{1 + \phi(e^{\lambda} - 1 + \delta)} + (1-\delta), \tag{9}$$

and

$$\widetilde{B}(e^{\lambda} - e^r) = A\widetilde{K}^{\alpha} - \widetilde{K}(e^{\lambda} - 1 + \delta)\left[1 + \frac{\phi}{2}(e^{\lambda} - 1 + \delta)\right] - \widetilde{C}. \tag{10}$$

Again, there are three endogenous variables, $\widetilde{C}$, $\widetilde{K}$, and $\widetilde{B}$, constrained by two equations (9) and (10).[7] The non-uniqueness feature of the steady state that we discussed in Section 2 sustains in this production economy. In contrast to a conventional sample-dependent method, the application of the CESS method to solving for an equilibrium of this model requires no particular sample information.

To apply the CESS method to this production model,[8] firstly, for any time *s*, we log-linearize the Euler equations (6) and (7) around the CESS and get a pair of contingency rules of consumption and capital stock illustrated by equation (4) and

$$E_s r_t - r = \frac{1}{1+\phi(e^{\lambda}-1+\delta)} E_s[e^{-r}\alpha(\alpha-1)A\widetilde{K}^{\alpha-1}(k_{t+1}-k) + e^{-r}\alpha A\widetilde{K}^{\alpha-1}(a_{t+1}-a)$$

$$+\phi e^{2\lambda-r}(k_{t+2}-k_{t+1}) - \phi e^{\lambda}(k_{t+1}-k_t)], \quad t \geq s, \qquad (11)$$

where $k_t = \ln \widetilde{K}_t$ and $k = \ln \widetilde{K}$.

Equation (11) describes the equilibrium law of motion for the capital stock. The characteristic equation associated with equation (11) implies two eigenvalues. By the intermediate value theorem and from the sum of these two eigenvalues, there are one root lying between zero and one and another root lying outside the unit circle. Let θ_1 be the

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[7] In fact, $\widetilde{K}$ can be solved alone by equation (9), but there arises an infinite combination of $\widetilde{C}$ and $\widetilde{B}$ subject to one equation, (10).

[8] The detailed mathematical derivation of applying the CESS method to this version of the model is available upon request.

stable eigenvalue and θ_2 be the unstable one. The convergent path for the capital stock, incorporating with the least-square projections of r_t and a_t, is thus given by:

$$E_s\left[k_{t+1} - k - \theta_1(k_t - k)\right] = \frac{\rho_a e^{-r}\alpha A\widetilde{K}^{\alpha-1}}{(\theta_2 - \rho_a)\phi e^{2\lambda-r}} E_s(a_t - a) - \rho_r \frac{1+\phi(e^{\lambda} - 1 + \delta)}{(\theta_2 - \rho_r)\phi e^{2\lambda-r}} E_s(r_{t-1} - r). \tag{12}$$

And the decision rule of consumption at time *t* is likewise obtained as that in Section 2 and is illustrated by equation (5).

Secondly, the derived contingent consumption and capital stock from equations (5) and (12) are incorporated into the (log-) linearized intertemporal budget constraint, and with the steady-state restrictions, the CESS variables (*c*, *k* and $\widetilde{B}$) at time *s* are solved. That is, the three endogenous variables are solved with three equality constraints, equations (9), (10) and

$$\widetilde{B}_s = \widetilde{B} - e^{-r}d_1(k_s - k) - \left\{\frac{1}{1-\rho_r e^{\lambda-r}}\left[\widetilde{B} + \frac{e^{-r}\rho_r\widetilde{C}}{\gamma(1-\rho_r)}\right] + e^{-r}d_2\right\}(r_{s-1} - r) - e^{-r}d_3(a_s - a),$$

where d_i, $i = 1, 2, 3$, are functions of structural parameters only.

Once the values of *c*, *k*, and $\widetilde{B}$ at time *s* are obtained, following the same procedure as described in Section 2 gives the equilibrium sequences of consumption, capital stock and foreign assets and completes the CESS method.

