A Theory-based, State-dependent Phillips Curve and its Estimation

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Abstract

To explain the existing empirical irregularity about the slope of a Phillips curve, this paper provides a model of imperfect competition to show that the slope of a Phillips curve is shock-dependent. We empirically apply a state-space, Markov-switching model to examine the impact of inflation surprise on the unemployment gap, resulting in the state-dependent Phillips curve fitting quite well. Our empirical evidence indicates that an unexpected monetary expansion does produce effects in reducing unemployment rates, and that supply shocks should not be ignored in estimating the Phillips curve since they dominate demand shocks in several non-oil shock periods.

Keywords: Phillips Curve, Unemployment Gap, Inflation Surprise, State-space Markov-switching Model.

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1. Introduction

It is well known that a downward-sloping short-run Phillips curve implies a trade-off between the inflation rate and unemployment rate (Friedman, 1968). However, empirical findings point out that the slope of the Phillips curve depends on the selected sample periods.[1] Does any theoretical model explain the changes in the slope of a short-run Phillips Curve as displayed in the data? Can we find empirical support for our model that implies a slope-changeable Phillips curve? To answer these questions, this paper provides a theoretical model implying that the slope of the Phillips curve is shock-dependent. We then apply a state-space, Markov-switching model to investigate the empirical support of the theoretical model.

Although a stable, negative, short-run Phillips curve is crucial for producing inflation forecasts in policymaking institutions, there is no general consensus about the negative slope of the Phillips curve in empirical literature.[2] One of the

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[1] While early studies such as Samuelson and Solow (1960), Brechling (1968), and Hansen (1970) support the downward-sloping Phillips curve, studies including data after the 1970s generally find an unstable relationship between inflation and unemployment (Friedman, 1977 and Lucas and Sargent, 1978).

[2] Since the negative correlation between inflation and unemployment vanished in the 1970s, Lucas and Sargent (1978) and some other neoclassical and monetarist economists dismiss the Phillips curve as an "econometric failure on a grand scale." On the contrary, Keynesian macroeconomists, such as King and Watson (1994, p.160), argue that "the remarkable feature of the Phillips curve in the post-war period was its stability;" and Blinder (1997, p.241) states that "the empirical Phillips curve has worked

explanations to the previous empirical irregularity is that the relationship between unemployment and inflation is shock-dependent (King and Watson, 1994), which means that the slope of a Phillips curve relies on the dominance of the demand or supply shocks. Therefore, the conventional single-equation regression based on the state independent framework is not appropriate.

Apart from neglecting the state-dependent characteristic of the Phillips curve, there are several other drawbacks for conventional empirical studies when investigating the Phillips curve. First, many articles assume that forecast errors of inflation are homoscedastic, which is challenged in the literature. The homoscedastic assumption is debatable since several studies point out significant heteroscedasticity in the U.S. inflation rate (Engle, 1983, Jansen, 1989, Evans, 1991, and Brunner and Hess, 1993). ARCH or GARCH-type models provide estimates of how the conditional variance of inflation varies over time within a given structure and therefore they ignore the possibility of structural changes caused by changing regimes. A Markov switching heteroskedasticity model is an alternative that allows regime changes in the variance structure. Instead of applying ARCH-type models to estimate the time-varying conditional variance of inflation within a given structure, this paper takes into account the influence of regime changes on inflation's variance.

amazing well for decades in the United States."

We apply the state-dependent conditional heteroscedasticity approach provided by Brunner and Hess (1993) and Evans and Wachtel (1993) to estimate the forecasting errors of inflation.

The second drawback is a constant natural rate assumption. The unemployment gap is the difference between the unemployment rate and the natural rate. Conventional literature assumes a constant natural rate, which is challenged theoretically and empirically (Gordon, 1997). The existence of unionization, unemployment insurance programs, and labor force demographics leads to a highly persistent unemployment rate. We therefore assume that the unobservable natural rate is time-varying and behaves in accordance with the hysteresis of unemployment rates.[3]

Recent empirical works of a regime-switching approach on the Phillips curve have focused on whether or not the relationship between inflation and unemployment rates is non-linear (Ferri et al., 2001) or whether the trade-off relationship is state-dependent (Ho, 2000). The previously-mentioned literature discusses the relationship between unemployment and inflation rather then the unemployment gap and inflation surprise. They therefore fail to take account of expectations in a

[3] Hysteresis means that temporary shocks have permanent effects on the level of unemployment. Blanchard and Summers (1986) point out "unemployment exhibits hysteresis when current unemployment depends on past values with coefficients summing to one."

framework with factor market imperfection, which is the new view of the Phillips curve.

According to the previous discussion, it is quite interesting to provide a theoretical foundation for a shock-dependent Phillips curve which is consistent with our observation from data, and then to provide empirical support for such a curve. Theoretically, we modify the imperfect competitive model in Blanchard and Fischer (1989) so as to provide a shock-dependent Philips curve. The important implication of our model is that the Phillips curve is downward sloping when demand shocks dominate contemporary supply shocks. While demand shocks are dominated by supply shocks, the Phillips curve may be upward sloping.

This paper does not try to separate "supply" shocks and "demand" shocks, but instead tries to identify "supply shock dominating" or "demand shock dominating" in terms of their effects on unemployment rates. Rather than including proxies for supply shocks in a regression as done by previous Phillips curve estimations, a non-linear specification is used to characterize the state-dependent property of the Phillips curve equation from our theoretical model. Hamilton's (1989, 1994) Markov-switching model is suitable for the estimation. Moreover, we apply a state-space, Markov-switching model to estimate the relationship between the unemployment gap and the forecast errors of inflation rates. The state-space specification is used to model the unobservable natural rate of unemployment.

