To Merge or to License:  
Implications of Information Sharing for  
Optimal Merger Policy  

by  

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Abstract  
We investigate the antitrust authority’s optimal merger policy in a duopoly model with cost asymmetry and asymmetric information regarding uncertain demand. Technology can be transferred either through a merger or a license, while market information can be shared only through a merger. We show that the optimal merger policy differs under Cournot and Bertrand competition. If firms compete in Bertrand fashion, mergers should never be allowed. If firms compete in Cournot fashion, there are three cases that depend on the degree of market fluctuations and the size of innovations. Specifically, if market volatility is low, mergers should not be allowed; if volatility is high, mergers should be allowed; and if volatility is in the intermediate range, mergers should be allowed only for large innovations. The driving force for our results lies in the opposing welfare effects of information sharing under price-setting and quantity-setting regimes.  

JEL classification: L44, L24, L13, D80  

Keywords: Horizontal mergers, Patent licensing, Information sharing, Demand uncertainty  

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1 INTRODUCTION

Mergers often enable firms to share their production technologies and market information. For example, the recent merger of Fiat and Chrysler, which was backed by the United States government, allows Fiat to obtain a large distribution network in America, while Fiat would make available to Chrysler its advanced, fuel-efficient technologies.\(^1\) The $7.4 billion acquisition of Sun Microsystems let Oracle gain ownership of two key Sun technologies, Java and Solaris, in addition to both sharing their existing customer bases and market demands. Amazon bought its rival, Zappos.com, for Zappos’s marketing expertise in online footwear retailing, while Zappos obtained access to Amazon’s resources and technology.\(^2\) The seemingly ideal mergers between a firm with better technologies and a firm with better market information are presumably profitable for the merging firms. The key antitrust question is whether or not it is also beneficial for social welfare.

Antitrust practitioners know very well that as far as technology transfer is concerned, mergers and licensing are two alternative means. For example, the U.S. Horizontal Merger Guidelines explicitly state: “The agency will not deem efficiencies to be merger specific if they could be preserved by practical alternatives that mitigate competitive concerns, such as … licensing.”\(^3\) In light of the view expressed in the Guidelines, the insightful study of Fauli-Oller and Sandonis (2003) investigates the welfare effects of mergers and licensing under Cournot and Bertrand competition, concluding that regardless of the competition style, mergers should be forbidden whenever two-part tariff licensing (i.e., licensing with a fee and per-unit royalty) is feasible. Their result justifies the view of the Guidelines. Interestingly, the subsequent contributions of González-Maestre and Peñarrubia (2005) and González-Maestre

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\(^1\) See, for example, The Economist 05/09/2009, p. 71.

\(^2\) See, for example, July 23, 2009, Wall Street Journal.

\(^3\) See, for example, Section 10 “Efficiencies” (footnote 13) of the 2010 U.S. Horizontal Merger Guidelines.
find in the context of Cournot competition that, if the superior technology to be licensed is not exogenously given but rather endogenously determined, then mergers may be welfare superior to licensing. As such, the view of the Guidelines might be too restrictive under Cournot competition.

The purpose of this paper is to extend this line of antitrust analyses from a deterministic environment to a world with market uncertainty, such that we can evaluate the welfare effects of mergers that allow the merging firms to share not only technologies but also market information. In particular, we consider a model in which technology transfer can occur either through a merger or a license, while market information can be shared only via a merger. Thus, the present paper is related to two strands of literature on mergers: (1) the above studies that compare the welfare effects of mergers and licensing under certainty; and (2) the information sharing implications of horizontal mergers, such as Gal-Or (1988), Stenbacka (1991), Wong and Tse (1997), Qiu and Zhou (2006), and Banal-Estañol (2007).

We find that even if two-part tariff licensing is feasible, the optimal merger policy differs under Cournot and Bertrand competition. In particular, if firms compete in Bertrand fashion, then mergers should never be allowed, whereas if firms compete in Cournot fashion, there are three cases depending on the degree of market fluctuations and the size of innovations. More precisely, if the degree of market fluctuations is small, then mergers should not be allowed; if the degree of market fluctuations is large, then mergers should be allowed; and if the degree of market fluctuations is in the intermediate range, then mergers should be allowed only for large innovations.

Our results thus yield interesting policy recommendations that can be compared to the contributions of the former antitrust studies. First, in contrast with Fauli-Oller and Sandonís (2003), we find that the competition style matters, and the view expressed in the Guidelines might be too restrictive for industries competing in Cournot fashion, yet be suitable for
industries competing in Bertrand fashion. Hence, we modify the results of Faulí-Oller and Sandonis (2003) under the consideration of information sharing taken into account. Second, similar to the contributions of González-Maestre and Peñarrubia (2005) and González-Maestre (2008), we also find conditions under which mergers are permissible when firms compete in Cournot fashion. In these two papers’ models, mergers may be welfare superior to licensing, because under some circumstances mergers lead firms to choose socially more desirable levels of endogenous innovations. In our model mergers may be welfare superior to licensing, because of the positive welfare effect of information sharing under quantity competition.

To understand the intuitions behind our results, it is helpful to think of firms’ responses upon observing “good news” about the demand. When outputs are the decision variables, only the informed firm increases its output if there is no information sharing, whereas both firms increase their outputs with information sharing. Thus, information sharing is good for welfare in the quantity-setting regime. By contrast, when prices are the decision variables, only the informed firm increases its price without information sharing, whereas both firms increase their prices with information sharing. Thus, information sharing is bad for welfare in the price-setting regime. Interestingly, these intuitions are entirely consistent with Vives (1984), who considers both a quantity-setting and price-setting differentiated duopoly and finds that the social value of information is positive under Cournot competition, but negative under Bertrand competition.

Gal-Or (1988) examines firms’ incentives for mergers (instead of the welfare effect of mergers) in a model with $n$ firms producing differentiated products and facing stochastic demand. She finds that under Cournot competition, firms have fewer incentives to merge

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4 She extends the seminal works by Salant et al. (1983) and Deneckere and Davidson (1985) in the contexts of Cournot and Bertrand competition, respectively, from a deterministic environment to an uncertain world with stochastic demand.
under uncertainty than under certainty, because mergers generate smaller informational advantages to insiders than outsiders. By contrast, under Bertrand competition, firms have even more incentives to merge under uncertainty, because mergers generate greater informational advantages to insiders than outsiders. While the focus of our paper differs from that of Gal-Or (1988), there is one thing in common: when the effects of information sharing under positive and negative shocks go in opposite directions, the effect of positive shocks dominates.\(^5\)

Qiu and Zhou (2006) consider an information structure different from that of Gal-Or (1988) and present that asymmetric information does create incentives for Cournot firms to merge. While our specification of demand uncertainty is in the spirit of Qiu and Zhou (2006), our study departs from theirs in two important respects. First, they assume zero production costs, while we consider cost asymmetry due to patentable process innovations. Thus, mergers in their model only allow firms to coordinate actions and share information, while mergers in our model further play the important role of technology transfer. Second, they focus on Cournot competition, while we consider both Cournot and Bertrand competition, such that we are able to derive contrasting implications of these two behavioral modes for an optimal merger policy.\(^6\)

Banal-Estañol (2007) considers a Cournot model in which firms face idiosyncratic uncertainty about costs. He finds a very interesting result that if information is privately observed, then mergers are socially less harmful due to informational gains. The present paper is complementary to the inspiring work of Banal-Estañol (2007) in two ways. First,

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\(^5\) We will have a further discussion in Section 5 about why this is the case in our model. We are grateful to an anonymous referee for suggesting us to refer to Gal-Or (1988) in order to explain contrasting results under Cournot and Bertrand in our model.

\(^6\) Qiu and Zhou (2006, p.54) explain why they employ a Cournot model. It is well known in the literature that in the absence of information sharing, a merger is profitable for merging firms under Bertrand competition, but not necessarily profitable under Cournot competition. To show that information sharing can make a difference, they need to employ a Cournot model, since mergers are profitable under Bertrand competition whether or not information is asymmetric.
under Cournot competition, we reinforce Banal-Estañol’s (2007) welfare result by considering a different kind of uncertainty namely, uncertainty regarding a common demand intercept. Second, by examining both Cournot and Bertrand competition, we find that the positive welfare result shown by Banal-Estañol (2007) under Cournot competition does not arise under Bertrand competition when uncertainty is about the demand intercept.

The remainder of the paper is organized as follows. Section 2 sets up our model, which is inspired by Faulí-Oller and Sandonís (2003), but with the introduction of uncertainty and asymmetric information. Section 3 characterizes equilibrium outputs and prices under Cournot and Bertrand competition, respectively. Section 4 analyzes the optimal licensing contract and firms’ incentives to merge. Section 5 investigates the optimal merger policy and derives our main results. Section 6 concludes with some remarks. All proofs are provided in Appendix A.