4.1. *Parameterizing the production economy*

One important role of analyzing an economy's real business cycles is to present the main features of the economy's real business cycles and to discuss the extent to which the features can be rationalized on the basis of a specific stochastic competitive equilibrium model. A common effort in this field for a small-open production economy is to replicate its empirical regularities by generating artificial data moments and by depicting the impulse response functions associated with innovations, such as productivity shocks or world real interest rate shocks. Because of applying the CESS method to solving the model without reference to any particular sample information, we define the data generated from the model by the CESS method to be the genuine data for our economy.

We perform simulation experiments on this small-open production economy and stimulate the model 100 times each with 100 observations to generate the distribution of the data. The statistic properties of these genuine data are compared with those generated by a sample-dependent method. The application of the sample-dependent method to our model means that a sample mean of the consumption-to-GDP ratio is employed to solve the transformed steady-state consumption (c). Together with the determination of k in equation (9), once c is determined, $\widetilde{B}$ in equation (10) can be solved. The sequences of consumption, capital stock, and foreign assets can then be generated through equations (5) and (12), and the budget constraint (8).

The parameterization of the simulation is as follows. We set the subjective discount rate $\beta = 0.96$, the capital share $\alpha = 0.4$, the long-run economic growth rate $\lambda = 0.02$, and the depreciation rate of capital stock $\delta = 0.1$. The parameters governing the temporary technology shock and the world real interest rate are set as: $A = 1$, $\rho_a = 0.95$, $\rho_r = 0.5$, $\sigma_a = \sigma_r = 0.05$, and the covariance between a_t and r_t is equal to zero. The baseline values of the coefficient of relative risk aversion (γ) and the coefficient of the adjustment cost of investment (ϕ) are set to 1 and 0.6, respectively.

All of the parameters chosen are plausible and are often used in the studies of real business cycles. In particular, the parameterization implies that our small-open economy faces a world interest rate of 6 percent in the long-run. Furthermore, we assume that the initial capital stock is equal to its unique long-run equilibrium, while initial net foreign assets owned by the small-open economy are set to zero. Finally, the simulation is also performed under the alternative parameterization of $\gamma = 0.5$ and 2, $\phi = 0.3$ and 0.9, and $\widetilde{B}_s = 2$ and -2. Since the simulation results are not sensitive to these parameters, we report the results generated from the baseline values only.

Figure 3 presents the simulation results of the means of the primary variables of interest. Figures 3(a), 3(b), 3(c) and 3(d) plot the means of the following four variables with 2 standard errors around each of them: the ratios of consumption to GDP, investment to GDP, trade balance to GDP, and foreign assets to GDP. It is obvious from these figures that all variables except the investment-to-GDP ratio are with ever-increasing standard errors that are identified as one character for those series being non-stationary.[9] The finding of non-stationarity of the consumption-to-GDP ratio implies a possibility of there being a variety of sample means associated with the genuine data. Since the application of a sample-dependent method relies on a particular sample chosen, the results obtained by this method could be very different due to a different selection of samples. This limitation of the sample-dependent method can be removed by using the CESS method, because it is sample independent in essence. We will make this point clearer by further comparing the simulation results of the impulse response functions between these two methods, and by illustrating a diversity of co-movement between consumption and GDP when the sample-dependent method is employed.

4.2. *The impulse response functions*

[9] The reason that the investment-to-GDP ratio series is stationary with stable standard errors is because leisure is not considered as a decision variable in our simple model. If leisure did play a role in the agent's utility function, then the standard errors of the investment-to-GDP ratio would grow too.

Let us consider the impulse response of consumption, trade balance and foreign assets to exogenous innovations,[10] for example, a temporary one standard error decline in the world real interest rate as illustrated in Figure 4(a). Figures 4(b), 4(c), and 4(d) depict the effects on those variables solved both by the CESS method and by the sample-dependent method. The key difference in these two methods is that the sample mean of the consumption-to-GDP ratio must be given first for the sample-dependent method, but not for the CESS method. From our simulation results using the CESS method, which we intend to refer to the genuine data, the mean of the consumption-to-GDP ratio is 0.746 and the standard error is 0.089.