To get rid of Pagan's (1984) critique on the two-step estimation method, we estimate the model simultaneously by applying the method suggested by Kim (1993a).[4] Our empirical evidence supports the theoretical implication that unexpected monetary expansion is helpful for reducing unemployment and that supply shocks should not be ignored in estimating the Phillips curve

The remainder of this paper is arranged as follows. In Section 2 we present a theoretical model of unemployment to show that the effects from inflation surprise on the unemployment gap are state-dependent. Section 3 introduces a state-space, Markov-switching model to estimate the state-dependent Phillips curve. The empirical investigation is given in Section 4 and we adopt a joint estimation method to estimate the model simultaneously. Section 5 concludes.

2. The Model for a State-dependent Phillips curve

Gordon (1997) states "the resurrection of a new breed of post-1975 Phillips curve estimates that incorporated a vertical long-run trade-off and included

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[4] Kim (1993) assumes that real GNP consists of the sum of two independent unobserved components: one following a random walk with drift, which evolves according to a two-state Markov process, and the other following an autoregressive process. The cyclical component of real GNP is affected by a conditional forecast error of monetary growth, which can be estimated from a monetary growth equation with Markov-switching heteroscedasticity. Kim provides the method of simultaneous estimation by maximizing the joint probability function of the state-space Markov-switching model.

supply-shock variables that allowed the model to generate either a negative or positive correlation between inflation and unemployment, depending on whether the economy was subjected to demand or supply shocks." To express these properties in some details, a simple model useful in the analysis of economic fluctuations in the presence of nominal rigidities is modified from Blanchard and Fischer (1989, p.518). This model is given as follows:

$$y_t^d = \beta(m_t - p_t + v_t), \quad \beta > 0, \tag{1}$$

$$y_t^s = a_t + \alpha n_t^d, \quad 0 \le \alpha \le 1, \tag{2}$$

$$n_t^d = -\gamma(w_t - p_t - a_t), \quad \gamma > 0, \tag{3}$$

$$n_t^s = \delta(w_t - p_t), \quad \delta > 0, \tag{4}$$

$$w_t \mid E_{t-1}n_t^d = E_{t-1}n_t^s, \quad n_t = n_t^d, \tag{5}$$

$$y_t^d = y_t^s. \tag{6}$$

Here, y_t, n_t, w_t, m_t, p_t, v_t, and a_t are the logarithms of aggregate output, employment, the nominal wage, the monetary aggregate, the price level, demand shocks, and supply shocks, respectively. Term $E_s x_t$ denotes the expectation of variable x_t conditional on the information set at time *s*. Constants are ignored for notational simplicity.

Equation (1) is an aggregate demand function or simply the Clower constraint. In the case of constant interest rates, term β can be interpreted as the inverse of the income elasticity of money demand. Equations (2) and (3) state that output supply is a function of labor demand which in turn is a function of real wage and a technology shock. These two equations can be derived from profit maximization under perfect competition. Labor supply as described in Equation (4) is a function of real wages, while equation (5) specifies the nature of the nominal rigidity. The nominal wage is set to equal expected labor demand and supply, and its behavior can be given by non-competitive interpretations. Finally, Equation (6) defines the equilibrium of the goods market.

To derive the Phillips curve, let us further separate the unemployment into two parts — structural unemployment (natural rate) and non-structural unemployment (unemployment gap). Assume that:

$$u_t - u_t^n = \theta(E_{t-1}n_t - n_t), \quad \theta > 0, \tag{7}$$

where u_t is the unemployment rate at time t and u_t^n is its natural rate.[5] The interpretation of Equation (7) is obvious: if the actual employment equals the expected equilibrium employment, then there is no involuntary unemployment and the unemployment rate thus equals its natural rate.

After some simple manipulation on Equations (1)-(7), we can derive the

[5] Instead of assuming Equation (7), textbooks usually assume that $u_t = -\theta n_t$. In this case the equation of the Phillips curve is much more complicated than that in (9) and (11), but the state-dependent characteristic of the Phillips curve is not affected, however.

following equation:

$$u_t - u_t^n = -\theta\gamma(a_t - E_{t-1}a_t + \pi_t - E_{t-1}\pi_t)\,, \tag{8}$$

where $\pi_t \equiv p_t - p_{t-1}$. Note that, since the inflation rate is an endogenous variable, Equation (8) is not a solution for the unemployment gap. The correlation of the unemployment gap and inflation surprise is determined by exogenous shocks. A positive supply shock reduces unemployment via the labor demand equation, on the one hand, and increases the commodity supply, on the other hand. The increase in commodity supply will decrease price and inflation level; this in turn decreases labor demand and leads to higher unemployment. Therefore, the correlation between the unemployment gap and inflation surprise is ambiguous when a supply shock occurs. On the contrary, demand and monetary shocks only affect the unemployment gap through their effects on inflation, and in this case the correlation between the unemployment gap and inflation surprise is definitely negative.