2 THE MODEL

We consider an industry with two firms, 1 and 2, each producing a differentiated product. We denote firm $i$’s output and price by $q_i$ and $p_i$, respectively, where $i = 1, 2$. The duopoly’s inverse demand functions are given by:

$$p_i = 1 + \theta - q_i - bq_j, \quad i, j = 1, 2, i \neq j,$$

where the parameter $b \in (0,1)$ denotes the degree of product differentiation between goods 1 and 2, and $\theta$ is a random variable capturing the common demand uncertainty facing the duopoly. Assume that $\theta$ is distributed over $[\theta_L, \theta_U]$, where $\theta_L < 0 < \theta_U$, with $E(\theta) = 0$ and $Var(\theta) = \sigma^2 = E(\theta^2)$. Note that the magnitude of $\sigma^2$ captures the degree of demand uncertainty.

As in Vives (2002) and Banal-Estañol (2007), we will impose further assumptions on $\theta$ later, so as to ensure
fluctuations in the product market. A larger value of $\sigma^2$ is associated with a higher volatility in the market.

To evaluate the welfare effects of a merger that plays the roles of information sharing and technology transfer, we make the following assumptions. First, before production takes place, only firm 2 is fully informed of the realization of $\theta$, while firm 1 is not unless it has merged with firm 2. Second, firm 2 has a constant marginal cost of production, $c$, where $c > 0$, while firm 1 has a proprietary cost-reducing technology, which allows it to produce good 1 at a lower marginal cost, which we assume, without loss of generality, to be zero. In the status quo of our model, firm 1 is more efficient in production, while firm 2 has an advantage in market information.

Note that the magnitudes of $\sigma^2$ and $c$ are related to the extents of asymmetries between these two firms in the status quo. In particular, larger values of $\sigma^2$ are associated with larger extents of asymmetric information between the duopoly (since if $\sigma^2 = 0$, then there is no asymmetric information at all). The parameter $c$ represents not only firm 2’s cost, but also the magnitude of cost reduction associated with firm 1’s superior technology. Thus, larger values of $c$ can be interpreted as larger innovations.

In passing, we would like to emphasize that the above modeling of one firm having better technology and the other firm having better market information is for illustrative purposes, but not restrictive. As we show in Appendix B, our results are robust to an alternative modeling environment in which one firm (say firm 1) simultaneously has better technology and better market information.

We model interactions between the antitrust authority and the duopoly as a three-stage game. In the first stage, the antitrust authority decides whether or not to allow a merger between the two firms. In the second stage, these two firms have the possibility to merge (if that both firms or plants remain active for all realizations of $\theta$.
allowed in the first stage) or to sign a two-part tariff licensing contract for technology transfer. Following former studies such as Fauli-Oller and Sandonis (2003), González-Maestre and Peñarrubia (2005), and González-Maestre (2008), we depict the licensing game as: First, firm 1 offers a contract, \((f, r)\), to firm 2 on a take-it-or-leave-it basis, where \(r\) is a per-unit charge and \(f\) is a flat lump-sum fee, and then firm 2 decides whether or not to accept the contract.\(^8\) During the transition from the second to the third stage, the uncertainty regarding \(\theta\) is realized. Firm 2 learns the exact value of \(\theta\), whereas firm 1 remains ignorant unless it has merged with firm 2. Note that firm 1 can acquire information about \(\theta\) only through a merger with firm 2, while firm 2 can acquire firm 1’s superior technology either via a merger or by a licensing contract.\(^9\) Given the cost functions and possible asymmetric information about \(\theta\) inherited from the second stage, production takes place in the third stage, in which we will consider both Cournot and Bertrand competition.

It has been well known from Singh and Vives (1984) that the linear inverse demand functions given in (1) are derived from the optimization problem of a representative consumer seeking to maximize \(U(q_1, q_2) = p_1 q_1 - p_2 q_2\), where the utility function \(U\) takes the following quadratic form in our model:

\[
U(q_1, q_2) = (1 + \theta)(q_1 + q_2) - \frac{1}{2} q_1^2 - \frac{1}{2} q_2^2 - b q_1 q_2.
\]

\(^8\) We consider two-part tariff contracts for two reasons. First, in an earlier version of the paper we allow firm 1 to choose among fixed-fee, royalty, and two-part tariff contracts. It is not surprising that the two-part tariff contract dominates the other two and is chosen endogenously. Second, according to Rostocker (1984), a two-part tariff is the most prevalent form of licensing as compared to a fixed fee or royalty alone. Other empirical studies such as Calvert (1964), Taylor and Silberston (1973), and Macho-Stadler et al. (1996) also document support for the prevalence of two-part tariff licensing.

\(^9\) We assume this in order to focus on the effects of information sharing and compare our results with Fauli-Oller and Sandonis (2003). Banal-Estañol (2007) also assumes that firms cannot credibly exchange their private information or transmit it to their competitors, so that the only credible commitment device for sharing information is a merger. He convincingly argues that in practice alliances among competing firms to share private information may be illegal, and that the revelation of information is difficult to verify. He shows (p. 43) that if firms can choose between mergers and informational coalitions, then they will prefer to merge.
We assume that the antitrust authority’s objective function is to maximize social welfare, which is defined as the sum of consumer surplus and the total profits in the industry. Because firm 1’s marginal cost is zero, social welfare can be rewritten as:

\[ W(q_1, q_2) = U(q_1, q_2) - c_2 q_2, \]  

(3)

where \( c_2 = 0 \) if a merger occurs or a licensing contract is signed, and \( c_2 = c \) otherwise. From (1), we derive the following demand functions:

\[ q_i(p_i, p_j) = \frac{1 + \theta}{1 + b} - \frac{p_i}{1 - b^2} + \frac{bp_j}{1 - b^2}, \quad i, j = 1, 2, i \neq j. \]  

(4)

The appropriate equilibrium concept for the entire three-stage game is that of the subgame-perfect Nash equilibrium. However, unless a merger occurs in the second stage, the equilibrium in the third stage is that of the Bayesian Nash equilibrium, because firms 1 and 2 possess asymmetric information regarding the common demand intercept. It is worth mentioning that the solution concepts employed herein are similar to those employed in Vives (1984). In what follows, we will solve the model backwards by first analyzing equilibrium outputs and prices in the third stage, followed by licensing and merger decisions in the second stage, and then the authority’s optimal merger policy in the first stage.

3 EQUILIBRIUM OUTPUTS AND PRICES IN THE THIRD STAGE

3.1. The merged entity

As in Faulí-Oller and Sandonís (2003) and Qiu and Zhou (2006), when two firms merge, the merged entity coordinates the outputs and prices of its two differentiated products in order to maximize its total profits. Given that the merging firms can share their information and technology, the merged entity solves:
\[
\max_{q_1, q_2} \pi^M = (1 + \theta - q_1 - b q_2)q_1 + (1 + \theta - q_2 - b q_1)q_2. \tag{5}
\]

Solving (5) yields the following equilibrium outputs, \(q^M_i\), prices, \(p^M_i\), and total profits, \(\pi^M\):

\[
q^M_i = \frac{1 + \theta}{2(1 + b)}, \quad p^M_i = \frac{1 + \theta}{2}, \quad \pi^M = \frac{(1 + \theta)^2}{2(1 + b)}, \quad i = 1, 2
\tag{6}
\]

### 3.2. Cournot competition

We now consider the case where a merger does not occur, but a licensing contract, \((f, r)\), is signed in the second stage, with firms competing in Cournot fashion in the third stage. Given that \(f\) is irrespective of firm 2’s output and \(r\) is the per-unit charge based on firm 2’s output, the duopoly’s maximization problems in the third stage are:

\[
\begin{align*}
\max_{q_1} & \quad E[(1 + \theta - q_1 - b q_2)q_1 + r q_2], \\
\max_{q_2} & \quad [(1 + \theta - q_2 - b q_1) - r]q_2.
\end{align*} \tag{7}
\]

Note that the expectation operator \(E(\cdot)\) enters firm 1’s but not firm 2’s problem, because firm 2 is fully informed of \(\theta\) when choosing \(q_2\). Solving simultaneously yields the following:

\[
\begin{align*}
q^C_1(r) &= \frac{2 - b + b r}{4 - b^2}, \quad q^C_2(r) = \frac{2 - b - 2 r + \theta}{2}, \\
p^C_1(r) &= \frac{2 - b + b r}{4 - b^2} + \frac{(2 - b) - r}{2} \theta, \quad p^C_2(r) = \frac{2 - b + r (2 - b^2)}{4 - b^2} + \frac{\theta}{2}, \\
\pi^C_1(r) &= p^C_1(r)q^C_1(r), \quad \pi^C_2(r) = [p^C_2(r) - r]q^C_2(r),
\end{align*} \tag{8}
\]

where the superscript \(C\) stands for Cournot, \(\pi^C_1(r)\) is firm 1’s production profits excluding licensing revenues, and \(\pi^C_2(r)\) is firm 2’s profits net of royalty payments. Note that firm 1’s output is independent of \(\theta\), while firm 2’s output is positively correlated with \(\theta\), because firm 2 knows the realization of \(\theta\), while firm 1 only knows \(\theta = E(\theta) = 0\) when choosing the outputs.