According to the true value of a consumption-to-GDP ratio, studies using the sample-dependent method have a probability of 95 percent to select a sample mean of it from a value lying between 0.568 and 0.924. From Figure 4(c), it is evident that the impulse response of consumption obtained by that method is closer to the true impulse response obtained by the CESS method if a sample mean near 0.746 is used. However, if a sample mean higher than 0.746 is picked up, then the effect of a negative world interest rate shock on consumption is exaggerated. Furthermore, if a sample mean sufficiently lower than 0.746 is picked up, then the direction of the effect of a negative world interest rate shock on consumption is even reversed. Responses of trade balance in Figure 4(b) and foreign asset holdings in Figure 4(d) behave likewise. The evidence

[10] Since the steady-state transformed capital stock is uniquely determined in equation (9), the impulse responses of the capital stock, investment, and output obtained by either a sample-dependent method or by the CESS method are the same, and thus are trivial for comparison.

confirms that different sample means will lead to a very different prediction of the business cycles of a small-open economy by using the sample-dependent method.[11]

4.3. *Co-movement of consumption and GDP*

To show that the statistical properties of the generated data would change a lot due to a different choice of samples, we take two transformed variables, (log) GDP and (log) consumption, as the example and present some co-movement between them. Table 1 reports the means and standard errors of the correlation coefficients calculated from the sample-dependent method for three different sample means of the consumption-to-GDP ratio.

Table 1. Co-movement (the sample-dependent method)

Sample mean chosen $\frac{C_t}{Y_t}$	Correlation between consumption and GDP
0.924	0.410 (0.440)
0.746	0.555 (0.323)
0.568	0.688 (0.167)

The standard deviation of the correlation is given in parentheses.

[11] The impulse responses to a productivity shock show only a slight difference in comparison of the CESS method and the sample-dependent method due to the negligence of leisure.

From Table 1, we see how the choice of sample means affects the correlation between consumption and GDP of the data. For example, consumption may be procyclical as the Keynesian asserts or acyclical as the permanent-income-hypothesis implies, depending on what sample we use.

In fact, the differences in statistical properties of different choices of samples may be enlarged if the initial foreign assets owned by the small-open economy deviate from the baseline value. It could also be the case if leisure is reconsidered in the felicity, or a more volatile sample of trade balance is used to calculate the "steady-state" value.

5. Conclusion

The distinct character of the real business cycles model of small-open economies with a standard time-separable preference assumption is the non-uniqueness of its steady state. Most studies follow the log-linear approximation method suggested in King et al. (1988) to solve a stochastic competitive equilibrium of the model. They must therefore assume additional restrictions to resolve this non-uniqueness problem. A common approach assumes that the time-varying steady-state consumption-to-GDP ratio, trade balance-to-GDP ratio, and the net foreign assets-to-GDP ratio are all constant and equal to their sample means, respectively. Since these ratios are non-stationary in essence, the empirical finding from such a sample-dependent method could be very sensitive to the sample, and could even lead to an incorrect implication. One purpose of this paper is to support this view by showing the simulation results from a different selection of samples. A different sample of an economy concludes sufficiently different implications on the properties of the economy's business cycles.

This paper goes further to introduce a sample-independent approach to resolve the unsatisfactory and unnecessary reliance of the outcome on a particular choice of sample. In the simple version of our model, it clearly shows the dependence of the steady state on its initial conditions. Since the conditional expected steady state (CESS) is unique at any point of time, we can solve it together with other contemporary endogenous variables without making any additional assumption. The proposed CESS method is consistent with, and can generate business cycles statistics in small-open economies at the same standpoint as, those methods widely adopted by studies of real business cycles in closed economies. More importantly, the application of the CESS method to a more general version of the model supports the superiority of the CESS method over the sample-dependent method.

