Private Tutoring, Wealth Constraint, and Higher Education

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Abstract

We show that when the effectiveness of *private tutoring* (PT) exceeds a PT-effectiveness threshold, poor families become disadvantaged in sending their children to college and, thus, inequity arises. At the same time, the average spending on PT of the poor families is smaller than that of the rich families. Policies of tax and transfer are able to amend the inequity, but student loans for college tuition and more establishments of colleges may have limited effects on reducing inequity.

Key words: Private Tutoring, Incentive Constraint, Wealth Constraint

JEL classification: I21, I24, J24

1 Introduction

Admissions to higher education (hereafter referred to as colleges) in many countries are critically determined by scores from a highly competitive nation-wide entrance examination. This kind of admission procedure tends to be accompanied by a large industry of private tutoring (PT).[1] PT offers classes in academic subjects taught in mainstream schools and is offered outside school hours (Bray et al., 2014). Many parents spend a considerable sum of money on PT, hoping that their children can excel in entrance examinations and therefore have a better financial and/or social perspective after graduation from college.[2] Since PT may improve students' performances in the college entrance exam and rich families can afford greater amounts of PT than poor families, it raises a concern of social inequity (Bray, 2011; Buchmann, 2002; Dawson, 2010). In this paper, we analyze theoretically how parents decide on their PT expenditure and, particularly, investigate under what circumstances PT causes unequal probabilities of getting children placed in colleges between rich families and poor families. Once these conditions are understood, we examine several government policies to see whether they are able to alleviate the extent of inequity between the rich and the poor.

[1]For example, Japan's entrance examinations exclusively determine whether students are admitted to a university. In South Korea, admissions are determined by a weighted average of high school grades (20%) and the score from the national entrance examination (80%). Bray et al. (2014) points out that "in Korea..., a 2008 survey found 72.5 per cent of middle school students and 60.5 per cent of general high school students receiving tutoring. In Japan, a 2007 survey found that 65.2 per cent of Grade 9 students attended tutorial centers, and that in addition 6.8 per cent of Grade 9 students received tutoring at home". The phenomenon is also observed in Turkey (Tansel & Bircan, 2005), Hong Kong (Bray & Kwok, 2003), Taiwan (Lin & Chen, 2006), and elsewhere (Bray, 2009, 2011; Dang & Rogers, 2008).

[2]According to Bray et al. (2014), in Hong Kong the mean overall cost of PT per student is around 8.7% of estimated family incomes, and the proportion is much greater for low-income households. Kim and Park (2010) document that the average household in South Korea spends 8–9% of total expenditure on PT. Dang and Roger (2008) point out that spending on private tutoring approaches the level of spending on the formal public school system in many countries. Kim and Lee (2001) find in South Korea that households spent 2.9% of GDP on private tutoring in 1998—almost as much as the 3.4% of GDP that the public sector allocates to formal education. Finally, Tansel and Bircan (2006) present that in Turkey households spend more than 1.4% of GDP on private tutoring—close to the 2.0% the country spends on public education.

Even though college entrance examinations and PT are widely practiced in many countries, as far as we know there is only one theoretical article by Kim (2007) that studies the role of PT played in determining who can go to college. Kim (2007) uses a two-player non-cooperative game framework to explain how parents strategically choose their spending on PT for their children in order to compete for one admission to higher education. He finds that parents and students who come from disadvantaged backgrounds usually end up giving up the PT game. In addition, he argues that the nature of the non-cooperative game leads to inefficient investments on PT.[3]

In this paper we take an entirely different approach to study the role of PT. This paper contributes to the literature in several aspects. First, instead of modelling interdependent strategies as in Kim (2007), we emphasize how parents' incentive and wealth constraints determine their investment on PT, given that only N students, $N \in (0,1)$, can be admitted to colleges at any time. By considering a fixed number of college admissions, we introduce competition among children in the sense that a young agent can pass the entrance examination only if his test score ranks among the top N students. The smaller is N, the higher the intensity of competition on a college entrance exam will be. Thus, we are able to explicitly derive the condition under which some disadvantaged families in terms of being wealth-constrained cannot participate in PT as they wish to.

Second, our results bear some empirical implications. For example, our analysis suggest that the PT-effectiveness threshold and the disparity of average spending on PT between different wealth groups can serve as useful indicators for testing whether the poor families are wealth-constrained in investing on PT.

Finally, we can discuss whether various government policies, such as taxes and transfers, student loans, and establishing more colleges, can reduce the inequity. For example, by introducing the number of college admissions into our model, we can discuss the impact of a change in N on the investment on PT and the extent of inequity. We find that,

[3] In a very different perspective from our model, Biswal (1999) proposes a theoretical explanation to the existence of PT in developing countries. He argues that PT is a form of corruption where public school teachers have monopolistic power to promote expenditures on PT in order to compensate for their low salaries.

in general, a larger N may stimulate even more investment on PT and it cannot amend the inequity.

We conduct our analysis in a two-period overlapping-generations model where each household consists of one old parent and one young child. Children are born with different innate abilities that exhibit a uniform distribution. Any two generations' distributions of innate abilities are identical but independent to each other. An old agent who is either a skilled worker or an unskilled worker chooses household consumption and expenditure on his child's education so as to maximize household utility given his life-time wealth and his child's innate ability. Any parent is aware of his child's innate ability when making decisions. A skilled worker, earning a higher wage rate than an unskilled worker, must be a college graduate. Passing the entrance exam is the only way to be admitted into a college. To pass the exam, the ability of a young agent must be no less than an endogenously determined ability threshold level. At any time, the number of students who pass the entrance exam must equals the number of college admissions, N.

A young agent's ability when taking the college entrance exam could be enhanced by participating in PT and becomes higher than his innate ability, which attracts parents to invest on their children's PT. An investment on PT can either succeed (child's ability becomes higher than his innate ability) or fail (child's ability remains unchanged) with a certain probability. Those young agents, who have an innate ability no less than the ability threshold or who experience successful PT, go to college and become skilled workers when they are old. The assumption of an uncertain effect of PT on students' abilities is motivated by empirical studies that have yet to deliver clear conclusions about the relationship between PT and academic achievement.[4]

Given that PT is available, it may emerge in the equilibrium that an old agent is willing to send his child to college but cannot afford the required educational expenses. When this happens, we say that this old agent is wealth-constrained. In general, poor families are more likely to be wealth-constrained and rich families have sufficient wealth to

[4]See Dang and Rogers (2008) for a survey on the impact of private tutoring on academic achievement. For example, some studies report a positive impact in Japan (Stevenson & Baker, 1992), Kenya (Buchmann, 2002), Vietnam (Dang, 2007), while some find a negative impact in South Korea (Lee, Kim, & Yoon, 2004) and Singapore (Cheo & Quah, 2005).

invest on PT as they wish to. Our results are consistent with the empirical observations that parental education levels and family incomes positively affect the probability of a child to receive PT.[5] To derive the condition under which poor families encounter such a disadvantage relative to rich families due to PT, we establish a PT-effectiveness threshold. We find that when the effectiveness of PT (measured by the probability of a successful investment on PT) exceeds the PT-effectiveness threshold, poor families are wealth-constrained. In general, the larger the wealth gap is between the rich and the poor, the lower is the PT-effectiveness threshold. Our result implies that when the PT-effectiveness threshold is relatively low, the inequity issue emerges. Therefore, we go on to discuss policies of taxes and transfers that may raise the PT-effectiveness threshold and/or decrease the effectiveness of PT so as to amend the inequity between the rich and the poor.

To highlight the role of wealth constraints, our model is based on an economic environment where households can save at an exogenous world interest rate, but they cannot borrow at all.[6] This may be because the young agents face a borrowing rate that is prohibitively high so that borrowing is never worthwhile for them to finance their education.[7] The borrowing restriction is suitable for the households where children are not financially independent. In particular, young agents' economic status before going to

[5]As Kim and Park (2010) point out, "Previous studies demonstrate that the most powerful factors determining the demand for PT are parental education and household income (Stevenson and Baker 1992; Kim and Lee 2001; Davies 2004; Tansel and Bircan 2006; Dang 2007). The influences of these variables are coherent across countries and quite robust to the models used."