For the simplicity of describing a slope-changeable Phillips curve, let us first assume that there exist only monetary or demand shocks, i.e., $a_t = E_{t-1}a_t$. Under this assumption, the Phillips curve can be derived as follows:

$$u_t = u_t^n - \gamma\theta(\pi_t - E_{t-1}\pi_t)\,. \tag{9}$$

Obviously, the previous equation indicates a downward-sloping Phillips curve, which implies that there is a trade-off between inflation surprise ($\pi_t - E_{t-1}\pi_t$) and the unemployment gap ($u_t - u_t^n$).

When there are only supply shocks (i.e., $v_t = E_{t-1}v_t$, $m_t = E_{t-1}m_t$), it can be shown that the unexpected inflation change is a function of an unpredicted supply shock as follows:

$$\pi_t - E_{t-1}\pi_t = -\frac{1+\alpha\gamma}{\alpha\gamma+\beta}(a_t - E_{t-1}a_t)\,. \tag{10}$$

Combining equations (8) and (10) yields a Phillips curve with an alternative slope:

$$u_t = u_t^n + \frac{\gamma\theta(\beta-1)}{1+\alpha\gamma}(\pi_t - E_{t-1}\pi_t)\,. \tag{11}$$

According to Equation (11), the Phillips curve under a supply shock could be upward sloping or downward sloping, depending on the income elasticity of money demand. If the elasticity is less than one, i.e., $\beta > 1$, then there is a positive relationship between the unemployment gap and the forecasting error of inflation.[6]

In sum, our model explains the reason why the slope of a short-run Phillips curve depends on whether the economy was subject to demand or supply shocks. The short-run implication of this Phillips curve also confirms conventional wisdom that an unexpected monetary expansion (just like a positive demand shock) is helpful for reducing unemployment. In addition, when all the actual variables equal their

[6] The empirical studies for the income elasticity of money demand in the U.S. are mixed. Some articles find that the elasticity is significantly greater than one (Mulligan and Sala-i-Martin, 1992); others conclude that it equals one (Stock and Watson, 1993 and Hoffman et al., 1995); still others argue that it is less than one (Ball, 2001).

expected counterparts, a long-run Phillips curve is vertical at the natural unemployment rate and it is consistent with any level of inflation rates.

3. A State-space, Markov-switching Econometric Model

In this section we present a state-space, Markov-switching model and provide the reason for adopting this model in our estimation. From the theoretical model in Section 2 we know that the effects of an inflation surprise on the unemployment gap are either negative or positive, depending on the dominance of demand or supply shocks. This justifies our using a Markov-switching specification on the slope coefficient of inflation surprise on non-structural unemployment.

As for structural unemployment, which is similar in spirit to the natural rate of unemployment, many studies find that the natural rate of unemployment is time-varying rather than constant (Ball, 1996; Gordon, 1997; Staiger et al., 1997, and Laubach, 2001). To capture the hysteresis in the structural unemployment, we assume that there is a unit root in the process of the natural rate as in Gordon (1997), Apel and Jansson (1999a, b), and Ireland (1999). The unit-root assumption is also supported by our empirical finding in which we fail to reject the unit-root hypothesis for the U.S. unemployment rate. Therefore, the specification for unemployment in this paper is given as follows:

$$u_t = u_t^n + u_t^g, \tag{12}$$

$$u_t^n = u_{t-1}^n + e_t^n, \qquad e_t^n \sim \text{i.i.d. } N(0, \sigma_n^2), \tag{13}$$

$$u_t^g = \sum_{i=1}^{l} \rho_i u_{t-i}^g + \phi_{st} e_t^\pi + e_t^g, \quad e_t^g \sim \text{i.i.d. } N(0, \sigma_g^2), \tag{14}$$

$$\phi_{st} = \phi_0 + (\phi_1 - \phi_0) S_t^u, \quad S_t^u = 0, 1, \tag{15}$$

$$\text{with } \begin{bmatrix} \Pr(S_t^u = 0 \mid S_{t-1}^u = 0) & \Pr(S_t^u = 1 \mid S_{t-1}^u = 0) \\ \Pr(S_t^u = 0 \mid S_{t-1}^u = 1) & \Pr(S_t^u = 1 \mid S_{t-1}^u = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}. \tag{16}$$

The variables u_t^n and u_t^g in Equation (12) are the natural rate of unemployment and the unemployment gap, respectively. The fact that both variables are unobservable justifies our using a state-space specification for our model. In addition, the residuals e_t^n and e_t^g are assumed to follow an identical, independent normal distribution with zero mean and variance σ_n^2 and σ_g^2, respectively. The regressor e_t^π in Equation (14) is the conditional forecast error of inflation. Equation (11) theoretically indicates the state-dependent property for the exact relation between the cyclical unemployment and the unexpected inflation, $u_t^g = \phi_{st} e_t^\pi$. The lagged cyclical unemployment rates and the error term (e_t^g), which may reflect the persistence of backward-looking expectations and measurement errors, respectively, are included in Equation (14) for empirical purposes.

The slope of the Phillips curve is state-dependent and is denoted by ϕ_{st}, which equals ϕ_0 if $S_t^u = 0$ and ϕ_1 if $S_t^u = 1$. The state S_t^u is given by the two-state, first-order Markov process with the transition probability described above. The transition probability p_{ij} describes the probability that state *i* will be followed by state *j*.