If neither a merger nor licensing occurs in the second stage, then the status quo prevails
with \( c_2 = c \). Substituting \( c \) for \( r \) in (8) yields the status quo Cournot equilibrium:

\[
q_{1,\text{sq}}^{C} = \frac{2-b+b\,c}{4-b^2}, \quad q_{2,\text{sq}}^{C} = \frac{2-b-2\,c}{4-b^2} + \frac{\theta}{2},
\]

\[
p_{1,\text{sq}}^{C} = \frac{2-b+b\,c + (2-b)\,\theta}{4-b^2} + \frac{\theta}{2}, \quad p_{2,\text{sq}}^{C} = \frac{2-b+c(2-b^2)}{4-b^2} + \frac{\theta}{2},
\]

\[
\pi_{1,\text{sq}}^{C} = p_{1,\text{sq}}^{C} q_{1,\text{sq}}^{C}, \quad \pi_{2,\text{sq}}^{C} = (p_{2,\text{sq}}^{C} - c)q_{2,\text{sq}}^{C},
\]

where the second superscript, \( \text{sq} \), is for the status quo. As in Vives (2002) and Banal-Estañol (2007), to avoid boundary problems in which firms become inactive under some realizations of \( \theta \), we impose further assumptions on the relationships among our parameters. In particular, to ensure that in the status quo Cournot equilibrium the high-cost firm 2 remains active for all realizations of \( \theta \), we require that

\[ q_{2,\text{sq}}^{C}(\theta = \theta_{L}) > 0, \]

which holds iff \( c < (2-b)/2 + \theta_{L}(4-b^2)/4 \). Note that \( \overline{\theta} \) > 0 iff \( \theta_{L} > -2/(2+b) \). Thus, we assume \( c < \overline{\theta} \) and \( \theta_{L} > -2/(2+b) \). In other words, firm 2’s cost cannot be too high and the worst state cannot be too negative, such that firm 2 always finds it optimal to produce a positive amount in the status quo.11

### 3.3. Bertrand competition

We now perform a similar analysis for the Bertrand regime. With a two-part tariff contract signed in the second stage, the maximization problems in the third stage are:

\[
\begin{align*}
\max_{p_1} & \quad E[p_1 q_1(p_1, p_2) + r q_2(p_1, p_2)], \\
\max_{p_2} & \quad [p_2 q_2(p_1, p_2) - r q_2(p_1, p_2)],
\end{align*}
\]

where the demand function \( q_i(p_1, p_2) \) is given in (4). Note that, unlike in the Cournot regime, firm 1’s decision variable here, \( p_1 \), affects not only its expected production profits, but also the profits of firm 2. The maximization problems in the status quo are \( \max E[(1 + \theta - q_1 - b q_2) q_1] \) and \( \max (1 + \theta - q_1 - b q_1 - c) q_2 \) for firms 1 and 2, respectively. Firm 1’s best response function here is the same as that for (7), because the additional term in the maximand of (7), \( r q_2 \), does not depend on \( q_1 \) and will disappear in its first-order condition. Firm 2’s best response function here is similar to that for (7), except that we have \( c \) instead of \( r \).

\[11 \text{ With } b \in (0, 1), \text{ the assumption for } \theta_{L} > -2/(2+b) \text{ implies that } \theta_{L} > -2/3. \text{ Given } -2/3 < \theta_{L} < 0, \text{ we have } \inf \overline{\theta} = (1-b)(2-b)/6 \text{ and } \sup \overline{\theta} = (2-b)/2. \]
\[ E[p_1q_1(p_1, p_2)], \text{ but also the expected royalty revenue, } E[rq_2(p_1, p_2)]. \text{ Therefore, as} \]

Fauli-Oller and Sandonis (2003) insightfully point out, firm 1 now has an incentive to set a higher price so as to increase firm 2’s demand and the resulting licensing revenue, \( rq_2(p_1, p_2). \) Thus, the employment of a royalty in the Bertrand regime not only softens price competition by raising firm 2’s marginal cost, but also provides the two firms with a collusive device in setting prices. Solving simultaneously yields the following results with the superscript \( B \) for Bertrand:

\[
P_1^B(r) = \frac{2 - b - b^2 + 3b}{4 - b^2}, \quad P_2^B(r) = \frac{2 - b - b^2 + (2 + b^2)r + (1-b)\theta}{2}, \quad \\
q_1^B(r) = \frac{(2 + b) - (1 + b)br}{(1 + b)(4 - b^2)} + \frac{(2 + b)\theta}{2(1 + b)}, \quad q_2^B(r) = \frac{(2 + b) - 2(1 + b)r}{(1 + b)(4 - b^2)} + \frac{\theta}{2(1 + b)}, \quad (11) \\
\pi_1^B(r) = p_1^B(r)q_1^B(r), \quad \pi_2^B(r) = [p_2^B(r) - r] q_2^B(r). \]

Again, note that firm 1’s equilibrium price does not vary with \( \theta \) while firm 2’s price does.

If neither a merger nor licensing occurs in the second stage, then the duopoly solves:

\[
\max_{p_1} E[p_1q_1(p_1, p_2)], \\
\max_{p_2} [p_2q_2(p_1, p_2) - cq_2(p_1, p_2)]. \quad (12)
\]

Solving simultaneously yields the following Bertrand status quo equilibrium:

\[
P_1^{B, sq} = \frac{(1 - b)(2 + b) + b}{4 - b^2}, \quad P_2^{B, sq} = \frac{(1 - b)(2 + b) + 2c + (1-b)\theta}{4 - b^2}, \quad \\
q_1^{B, sq} = \frac{(1 - b)(2 + b) + b}{(1 - b^2)(4 - b^2)} + \frac{(2 + b)\theta}{2(1 + b)}, \quad q_2^{B, sq} = \frac{(1 - b)(2 + b) - (2 - b^2)c}{(1 - b^2)(4 - b^2)} + \frac{\theta}{2(1 + b)}, \quad (13) \\
\pi_1^{B, sq} = p_1^{B, sq}q_1^{B, sq}, \quad \pi_2^{B, sq} = (p_2^{B, sq} - c) q_2^{B, sq}. \]

Note that, unlike in the Cournot regime, the status quo Bertrand prices here, \( p_1^{B, sq} \), are no longer analogous to \( p_1^B(r) \) with \( r \) replaced by \( c \), because the strategic effect of \( p_1 \) on firm
1’s licensing revenues is present under two-part tariff licensing (refer to (10)), but absent in the status quo (see (12)). We have \( q_{z,\text{sq}}^b(\theta = \theta_L) > 0 \iff c < (2 - b^3)/(2 - b^3) \) +
\[(1 - b)(4 - b^3)\theta_L / 2(2 - b^3) \equiv \bar{c}^b, \text{ where } \bar{c}^b > 0 \iff \theta_L > -2/(2 - b).\] Thus, for firm 2 to remain active in the status quo under all realizations of \( \theta \) we need to impose an upper bound, \( \bar{c}^b \), on firm 2’s cost, as well as a lower bound, \(-2/(2 - b)\), on the worst state.\(^{12}\)

4 EQUILIBRIUM OUTCOME IN THE SECOND STAGE

We first consider the licensing game in the second stage. Under two-part tariff licensing, both instruments, \( f \) and \( r \), are adopted to extract the increase in profits for the licensee generated by the use of the efficient technology. Thus, the patent holder solves:

\[
\max_{r,f} E[\pi_j^f(r) + r q_{z,j}(r) + f]
\]
\[
\text{s.t. } f \leq \pi_j^f(r) - \pi_{z,j}\text{sq,}
\]
\[
r \leq c,
\]

where \( j = C \) or \( B \). Note that the second constraint implies that the right-hand side of the first constraint is non-negative. It is straightforward to see that the first constraint is binding at the optimal solution. Thus, we can rewrite the above maximization problem as:

\[
\max_r E[\pi_j^f(r) + r q_{z,j}(r) + \pi_j^f(r) - \pi_{z,j}\text{sq}]\]
\[
\text{s.t. } r \leq c.
\]

For Cournot competition, by using the expressions in (8) and (9), we can solve (15) as:

\[
r^* = \min\{c, r^C\}, \text{ where } r^C = \frac{b(2 - b^2)}{2(4 - 3b^2)} \text{ and } f^* = \pi_{z,C}^C(r^*) - \pi_{z,C}\text{sq}.
\]

\(^{12}\) With \( b \in (0, 1) \), the assumption for \( \theta_L > -2/(2 - b) \) implies that \( \theta_L > -1 \). Given \(-1 < \theta_L < 0\), we have \( \inf \bar{c}^b = b(1 - b)(2 + b)/2(2 - b^2) \text{ and } \sup \bar{c}^b = (1 - b)(2 + b)/(2 - b^2).\)
Under Bertrand competition, with the expressions in (11) and (13), we solve (15) to obtain:\textsuperscript{13}

\[ r^* = \min\{c, r^B\}, \text{ where } r^B = \frac{b(2+b)^2}{2(4+5b^2)} \text{ and } f^* = \pi^B_j(r^*) - \pi^{*\text{sq}}_j. \tag{17} \]

To see whether firms have incentives to merge when licensing is an option, we compare the industry’s total profits under mergers, licensing, and the status quo. The following result is obtained:

**Lemma 1:** We have \( E(\pi^{M*}) > E[\Pi^j(r^*)] > E(\pi^{*\text{sq}}_j + \pi^{j\text{sq}}_j), j = C \text{ or } B, \) where
\[
\Pi^j(r^*) = [\pi^j_1(r^*) + r^*q^j_2(r^*) + f^*] + [\pi^j_2(r^*) - f^*],
\]
in which the first bracket is the patentee’s payoff and the second bracket is the licensee’s payoff under licensing.