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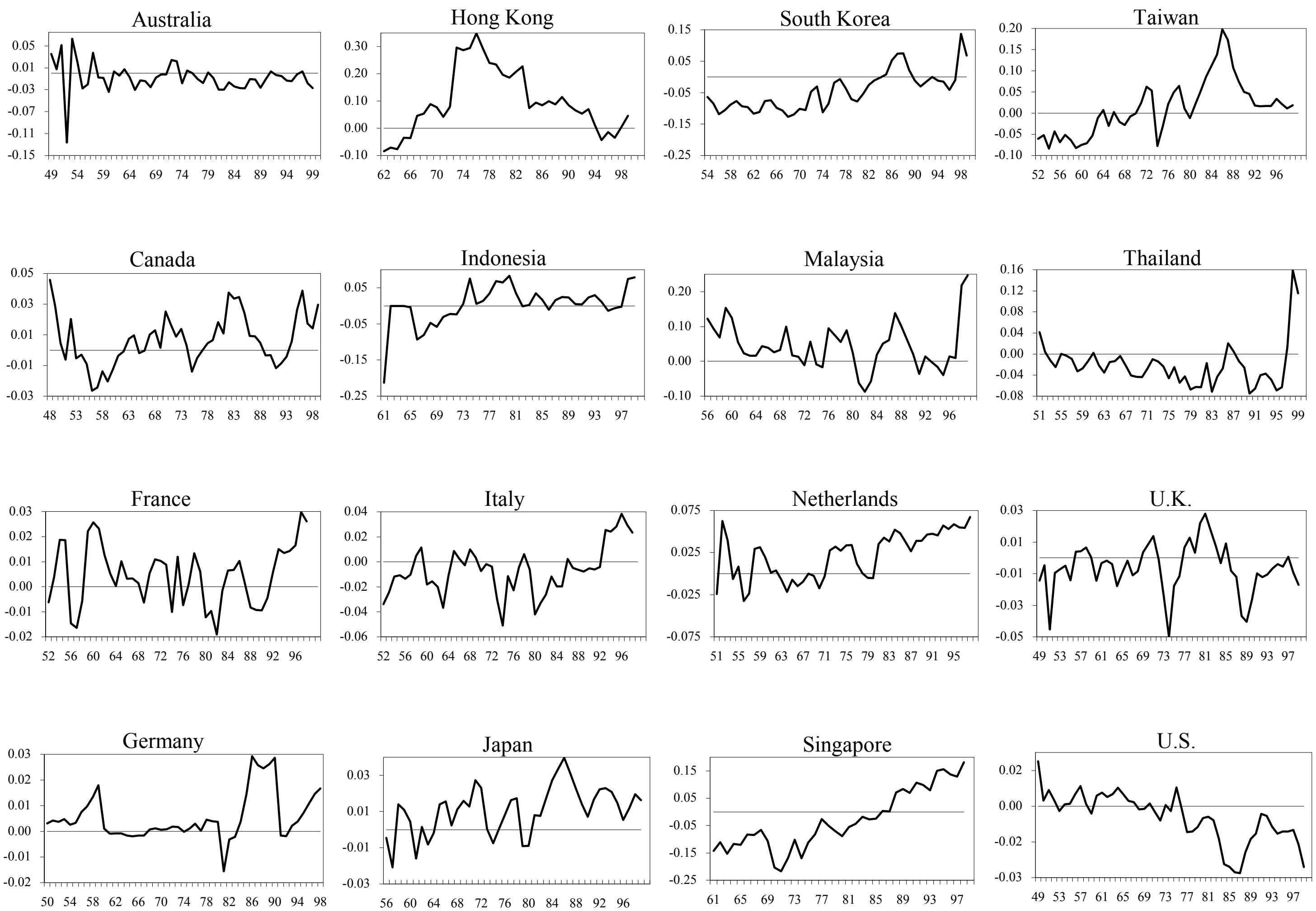