Since the conditional forecast error of inflation, e_t^{π}, in Equation (14) is not observable, we therefore specify an inflation equation to complete the model. There are many empirical works that support heteroscedasticity in the U.S. inflation rate (Engle, 1983, Jansen, 1989, Evans, 1991, and Brunner and Hess, 1993). In order to take into account the influence on the uncertainty of possible regime changes, we apply a state-dependent conditional heteroscedasticity framework, provided by Brunner and Hess (1993), Evans and Wachtel (1993), and Kim (1993b), to describe the dynamics of inflation rates. Our specification for the inflation rate is given as follows:

$$\pi_t = \lambda_0 + \sum_{i=1}^{k} \lambda_i \pi_{t-i} + e_t^{\pi}, \quad e_t^{\pi} \sim N(0, \sigma_{st}^2), \tag{17}$$

$$\sigma_{st}^2 = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2) S_t^{\pi}, \quad S_t^{\pi} = 0, 1, \tag{18}$$

$$\text{with} \begin{bmatrix} \Pr(S_t^{\pi} = 0 \mid S_{t-1}^{\pi} = 0) & \Pr(S_t^{\pi} = 1 \mid S_{t-1}^{\pi} = 0) \\ \Pr(S_t^{\pi} = 0 \mid S_{t-1}^{\pi} = 1) & \Pr(S_t^{\pi} = 1 \mid S_{t-1}^{\pi} = 1) \end{bmatrix} = \begin{bmatrix} q_{00} & q_{01} \\ q_{10} & q_{11} \end{bmatrix}. \tag{19}$$

The residual e_t^{π} in Equation (17) is assumed to follow a normal distribution with zero mean and a state-dependent variance σ_{st}^2, which equals σ_0^2 if $S_t^{\pi} = 0$ and equals σ_1^2 if $S_t^{\pi} = 1$; state S_t^{π} is given by the two-state, first-order Markov process with the transition probability described above. The transition probability q_{ij} describes the probability that state *i* will be followed by state *j*. For identification purposes, disturbances e_t^n, e_t^g, and e_t^{π} are assumed to be orthogonal to each other. Under the assumption of no heteroscedasticity (i.e., $\sigma_0^2 = \sigma_1^2$), the model for inflation rates reduces to a conventional autoregressive process. Furthermore, if σ_{st}^2 follows a Markov chain, then the sum of q_{00} and q_{11} must be distinct from one.

The inflation surprise can be conventionally estimated from Equations (17)-(19). Having the estimated inflation surprise from the previous step, one can estimate the influence of inflation surprise on unemployment (Evans and Wachtel, 1993). However, Pagan (1984) points out that this two-step approach may lead to a bias in the estimated standard deviations in the second step. To avoid the criticism of Pagan and to take into account the state-dependent and latent properties in the inflation surprise and the unemployment gap, we estimate the systems (12)-(19) simultaneously by modifying Kim's (1993a) methodology.

4. Data and Empirical Investigation

4.1 Data description

Seasonally-adjusted quarterly data of the unemployment rate and the consumer price index for the U.S. are obtained from the *Bureau of Labor Statistics*. Inflation rates are computed from the price index. The sample period starts from 1948Q1 and ends in 2003Q2.

To provide a first visual impression about the breakdown of a naive Phillips curve, Figures 1 and 2 illustrate the relationship between the rate of unemployment and inflation. The horizontal axis in Figure 1 reflects the unemployment rate for each quarter from 1948Q1 through 1973Q4 and from 1974Q1 through 2003Q2 in Figure 2, while the vertical axis indicates the contemporary inflation rates. The lines in these figures are the linear regression lines through these data. It can be seen that the line in Figure 1 is clearly downward sloping, which represents a significant negative relationship between the unemployment rate and the inflation rate before the first oil price shock. The line in Figure 2, however, is upward sloping with an insignificant regressive coefficient. This evidence explains why quite a few authors have devoted exhaustive efforts to re-estimate the Phillips curve with various techniques since the 1980s.[7]

To justify the unit-root specification of the natural rate of unemployment (and therefore the unemployment rate), we apply two tests provided by Dickey and Fuller (1979) and Elliott et al. (1996) to examine the unit-root hypothesis of the U.S. unemployment rate (hereafter, the ADF test and DF-GLS test). The model we adopt is the one without a deterministic trend. As shown by Ng and Perron (2001), techniques of selecting the lag order are crucial in unit-root tests. They point out that modified information criteria leads to substantial size improvement over standard

[7] Different econometric models are discussed in King and Watson (1994).

information criteria in unit-root tests. We therefore select the lag length of the model using the modified Akaiki Information Criterion (MAIC).[8] With a maximum length up to 6, the optimal lag order decided by MAIC is four. The statistics of the ADF and DF-GLS tests for unemployment rates are -2.64 and -1.66, respectively, which are larger than their corresponding 5% critical values: -2.88 and -1.95. Our findings fail to reject the unit-root hypothesis for the U.S. unemployment rate at the 5% level of significance.

To justify our specification of heteroscedasticity in the inflation equation, we test whether the estimated residuals from the OLS are heteroscedastic. The Lagrange multiplier test of Engle (1982) is applied to examine the existence of autoregressive conditional heteroscedasticity (ARCH) in the estimated residuals. We regress an AR(4) representation for the quarterly inflation rate and then construct the ARCH(1) and ARCH(4) statistics from the estimated residuals, which has a χ^2 distribution with degrees of freedom being one and four, respectively. The ARCH(1) and ARCH(4) statistics are respectively 61.61 and 77.52, which reject the hypothesis of no ARCH in the estimated residuals at the 1% level of significance. This finding indicates the need for a heteroscedasticity specification for the inflation equation. In this paper we assume a regime-switching variance in the inflation equation.