Lemma 1 establishes that the industry’s expected total profits are higher under mergers than under licensing. When mergers are allowed, how the gains from mergers (i.e., the difference between the monopoly profit and the sum of the duopoly profits under licensing) are divided will presumably depend on the actual bargaining process between these two firms, whether or not mergers indeed occur and if they occur. To deal with this, we follow the approach of previous contributions by Stenbacka (1991), Wong and Tse (1997), and González-Maestre and Peñarrubia (2005) to assume that, if mergers are allowed, firm 1 can present a merger proposal to firm 2 by means of a takeover offer. In particular, firm 1 can compensate firm 2 exactly by firm 2’s reservation payoff (i.e., firm 2’s profits in the case of

\textbf{13} We can show that \( r^C < \sup \pi^C \), such that in the Cournot regime, \( r^* = c \) and \( r^C \) for \( c \in (0, r^C] \) and \( (r^C, \sup \pi^C) \), respectively. Moreover, we can show that \( r^B < \sup \pi^B \) \( \iff b < 0.7865 \). Thus, in the Bertrand regime, if \( 0 < b < 0.7865 \), we have \( r^* = c \) and \( r^B \) for \( c \in (0, r^B] \) and \( (r^B, \sup \pi^B) \), respectively; and if \( 0.7865 \leq b < 1 \), we have \( r^* = c \) for all \( c \leq \sup \pi^B \).
duopoly competition under licensing) so that firm 2 agrees to merge.\textsuperscript{14} Thus, when the antitrust authority considers its optimal merger policy in the first stage, it anticipates that firms will merge as long as mergers are allowed. Finally, note that regardless of the competition style and the antitrust policy, in equilibrium firm 2 always obtains access to firm 1’s superior technology (such that $c_2 = 0$), either through a merger or by a licensing contract.

5 OPTIMAL MERGER POLICY IN THE FIRST STAGE

In the absence of market uncertainty and asymmetric information about demand, Faulí-Oller and Sandonis (2003) compare society’s welfare under mergers with that under two-part tariff contracts, and find that social welfare is unambiguously lower under mergers for both Cournot and Bertrand competition. They then suggest that mergers should be forbidden regardless of the competition style whenever two-part tariff licensing is feasible. In their model, both mergers and two-part tariff licensing contracts result in efficiency losses to society. In particular, while the anticompetitive effects of mergers are detrimental, two-part tariff contracts also result in distortions – they distort the licensee’s output under Cournot competition (by raising the licensee’s actual cost through a royalty) and distort the patent holder’s own price under Bertrand competition (in order to increase the licensee’s output and thus the derived royalty revenue). Because the distortion caused by mergers exceeds that caused by two-part tariff contracts, the final result is to forbid mergers.

In our model with demand uncertainty and asymmetric information, in addition to the above welfare effects, there is a third welfare effect at work – the information sharing effect associated with mergers. Thus, the optimal merger policy in our model may differ from that

\textsuperscript{14} This is one possible way for merging firms to divide their gains from mergers. Another possibility is to let the better informed firm 2 make the merger proposal by paying firm 1 its reservation payoff (i.e., firm 1’s production profits plus licensing revenues). The point here is that as long as the post-merger monopoly profit is greater than the pre-merger sum of the licensor’s and the licensee’s profits, then both firms can find a way to divide the gains from mergers, such that they do have incentives to merge. We would like to thank Professor Picard (the Editor) and one anonymous referee for drawing our attention to discuss this.
in Faulí-Oller and Sandonis (2003), depending on the direction of this additional welfare
effect. More precisely, if the welfare effect of information sharing is positive, then mergers
under uncertainty will be less harmful and may even become welfare superior to licensing.
They should thus be allowed under some circumstances. On the other hand, if the welfare
effect of information sharing is negative, then mergers under uncertainty will be even more
undesirable and thus should be forbidden as in Faulí-Oller and Sandonis (2003). We will
show below that the former case arises in the quantity-setting regime, while the latter case
occurs in the price-setting regime.

5.1. Cournot competition

Anticipating that either a merger or a two-part tariff licensing contract will prevail in the
second stage, the antitrust authority determines its optimal merger policy in the first stage by
comparing expected social welfare under mergers with that under licensing. We first consider
the Cournot regime (i.e., if mergers are not allowed, then firms sign a licensing contract in the
second stage and compete by choosing outputs in the third stage). Plugging the relevant
expressions for outputs in (6) and (8) into the welfare function in (3), and by direct
computations, we have:

\[
E(W^M - W^C) = \left[ -\frac{b(4 + b)}{4(1 + b)(2 + b)^2} + \frac{r}{(2 + b)^2} + \frac{(4 - 3b^2)r^2}{2(4 - b^2)^2} \right] + \frac{3(1-b)}{8(1+b)} \sigma^2,
\]

(18)

where \( W^M \) is the welfare under mergers, \( W^C \) is the welfare under licensing in the Cournot
regime, and \( E(W^M - W^C) \) is the expected welfare differential between mergers and
licensing in the quantity-setting regime.

The expected welfare differential can be broken into two parts. The first part is
completely independent of market volatility, \( \sigma^2 \), and is similar to the welfare difference
derived in the deterministic model of Faulí-Oller and Sandonis (2003). Given the expressions
of \( r^* \) in (16), we can show that this part is negative as in Fauli-Oller and Sandonís (2003). The second part measures the welfare effect of information sharing associated with mergers, which is positive and positively correlated with \( \sigma^2 \).\(^{15,16}\) Thus, the more (less) volatile the market demand is, the larger (smaller) the value of \( \sigma^2 \) and the greater (smaller) the positive term will be. Letting \( \sigma^2 = b(4 + b) / 3(4 - 3b^2) \) and \( \tilde{\sigma}^2 = 2b(4 + b) / 3(1 - b)(2 + b)^2 \), where \( \sigma^2 < \tilde{\sigma}^2 \), we have:

\[
\sigma^2 = b(4 + b) / 3(4 - 3b^2) \quad \text{and} \quad \tilde{\sigma}^2 = 2b(4 + b) / 3(1 - b)(2 + b)^2,
\]

**Proposition 1:** When two-part tariff licensing is an alternative means to transfer a superior technology, the optimal merger policy under Cournot competition is characterized by:

(a) If \( \sigma^2 < \tilde{\sigma}^2 \), mergers should be forbidden.

(b) If \( \sigma^2 \leq \sigma^2 < \tilde{\sigma}^2 \), mergers should be allowed for large innovations and be forbidden otherwise.

(c) If \( \sigma^2 \leq \sigma^2 \), mergers should be allowed.

The driving force for Proposition 1 is that the welfare effect of information sharing is positive when quantities are the choice variables. While the Cournot firms do not share information, this positive welfare effect of information sharing is absent under licensing, but present under mergers. Therefore, mergers may be welfare superior to licensing when demand is uncertain. The reasons why information sharing is good for welfare in the quantity-setting regime can be discussed as follows.

Recall that if there is no information sharing, then only product 2’s output varies

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\(^{15}\) This separation of two parts is reminiscent of Gal-Or’s (1988) model with stochastic demand. She finds that for both Cournot and Bertrand competition, the changes in expected profits due to mergers can be broken into two parts. One part is completely independent of the precision of information, which is identical to the changes derived in a deterministic environment by Salant et al. (1983) and Deneckere and Davidson (1985) under Cournot and Bertrand competition, respectively. The second part depends on the precision of information and captures the informational advantage or disadvantage of mergers in her model.