Figure 1. Trade Balance-to-GDP Ratio

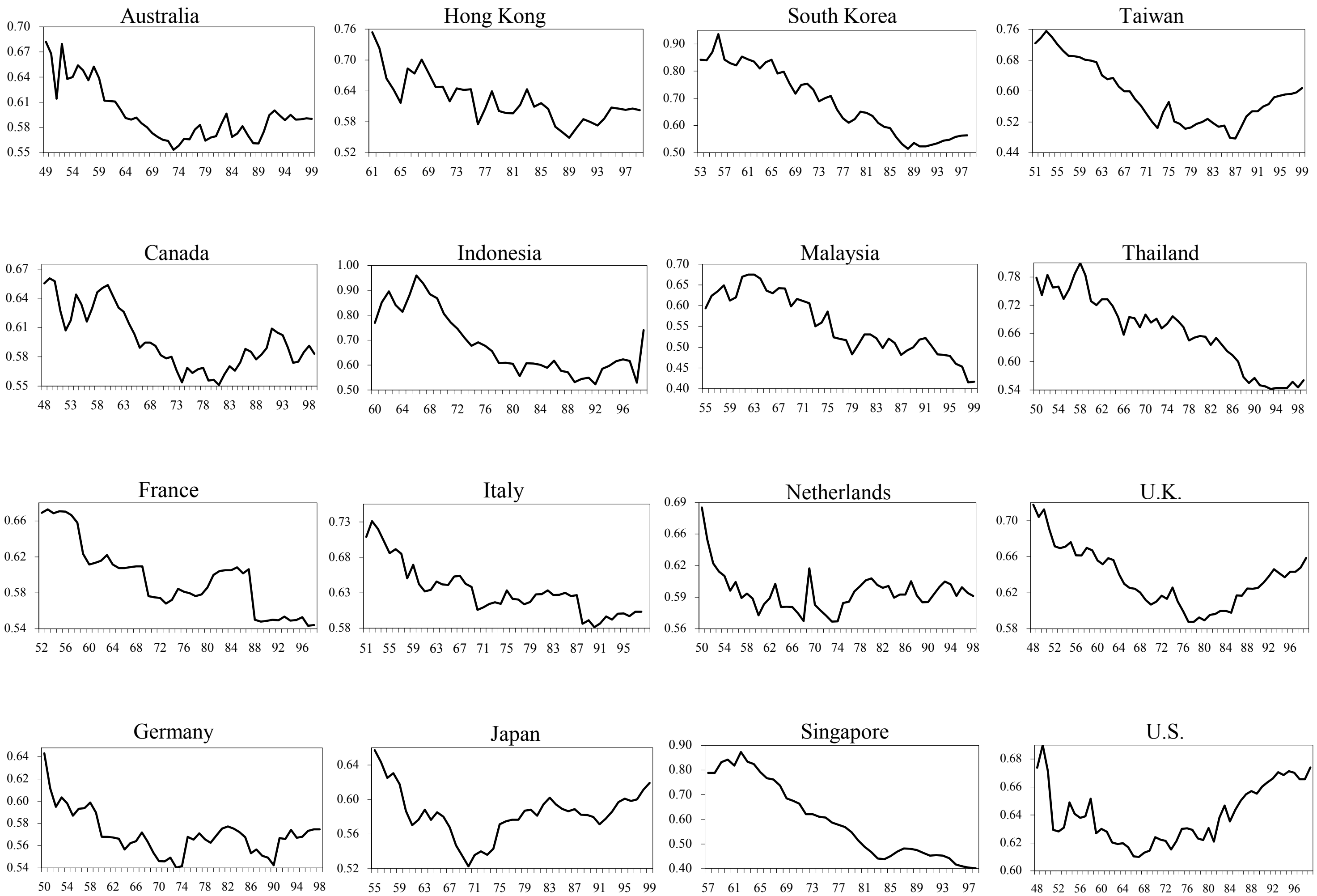