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[8] The optimal log order remains the same if the modified Schwarz Information Criterion is used.

4.2 Model estimation and empirical results

From the implications of the theory in Section 2, the observation of unemployment and inflation can be thought of as drawing from the mixtures of two normal distributions. The states in each period for the two variables are assumed to independently switch between two regimes according to their transition probabilities. The empirical procedure includes two parts. First, we estimate the state-space, Markov-switching model simultaneously. Second, after estimating the model with the maximum likelihood method, the smoothed probabilities are derived by Hamilton's (1989) full-sample smoother. In addition, the smoothed estimate of the natural rate of unemployment can be calculated given the estimated probability and the estimates from the model.

In order to take into account the distributed lag effects on unemployment and inflation, we set the lag orders in Equations (14) and (17) to be 2 and 4, respectively.[9] The joint estimations of the generalized model in Equations (12)-(19) are reported in Table 1.

Findings from the upper panel of Table 1 indicate that there are two different shapes of Phillips curves in the sample. The estimated slope coefficient for state 0

[9] Note that there is no easy way to determine the lag orders in the Markov-switching model. The lag orders in autoregressive models for the unemployment rate and inflation rate, which are determined according to the Schwarz Bayesian criterion, are therefore chosen as the lag length used in this paper.

(ϕ_0) is -0.115, which is significantly less than zero. It indicates that a 1 percent increase in the inflation rate caused by an unexpected monetary expansion leads to a fall in the unemployment gap of 0.12 percent. This also implies that state 0 is dominated by demand shocks and supports a downward-sloping Phillips curve. However, the Phillips curve is upward sloping in state 1 since the estimated slope coefficient (ϕ_1) is 1.005, which implies that state 1 is dominated by supply shocks. Therefore, our results appear to solve the empirical puzzle on the slope of the Phillips curves that is found by previous research.

Both of the estimated autoregressive coefficients (ρ_1 and ρ_2) in the unemployment gap equation, Equation (15), are significant. In addition, the estimated conditional standard deviation for the natural rate of unemployment σ_n is 0.117. This estimate is very close to the reasonable value of 0.1 considered in Gordon (1997). Estimates of the transition probabilities in the unemployment equation are 0.952 and 0.924 for p_{00} and p_{11}, respectively, which indicate that both states are highly persistent.

From the middle panel of Table 1 we find that the sum of the autoregressive coefficients, λ's, in the inflation rate equation is well less than one. This estimated result is consistent with the stationarity of the U.S. inflation rate in the sample period.[10] The estimate of σ_0 is 0.766 and is larger than that of σ_1, which in turn implies that state 0 is more volatile than state 1 for inflation rates. Finally, the estimated transition probabilities for inflation are 0.947 and 0.976 for q_{00} and q_{11}, respectively, which indicate very slow changes in the states for different inflation variances.

After estimating the model simultaneously with the maximum likelihood method, we next investigate the appropriateness of our state-space, Markov-switching specification. We perform LM tests to examine the null hypothesis of no autocorrelation and ARCH effects in estimated residuals using the method suggested by Hamilton (1996). In addition, we apply the Wald test to examine the hypotheses of independent state variables, i.e., $p_{00} + p_{11} = 1$ and $q_{00} + q_{11} = 1$. If our model is correctly specified, then we should find that the Wald test rejects the hypothesis of no Markov chain, but the LM tests fail to reject the hypothesis of no serial correlation and of no ARCH effects in the estimated residuals, respectively.

The Wald statistic, listed in the lower panel of Table 1, indicates that the hypothesis of no Markov chain is rejected at the 5% level of significance. The LM test fails to reject the hypothesis of no serial correlation at the conventional level of significance. However, the hypothesis of no ARCH effects in the estimated residuals from inflation equation is rejected at the 5% level but fails to be rejected at the 1%

[10] The unit root hypothesis for the U.S. inflation rate is rejected at the 5% level of significance by both ADF and DF-GLS tests.

level.

It is worth noting that Hamilton (1996) and Driffill and Sola (1998) recommend using the 1% critical values of the F distribution as a rough guide for the 5% finite-sample critical values in performing LM tests. Based on their suggestion, it is reasonable for us to claim that our empirical results fail to reject the hypothesis of no autocorrelation and of no ARCH effects in estimated residuals at the 5% level of significance if finite sample critical values are used. In sum, findings from diagnostic checks appear to support the appropriateness of our state-space, Markov-switching specification.

Figures 3 and 4 plot the unemployment rate series and the smoothing probability estimated from the model. Figure 4 points out that the U.S. economy was dominated by state 0 during the periods of 1950-1951, 1956, 1962-1969, 1971-1973, 1976-1979, and 1984-2003. According to our model, these periods correspond to when demand shock dominated. It is not surprising to see that the periods with oil price shocks are dominated by supply shocks (state 1). However, as displayed in Figures 3 and 4, several periods before 1962 (with a volatile unemployment rate) are also dominated by supply shocks. In addition, our empirical results indicate that supply shocks have dominating effects on the U.S. unemployment rate during the period 1980-1983, which is close to the periods of Volker's deflation policy and Kemp and Roth's supply-side policy. These findings indicate the importance of including supply shocks in analyzing a Phillips curve.