\(^{16}\) Our second part in (18) is negatively correlated with \( b \). The implication of this will be pursued in Corollary 1.
positively with the uncertain demand intercept, $\theta$, while product 1’s output is independent of $\theta$. Hence, when a positive value of $\theta$ is observed, only product 2’s output increases in the absence of information sharing, whereas both products’ outputs expand with information sharing. Thus, information sharing is welfare-improving under positive shocks (which are associated with higher demand under larger demand intercepts). On the other hand, if a negative shock is observed, only product 2’s output drops without information sharing, while both products’ outputs contract with information sharing. Hence, information sharing is welfare-impairing under negative shocks (which are associated with lower demand under smaller demand intercepts).

Since both products are produced at zero costs (whether mergers or licensing occur), the area under a demand curve represents the change in welfare associated with a change in output. With demand curves being higher (with larger intercepts) under positive shocks, society benefits from a more significant increase in welfare due to a given amount of output expansion (i.e., a larger trapezium under a higher demand curve) than a reduction of welfare due to the same amount of output contraction (i.e., a smaller trapezium under a lower demand curve). As such, the positive effect under positive shocks prevails over the negative effect under negative shocks. In other words, the net welfare effect of information sharing is positive in the quantity-setting regime.\(^{17}\)

To visualize the above discussion for intuitions, let us consider a simplified graphical representation. Because we have two products, to illustrate the welfare changes under positive and negative shocks in a two-dimensional figure, we derive from equation (1) a

\(^{17}\)Interestingly, Gal-Or (1988) also finds that the effect of positive shocks prevails over the effect of negative shocks when they go in the opposite directions in her model with both Cournot and Bertrand competition. For instance, she finds that under a Cournot regime, the merging firms will respond less aggressively to a change in demand. The non-merging firms’ market share expands under positive shocks and declines under negative shocks. Because prices are higher under positive shocks, the non-merging firms benefit from a more significant increase of expected profits under positive shocks than a reduction of profits under negative shocks. Thus, the effect of positive shocks dominates, so that the net effect is to increase the expected profits of the non-merging firms.
hypothetical “average” inverse demand curve of the two products, \( \bar{p} = 1 + \theta - (1 + b)\bar{q} \), where 
\( \bar{p} = (p_1 + p_2)/2 \) and \( \bar{q} = (q_1 + q_2)/2 \). The curves \( GG' \) and \( BB' \) denote the average demand curves under a good state (with \( \theta = \hat{\theta} > 0 \)) and a bad state (with \( \theta = -\hat{\theta} < 0 \)), respectively.

\[
\bar{p} = 1 + \theta - (1 + b)\bar{q}
\]

When a good state is observed by firm 2 and there is no information sharing, the average output is given by \( \bar{q}_g^{NS} \) (the superscript \( NS \) is for “no sharing”), which is lower than the average output with information sharing, \( \bar{q}_g^S \) (the superscript \( S \) is for “sharing”). Similarly, \( \bar{q}_b^{NS} \) and \( \bar{q}_b^S \) represent the average output under a bad state without and with information sharing, respectively. The welfare gain from information sharing under a good state is given by the cross-shaded area, \( N_g \bar{q}_g^{NS} \bar{q}_g^S S_g \), and the welfare loss due to information sharing under a bad state is given by the dotted area, \( S_b \bar{q}_b^S \bar{q}_b^{NS} N_b \). Clearly, the gain in a good state is larger than the loss in a bad state, such that the net welfare effect is positive. It is worth mentioning that the actual net welfare effect of information sharing is even larger than the effect shown.
in this simplified figure with prices and outputs expressed in average terms.18

Given that the positive welfare effect of information sharing is positively correlated with the degree of market fluctuations, it follows that the less (more) volatile the market demand is, the smaller (greater) the positive effect of information sharing will be, and the less (more) likely that mergers will be approved. These are the intuitions for the results in Parts (a) and (c). As for the result in (b), the economic intuition lies in that larger innovations are associated with higher royalties, which result in more efficiency losses associated with output distortions, such that mergers are more likely to be preferred as licensing is less desirable.

Before ending this subsection, it is interesting to analyze the relationship between the critical values in Proposition 1 and the degree of product differentiation, $b$. Differentiating $\sigma^2$ and $\bar{\sigma}^2$ with respect to $b$ yields the following result:

**Corollary 1:** We have $d\sigma^2 / db > 0$ and $d\bar{\sigma}^2 / db > 0$.

The positive sign of the derivative of $\sigma^2$ with respect to $b$ means that the smaller the value of $b$ is (i.e., the more differentiated the two products are), the smaller is $\sigma^2$, and thus the smaller is the parameter space described in Part (a) of Proposition 1, in which mergers

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18 To isolate the information sharing effect, we assume $c_2 = 0$ to compute the values of outputs given in Figure 1. From (9) and with $c_2 = 0$, we have: $q_{1g}^{NS} = q_{1b}^{NS} = 1/(2+b)$, $q_{2g}^{NS} = 1/(2+b) + \hat{\theta}/2$, and $q_{2b}^{NS} = 1/(2+b) - \hat{\theta}/2$. If firms share information, they maximize their respective profits, $(1 + \theta - q_1 - bq_2)q_1$ and $(1 + \theta - q_2 - bq_2)q_2$. Solving simultaneously yields: $q_1^S = q_2^S = (1 + \hat{\theta})/(2+b)$ and $q_{1b}^S = q_{2b}^S = (1 - \hat{\theta})/(2+b)$. From these, we have $\bar{q}_1^S = (1 + \hat{\theta})/(2+b)$, $\bar{q}_b^S = (1 - \hat{\theta})/(2+b)$, $\bar{q}_{NS}^S = 1/(2+b) + \hat{\theta}/4$, and $\bar{q}_{bNS}^S = 1/(2+b) - \hat{\theta}/4$. The actual welfare gain from information sharing under a positive shock is $A = W(q_{1g}^S,q_{2g}^S) - W(q_{1b}^{NS},q_{2b}^{NS})$, and the welfare loss from information sharing under a negative shock is $B = W(q_{1b}^{NS},q_{2b}^{NS}) - W(q_{1b}^S,q_{2b}^S)$, such that the actual net welfare effect of information sharing is $\Delta W = A - B$. In Figure 1 the cross-shaded area is $A = W(q_{1g}^S,q_{2g}^S) - W(q_{1b}^{NS},q_{2b}^{NS})$, the dotted area is $B = W(q_{1b}^{NS},q_{2b}^{NS}) - W(q_{1b}^S,q_{2b}^S)$, and the net welfare effect is $\Delta W = A - B'$. It can be shown that $\Delta W = \Delta \bar{W} + K$, where $K = (28 - 8b - 9b^2 - b^3)\hat{\theta}^2 / 16(2+b)^2 > 0$, such that $\Delta W > \Delta \bar{W}$.
should be prohibited. The positive sign of the derivative of $\sigma^2$ with respect to $b$ means that the smaller the value of $b$ is (i.e., the more differentiated the two products are), the smaller is $\sigma^2$, and thus the larger is the parameter space described in Part (c) of Proposition 1, in which mergers should be permitted regardless of innovation sizes.

The results described above are appealing intuitively. When the two products are more differentiated (with smaller values of $b$), the pre-merger markets are less competitive, such that mergers have smaller anticompetitive effects and thus are more likely to be permitted. Conversely, when the two products are closer substitutes (i.e., with larger values of $b$), then mergers are less likely to be permitted, because the pre-merger markets are more competitive such that the anticompetitive effects associated with mergers are larger. In the extreme, when $b$ approaches 1, the value of $\sigma^2$ goes to infinity, so that the parameter space under which mergers should be permitted regardless of innovation sizes disappears. This last result is also observed from equation (18) where the second positive term (due to a positive information sharing effect) goes to zero when $b$ goes to 1.

5.2. Bertrand competition

We now turn to the Bertrand regime. Substituting the relevant terms for outputs given in (6) and (11) into the welfare function in (3) and by direct computations, we have:

$$E(W^M - W^B) = \left[ -\frac{b(4-3b)}{4(1+b)(2-b)^2} + \frac{(1-b)r}{(2-b)^2} + \frac{(4+5b^2)r^2}{2(4-b^2)^2} \right] - \frac{\sigma^2}{8}, \quad (19)$$

where $W^B$ is the welfare under licensing in the Bertrand regime, and $E(W^M - W^B)$ is the expected welfare differential between mergers and licensing in the price-setting regime.

The expected welfare differential can again be broken into two parts. The first part is completely independent of market volatility and is similar to the welfare differential derived in Fauli-Oller and Sandonís’s (2003) deterministic model. Given the expressions of $r^*$ in
(17), it is straightforward to show that this part is negative as in Fauli-Oller and Sandonis (2003). The second part, which is negative, measures the welfare effect of information sharing associated with mergers when firms compete in the Bertrand fashion. Again, if there is no market volatility (i.e., $\sigma^2 = 0$), then mergers are welfare inferior to two-part tariff licensing as shown by Fauli-Oller and Sandonis (2003). With market uncertainty and information sharing, mergers become even more undesirable than under certainty. We thus have:

**Proposition 2:** When two-part tariff licensing is an alternative means to transfer a superior technology, mergers should never be allowed under Bertrand competition.