Figure 2. Consumption-to-GDP Ratio

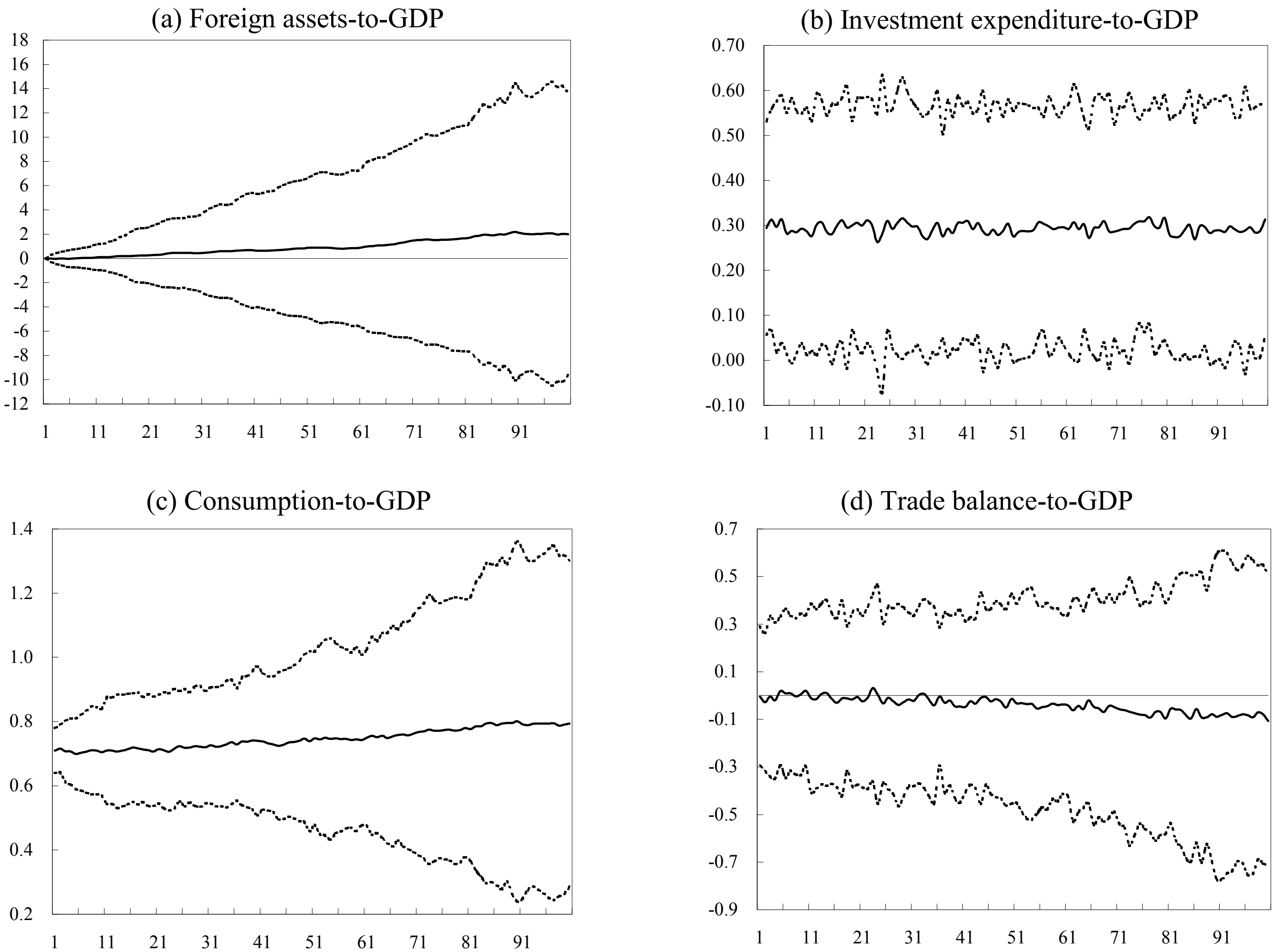

Figure 