Figures 5 and 6 present the U.S. inflation rate and its smoothing probability. From Figure 6, we find that inflation is more volatile (state 0) during the periods of 1949-1951, 1973-1983, and 1986 than that in other periods. These periods are basically in accordance with the Korean War, two oil price shocks, financial deregulation starting from 1979, the switch of the Fed's monetary policy target in 1982, and the G-5 Plaza Accord in 1985.

The specification of the natural rate of unemployment in our paper does not forbid negative values. Nevertheless, as shown in Figure 7, the estimated natural rates (bold line) constructed by the smoothing probability, ranging from 3.31 to 6.43 percent, are all positive. Given the estimated standard deviation (σ_n) of 0.117, the conditional probability of observing a negative value for u_t^n is less than 0.001 and thus is negligible.

The main periods when actual unemployment was above its natural rate are 1949-1951, 1954, 1958-1961, 1970-1988, 1991-1994, and after 2002. The oil price shocks in the 1970s caused high inflation and led to a high unemployment rate. However, accommodating monetary policies are crucial in explaining the negative unemployment gaps in the late 1970s. Furthermore, the contractionary fiscal and monetary policies in the early 1980s, along with a global depression, led to another high unemployment gap. More recently, the period from 1995 to 2000 has been the longest period for a negative unemployment gap since WWII. After two decades of high employment, this may reflect the expansion of the U.S. economy in the late 1990s.

Cosimano and Jansen (1988) point out that the high variance of U.S. inflation data before 1954 may be artifact of the data collection procedure for CPI. It therefore is reasonable to conjecture that the finding of ARCH effects in the U.S. inflation rate may be due to the inclusion of data before 1954. To answer the previous question, we exclude data before 1954 and then re-estimate the model to see whether our empirical results are significantly affected by this change.[11] To justify the specification of heteroscedasticity in the inflation equation when data before 1954 are removed, we apply Engle's (1982) LM statistic to examine the existence of ARCH effects in estimated residuals from OLS. The ARCH(1) statistic is 4.313, which is larger than its 5% critical value. This justifies the specification of heteroscedasticity in variance even though the data before 1954 are removed.

From the upper and middle panels of Table 2, it can be shown that the main results of the state-dependent Phillips curve are unaffected. State 0 corresponds to a

[11] We thank a referee for this suggestion.

negative-sloped Phillips curve while state 1 corresponds to a positive-sloped one.[12] Our state-space Markov-switching model explains the unstable Phillips curve relationship for the period 1954Q1-2003Q2.

The mis-specification tests for the case of excluding data before 1954 are reported in the lower panel of Table 2. The Wald statistics indicate that the hypothesis of no Markov chain is strongly rejected. Furthermore, all of the LM tests fail to reject the null hypothesis of no autocorrelation and of no ARCH effects in the estimated residuals at the 5% level of significance. In sum, findings from Table 2 indicate that our empirical findings in Table 1 are not significantly affected if data before 1954 are excluded.

5. Conclusions

In the 1950s and 1960s, economists believed that the slope of the Phillips curve was negative. However, more recent empirical findings indicate that the slope of the Phillips curve depends on the selected sample period. To explain the existing empirical irregularity, King and Watson (1994) argue that the relationship between

[12] We find little difference in the estimated smoothing probability of the unemployment rate rather than that of the inflation rate when data before 1954 are removed. We find additional periods of state 0, except for those shown in Figure 6. They are the periods in 1958-1961, 1989-1990, and 2002-2003, in which the volatility of inflation was mild. We do not report the figures of estimated smoothing probability in the text when data before 1954 are excluded, but they are available upon request from the authors.

unemployment and inflation depends on shocks. In this paper we provide a simple theoretical framework based on an imperfect competitive labor market model to show that the slope of the Phillips curve is shock dependent. The short-run implication of the Phillips curve confirms conventional wisdom that an unexpected monetary expansion (just like a positive demand shock) is helpful for reducing unemployment. In addition, when all the actual variables equal their expected counterparts, a long-run Phillips curve is vertical at the natural unemployment rate and it is consistent with any level of inflation rates.

We apply a state-space, Markov-switching model to examine the impact of inflation surprise on the unemployment gap. After estimating the inflation equation and unemployment equation simultaneously, we obtain important findings as follows. The impact of inflation surprises on unemployment gaps is negative when demand shocks dominate, but it is positive when supply shocks dominate. Therefore, even though an unexpected monetary expansion produces effects in reducing unemployment rates, shocks from the supply side must not be ignored in the estimation of a Phillips curve.

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Table 1. Joint Estimation Results (whole sample)

A. Unemployment Equations: (12)-(16)

σ_n	ρ_1	ρ_2	ϕ_0	ϕ_1	σ_g	p_{00}	p_{11}
0.117*	1.659*	-0.688*	-0.115*	1.005*	0.138*	0.952*	0.924*
(0.027)	(0.064)	(0.053)	(0.043)	(0.225)	(0.034)	(0.023)	(0.041)

B. Inflation Equations: (17)-(19)

λ_0	λ_1	λ_2	λ_3	λ_4	σ_0	σ_1	q_{00}	q_{11}
0.151*	0.581*	0.134*	0.294*	-0.201*	0.766*	0.274*	0.947*	0.976*
(0.026)	(0.047)	(0.056)	(0.053)	(0.045)	(0.076)	(0.020)	(0.036)	(0.016)

C. Mis-specification Tests

	Unemployment equation	Inflation equation
Wald test for i.i.d. switching regression $\sim \chi^2(1)$	263.693**	414.649**
LM test for autocorrelation $\approx$ F(1, 204)	0.635	0.288
LM test for ARCH $\approx$ F(1, 204)	3.051	4.156*

Notes: 1. Numbers in parentheses are standard errors of estimates.