[6]That credit market imperfections and limited borrowing opportunities from privately provided credit play crucial roles in human capital investments has long been discussed by economists (for example, Boldren and Montes (2005), Jacobs (2007) and Cremer et. al. (2011)). Intuitively speaking, when the young agents cannot pledge their future skills or labor as collateral and investment in human capital involves risks, they will be subject to borrowing constraint for financing education. Taking borrowing restrictions into account, many studies assume an exogenous borrowing constraint for human capital investments (for example, Aiyagari, Greenwood, and Seshadri (2002), Cordoba and Ripoll (2009), Keane and Wolpin (2001) and Galor (2012)).

[7]We explain the reason by comparing our model with Galor (2012). Galor (2012) assumes children receive bequest from their parents when they are young and make educational decisions by themselves. Any young agent can go to college if he pays the tuition which may be financed by borrowing in the financial market and repaid after he graduates. In contrast, in this paper, parents decide on their children's' educational investment and there is no bequest.

college and/or before entering the job market prevents them from financing their private tutoring. In addition, since a parent is an old agent who will die in the next period, any lending to an old agent has no way to enforce loan repayments from the borrower in the next period. Thus, no old agent can borrow in the financial market. If the households are allowed to borrow up to a certain limit, our results will be affected only quantitatively and remain intact qualitatively as long as not all individuals can borrow sufficient amounts to fulfill their consumption and investment plans, i.e., some poor agents remain wealth-constrained particularly.

Alternatively, we later on allow for the young agents, rather than old agents, to finance their college tuitions by borrowing student loans.[8] Since our model is a two-period OG model, we cannot rely on the old agents to lend to the young agents. However, because the environment under study is a small open economy, an exogenous credit provision for the student loans is possible, such as borrowing from abroad through government guarantee. This specification simplifies the supply side of credits for the student loans, but it facilitates us to generate a scenario illustrating the essential role of PT played in wealth constraints and in the inequity issue. We can show that even if students have free access to loans for paying full college tuition and the loans involve no default risks, the inequity issue remains unsolved.

The rest of the paper proceeds as follows. In section 2 we introduce the model and derive the steady-state equilibrium. In section 3 policy implications are discussed. Finally, we summarize the contribution of this paper and make concluding remarks in section 4.

[8]Similar idea has been studied by Boldren and Montes (2005). They consider a three-period overlapping generation model so that the young generation can live with an older generation – the middle aged – which still has one period of life to go. Since the young can borrow from the middle aged to finance their college tuition and at the same time the middle aged can increase their wealth at their last period of life through lending to the young, this creates more opportunities for the young to finance tuitions. Still, in their model older generations play the role of savers instead of borrowers in the credit market.

2 The Model

Consider a small open overlapping-generations economy in which economic activity extends over infinite discrete time. Each generation lives for two periods and has a population of one. A household consists of one old parent and one young child. The innate abilities of young agents are distributed uniformly on the interval $[0, 1]$, which are independent of their parents' abilities.

Young agents go to school for free education in the first half of their young life so as to be qualified as unskilled workers. They may further go to college in the second half of their young life. The tuition for college per student is c, paid by the parents. There are N, $N \in (0, 1)$, admissions to college at a time so that not every young agent can go to college. To study in a college, a young agent at time $t+1$ must pass the college entrance examination, which requires an ability to be equal to or higher than an ability threshold level $\overline{h}_{t+1}$. Receiving PT may raise a young agent's ability level before taking the college entrance exam. Parents decide whether to invest on their children's PT and parents are aware of their children's innate abilities when making the decisions.

All college graduates become skilled workers when they are old. Alternatively, if a young agent fails to pass the entrance exam then he starts to work in the middle of the period as an unskilled worker and remains unskilled when he is old. Let superscripts s and u attached in notations refer to skilled workers and unskilled workers, respectively, and denote old workers by type-j, $j = u$, s. A type-j family is headed by a type-j old worker and the fraction of type-j old workers at time t is γ_t^j, $j = u$, s.

2.1 Production

The country is a small open economy. In every period the economy produces one single homogeneous good Y in a perfectly competitive environment. There are two technologies to produce the good Y : one is by skilled labor services and physical capital, the other is by unskilled labor services. The stock of physical capital in every period is formed by aggregate domestic savings in the preceding period, net of international lending. All firms can borrow and lend at the world one-period interest rate r, $r \in (0, 1)$. Physical capital depreciates fully within a period. Each skilled (unskilled) worker can offer one unit of skilled (unskilled) labor services by working one time period at a domestic firm.

There are two sectors, S and U, producing the good Y which can be used for consumption or investment. In sector S, investing K_t units of capital and employing L_t^s units of skilled labor services at time t yield Y_t^s units of goods according to:

$$Y_t^s = F\left(K_t, L_t^s\right) = L_t^s f\left(k_t\right),$$

where $k_t \equiv K_t/L_t^s$ is the capital-labor ratio. In sector U, L_t^u units of unskilled labor services are employed to produce Y_t^u according to:

$$Y_t^u = aL_t^u.$$

Profit-maximization of these two sectors leads to:

$$r_t = f'\left(k_t\right) \equiv r(k_t),\ w_t^s = f(k_t) - f'(k_t)k_t \equiv w^s(k_t),\ \text{and } w_t^u = a \equiv w^u,$$

where w_t^s and w_t^u are the one-period wage rate of skilled workers and unskilled workers, respectively, and r_t is the one-period rental rate of physical capital. Given that the world interest rate r is constant over time to this small open country, we have $r_t = r$. Therefore, we can write

$$k_t = f'^{-1}\left(r\right) \equiv k \ \text{ and } \ w_t^s = w^s(k) \equiv w^s.$$

The production setup of the small open economy is based on Galor (2012). Several features of the production setup permit a simplified economic environment but there still exist sufficient economic interactions in the environment, allowing us to focus on how parents with different wealth levels invest on their children's' private tutoring. Because the country is a small open economy, domestic interest rate is exogenously determined by the world interest rate. Therefore, the capital stock k_t is pinned down easily by $k_t = f'^{-1}\left(r\right)$, which leads to a constant wage rate of skilled workers by $w_t^s = w^s(k)$. The constant wage rate of unskilled workers, w^u, is derived directly by the coefficient of marginal labor productivity a. That sectors S and U produce the same final good Y with different technologies implies the skills acquired in colleges to work with physical capital raises labor productivity, offering an explanation to why w^s is larger than w^u. The production side of the economy generates $w^s > w^u$, facilitating a financial incentive for people to pursue higher education. Moreover, since there is only one final good Y, we need not to deal with any relative price of final goods, which simplifies households' consumption choices. After all, in this paper, we are interested in the division of a parent's wealth between consumption and investment in education, not in the composition of consumption.

There is one feature of the production setup here which differs from Galor (2012), that is, the young unskilled workers are accommodated into productions in the middle of a time spell. In this paper, a young agent born at time t who does not go to college starts to work in the middle of time t in sector U. Recall that each worker can offer one unit of labor service during one unit of time spell. Since the young unskilled worker only work a half units of a time spell in sector U where only unskilled labor services with a constant marginal productivity are employed as input, we can assume that the young unskilled worker offers 1/2 units of unskilled labor services and, therefore, earns $w_t^u/2$ in time t.

2.2 Private Tutoring

Let $e^{i_{t+1}^j}$ denote the investment on PT by a generation-t type-j parent at time $t+1$ given that his child's innate ability is i_{t+1}^j. The old agent finances $e^{i_{t+1}^j}$ through his wealth. Let $h^{i_{t+1}^j}$ denote the child's ability when taking the entrance exam. If the parent does not spend on PT then $e^{i_{t+1}^j} = 0$ and $h^{i_{t+1}^j} = i_{t+1}^j$. If the parent invests $e^{i_{t+1}^j}$ on PT then his child's ability becomes

$$h^{i_{t+1}^j} = \begin{cases} i_{t+1}^j & \text{with probability} \quad 1-\phi, \\ i_{t+1}^j + \delta e^{i_{t+1}^j} & \text{with probability} \quad \phi, \qquad \phi \in [0,1], \end{cases} \tag{1}$$

where ϕ is the probability of a successful investment on PT, measuring the effectiveness of PT. The larger the probability ϕ is, the more effective the PT becomes. Note that $\delta > 0$ is the transformation parameter of expenditure on PT. Without loss of generality, we normalize it to be unity, $\delta = 1$.

