Why Use Markov-switching Models in Exchange Rate Prediction?

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Abstract

A large amount of empirical studies finds the superiority of the regime-switching model in generating the process of exchange rates and in forecasting future exchange rates. This paper justifies the use of Markov-switching models by showing that this kind of time series process is consistent with the most popular exchange rate regime in the world — the dirty floating exchange rate regime. The theoretical implication of exchange rate determination indicates that a higher probability of a central bank's future interventions raises the rational expectations discrepancy between the exchange rate and its fundamentals, even though the bank does not step in the foreign exchange market during that period.

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1. Introduction

Engel and Hamilton (1990)'s pioneering work provides evidence that a Markov-switching model of exchange rates outperforms the random walk one. Plenty of followers use regime-switching models in exchange rate estimation and forecasting, and most of them find that these kinds of models either fit exchange rate data well or generate superior forecasts to a random walk model or other models.[1] A question hence arises: Is there an explicit economic explanation that could account for the Markov-switching process of exchange rates?

It seems natural to begin by building a new model according to the exchange rate regime in reality. The most popular exchange rate arrangement nowadays is a regime between undisputed floats and undisputed pegs. This can be described as a dirty float or a managed float. Many major industrial countries have taken an active role in the management of their currencies since the mid-1980s and many other economies that claim they are floating actually manage their exchange rates, too.[2] For these countries, while their exchange rates are basically allowed to move in response to market forces, their central banks at times do intervene to prevent undesirable movements in the exchange rate or to arrive at some desired currency values if the change in foreign reserves is allowable.

This paper tries to propose a rational expectations model of exchange rate determination under a dirty float regime and to solve the exchange rate process. With the monetary authority's stochastic interventions, the exchange rate sometimes is

[1] See, e.g., Engel (1994) or Kirikos (2000) for a discussion.

[2] "Double fears" (i.e., fear of floating and fear of pegging) make the deed of the exchange rate regime adopted by an economy different from the nominal regime it claims to be; see Calvo and Reinhart (2002) and Levy-Yeyati and Sturzenegger (2002).

endogenously determined by market fundamentals and expectations, while sometimes manipulated by the central bank. Stochastic interventions by the monetary authority change market expectations and therefore the process of exchange rates. The uncertainty of future intervention enlarges the differential between the exchange rate and its fundamentals. Specifically, a higher probability of a central bank's future interventions raises the rational expectations discrepancy between the exchange rate and its fundamentals, even though the bank does not step in the foreign exchange market during that period. The theoretical-implied exchange rate adjustment can then be estimated by a Markov-switching model.

While this article provides a rationale for using Markov-switching models in explaining exchange rate dynamics, the research works in Kirikos (2002, 2004) and Sarno, Valente, and Wohar (2004) appear to be another approach for doing so. In the latter studies, market fundamentals themselves are regime switching. Kirikos (2002, 2004) both consider a policy instrument fundamental, which is the monetary authority's interest rate differential management, and use the uncovered interest parity to derive the exchange rate change process. Sarno, Valente, and Wohar (2004) focus on the deviation of the nominal exchange rate from its fundamental that is assumed to be the differential between the contemporary logarithm of relative money supply and the contemporary logarithm of relative income. However, under the assumption of rational expectations it is not easy to understand why the transition probability is irrelevant in the relation between the exchange rate and its fundamentals for the case of the fundamentals themselves following a Markov-switching process.

Section 2 of this article describes the rational expectations models of exchange rate determination under a dirty floating regime. Specifically, we can consider under-valuation operations that many monetary authorities of export-led Asian countries have adopted. After solving the model, we show that the parameters, especially the constant term and variance, of the exchange rate (or depreciation rate) are state-dependent. The analytical solution therefore justifies estimating the exchange rate process by a regime-switching model. Finally, several concluding remarks are given in the last section.