Proposition 2 generalizes Fauli-Oller and Sandonis’s (2003) result to an environment with random demand. The driving force for our result is that in a world with market uncertainty, the welfare effect of information sharing is negative when prices are the choice variables. The intuition for this force is as follows. When a positive (negative) shock is observed, only product 2’s price is raised (reduced) without information sharing, while both products’ prices are increased (decreased) with information sharing. Thus, information sharing is detrimental under positive shocks and beneficial under negative shocks. The welfare loss due to raising prices (output contraction) under positive shocks is associated with a larger trapezium under a higher demand curve with a larger intercept. The welfare gain due to cutting prices (output expansion) under negative shocks is associated with a smaller trapezium under a lower demand curve with a smaller intercept. Thus, the effect under positive shocks dominates, leaving the net welfare effect of information sharing negative. While the Bertrand firms do not share information, this negative welfare effect is absent under licensing, but present under mergers. Therefore, mergers are even more undesirable...
under uncertainty with information sharing.

6 CONCLUSION

We have investigated the antitrust authority’s optimal merger policy in a differentiated duopoly model with cost asymmetry (due to exogenously given process innovations) and asymmetric information about uncertain demand. Mergers allow firms to share market information and technological knowledge, while licensing is an alternative way of transferring a superior technology. We first show that, regardless of the competition styles, if allowed, the duopoly firms indeed have incentives to merge, and they prefer mergers to two-part tariff licensing.

We next show that the optimal merger policy under uncertainty indeed differs under Cournot and Bertrand competition. In particular, under Cournot competition, we find that if demands are sufficiently volatile, then mergers are welfare superior to two-part tariff licensing and should be allowed accordingly; if demand fluctuations are in the intermediate range, then mergers are still welfare superior to licensing for large innovations; and only when demand fluctuations are small are mergers welfare inferior to two-part tariff licensing and should be forbidden. By contrast, if firms compete in Bertrand fashion, then mergers become even more undesirable under uncertainty than under certainty, and thus should be forbidden regardless of the degree of market volatility and the sizes of innovations. Our results yield interesting policy implications that can be compared to the contributions of Faulí-Oller and Sandonis (2003), González-Maestre and Peñarrubia (2005), and González-Maestre (2008), shedding light on a common antitrust view expressed, for example, in the U.S. Horizontal Guidelines.

It is worth noting that, even though this paper has employed a set-up in which one firm has better technology while the other firm has better market information, however, as
Appendix B shows, our results hold true qualitatively under an alternative modeling environment in which one firm simultaneously has better technology and market information. The reason is that the driving force for our results lies in the welfare effect of information sharing. More precisely, the welfare effect of information sharing is positive when quantities are the decision variables, while the effect is negative when prices are the decision variables. The positive (negative) welfare effect comes from the fact that, after information sharing, both firms get to increase their outputs (prices) upon the observation of good news about demand. It really does not matter which firm has information before sharing information.

Given that our theoretical findings and policy implications are related to some structural characteristic, such as competition style, and exogenously determined factors, such as the degree of market volatility and the size of innovations, it is helpful to discuss how to empirically identify these characteristics and factors. First, regarding the competition style, the seminal work of Kreps and Scheinkman (1983) suggests to distinguish Cournot competition from Bertrand competition by production capacity. They show that firms are Bertrand-like if they are unconstrained by capacity, while they are Cournot-like if constrained by capacity. Haskel and Martin (1994) test the relationship between profits and capacity constraints, finding empirical support for the conclusion of Kreps and Scheinkman (1983). Brander and Zhang (1990) investigate the market conduct of the American airline industry using a set of duopoly airline routes. They estimate the mean conduct parameter of each airline to draw inferences about which competition modes (Cournot, Bertrand, or Cartel) are supported by their data. Next, regarding the degree of market uncertainty, Showalter (1999) uses the deviation from a firm’s expected sales trend as a proxy for market uncertainty. As for the size of innovations, Liu and Shumway (2009) extend the non-parametric methods developed by Varian (1984) and Chavas et al. (1997) to estimate the change of marginal costs induced by technology innovations. Their method can be applied to measure the magnitude of
cost reduction associated with the adoption of a superior technology.

Lastly, this paper takes up the optimal merger policy in a duopoly model, taking into account market uncertainty. Our results are therefore directly comparable to former contributions, such as Faulí-Oller and Sandonís (2003), González-Maestre and Peñarrubia (2005), and González-Maestre (2008), who also employ duopoly models to analyze an optimal merger policy, but in the context of deterministic demand. The important driving forces for our results namely, the welfare effects of information sharing under both Cournot and Bertrand competition are in close relation to the contributions of Vives (1984), who considers a differentiated duopoly as well. The present paper shows that, even under such a restrictive environment with only two firms, mergers may still be welfare superior to licensing under some circumstances. Nevertheless, it would be interesting to extend our analysis from a duopoly framework to an oligopoly with \( n \) firms competing in the industry. This is a direction which may be worthwhile pursuing in the future.
APPENDIX A

Proof of Lemma 1

We have \( \Pi'(r) = [\pi'_1(r) + rq'_1(r) + f] + [\pi'_2(r) - f] = p'_1(r)q'_1(r) + rq'_2(r) + [p'_2(r) - r]q'_2(r) = p'_1(r)q'_1(r) + p'_2(r)q'_2(r). \) In the case of Cournot competition, by using expressions in (8), we have \( \Pi^C(r) = (2 - b + br)^2 / [(4 - b^2)^2] + (2 - b + 2r - b^2 r)(2 - b - 2r) / [(4 - b^2)^2] + \{(2 - b + br) / [2(2 + b)] + [2 - b + (2 - b^2)r] / [2(4 - b^2)] + (2 - b - 2r) / [2(4 - b^2)]\} \theta + \theta^2 / 4. \)

Given \( E(\theta) = 0, \) we have \( E[\Pi^C(r)] = E\{(2 - b + br)^2 / [(4 - b^2)^2] + (2 - b + 2r - b^2 r)(2 - b - 2r) / [(4 - b^2)^2]\} + \sigma^2 / 4. \) Substituting the unconstrained solution \( r^* = r^C \) into \( E[\Pi^C(r)] \) and after rearranging, we have \( E[\Pi^C(r^*)] = (8 - 8b + b^2) / [4(4 - 3b^2)] + \sigma^2 / 4. \) From (6) we have \( E(\pi^M) = \) \( E[\Pi^C(r^*)] = b^2(1 - b) / [4(1 + b)(4 - 3b^2)] + (1 - b)\sigma^2 / [4(1 + b)], \) which is positive.

In the case of Bertrand competition, using expressions in (11), we have \( E[\Pi^B(r)] = \) \( E\{2(1 - b) / [(1 + b)(2 - b^2)] + br / [(2 - b^2)] - (4 + 5b^2)r^2 / [(4 - b^2)^2]\} + (1 - b)\sigma^2 / [4(1 + b)]. \)

Substituting the unconstrained solution \( r^* = r^B \) into \( E[\Pi^B(r)] \) and after rearranging, we have \( E[\Pi^B(r^*)] = 2(1 - b) / [(1 + b)(2 - b^2)] + b^2(2 + b)^2 / [4(2 - b^2)(4 + 5b^2)] + (1 - b)\sigma^2 / [4(1 + b)]. \) Direct computations yield that \( E(\pi^M) - E[\Pi^B(r^*)] = b^2(1 - b) / [4(1 + b)(4 + 5b^2)] + \sigma^2 / 4, \) which is positive for sure.

We next compare the industry’s profits under two-part tariff licensing with that under the status quo. The status quo industry’s profits are \( \Pi^{1,eq} = \pi^{1,eq}_1 + \pi^{1,eq}_2, \) where the expressions for \( \pi^{1,eq}_1 \) and \( \pi^{1,eq}_2 \) are in (9) and (13) for \( j = C \) and \( B, \) respectively. It is straightforward to show that the inequality \( E[\Pi^1(r^*)] > E(\Pi^{1,eq}), j = C, B, \) holds true both for \( r^* = c \) and for \( r^* = r^i. \) Q.E.D.
Proof of Proposition 1

Equation (18) gives the expression for $E(W^M - W^C)$. It is straightforward to show that $E(W^M - W^C) > 0$ holds true for $\sigma^2 > 2(2+2b-3b^2)/3(1-b)(4-3b^2) \equiv \hat{\sigma}^2$. For $\sigma^2 \leq \hat{\sigma}^2$, we have $E(W^M - W^C) > 0$ iff $r^* > k$, where $k \equiv [-8+2(3-b)b^2+(4-b^2)\sqrt{s}] / 2(1+b)(4-3b^2)$ with $s \equiv 2(1+b)(2+2b-3b^2) - 3(1-b^2)(4-3b^2)\sigma^2$ (noting that $s \geq 0$ for $\sigma^2 \leq \hat{\sigma}^2$). It can be shown that $k > 0$ iff $\sigma^2 \leq 2b(4+b)/3(1-b)(2+b)^2 \equiv \bar{\sigma}^2$, where $\hat{\sigma}^2 < \bar{\sigma}^2$. Thus, for $\hat{\sigma}^2 < \sigma^2 \leq \hat{\sigma}^2$, we have $k < 0$ such that $r^* > k$ holds and thus $E(W^M - W^C) > 0$. This completes our proof for Part (c).