Only if $h^{i_{t+1}^j} \geq \overline{h}_{t+1}$, the young agent can go to college. It is assumed that any generation-t type-j old agent has perfect foresight of the threshold level $\overline{h}_{t+1}$ and is aware of his child's innate ability i_{t+1}^j.[9] Therefore, we can focus on the uncertainty or the

[9]We can assume that by the stage of free education where all young agents must participate, the

effectiveness of PT in this paper.[10] We specify $\widetilde{e}^{i^j_{t+1}}$ to be the minimum investment on PT so that $h^{i^j_{t+1}}$ is no less than the ability threshold $\overline{h}_{t+1}$,

$$\widetilde{e}^{i^j_{t+1}} = \begin{cases} \overline{h}_{t+1} - i^j_{t+1} & \text{for } i^j_{t+1} < \overline{h}_{t+1}, \\ 0 & \text{for } i^j_{t+1} \geq \overline{h}_{t+1}. \end{cases} \tag{2}$$

2.3 Households

2.3.1 Preferences and Wealth Levels

Consider a generation-t type-j old agent who has a child with innate ability i^j_{t+1}, $j = s, u$. His preference is presented by

$$U^{i^j_{t+1}} = C^{i^j_{t+1}} + W^{i^j_{t+1}}, \tag{3}$$

where $C^{i^j_{t+1}} \geq 0$ is household consumption and $W^{i^j_{t+1}}$ is the life-time wealth of his child. The old agent cares equally about the family's current living standard (represented by $C^{i^j_{t+1}}$) and its future prosperity (represented by $W^{i^j_{t+1}}$). The old agent maximizes (3) subject to his life-time wealth.

It is assumed that any agent can save at the interest rate r but he cannot borrow in the credit market. A skilled old agent has a life-time wealth of w^s which is wage earned after graduation from college. An unskilled agent earns $w^u/2$ when he is young and the wage is saved at the world interest rate r. Thus the life-time wealth of an unskilled old equals to his wage at old plus his wealth from saving at young, that is, $w^u + (1+r)\, w^u/2$. Since $w^j_t = w^j$ for $j = s, u$, we define the life-time wealth of a type-j old agent by ω^j where[11]

$$\omega^u \equiv Rw^u,\ \omega^s \equiv w^s \text{ and } R \equiv 1 + (1+r)/2.$$

society can have sufficient information to generate a prediction on the ability threshold which is correct in the equilibrium.

[10]When $\overline{h}_{t+1} \geq 1$, every young agent at time $t+1$ receives PT and may fail to go to college even if his innate ability is one, the highest innate ability. When $\overline{h}_{t+1} < 1$, those young with innate abilities larger than $\overline{h}_{t+1}$ do not receive PT and go to college for sure. It may be argued that no one can be sure about passing the college entrance exam. This situation occurs if there is a risk involved in taking the entrance exam itself. However, if everyone is subject to the same uncertainty in taking the exam, this will not raise inequity issue and will not affect our results qualitatively. Since we intend to uncover the importance of PT in this paper, we only specify risks associated with PT itself.

[11]We specify that the wage $w^u/2$ earned by a unskilled young worker at time t generates an interest

Assumption 1. $\omega^u < \omega^s$ and $\omega^u \geq c$.

We assume $\omega^u < \omega^s$ so that higher education is attractive to students and assume $\omega^u \geq c$ so that every old agent can afford college tuition. We intend to feature an environment where a parent unable to send his child to college is not because of the college tuition. Given $\omega^u \geq c$, if there is no PT or if PT cannot affect students' abilities at all, i.e., $\phi = 0$, there will be no inequity.

If $\phi = 0$, a type-j generation-t old agent whose child's innate ability is no less than the ability threshold $\overline{h}_{t+1}$ decides to pay for the college tuition only if $\omega^s - \omega^u \geq c$. For this old agent, paying c generates a utility of $\omega^j - c + \omega^s$; not paying c (which means that his child will not go to college) results in a utility of $\omega^j + \omega^u$. Utility-maximization implies that the old agent pays c only if $\omega^s - \omega^u \geq c$, that is to say, if $\omega^s - \omega^u < c$, no one goes to college. Therefore, when PT is available and $\phi > 0$, the difference between ω^s and ω^u must be larger than c so that this old agent will have an incentive to pay for c and/or his child's PT. Since we are interested in the environment where the measure of college students is positive, we specify Assumption 2.

Assumption 2. $0 < \omega^s - \omega^u - c \leq 1$.

Assumption 2 not only ensures $\omega^s - \omega^u \geq c$ but also serves to scale the values of parameters. Since we specify the value of an innate ability to belong to $[0, 1]$ and the ability threshold will generally not being far from the interval of $[0, 1]$, Assumption 2 restricts the values of parameters so that all agents encounter nontrivial constraints when making optimal choices. This nontrivial-constraints argument will be made clear in the following subsection.

2.3.2 Incentive Constraints and Wealth Constraints

Consider a generation-t type-j old agent who has a child with innate ability $i^j_{t+1} < \overline{h}_{t+1}$, $j = u, s$. Clearly, his child cannot pass the college entrance exam without an investment

revenue of $rw^u/2$ at time $t+1$. It can be argued that the interest revenue of $w^u/2$ is less than $rw^u/2$, for example, $\frac{r}{\theta}w^u/2$ and $\theta < 1$, since he enters the job market in the middle of a time period. Then, we can define $R_\theta \equiv 1 + \frac{1}{2}\left(1 + \frac{r}{\theta}\right) < R$. However, the difference between R_θ and R can be negligible. What is required in the later analysis is $R > 1$. Therefore, it does not matter if we use R_θ or R.

of $\widetilde{e}^{i^j_{t+1}}$ on PT. The old agent is able to pay $\widetilde{e}^{i^j_{t+1}}$ and c if his wealth is large enough, i.e., $\omega^j \geq \widetilde{e}^{i^j_{t+1}} + c$. This condition, together with (2) and $i^j_{t+1} < \overline{h}_{t+1}$, results in

$$\overline{h}_{t+1} - \left(\omega^j - c\right) \leq i^j_{t+1} < \overline{h}_{t+1},$$

which is the wealth constraint of the type-j old agent. The old agent is willing to pay both $\widetilde{e}^{i^j_{t+1}}$ and c if the expected utility level after paying them, i.e., $(1-\phi)\left(\omega^j - \widetilde{e}^{i^j_{t+1}} + \omega^u\right) + \phi\left(\omega^j - \widetilde{e}^{i^j_{t+1}} - c + \omega^s\right)$, is no less than the utility level without paying them, i.e., $\omega^j + \omega^u$. We can, therefore, write

$$\overline{h}_{t+1} - \phi\left(\omega^s - \omega^u - c\right) \leq i^j_{t+1} < \overline{h}_{t+1},$$

as the incentive constraint for this type-j old agent.[12] The lower bounds of wealth constraints and incentive constraints matter very much in our analysis followed. We thus define x^j and y to make the expressions regarding these lower bounds more concise,

$$x^j \equiv \phi\left(\omega^j - c\right) \quad \text{and} \quad y \equiv \phi^2\left(\omega^s - \omega^u - c\right),\ j = s, u,$$

where $x^j \geq 0$ and $y \geq 0$ according to Assumptions 1 and 2. We then respectively express the wealth constraint and the incentive constraint of the type-j old agent, $j = s, u$, as

$$\overline{h}_{t+1} - \frac{x^j}{\phi} \leq i^j_{t+1} < \overline{h}_{t+1},\ j = s, u, \tag{WCj}$$

and

$$\overline{h}_{t+1} - \frac{y}{\phi} \leq i^j_{t+1} < \overline{h}_{t+1}. \tag{ICj}$$

Definition 1. ICj **dominates** WCj if the lower bound of ICj is larger than the lower bound of WCj, $j = s, u$, and vice versa.