2. Rational expectations models under dirty floating regime

In both traditional models of exchange rate determination based upon aggregate functions and the recent micro-founded open economy models, an equilibrium exchange rate in the foreign exchange market without central bank intervention can be described as:

$$e_t = \alpha_0 + \beta E_t^n e_{t+1} + \alpha_1 F_t \,, \quad |\beta| < 1 \,, \tag{1}$$

where e_t is the logarithm of the exchange rate, F_t denotes the fundamental variables, and E_t^n is the expectations operator based on the information set when the central bank does not intervene at time *t*. The components of F_t obviously depend on the model employed by researchers. Equation (1) is very general and it describes that the exchange rate in period *t* is affected by economic fundamentals as well as the expectation of the exchange rate in the next period.

Let us consider the exchange rate under a pure float regime as the benchmark. Without loss of generality, assume that F_t follows an AR(1) process as follows:

$$F_t = a + bF_{t-1} + \varepsilon_t \,, \quad |\, b \,| < 1 \,, \tag{2}$$

where ε_t is a serial-uncorrelated disturbance. The exchange rate solved forward is hence as follows:

$$
\begin{aligned}
e_t^* &= \frac{\alpha_0}{1-\beta} + \alpha_1 F_t + \alpha_1 E_t^n \sum_{i=1}^{\infty} \beta^i F_{t+i}, \\
&= c + \frac{\alpha_1}{1-\beta b} F_t,
\end{aligned}
\tag{3}
$$

where e_t^* is the fundamental rate of exchange and the constant term $c \equiv [\alpha_0 + \alpha_1 a\beta/(1-\beta b)]/(1-\beta)$.

In a country with a dirty float regime, at any point of time the central bank switches stochastically between intervening and non-intervening in the foreign exchange market.[3] However, whether or not the central bank (non-) intervenes will depend on a persistently changing economic environment. Thus, it seems better to model the central bank's intervention behavior as a Markov chain rather than to assume independent (non-) interventions.

Let q_0 be the probability that the central bank continuously intervenes in the foreign exchange market at time $t+1$ if it intervenes at time *t*, and let q_1 be the probability that the central bank still allows the exchange rate to freely adjust at time $t+1$ if it does not intervene in the foreign exchange market at time *t*. On the one hand, the expected exchange rate under intervention is the following:

$$
E_t^i e_{t+1} = (1-q_0) E_t^i e_{t+1}^n + q_0 E_t^i e_{t+1}^i,
$$

where e_t^i is the exchange rate under intervention at time *t*, e_t^n is the market-determined exchange rate at time *t*, and E_t^i is the expectations operator based

[3] For example, Taiwan's central bank has this message on its website: "the New Taiwan dollar exchange rate is determined by market forces, which is in line with the policy of exchange rate liberalization. Only when the foreign exchange market is disrupted by seasonal or irregular factors will the Bank step in." However, the central bank never defines what is an irregular factor or to what degree of irregularity it will intervene.

upon the information set when the central bank intervenes at time *t*. On the other hand, in the case of non-intervention the expected exchange rate is as follows:

$$E_t^n e_{t+1} = (1-q_1)E_t^n e_{t+1}^i + q_1 E_t^n e_{t+1}^n. \tag{4}$$

There are many countries conducting direct intervention, especially for those countries whose currency is not a major trading one. Direct interventions in the foreign exchange market are practically more effective in influencing the terms of trade rather than controls of other monetary policy targets. In fact, many central banks of export-led countries step in the foreign exchange market and keep their currencies undervalued in order to stimulate their exports and economic growth. For example, Yang and Shea (2005) summarized that the central bank of Taiwan has in mind a strategy of repressing the New Taiwan dollar to promote exports.

Assume that when the central bank intervenes, it sets its target as follows:

$$e_t^i = d + e_t^*, \tag{5}$$

where *d* is a discretionary parameter. If the central bank wants its money to be under-valued/over-valued, then the parameter *d* is positive/negative; otherwise, if the central bank just wants to target the value of its money to be consistent with economic fundamentals, then *d* is zero. Moreover, a disturbance u_t can be added to equation (5) for empirical purpose and to capture the fact that a discretionary central bank pegs its parity with a band.