We next consider the parameter range with $0 \leq \sigma^2 \leq \bar{\sigma}^2$, where $0 < k < \sup \bar{\sigma}^2$. Direct computations indicate that $r^C \geq k$ iff $\sigma^2 \geq b(4+b)/3(4-3b^2) \equiv \bar{\sigma}^2$, where $\sigma^2 < \bar{\sigma}^2$. Thus, for $0 \leq \sigma^2 < \bar{\sigma}^2$, we have $r^* = \min\{c, r^C\} \leq r^C < k$, such that $E(W^M - W^C) < 0$. This completes our proof for Part (a).

Lastly, for $\sigma^2 \leq \sigma^2 < \bar{\sigma}^2$, we have $r^C \geq k$. There are two sub-cases here: (i) If $c \geq k$ (i.e., the case of large innovations), then $E(W^M - W^C) \geq 0$ holds true, because $c \geq k$ and $r^C \geq k$ imply $r^* = \min\{c, r^C\} \geq k$; and (ii) If $c < k$, then $r^* = \min\{c, r^C\} < k$, such that $E(W^M - W^C) < 0$. This completes our proof of Part (b). Q.E.D.

Proof of Proposition 2

It suffices to show that the sign of the set of brackets in (19) is negative. Direct computations indicate that this is true iff $h_1 < r^* < h_2$, where $h_1 \equiv [-2(1-b^2)(2+b)^2-(4-b^2)\sqrt{t}] / 2(1+b)(4+5b^2) < 0$, $h_2 \equiv [-2(1-b^2)(2+b)^2+(4-b^2)\sqrt{t}] / 2(1+b)(4+5b^2) > 0$, and $t \equiv 2(1+b)(2+6b-b^2+2b^3) > 0$. It is straightforward to show that $r^B < h_2$. Thus, $r^* =
min\{c, r^B\} \leq r^* < h_2, \text{ such that } r^* < h_2 \text{ holds true.} \quad \text{Q.E.D.}

Proof of Corollary 1

Given \( \sigma^2 \equiv b(4 + b) / 3(4 - 3b^2) \) and \( \bar{\sigma}^2 \equiv 2b(4 + b) / 3(1 - b)(2 + b)^2 \), it is straightforward to show that \( d\sigma^2 / db = 4(4 + 2b + 3b^2) / 3(4 - 3b^2)^2 > 0 \) and \( d\bar{\sigma}^2 / db = 2(8 + 6b^2 + b^3) / 3(1 - b)^2(2 + b)^3 > 0 \). \quad \text{Q.E.D.}
APPENDIX B

Firm 1 has better technology and better information

Suppose that during the transition from the second to the third stage, it is firm 1 that learns the exact realization of $\theta$, whereas firm 2 remains ignorant unless it has merged with firm 1. Note that firm 2 can now acquire firm 1’s superior technology either via a licensing contract or merger, whereas it can acquire information about $\theta$ only through a merger with firm 1. We solve this alternative model as follows. To distinguish from the solutions in the original model, in this Appendix we denote all the optimal solutions with a “∧” above variables.

1. Equilibrium output and prices in the third stage

1.1: The merged entity

If a merger occurs in the second stage, then in the third stage the merged entity solves:

$$\max_{q_1, q_2} \pi^M = (1 + \theta - q_1 - bq_2)q_1 + (1 + \theta - q_2 - bq_1)q_2.$$  \hspace{1cm} (A1)

Note that this problem is exactly the same as in the original model. That is, the pre-merger information structure (i.e., which firm has private information about $\theta$) does not affect the merged entity’s post-merger decision making. The optimal solution to (A1) is in equation (6).

1.2: Cournot competition

If a merger does not occur, but a two-part tariff licensing contract is signed in the second stage, then under Cournot competition, the duopoly solves the following problem in the third stage:

$$\max_{q_1} \ (1 + \theta - q_1 - bq_2)q_1 + r q_2,$$

$$\max_{q_2} \ E[(1 + \theta - q_2 - bq_1 - r)q_2].$$ \hspace{1cm} (A2)

Note that it is now firm 2 (rather than firm 1) that needs to make an expectation regarding its
profits (net of licensing payments). The solution is given by:

\[
\begin{align*}
\hat{q}_1^C(r) &= \frac{2 - b + br}{4 - b^2} + \frac{\theta}{2}, \\
\hat{p}_1^C(r) &= \frac{2 - b + br}{4 - b^2} + \frac{\theta}{2}, \\
\pi_1^C(r) &= p_1^C(r) q_1^C(r),
\end{align*}
\]

(A3)

Interestingly, compared to the Cournot outputs in the original model (i.e., \(q_1^C(r)\) and \(q_2^C(r)\) in equation (8)), \(q_1^C(r)\) has an additional term \(\theta/2\), while \(q_2^C(r)\) drops this term, so that the total output remains the same (i.e., \(q_1^C(r) + q_2^C(r) = q_1^C(r) + q_2^C(r)\)).

If neither a merger nor licensing occurs in the second stage, then the status quo solution in the third stage is given by:

\[
\begin{align*}
\hat{q}_1^{C,\text{sq}} &= \frac{2 - b + bc}{4 - b^2} + \frac{\theta}{2}, \\
\hat{p}_1^{C,\text{sq}} &= \frac{2 - b + bc}{4 - b^2} + \frac{\theta}{2}, \\
\pi_1^{C,\text{sq}} &= p_1^{C,\text{sq}} q_1^{C,\text{sq}},
\end{align*}
\]

(A4)

Again, compared to the status quo solution in the original model (in equation (9)), \(\hat{q}_1^{C,\text{sq}}\) here adds the term \(\theta/2\) while \(\hat{q}_2^{C,\text{sq}}\) drops this term, leaving the total output unchanged.

To ensure that both firms are active in the status quo Cournot equilibrium, we impose assumptions on the relationship among our parameters (just like what we did in the original model). To ensure that the high-cost firm 2 is active, we require \(c < (2 - b) / 2 \equiv c^C\). To ensure that the informed firm 1 produces under all realizations of \(\theta\), we require that \(\hat{q}_1^{C,\text{sq}}(\theta = \theta_L) > 0\), which holds true iff \(c > -(2 - b)[2 + (2 + b)\theta_L] / 2b \equiv c^C\). Because \(c^C > c^C\) iff \(\theta_L > -1\). We also assume \(\theta_L > -1\). In other words, the worst state cannot be too negative and firm 2’s cost cannot be too high, so that both firms are active in the status quo.
### 1.3: Bertrand competition

If a merger does not occur, but a licensing contract is signed in the second stage, then under Bertrand competition, the duopoly solves the following in the third stage:

\[
\begin{align*}
\max_{p_1} & \quad p_1q_1(p_1, p_2) + rq_2(p_1, p_2), \\
\max_{p_2} & \quad E[p_2q_2(p_1, p_2) - rq_2(p_1, p_2)].
\end{align*}
\] (A5)

The solution is given by:

\[
\begin{align*}
\hat{p}_1^\theta (r) &= \frac{2 - b - b^2 + 3b}{4 - b^2} + \frac{r(1-b)\theta}{2}, \\
\hat{p}_2^\theta (r) &= \frac{2 - b - b^2 + (2 + b^2)r}{4 - b^2}, \\
\hat{q}_1^\theta (r) &= \frac{(2 + b) - (1 + b)br}{(1 + b)(4 - b^2)} + \frac{\theta}{2(1 + b)}, \\
\hat{q}_2^\theta (r) &= \frac{(2 + b) - 2(1 + b)r}{(1 + b)(4 - b^2)} + \frac{(2 + b)\theta}{2(1 + b)}, \\
\hat{\pi}_1^\theta (r) &= p_1^\theta (r) q_1^\theta (r), \\
\hat{\pi}_2^\theta (r) &= [p_2^\theta (r) - r] q_2^\theta (r). \\
\end{align*}
\] (A6)

Interestingly, compared to the solution in the original model (in equation (11)), \(\hat{p}_1^\theta (r)\) here adds the term \((1-b)\theta / 2\), while \(\hat{p}_2^\theta (r)\) drops this term. As for outputs, the first terms of \(\hat{q}_1^\theta (r)\) and \(\hat{q}_2^\theta (r)\) are the same as the first terms of \(q_1^\theta (r)\) and \(q_2^\theta (r)\), respectively, while their second terms are reversed. As a result, the total output here, \(\hat{q}_1^\theta (r) + \hat{q}_2^\theta (r)\), remains the same as the total output in the original model, \(q_1^\theta (r) + q_2^\theta (r)\).