Definition 2. ICj or WCj, $j = s, u$, is **trivial** (**nontrivial**) if the lower bound for i^j_{t+1} is less than zero (no less than zero).

Given that $i^j_{t+1} \in [0, 1]$, if $\overline{h}_{t+1} < x^j/\phi$, then WCj is trivial and every type-j old agent is able to pay $\widetilde{e}^{i^j_{t+1}}$ and c; if $\overline{h}_{t+1} < y/\phi$ then ICj is trivial and every type-j old agent is willing to pay $\widetilde{e}^{i^j_{t+1}}$ and c. We will focus on the environment where every dominating constraint is nontrivial.[13]

[12] The incentive constraint is derived by $(1-\phi)\left(\omega^j - \widetilde{e}^{i^j_{t+1}} + \omega^u\right) + \phi\left(\omega^j - \widetilde{e}^{i^j_{t+1}} - c + \omega^s\right) \geq \omega^j + \omega^u$, $\widetilde{e}^{i^j_{t+1}} = \overline{h}_{t+1} - i^j_{t+1}$ and $i^j_{t+1} < \overline{h}_{t+1}$.

[13] Now we can also tell the importance of Assumption 2 regarding nontrivial constraints. By assuming $y/\phi = \phi\left(\omega^s - \omega^u - c\right) \in (0, 1]$ for $\phi \in (0, 1]$, we can generate nontrivial ICj and WCj even when $\overline{h}_{t+1} < 1$.

Definition 3. A type-j old agent is **wealth-constrained** if WCj dominates ICj, $j = s, u$.

Since the optimal choice will only be determined by the dominating constraint, we can neglect the dominated constraint.[14] In addition, we can show that ICs dominates WCs and the rich will never be wealth-constrained.[15] This is because the wealth level of a skilled worker is sufficiently large by Assumption 2. A dominating ICs simplifies our analysis, allowing us to focus on the case(s) where PT makes unskilled old (poor) agents wealth-constrained. Furthermore, WCj and ICj indicate that $y < x^j$ when ICj is dominating and $y > x^j$ when WCj is dominating. Since WCs never dominates, for convenience, we define x by

$$x \equiv x^u.$$

By equating the lower bounds of WCu and ICu for the unskilled old agent, we define the PT-effectiveness threshold level ϕ^* as follows,

$$\phi^* \equiv \frac{\omega^u - c}{\omega^s - \omega^u - c},$$

so that we have

$$\begin{cases} \text{WCu dominates ICu and } y > x \text{ if } \phi > \phi^*; \\ \text{ICu dominates WCu and } y < x \text{ if } \phi < \phi^*. \end{cases} \tag{4}$$

This says that the lower the PT-effectiveness threshold level ϕ^* is, the more likely the unskilled old agent is wealth-constrained.[16] It is straightforward to show that, given

[14]For example, once an agent is wealth-constrained, his incentive constraint is dominated. He will pay for his child's PT and college tuition if he can afford the expense. Since whenever he can afford the educational expense he is also willing to pay for it, we can neglect his incentive constraint.

[15]By the lower bounds of WCs and ICs, we derive $(\omega^s - c) - \phi(\omega^s - \omega^u - c) = (1-\phi)(\omega^s - \omega^u - c) + \omega^u > 0$.

[16]It may be argued that the marginal effect of PT on children's ability is not constant as specified in (1). For example, we can specify

$$h^{i^j_{t+1}} = \begin{cases} i^j_{t+1} & \text{with probability} \quad 1-\phi, \\ i^j_{t+1} + \left(e^{i^j_{t+1}}\right)^\theta & \text{with probability} \quad \phi, \quad \phi \in (0,1]. \end{cases}$$

Then, following the same analysis mentioned above, we can derive the wealth constraint as $\overline{h}_{t+1} - \left(\omega^j - c\right)^\theta \leq i^j_{t+1} < \overline{h}_{t+1}$ and the incentive constraint as $\overline{h}_{t+1} - [\phi(\omega^s - \omega^u - c)]^\theta \leq i^j_{t+1} < \overline{h}_{t+1}$. If $\theta > 1$ (if $\theta \in (0,1)$), the marginal effect of PT on children's ability is increasing (decreasing). In addition, let $x^* \equiv \phi\left(\omega^j - c\right)^\theta$ and $y^* \equiv \phi[\phi(\omega^s - \omega^u - c)]^\theta$. Since j^* is a monotone transformation of j, $j = x, y$, the

$\phi^* < 1$, ϕ^* increases in ω^u but decreases in ω^s and c.[17] From this perspective, widening the wealth gap between the rich and the poor or raising college tuition tends to make the poor become wealth-constrained.

2.4 Equilibrium

An equilibrium is a sequence of $\left\{C^{i_t^j}, e^{i_t^j},\ \overline{h}_t\right\}$ for $j = s, u$, $i_t^j \in [0, 1]$ and $t = 1, 2, ..$, which satisfies the following (i), (ii) and (iii).

(i) Each generation-$(t-1)$ type-j old agent who has lifetime wealth ω^j and a child with an innate ability of i_t^j chooses his investment on PT $e^{i_t^j}$ and consumption $C^{i_t^j}$ given $\overline{h}_t$ to maximize expected utility $U^{i_{t+1}^j}$.

(ii) Only if $i_t^j < \overline{h}_t$ and i_t^j satisfies both of WCj and ICj, $e^{i_t^j} = \widetilde{e}^{i_t^j} > 0$, where $\widetilde{e}^{i_t^j}$ satisfies (2) ; otherwise, $e^{i_t^j} = 0$. If $e^{i_t^j} = 0$ then $C^{i_t^j} = \omega^j$; if $e^{i_t^j} > 0$ then $C^{i_t^j} = \omega^j - \widetilde{e}^{i_t^j} - c$ with a probability of ϕ and $C^{i_t^j} = \omega^j - \widetilde{e}^{i_t^j}$ with a probability of $(1 - \phi)$.

(ii) Given $\left\{e^{i_t^j}\right\}$, the measure of generation-$(t-1)$ old agents who has a child with $i_t^j + e^{i_t^j} \geq \overline{h}_t$ is N. N is the sum of admissions to college.

To derive the equilibrium, we consider several possible cases, Case z, $z = A$, B, C, D. Let e_t denote the total spending on PT at time t in the equilibrium and let $\overline{h}_z$ and $\overline{e}_z$ respectively denote the steady state value of $\overline{h}_t$ and of e_t in Case z. One special feature of the equilibrium is that no matter the economy is or is not at the steady state when $t = 1$, it reaches the steady-state equilibrium when $t = 2$. The value of γ_0^s and γ_0^u (which are exogenously given at $t = 1$ and respectively is the population of type-s and the population of type-u generation-0 old agents) will affect the value of $\overline{h}_1$. However, since $\overline{h}_2$ is independent of $\overline{h}_1$ and since $\gamma_t^s = N$ and $\gamma_t^u = 1 - N$ for $t \geq 1$, we know that the equilibrium values of $\overline{h}_2$ and $\overline{h}_t$ for $t > 2$ will be determined by the same variables of the same values in the same way so that $\overline{h}_2$ is the steady-state value of the ability threshold. Therefore, in the following Cases, we focus on the steady-state equilibriums.

results derived from WCj and ICj will be preserved qualitatively if we replace x/ϕ and y/ϕ respectively by x^*/ϕ and y^*/ϕ in WCj and ICj. The non-constant marginal effect of PT on children's abilities may only affect our results quantitatively but not qualitatively.

[17] Since $\phi^* < 1$ and $\omega^u > c$, we have $\omega^s > 2\omega^u > 2c$. Thus, $\frac{\partial\phi^*}{\partial\omega^u} = \frac{\omega^s - 2c}{(\omega^s - \omega^u - c)^2} > 0$, $\frac{\partial\phi^*}{\partial\omega^s} = \frac{-(\omega^u - c)}{(\omega^s - \omega^u - c)^2} < 0$ and $\frac{\partial\phi^*}{\partial c} = \frac{2\omega^u - \omega^s}{(\omega^s - \omega^u - c)^2} < 0$.