If the central bank does not intervene, then the market-determined exchange rate (e_t^n) affected by the market fundamentals and the expected exchange rate for the next period is:

$$e_t^n = \alpha_0 + \beta E_t^n e_{t+1} + \alpha_1 F_t. \tag{1'}$$

Combining equations (1'), (2), (4), and (5), we can solve the equilibrium exchange rate under the dirty floating regime:[4]

$$e_t^n = \frac{\beta(1-q_1)d}{1-\beta q_1} + e_t^* . \tag{6}$$

Note that in the stochastic intervention model the equilibrium exchange rate differs from its fundamental rate even though the central bank does not intervene in the foreign exchange market, which is a fact that has been neglected in empirical studies. The uncertainty of future intervention enlarges the differential between the exchange rate and its fundamentals. Specifically, a higher probability of a central bank's future interventions raises the rational-expectations discrepancy between the exchange rate and its fundamentals, even though the bank does not step in the foreign exchange market during that period.

A disturbance term v_t - which may be interpreted as measurement errors, fads, or even rational bubbles - may in practice be added to the equilibrium exchange rate equation. Evans (1991) shows that even when v_t is a periodically collapsing bubble, asset prices will not appear to be more explosive than their fundamentals. Assume that intervention and non-intervention states follow a first-order Markov chain. A regime-switching model can thus be applied to estimate the exchange rate dynamics. In the simple case where F_t is an AR process and u_t and v_t are white noises, measured e_t^n has the same AR coefficients as e_t^i except for constant terms and variances.

Most researchers estimate the regime-switching model for the change of the logarithmic exchange rates. This can be rationalized by considering non-stationary fundamentals. If a discretionary central bank pursues a more rapid or slowly

[4] The details for the derivation are in the Appendix.

devaluation rate, say,

$$\Delta e_t^i = d_1 + \Delta e_t^*, \tag{7}$$

then it is easy to show that

$$e_t^i = d_0 + d_1 t + e_t^*, \tag{7'}$$

where $d_0 = e_0^i - e_0^*$. Assuming that ΔF_t is an AR(1) process and after combining equations (1'), (4), and (7'), the equilibrium exchange rate under the dirty floating regime is solved forward as

$$e_t^n = d_0' + d_1' t + e_t^*.$$

Rewriting this can be described by:

$$\Delta e_t^n = d_1' + \Delta e_t^*, \tag{8}$$

where d_0' and d_1' are functions of the structural parameters.

For empirical purpose, disturbance terms u_t and v_t can be added to equations (7) and (8), respectively. Note that the depreciation rate of exchange does switch between two linear stochastic processes, depending on the presence or absence of intervention. In the simple case where F_t follows an integrated AR process and disturbances u_t and v_t are white noises, measured Δe_t^n has the same AR coefficients as Δe_t^i except for constant terms and variances. In the more general cases for F_t, u_t, and v_t, the depreciation rates Δe_t^n and Δe_t^i follow different ARMA processes and may be approximated by autoregressive representations with different lag orders and different coefficients. Empirically, any ARIMA process can be approximated by an integrated AR process with a lag length that is sufficiently long.

3. Concluding remarks

This paper justifies the use of Markov-switching models by showing that the empirical exchange rate process is indeed implied by the most popular exchange rate regime in the world — the dirty floating exchange rate regime. We build rational expectations models of the dirty float regime and derive the law of motion for the exchange rate. The theoretical process of the exchange rate ties closely into its empirical time-series process. Moreover, the implied exchange rate process is state-dependent and approximated by an autoregressive representation in each state. Differences between the exchange (depreciation) rate time series processes with and without central bank interventions can lie solely in the constant term and variance or in every regression coefficient and variance, depending on the complexity of the underlined models.

One attached merit of this article is that it provides a method for detecting a central bank's direct intervention behaviors. Unlike monetary authorities in a few industrialized countries that have recently released their high frequency data for intervention, most central banks do not reveal their intervention data. When intervention data are not available, the estimated smooth probability of the regime-switching model provides a good inference for whether a central bank has conducted interventions in a specific period or not.