2. * and ** indicate significance at the 5% and 1% levels, respectively.

Table 2. Joint Estimation Results (sub-sample)

A. Unemployment Equation: (12)-(16)

σ_n	ρ_1	ρ_2	ϕ_0	ϕ_1	σ_g	p_{00}	p_{11}
0.113*	1.648*	-0.679*	-0.117[a]	0.831*	0.125*	0.962*	0.936*
(0.021)	(0.062)	(0.051)	(0.066)	(0.190)	(0.027)	(0.026)	(0.035)

B. Inflation Equation: (17)-(19)

λ_0	λ_1	λ_2	λ_3	λ_4	σ_0	σ_1	q_{00}	q_{11}
0.137*	0.555*	0.092*	0.289*	-0.110*	0.499*	0.207*	0.940*	0.942*
(0.027)	(0.050)	(0.073)	(0.061)	(0.053)	(0.045)	(0.020)	(0.039)	(0.033)

C. Mis-specification Tests

	Unemployment equation	Inflation equation
Wald test for i.i.d. switching regression $\sim \chi^2(1)$	311.471**	198.204**
LM test for autocorrelation $\approx$ F(1, 176)	1.423	0.166
LM test for ARCH $\approx$ F(1, 176)	0.526	1.701

Notes: 1. Numbers in parentheses are standard errors of estimates.

2. [a], *, and ** indicate significance at the 10%, 5%, and 1% levels, respectively.

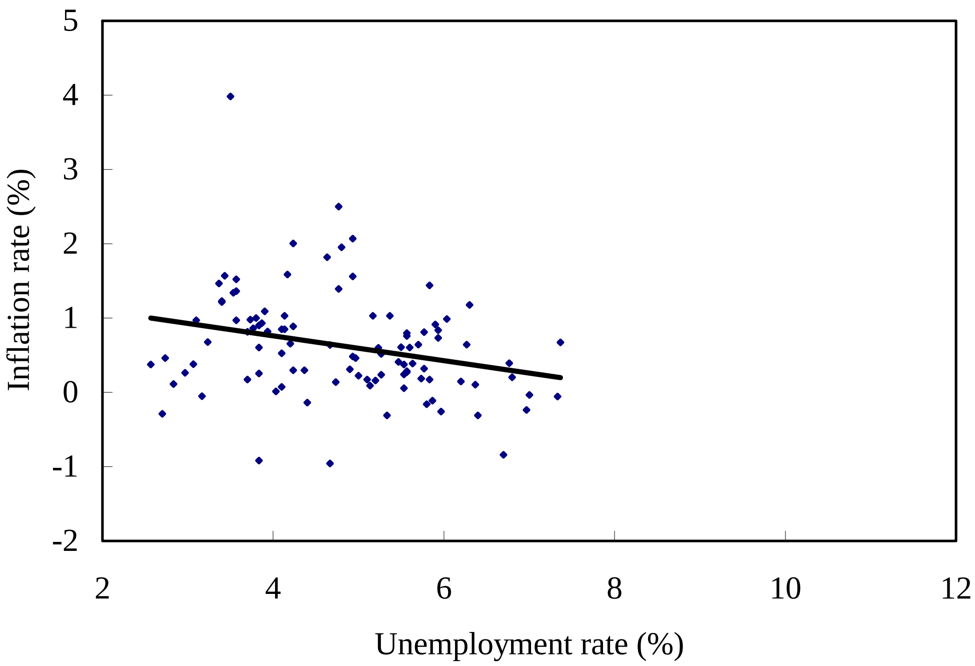

Figure 1. Naive Phillips Curve (1948Q1-1973Q4)

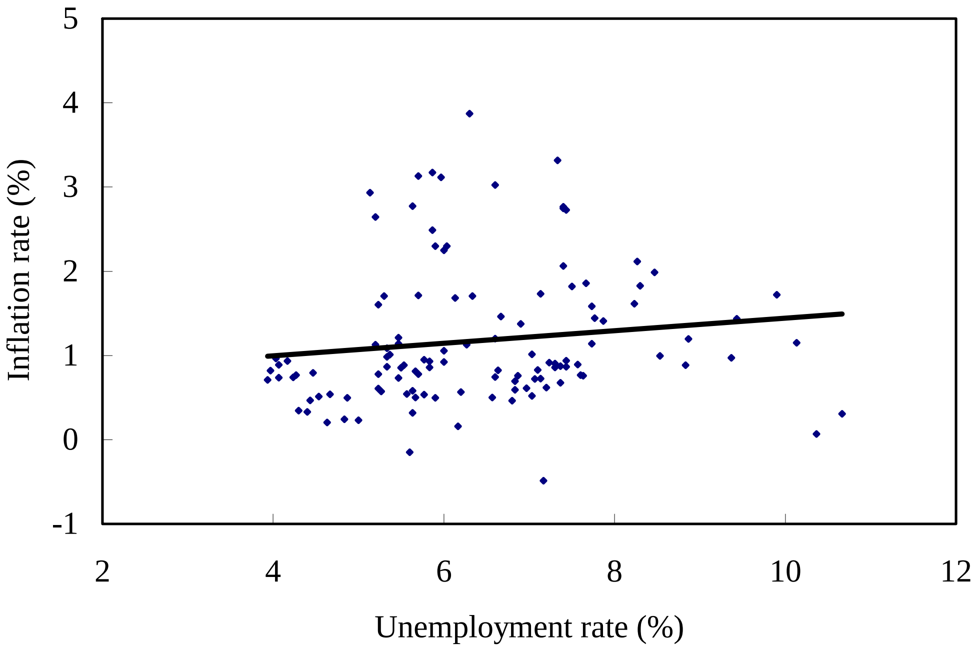

Figure 2. Naive Phillips Curve (1974Q1-2003Q2)