The equilibrium depends on how large the number of college admissions is. To characterize the size of N, we firstly work on the solution of $\overline{h}_t$ at time t given N; secondly pin down the the range of N respectively for $\overline{h}_t \geq 1$ and for $\overline{h}_t < 1$; at last, we say that a level of N is small (large) when it induces a steady-state ability threshold level which is greater than or equal to one (less than one).

Case A: Given that $\phi \leq \phi^*$ and N Is Large.

Given $\phi \leq \phi^*$, the wealth constraint is dominated by the incentive constraint for all families and $y \leq x$. Given a large N, the steady-state ability threshold should be less than one, i.e., $\overline{h}_A < 1$.

Consider time t, $t \geq 2$, when $\gamma^s_{t-1} = N$ and $\gamma^u_{t-1} = 1 - N$. If $\overline{h}_t < 1$ in the equilibrium, we also have $\overline{h}_t - \frac{y}{\phi} < 1$. In addition, $0 \leq \overline{h}_t - \frac{y}{\phi}$ is required for nontrivial ICs and ICu. The total number of children go to college at time t is $\left(1 - \overline{h}_t + y\right)$, which includes those who receive PT and pass the entrance exam, i.e., $\phi\left(\gamma^s_{t-1} + \gamma^u_{t-1}\right)\frac{y}{\phi}$, and those who pass the exam without PT, i.e., $\left(\gamma^s_{t-1} + \gamma^u_{t-1}\right)\left(1 - \overline{h}_t\right)$. Thus, the composition of population for the skilled and unskilled workers features $\gamma^s_t = y + 1 - \overline{h}_t = N$, which implies $\overline{h}_t = 1 + y - N$ for $t \geq 2$. Thus, the economy is at its steady state at time 2 and we can write

$$\overline{h}_A = 1 + y - N.$$

To ensure $\overline{h}_A < 1$ and $\overline{h}_A - \frac{y}{\phi} \geq 0$, i.e., $\frac{y}{\phi} \leq \overline{h}_A < 1$, the parameters must satisfy[18]

$$y < N \leq 1 - \frac{y}{\phi}(1 - \phi) \text{ and } y < \phi.$$

The total spending on PT at the steady-state equilibrium, therefore, is derived as:

$$\overline{e}_A = \int_{\overline{h}_A - y/\phi}^{\overline{h}_A} \left(\overline{h}_A - i\right) di = \frac{y^2}{2\phi^2}.$$

[18]The parameters must satisfy $y < N$ (to ensure $\overline{h}_A < 1$), $y \leq \phi\left(\frac{1-N}{1-\phi}\right)$ (to ensure $\overline{h}_A - \frac{y}{\phi} \geq 0$) and $y < \phi$ (so that $\frac{y}{\phi} < 1$). When $N > \phi$, we have $y \leq \phi\left(\frac{1-N}{1-\phi}\right) < \phi < N$, and we derive $\phi < N \leq 1 - \frac{y}{\phi}(1-\phi)$ by $y \leq \phi\left(\frac{1-N}{1-\phi}\right)$ and $N > \phi$. When $N \leq \phi$, we have $y < N \leq \phi \leq \phi\left(\frac{1-N}{1-\phi}\right)$, and we derive $y < N \leq \phi$ by $y < N$ and $N \leq \phi$. In sum, we get $\begin{cases} \phi < N \leq 1 - \frac{y}{\phi}(1-\phi) & \text{for } N > \phi \\ y < N \leq \phi & \text{for } N \leq \phi \end{cases}$, and, in any case, $y < \phi$ holds.

Case B: Given that $\phi \le \phi^*$ and N Is Small.

In this case ICj, $j = s, u$, dominates, $y \le x$ and the steady-state ability threshold should be no less than one, i.e., $\overline{h}_B \ge 1$.

When $\overline{h}_t \ge 1$, all college students have received PT. The dominating ICj should be rewritten as $0 \le \overline{h}_{t+1} - \frac{y}{\phi} \le i^j_{t+1} \le 1,\ j = s, u$, which imply that $\left(\gamma^u_{t-1} + \gamma^s_{t-1}\right)\left(1 - \overline{h}_t + y/\phi\right)$ young agents receive PT and ϕ of them go to college. The composition of population implies $\gamma^s_t = \phi\left(1 - \overline{h}_t + y/\phi\right) = N$ so that $\overline{h}_t = 1 + \frac{y-N}{\phi}$ is derived. Thus, we can write

$$\overline{h}_B = 1 + \frac{y - N}{\phi}.$$

To derive $\overline{h}_B \ge 1$ and $0 \le \overline{h}_B - \frac{y}{\phi} < 1$, the parameters must satisfy

$$0 < N \le y.$$

Notice that, since $y \le \phi$ by Assumption 2, $N \le y$ implies $N \le \phi$. In addition, we get

$$\overline{e}_B = \int_{\overline{h}_B - y/\phi}^{1} \left(\overline{h}_B - i\right) di = \frac{y^2 - (y - N)^2}{2\phi^2}.$$

Case C: Given that $\phi > \phi^*$ and N Is Large.

A large N implies that the steady-state ability threshold should be less than one, i.e., $\overline{h}_C < 1$. When $\phi^* < \phi$, WCu dominates ICu for the unskilled old agents, ICs dominates WCs for skilled old agents and $y > x$. By WCu and ICs, we know that at time t, $t \ge 2$, $\gamma^s_t = \left(1 - \overline{h}_t\right) + \phi\left[(1 - N)\, x/\phi + Ny/\phi\right] = N$. Thus, we have

$$\overline{h}_C = 1 + x - N\left(1 + x - y\right).$$

Since $\overline{h}_C < 1$, we know that $\frac{x}{1+x-y} < N$ and $1 + x - y > 0$. If $1 + x - y \le 0$ then $\overline{h}_C \ge 1 + x$, contradicting $\overline{h}_C < 1$. In addition, $0 \le \overline{h}_C - \frac{y}{\phi}$ is required so that ICs and WCu are nontrivial given $y > x$. By $0 \le \overline{h}_C - \frac{y}{\phi}$, we derive $N \le 1 - \left(\frac{1}{1+x-y}\right)\frac{y}{\phi}\left(1 - \phi\right).$

Thus, the parameters must satisfy[19]

$$\frac{x}{1+x-y} < N \leq 1 - \left(\frac{1}{1+x-y}\right)\frac{y}{\phi}(1-\phi),\ 1+x-y > 0 \text{ and } y < \phi.$$

We also derive

$$\bar{e}_C = N\int_{\overline{h}_C - y/\phi}^{\overline{h}_C}\left(\overline{h}_C - i\right)di + (1-N)\int_{\overline{h}_C - x/\phi}^{\overline{h}_C}\left(\overline{h}_C - i\right)di = \frac{Ny^2 + (1-N)x^2}{2\phi^2}.$$

Case D: Given that $\phi > \phi^*$, N Is Small.

When $\phi > \phi^*$, ICs and WCu are dominating and $y > x$. Since we focus on $\overline{h}_D \geq 1$, ICs and WCu may feature that $0 \leq \overline{h}_D - \frac{y}{\phi} \leq 1 < \overline{h}_D - \frac{x}{\phi}$ and college students come from type-s families only or that $0 \leq \overline{h}_D - \frac{y}{\phi} < \overline{h}_D - \frac{x}{\phi} \leq 1$ and college students come from both types of families in the steady state.

When no type-u family affords PT and college tuition in the equilibrium, the composition of population indicates $\gamma_t^s = N\phi = N$ for $t \geq 2$. Thus, $\phi = 1$ is required. In addition, all skilled old agents must invest in PT. Thus, the lower bound of ICs is zero. In sum, in this equilibrium, $\phi = 1$, $\overline{h}_D \geq 1$ and $0 = \overline{h}_D - \frac{y}{\phi} \leq 1 < \overline{h}_D - \frac{x}{\phi}$ hold. Since $\overline{h}_D = \frac{y}{\phi} = y$ (by $\phi = 1$ and $\overline{h}_D - \frac{y}{\phi} = 0$) and $y \leq \phi$ (by Assumption 2), we know that $\overline{h}_D = y \leq 1$. Furthermore, $\overline{h}_D \leq 1$ and $\overline{h}_D \geq 1$ indicate $\overline{h}_D = 1$. Thus, the restriction of $1 < \overline{h}_D - \frac{x}{\phi}$ means $1 < 1 - x$. However, $1 < 1 - x$ contradicts $x \geq 0$.