The univariate Markov-switching model of exchange rates in this paper can obviously be extended. If we specify the fundamental variables (F_t) according to some specific theoretical model, then a simultaneous-equation model with observed fundamentals or a state-space model with unobserved fundamentals such as in Kim (1994) can be established. In addition, when embodying country-specific and more sophisticated intervention mechanisms, this kind of Markov-switching model could help capture some special characteristics of the considered exchange rates. However, such an extension may need more mathematical derivations, and possible empirical improvements will not be arrived at without any cost.

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Appendix

This appendix is presented for the derivation of the equilibrium exchange rate under the dirty floating regime in equation (6).

Firstly, substituting equations (4), (5), and (2) recursively into equation (1'), we have the relation for the exchange rate under a central bank's non-intervention:

$$
\begin{aligned}
e_t^n &= \alpha_0 + \beta E_t^n e_{t+1} + \alpha_1 F_t \\
&= \alpha_0 + \beta\left[(1-q_1)E_t^n e_{t+1}^i + q_1 E_t^n e_{t+1}^n\right] + \alpha_1 F_t \\
&= \alpha_0 + \beta\left[(1-q_1)\left(d + E_t^n e_{t+1}^*\right) + q_1 E_t^n e_{t+1}^n\right] + \alpha_1 F_t \\
&= \alpha_0 + \beta(1-q_1)d + \beta(1-q_1)\left(c + \frac{\alpha_1}{1-\beta b} E_t^n F_{t+1}\right) + \beta q_1 E_t^n e_{t+1}^n + \alpha_1 F_t \\
&= \alpha_0 + \beta(1-q_1)\left(d + c + \frac{\alpha_1 a}{1-\beta b}\right) + \beta q_1 E_t^n e_{t+1}^n + \alpha_1\left[1 + \frac{\beta(1-q_1)b}{1-\beta b}\right]F_t .
\end{aligned}
$$

Since the eigen root for the linear expectational difference equation lies out of the unit circle, the rational expectations solution for the exchange rate can be solved forward as:

$$
e_t^n = \frac{\kappa}{1-\beta q_1} + \alpha_1 \frac{1-\beta q_1 b}{1-\beta b} E_t^n \sum_{i=0}^{\infty} (\beta q_1)^i F_{t+i} ,
$$

where $\kappa \equiv \alpha_0 + \beta(1-q_1)(d + c + \alpha_1 a/(1-\beta b)) = \alpha_0 (1-\beta q_1)/(1-\beta) + \alpha_1 a\beta(1-q_1)/$ $(1-\beta)/(1-\beta b) + \beta(1-q_1)d$.

To further simplify the solution of the exchange rate we need to know the process of the fundamentals. Using equation (2), it is easy to show that:

$$
E_t^n \sum_{i=0}^{\infty} (\beta q_1)^i F_{t+i} = \frac{\beta q_1 a}{(1-\beta q_1)(1-\beta q_1 b)} + \frac{F_t}{1-\beta q_1 b} .
$$

Substituting the present value of the fundamentals into the above exchange rate equation we get

$$
\begin{aligned}
e_t^n &= \frac{\kappa}{1-\beta q_1} + \frac{\beta q_1 a}{1-\beta q_1}\frac{\alpha_1}{1-\beta q_1 b}\frac{1-\beta q_1 b}{1-\beta b} + \alpha_1 \frac{1-\beta q_1 b}{1-\beta b}\frac{F_t}{1-\beta q_1 b} \\
&= \frac{\alpha_0}{1-\beta} + \frac{\beta(1-q_1)d}{1-\beta q_1} + \frac{\beta\alpha_1 a}{(1-\beta b)(1-\beta)} + \alpha_1 \frac{F_t}{1-\beta b} \\
&= \frac{\beta(1-q_1)d}{1-\beta q_1} + e_t^* .
\end{aligned}
$$