3. Simulation Results

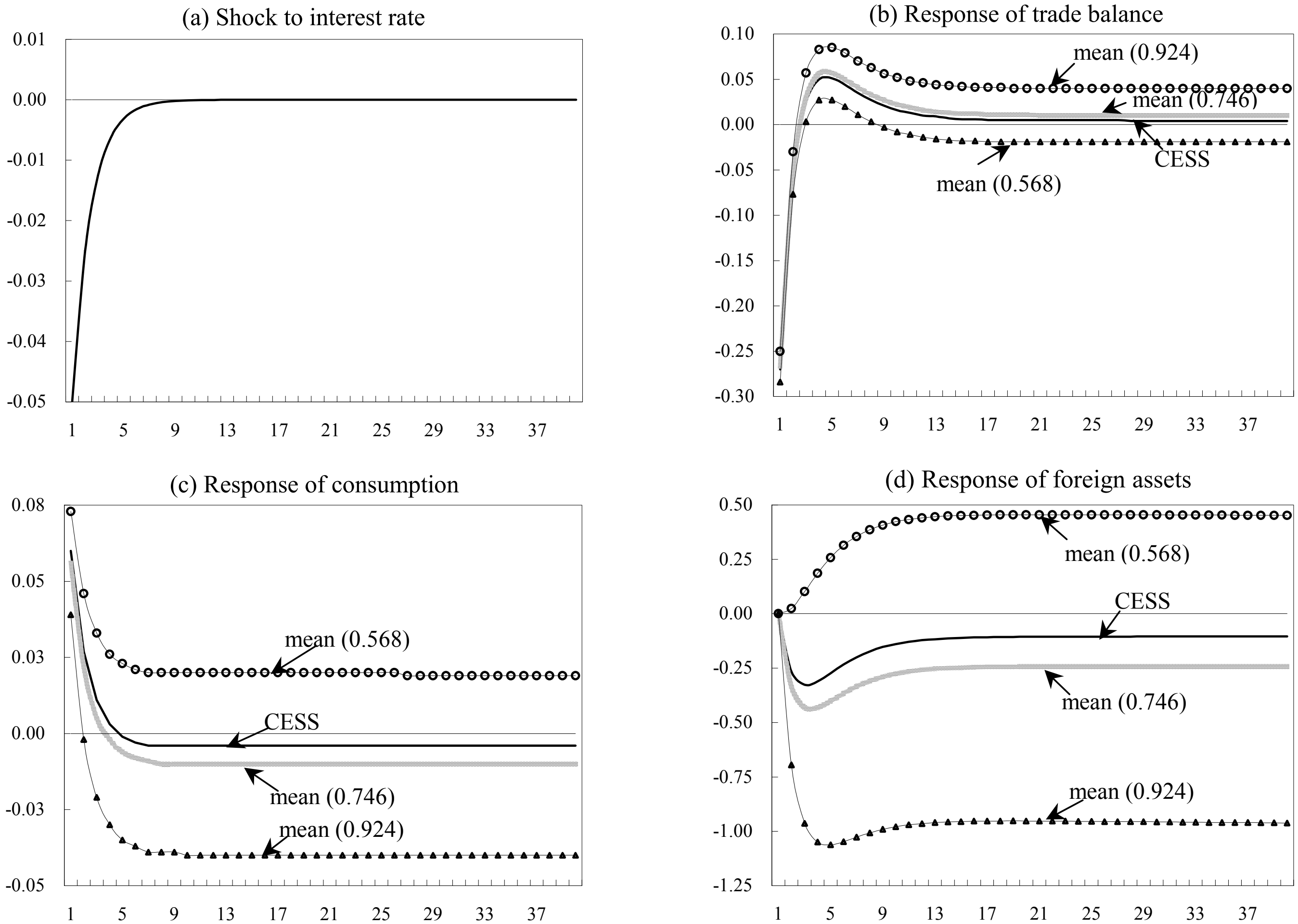

Figure 4. Impulse Responses