When college students come from both types of families and $\overline{h}_t \geq 1$ in the equilibrium, the composition of the population for the skilled and unskilled workers at time t, $t \geq 2$, is $\gamma_t^s = (1-N)\phi\left(1 - \overline{h}_t + \frac{x}{\phi}\right) + N\phi\left(1 - \overline{h}_t + \frac{y}{\phi}\right) = N$. Thus, we get

$$\overline{h}_D = 1 + \frac{x - N(1+x-y)}{\phi}.$$

For $\overline{h}_D \geq 1$ and $0 \leq \overline{h}_D - \frac{y}{\phi} < \overline{h}_D - \frac{x}{\phi} \leq 1$, the parameters must satisfy

$$0 < N \leq \frac{x}{1+x-y} \quad \text{and} \quad 1+x-y > 0.$$

[19] Consider the upper and lower bounds of N, i.e., $\frac{x}{1+x-y}$ and $1 - \left(\frac{1}{1+x-y}\right)\frac{y}{\phi}(1-\phi)$. By Assumption 2, we have $y \leq \phi$. It is easy to show that if $y < (=)\phi$ then $\left[1 - \left(\frac{1}{1+x-y}\right)\frac{y}{\phi}(1-\phi)\right] > (=)\frac{x}{1+x-y}$. Thus, $y < \phi$ is required in Case C.

The total spending on PT is derived as

$$\begin{aligned}\overline{e}_D &= N\int_{\overline{h}_D-\frac{y}{\phi}}^{1}\left(\overline{h}_D-i\right)di+(1-N)\int_{\overline{h}_D-\frac{x}{\phi}}^{1}\left(\overline{h}_D-i\right)di\\ &= \frac{Ny^2+(1-N)x^2-\left[x-N\left(1+x-y\right)\right]^2}{2\phi^2}.\end{aligned}$$

2.5 Discussion on the Steady-State Equilibriums

We summarize the steady state values of $\overline{h}_z$ and $\overline{e}_z$ and depict the ranges of ϕ and N for the above 4 Cases in Table 1 and Figure 1.[20]

[Insert Table 1 here]

[Insert Figure 1 here]

We first interpret Figure 1 for a given number of admissions N and see how the equilibrium changes when ϕ varies. For a given N, we find that the ability threshold rises as ϕ increases. Since a higher ability threshold requires a larger minimum investment on PT, $\widetilde{e}^{ij}_{t+1}$, a higher ϕ makes unskilled old agents become wealth-constrained. In Cases A and B, children with relatively high innate abilities receive PT no matter they are from the rich or from the poor families. In Cases C and D where WCu dominates, some children from the poor families cannot receive PT while other children with lower innate abilities than these poor children can because they are from the rich families. To let a young agent with a low innate ability become a skilled worker is more costly than to train one with a higher innate ability since the former situation requires a larger amount of investment on PT than the latter. Thus, a dominating WCu not only implies the inequity in receiving higher education, it also implies inefficiency in cultivating skilled workers.

We next discuss Figure 1 for a given effectiveness of PT ϕ and see how the equilibrium changes as N varies. In cases A and B where $\phi \leq \phi^*$, no family is wealth-constrained and the average spending on PT for any type of families is the same, i.e., $\frac{y^2}{2\phi^2}$ in Case A and $\frac{y^2-(y-N)^2}{2\phi^2}$ in Case B. This is because that as long as a child's innate ability is above

[20] Notice that Figure 1 features several properties among the variables: (1) If $\phi = \phi^*$ then $x = y$. (2) Since $\omega^s - \omega^u - c \leq 1$, we know that $y/\phi \leq 1$ and the line $y = 1 + x$ must be on the RHS of $\phi = 1$. (3) In addition, $\omega^s - \omega^u - c \leq 1$ also implies $x < \phi$ given $\phi^* < \phi$.

a certain level, he receives PT. Furthermore, all PT-receiving students are subject to an equal probability to become a college student, thus, both types of families share an equal proportion of children to participate in PT, leading to an equal average spending on PT.

For a given ϕ, we find that the number of admissions to colleges is inversely related to the ability threshold level ($\partial\overline{h}_j/\partial N < 0$, for $z = A, B$). Consider a sufficiently large N, for example, $N \geq y$ in Case A. As N increases in Case A, more parents with high-innate-ability children are no longer need to invest on PT, which helps to reduce the ability threshold. At the same time, a lower ability threshold draws in more parents to invest on PT though they actually spend less on PT as $\overline{h}_A$ declines. In aggregate, the spending on PT is kept stable at $\frac{y^2}{2\phi^2}$. Relative to Case A, N is smaller in Case B. A smaller N induces those parents with high-innate-ability children to invest more on PT in order to compete for the smaller number of admissions, which pushes up the ability threshold level and raises the expenditure of PT. The competition is rather intense reflected by $\overline{h}_B \geq 1$, as a result, those parents with lower-innate-ability children are no longer willing to invest on PT which would cost them a rather considerable amount of wealth, reducing the expenditure of PT. Since the latter change in the expenditure of PT dominates the former, the aggregate spending on PT declines as N becomes smaller.

Interestingly, results from Cases A and B indicate that the aggregate spending on PT is non-decreasing in the number of college admissions, i.e., $\partial\overline{e}_B/\partial N > 0$, $\partial\overline{e}_A/\partial N = 0$ and $\overline{e}_B < \overline{e}_A$. In essence, the ability threshold declines as N rises, which reduces the minimum required investment on PT and, therefore, attracts more investments on PT. If an educational policy is aimed at reducing the activities of PT through establishing more colleges or offering more placements for students in colleges, our result indicates that the implementation may actually enhance the activities of PT. In Figure 1, the upper bound of N which corresponds to an ability threshold valued at 1 is increasing in ϕ also indicates that as ϕ increases, investment on PT becomes more effective and, therefore, raising N may not make PT less attractive.

We now consider Cases C and D where ϕ is large and poor families are wealth-constrained. Similarly, we also get $\partial\overline{e}_j/\partial N > 0$, $z = C, D$ and $\overline{e}_D < \overline{e}_C$. Thus, no matter the poor families are wealth-constrained or not, the aggregate spending on PT is non-decreasing in the number of college admissions. Increasing N by expanding the size of colleges and/or increasing the number of colleges will increase the population of skilled workers, however, it still puts the remaining poor families in a disadvantaged status. The crux is that a dominating wealth constraint WCu will not be changed into a dominated one by any variation in N for a given ϕ. In essence, a variation in N changes the ability threshold, which further changes the minimum required investment on PT, $\widetilde{e}^{i^j_{t+1}}$, for the poor as well for the rich in the same way. Since all young agents are subject to the same ability threshold, if the poor families are wealth-constrained before the change in N, they will still be wealth-constrained after the change.

Different from Cases A and B, in Cases C and D, the average spending on PT between the two groups are not equal: $\frac{y^2}{2\phi^2}$ for the rich and $\frac{x^2}{2\phi^2}$ for the poor in Case C; $\frac{y^2-[x-N(1+x-y)]^2}{2\phi^2}$ for the rich and $\frac{x^2-[x-N(1+x-y)]^2}{2\phi^2}$ for the poor in Case D. The average spending on PT of the poor families is smaller than that of the rich families indicates that the rich can make better use of PT to send their children to colleges than the poor.