For the status quo solution under Bertrand competition, the duopoly solves:

\[
\begin{align*}
\max_{p_1} & \quad p_1q_1(p_1, p_2), \\
\max_{p_2} & \quad E[p_2q_2(p_1, p_2) - cq_2(p_1, p_2)].
\end{align*}
\] (A7)

The solution is given by:
Again, as compared to the status quo solution in the original model (in equation (13)), \( \hat{p}^{B, sq}_1 \) here adds the term \( (1-b)\theta / 2 \), while \( \hat{p}^{B, sq}_2 \) drops this term. The first terms of \( \hat{q}^{B, sq}_1 \) and \( \hat{q}^{B, sq}_2 \) are the same as the first terms of \( q^{B, sq}_1 \) and \( q^{B, sq}_2 \), respectively, while their second terms are reversed, leaving the total output in both models the same.

To ensure that both firms remain active in the status quo Bertrand equilibrium, we assume the following. First, we have \( \hat{q}^{B, sq}_2 (\theta = \theta_L) > 0 \) iff \( c < (1-b)(2+b)/(2-b^2) + (1-b)(4-b^2)(2+b)\theta_L / 2(2-b^2) \equiv \hat{c}_B \), where \( \hat{c}_B > 0 \) iff \( \theta_L > -2/(4-b^2) \). With \( b \in (0, 1) \), the last condition implies that \( \theta_L > -1/2 \). Next, it can be shown that \( \hat{q}^{B, sq}_1 (\theta = \theta_L) > 0 \) holds true for all \( \theta_L > -1/2 \). Thus, we assume \( \theta_L > -2/(4-b^2) \) and \( c < \hat{c}_B \), so as to guarantee that both firms remain active in the status quo Bertrand equilibrium.

2. Equilibrium outcome in the second stage

At the beginning of the second stage, firm 1 has not learned about the realization of \( \theta \). As the patent holder, firm 1 chooses the optimal fee, \( f \), and royalty, \( r \), by solving the following:

\[
\max_{r,f} \quad E[\tilde{\pi}_1^q(r) + r \tilde{q}_1^q(r) + f] \\
\text{s.t.} \quad f \leq \tilde{\pi}_1^q(r) - \tilde{\pi}_2^q, \\
r \leq c. 
\]

(A9)

Note that the expectation operator still enters firm 1’s maximization problem in this stage.

Firm 1’s maximization problem can be rewritten as:
\[
\max_r E[\pi_1^r(r) + r q_1^r(r) + \pi_2^r(r) - \pi_2^{eq}]
\]
\[s.t. \quad 0 \leq r \leq c.\] (A10)

For Cournot competition, by using the expressions in (A3) and (A4), we can obtain the solution to (A10) as follows:

\[\hat{r}^* = \min\{c, r^c\}, \quad \text{where} \quad r^c = \frac{b(2-b)^2}{2(4-3b^2)} \quad \text{and} \quad \hat{f}^* = \pi_2^* - \pi_2^{eq}.\] (A11)

Interestingly, the optimal royalty rate here, \(\hat{r}^c\), equals the optimal royalty rate in the original model, \(r^C\) (see equation (16)). However, the associated fees \(\hat{f}^*\) and \(f^*\) may be different.

Utilizing relevant expressions for \(\hat{f}^*\) in (A3) and (A4) and relevant expressions for \(f^*\) in equations (8) and (9), and by straightforward calculations, we can show that: (i) If \(r^* = \hat{r}^* = c\), then \(f^* = \hat{f}^* = 0\); and (ii) If \(r^* = \hat{r}^* < c\), then \(\hat{f}^* > (=, <) f^*\) for \(\theta < (=, >) 0\).

For Bertrand competition, by using the expressions in (A6) and (A8), we can obtain the solution to (A10) as follows:

\[\hat{r}^* = \min\{c, r^b\}, \quad \text{where} \quad r^b = \frac{b(2+b)^2}{2(4+5b^2)} \quad \text{and} \quad \hat{f}^* = \pi_2^b - \pi_2^{eq}.\] (A12)

Again, the optimal royalty rate here, \(\hat{r}^b\), equals the optimal royalty rate in the original model, \(r^B\) (see equation (17)). However, the associated fees \(\hat{f}^*\) and \(f^*\) may be different.

Utilizing relevant expressions for \(\hat{f}^*\) in (A6) and (A8) and relevant expressions for \(f^*\) in equations (11) and (13), and by straightforward calculations, we can show that: for both \(r^* = \hat{r}^* = c\) and \(r^* = \hat{r}^* < c\), \(\hat{f}^* > (=, <) f^*\) for \(\theta > (=, <) 0\).

The analogue of Lemma 1 also holds in this alternative model, because the expected

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industry’s profits under licensing in this model remain the same as in the original model (i.e.,
$$E[\hat{\Pi}^I(r)] = E[\Pi^I(r)]$$) and the expected values of the status quo industry’s profits are also the
same in both models (i.e., $$E(\hat{\Pi}^{I, sq}_r) = E(\Pi^{I, sq}_r)$$). To see this, note that
$$\hat{\Pi}^I(r) = \hat{p}_r^I(r)q_r^I(r) + \hat{p}_r^I(r)q_r^I(r) + \hat{p}_r^I(r)q_r^I(r)$$. In the case of Cournot competition, using expressions in (A3), we have
$$\hat{\Pi}^C(r) = \frac{(2 - b + 2r)^2}{(4 - b^2)^2} + (2 - b + 2r - b^2r)(2 - b - 2r)/(4 - b^2)^2 + \{(2 - b + br)/
[(4 - b^2)] + (2 - b - 2r)/(2(2 + b))]\theta + \theta^2/4$$, which is the same as $$\Pi^C(r)$$ except for the
coefficient of $$\theta$$. Since $$E(\theta) = 0$$, after taking expectations, we have
$$E[\hat{\Pi}^C(r)] = E[\Pi^C(r)]$$.

3. Optimal merger policy in the first stage

In the original model, we define social welfare in equations (2) and (3) as:
$$W(q_1^j, q_2^j) = U(q_1^j, q_2^j) - c_2q_2^j = (1 + \theta)(q_1^j + q_2^j) - (q_1^j + q_2^j)^2 / 2 + (1 - b)q_1^j q_2^j - c_2q_2^j$$ (A12)

where $$j = C, B$$. Similarly, social welfare in the present model can be written as:

$$\hat{W}(\hat{q}_1^j, \hat{q}_2^j) = U(q_1^j, q_2^j) - c_2q_2^j = (1 + \theta)(q_1^j + q_2^j) - (q_1^j + q_2^j)^2 / 2 + (1 - b)q_1^j q_2^j - c_2q_2^j$$ (A13)

Note that $$c_2 = 0$$ whether mergers or licensing occur in the second stage. Thus, the last terms
of (A12) and (A13) equal zero when we evaluate the welfare differential between mergers
and licensing. Next, as shown above, the total outputs and the optimal royalty rates in both
models are the same (i.e., $$\hat{q}_1^j(r) + \hat{q}_1^j(r) = q_1^j(r) + q_1^j(r)$$ and $$\hat{r}_1 = r^j$$ for $$j = C, B$$). Therefore,
(A12) and (A13) differ only by their third terms $$(1 - b)q_1^j q_2^j$$ and $$(1 - b)\hat{q}_1^j q_2^j$$. Although
\( q_1^t q_2^t \neq \hat{q}_1^t q_2^t \), their expectation values are the same. In particular, using the expressions for 
\( q_1^c(r) \) and \( q_2^c(r) \) in (8) and \( \hat{q}_1^c(r) \) and \( q_2^c(r) \) in (A3), and by direct computations, we have 
\[
E(q_1^c(r)q_2^c(r)) = E(\hat{q}_1^c(r)q_2^c(r)) = (2 - b + b \sigma^2 - b - br) / (4 - b^2)^2.
\]
Similarly, from the expressions for \( q_1^\theta(r) \) and \( q_2^\theta(r) \) in (11) and \( \hat{q}_1^\theta(r) \) and \( q_2^\theta(r) \) in (A6), it is straightforward to show that 
\[
E(q_1^\theta(r)q_2^\theta(r)) = E(\hat{q}_1^\theta(r)q_2^\theta(r)) = [2 + b - b(1 + b)r][2 + b - 2 \sigma^2 + (1 + b)^2].
\]
Thus, the expected welfare differences between mergers and licensing in both models are also the same (i.e., 
\( E(W_M^t - \hat{W}_t^t) = E(W_M^t - \hat{W}_t^t) \)).

The welfare results obtained in our original model are robust to this alternative modeling.
REFERENCES