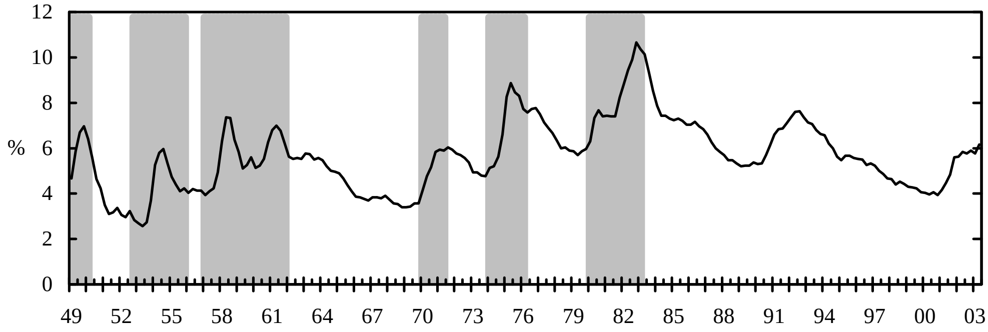

Figure 3. The U.S. Unemployment Rate

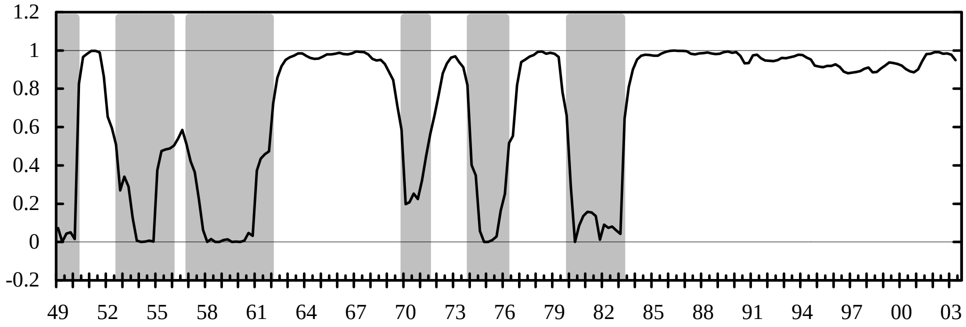

Figure 4. The Smoothing Probability of State 0
(Unemployment rate)

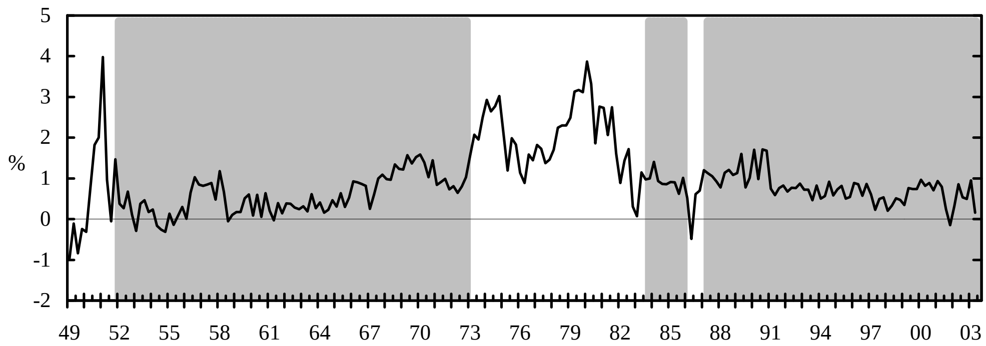

Figure 5. The U.S. Inflation Rate

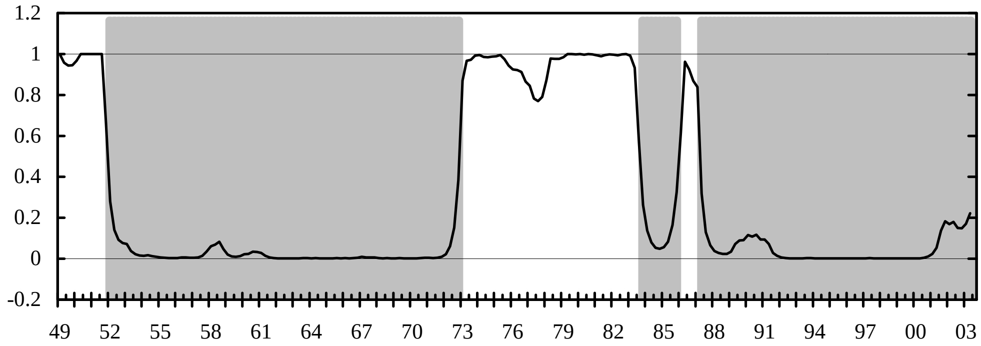

Figure 6. The Smoothing Probability of State 0
(Inflation rate)

Figure 7. The U.S. Unemployment Rate and its Natural Rate