2.6 Discussion on the Effectiveness of PT

The effectiveness of PT represented by ϕ is a constant, particularly, ϕ is independent of a parent's investment on PT in the previous analysis. An alternative specification might consider that if a parent spends more on PT then the investment will generate a higher probability for PT to succeed. Analogous to (1), we specify

$$h^{i^j_{t+1}} = \begin{cases} i^j_{t+1} & \text{with probability} \quad 1-\phi\left(1+\theta e^{i^j_{t+1}}\right), \\ i^j_{t+1}+\delta e^{i^j_{t+1}} & \text{with probability} \quad \phi\left(1+\theta e^{i^j_{t+1}}\right), \end{cases} \tag{5}$$

where $\phi\left(1+\theta e^{i^j_{t+1}}\right) \in [0,1]$, $\theta > 0$ and $\phi \in [0,1]$. If $\theta = 0$, equation (5) reduces back to (1). Since θ will not affect the wealth levels of old agents, the wealth constraint of a type-j old agent is still WCj. However, comparing (5) to (1), the positive relationship between $e^{i^j_{t+1}}$ and the expected value of $h^{i^j_{t+1}}$ is strengthened in (5) in the sense that the expected value of $h^{i^j_{t+1}}$ is larger in (5) than in (1) for a given $e^{i^j_{t+1}}$. Therefore, a larger θ stimulates a stronger incentive to invest on PT. Specifically, given (5), a type-j old agent with $i^j_{t+1} < \overline{h}_{t+1}$ is willing to pay both $\widetilde{e}^{i^j_{t+1}}$ and c if the expected utility level after paying is no less than the utility level without paying, i.e.,

$$\left[1-\phi\left(1+\theta\widetilde{e}^{i^j_{t+1}}\right)\right]\left(\omega^j-\widetilde{e}^{i^j_{t+1}}+\omega^u\right)+\phi\left(1+\theta\widetilde{e}^{i^j_{t+1}}\right)\left(\omega^j-\widetilde{e}^{i^j_{t+1}}-c+\omega^s\right) \geq \omega^j+\omega^u,$$

which implies an incentive constraint named ICCj as

$$\overline{h}_{t+1} - \frac{\phi\left(\omega^s - \omega^u - c\right)}{1 - \theta\phi\left(\omega^s - \omega^u - c\right)} = \overline{h}_{t+1} - \frac{y/\phi}{1 - \theta y/\phi} \leq i_{t+1}^j < \overline{h}_{t+1} \quad \text{for } 0 < \theta < 1. \qquad \text{(ICCj)}$$

In the constraint ICCj, the lower bound of i_{t+1}^j is decreasing in θ. Thus, more old agents are willing to pay for PT under the specification of (5) than under (1).

For $\omega^u - c > 0$, we derive $\partial\phi_j^*/\partial\theta < 0$, $j = u, s$, in particular, we get $\phi_u^* < \phi^*$. Therefore, both types of families might be wealth-constrained, i.e., it is possible that $\phi_u^* < \phi$ and $\phi_s^* < \phi$. Similar to (4), we can write, for $j = s, u$,

$$\begin{cases} \text{WCj dominates ICCj and if } \phi > \phi_j^*; \\ \text{ICCj dominates WCj and if } \phi < \phi_j^*, \end{cases}$$

where

$$\phi_j^* \equiv \frac{\omega^j - c}{\omega^s - \omega^u - c}\left[1 + \theta\left(\omega^j - c\right)\right]^{-1}$$

is derived by equating the lower bounds of WCj and ICCj. To derive the equilibrium, the analysis can be proceeding in a way similar to that in the previous subsections. However, if we also want to consider the scenarios where the rich families are wealth-constrained, we may want to add the case(s) with $\phi > \phi_s^*$.[21]

Although both types of families can be wealth-constrained, by

$$\phi_s^* - \phi_u^* = \frac{\left(\omega^s - \omega^u\right)/\left(\omega^s - \omega^u - c\right)}{\left[1 + \theta\left(\omega^s - c\right)\right]\left[1 + \theta\left(\omega^u - c\right)\right]} > 0,$$

we know that $\phi_s^* < \phi$ implies $\phi_u^* < \phi$, i.e., whenever the rich families are wealth-constrained, the poor families must be wealth-constrained; however, the reverse is not true. Thus, replacing (1) by (5) in this model does not change the rich families' advantage of more wealth over the poor families qualitatively. Recall that, in our previous analysis, only the poor families may be wealth-constrained, in addition, equation (1) has already implied a positive relationship between $e^{i_{t+1}^j}$ and the expected value of $h^{i_{t+1}^j}$. Therefore, no matter (1) or (5) applies, we expect that the properties of the equilibrium are qualitatively the same. The crux is that the rich families have an advantageous wealth status under both specifications.

[21] By the lower bounds of WCs and ICCs, we can derive that if $\theta > \left[\phi\left(\omega^s - \omega^u - c\right)\right]^{-1} - \left(\omega^s - c\right)^{-1} > 0$ (the second inequality comes from $\left(\omega^s - c\right) > \phi\left(\omega^s - \omega^u - c\right)$ as shown in footnote 12) then $\left(\overline{h}_{t+1} - \frac{x^s}{\phi}\right) > \left(\overline{h}_{t+1} - \frac{y/\phi}{1-\theta y/\phi}\right)$ and WCs dominates ICCs.

In the following, we stick to the specification of (1) to emphasize the role of ϕ as an overall measure of the effectiveness of PT and as an overall measure of the insufficiency of pre-college formal education in copying with the college entrance exam.

3 Policy Implications

3.1 Tax and Transfer

Our analysis suggests that to solve the inequity issue requires $\phi \leq \phi^*$, which can be achieved by raising ϕ^* or by reducing ϕ or by both ways.

[Insert Figure 2 here.]

According to Figure 1, ϕ^* is determined by the intersection of curve x and curve y. Therefore, if we can let curve x move upwards to curve $\widetilde{x}$ or curve y move downwards to curve $\widetilde{y}$, ϕ^* will increase to $\widetilde{\phi}$ as shown in Figure 2. Of course, $\widetilde{x}$ and $\widetilde{y}$ together will lead to an even larger PT-effectiveness threshold, for example, $\widetilde{\widetilde{\phi}}$ in Figure 2. Recall that $x \equiv \phi\left(\omega^u - c\right)$ and $y \equiv \phi^2\left(\omega^s - \omega^u - c\right)$, which give us

$$\frac{\partial x}{\partial \omega^u} = -\frac{\partial x}{\partial c} \text{ and } \frac{\partial y}{\partial \omega^s} = -\frac{\partial y}{\partial \omega^u} = -\frac{\partial y}{\partial c}.$$

Therefore, our model implies several ways to raise ϕ^* :

(1) Tax on the wealth of the rich and use the tax revenues to reduce the college tuition. This makes ω^u unchanged but both of ω^s and c decline, leading to a upward shift of curve x.[22]

(2) Tax on the wealth of the rich and transfer the tax revenues to the poor. This makes c unchanged, ω^s decrease and ω^u increase, leading to a upward shift of curve x and/or a downward shift of curve y.[23]

(3) A policy mixed the ways of (1) and (2) raises ϕ^*.

[22] Since in the steady state $\gamma_t^s = N$, the change in ω^s is equal to the change in c but in the opposite directions, i.e., $-\Delta\omega^s = \Delta c$. Thus, curve y will not shift.

[23] Suppose that the policy effectively imposes a tax rate of τ on ω^s. The wealth level of the skilled workers becomes $(1-\tau)\,\omega^s$ and that of the unskilled workers becomes $\omega^u + \frac{N\tau\omega^s}{1-N}$. If $N = 1/2$, y is unchanged.

The policy to reduce ϕ is less obvious in this paper since we treat ϕ exogenous without an explanation to how it is formed. However, since PT acts as supplementary education, we can think about the issue by looking at the formal educational system. To reduce the effectiveness of PT may be achieved by increasing the effectiveness of formal education and/or by strengthening the connection between formal education in schools and the sorting mechanism of the college entrance exam. These two ways can make the education in formal schools become more sufficient for students to copy with the entrance exam. Once raising ability level through PT to pass the college entrance exam becomes less possible, ϕ is effectively reduced.

The government can collect tax revenues from the rich and/or the poor families to finance the educational reform and/or reconstruction of the mechanism of college entrance exam. The taxes generally make curves x and y shift downwards and, thus, the changing in the PT-effectiveness threshold is ambiguous. However, if the rich families pay most of the taxes, we can expect that the taxes lead to an increase in ϕ^*. Once the taxes revenues lead to $\phi \leq \phi^*$, no family is wealth-constrained. Alternatively, as a small open economy, the government may consider finance the expense by borrowing in the world credit market.

3.2 Student Loans

In addition to shift curves x and y, the government can offer student loans which will create new and larger PT-effectiveness thresholds. Specifically, a generation-t young agent who passes the entrance exam can take on a student loan equal to the full college tuition at the middle of time t and repays the principal c plus the interests $\frac{1}{2}rc$ at time $t+1$.

We name those old agents who took on student loans when young, as type-c old agents. The notation attached with superscript u, s and c respectively refers to unskilled workers, skilled workers whose tuition were paid by parents, and skilled workers who took on student loans. Denote ω^c as type-c agent's wealth in his old life after repaying his student loans. We have:

$$\omega^c = \omega^s - \left(1 + \frac{r}{2}\right) c = (\omega^s - \omega^u - c) + \omega^u - \frac{r}{2}c > 0.$$

Assumptions 1 and 2 together with $r \in (0,1)$ imply $\omega^c > 0$ and, thus, all student loans will be fully repaid.

3.2.1 Wealth Constraints and Incentive Constraints

Consider a generation-t type-j old agent for $j = s, c, u$ who has a child with an innate ability i^j_{t+1}, $i^j_{t+1} < \overline{h}_{t+1}$. We first discuss on his wealth constraint. When student loans are available, the old agent is able to send his child to college if $\omega^j \geq \widetilde{e}^{i^j_{t+1}}$ and his investment on PT is successful. Thus, we define a wealth constraint WCj* by:

$$\overline{h}_{t+1} - \omega^j \leq i^j_{t+1} < \overline{h}_{t+1}. \tag{WCj*}$$

When WCj* is satisfied, the old is able to pay $\widetilde{e}^{i^j_{t+1}}$.[24]

We now study the type-j old agent's incentive constraint. According to his utility function, equation (3), we consider three possibilities, (a), (b) and (c):

(a) the old does not invest on his child's PT and the utility level is $\omega^j + \omega^u$,

(b) the old invests on PT but does not pay for college tuition c (i.e., if his child passes the entrance exam, the child will take on a student loan) and the utility level is $\omega^j - \widetilde{e}^{i^j_{t+1}} + (1-\phi)\,\omega^u + \phi\omega^c$, and

(c) the old pays both of $\widetilde{e}^{i^j_{t+1}}$ and c so that the utility level is $\omega^j - \widetilde{e}^{i^j_{t+1}} + (1-\phi)\,\omega^u + \phi\,(\omega^s - c)$.

As explained in Section 2, if the old prefers (c) to (a), ICj holds. Here, we can tell that when the old prefers (b) to (a), i^j_{t+1} must satisfy $i^j_{t+1} \in [0,1]$ and

$$\overline{h}_{t+1} - \phi\,(\omega^c - \omega^u) \leq i^j_{t+1} < \overline{h}_{t+1}, \tag{ICj*}$$

which is named his incentive constraint ICj*.

Notice that the old prefers (c) to (b) for sure since $\omega^s - c > \omega^c$. Thus, if ICj dominates WCj, the possibility (b) will never be realized. In the following, we will study on the environment where ICj dominates WCj for $j = s, u$, to focus on whether student loans

[24] Recall that a non-trivial WCu implies $\overline{h} \geq \omega^j - c$. Particularly, we consider $\overline{h} \geq \omega^j$ so that WCj* is also not trivial.

can amend the disadvantage of the *poorest* families.[25] We have shown ICs dominates WCs. To ensure ICc dominates WCc, we extend the assumption of $\omega^u \geq c$ to assuming $\omega^u \geq \left(1+\frac{r}{2}\right)c$.[26]

3.2.2 PT-Effectiveness Thresholds and the Population of Type-c Agents

To study how type-u families can use student loans to overcome their wealth constraint WCu to send their children to colleges, we define two new PT-effectiveness thresholds, ϕ^{***} and ϕ^{**}. ϕ^{***} is derived by equating the lower bounds of WCu* and ICu*; ϕ^{**} is defined by equating the lower bounds of WCu and ICu*. Figure 3 exhibits these PT-effectiveness thresholds, where

$$x^* \equiv \phi\omega^u \text{ and } y^* \equiv \phi^2\left(\omega^c - \omega^u\right).$$

Then we can write:

$$\phi^{***} \equiv \frac{\omega^u}{\omega^c - \omega^u} > \phi^{**} \equiv \frac{\omega^u - c}{\omega^c - \omega^u} > \phi^* = \frac{\omega^u - c}{\omega^s - \omega^u - c},$$

given $\omega^c > \omega^u$.[27]

[Insert Figure 3 here]

These PT-effectiveness thresholds lead to several results.

First, student loans can make a difference only if $\phi > \phi^{**}$. This is because, on the one hand, if $\phi^* \geq \phi$, no family is wealth-constrained and no one needs a student loan; on the other hand, if $\phi^* < \phi \leq \phi^{**}$, those type-$u$ old agents will not pay for PT if they cannot afford both of PT and college tuition. In this latter situation, the interest payment of student loans prevents the old agents from making use of the policy and their children will not take on student loans.

[25] It might be argued that student loans should serve disadvantaged groups, leading to greater access to higher education for them. In this model we focus on the environment where only children from the poorest families take on student loans, which is consistent with the purpose of serving the disadvantaged groups.

[26] Using the lower bounds of ICc and WCc, we get $(\omega^c - c) - \phi\left(\omega^s - \omega^u - c\right) = (1-\phi)\left(\omega^s - \omega^u - c\right) + \omega^u - \left(1+\frac{r}{2}\right)c \lesseqgtr 0$. Thus, assuming $\omega^u \geq \left(1+\frac{r}{2}\right)c$ ensures ICc dominates WCc.

[27] Notice that if $\omega^c \leq \omega^u$, no young agent will take on a student loan. To ensure $\omega^c > \omega^u$, we assume $\omega^s - \omega^u - \left(1+\frac{r}{2}\right)c \geq 0$ in addition to Assumption 1, $\omega^s - \omega^u - c \in (0,1]$.

[Insert Figures 4 and 5 here]

Second, if $\phi^{**} < \phi < \phi^{***}$, as shown in Figure 4, $(y^*/\phi - x/\phi)$ type-u old agents only afford to pay for PT and they are willing to do it. Therefore, a ϕ percent of their children who pass the entrance exam will take on student loans. At the same time, some type-u old agents are able to pay for PT only but they do not want their children to apply for student loans because of the attached financial burden of interest payment, thus, their children will not receive PT and become unskilled workers. When $\phi^{**} < \phi < \phi^{***}$, we can say that type-$u$ old agents are not wealth-constrained because those who are willing to invest on PT can fulfill their investments.

Third, if $\phi > \phi^{***}$, as shown in Figure 5, every type-u old agent will pay for his child's PT if it is affordable and $\phi\,(x^*/\phi - x/\phi)$ of their children will take on student loans. Here, some type-u old agents are willing to pay for PT and let their children apply for student loans after going to college, but they cannot even afford the investments on PT. That is to say, there still exist wealth-constrained families in the sense that children from these families might go to college if their parents can afford their PT.

Let $\overline{\gamma}^j$ denote the steady-state measure of population of type-j, $j = s, u, c$, agents. Since type-c agents emerge from the poorest families due to student loans and $\overline{\gamma}^u = 1 - N$, we can tell that

$$\overline{\gamma}^c = \begin{cases} 0 \text{ (no one take on a student loan)} & \text{for } \phi \leq \phi^{**}, \\ (1-N)\,(y^* - x) & \text{for } \phi^{**} < \phi < \phi^{***}, \\ (1-N)\,(x^* - x) & \text{for } \phi > \phi^{***}, \end{cases}$$

and the total student loans taken out is $c\overline{\gamma}^c$ which is increasing in ϕ. In general, a more effective PT induces more participation in PT and a higher demand for student loans, however, it is also more likely to raise inequity. We cannot judge whether the poorest families are no longer be wealth-constrained simply by the population of type-c agents or the total amount of student loans taken out.

In sum, if $\phi^{**} < \phi < \phi^{***}$ then student loans eliminate the inequity issue and no type-u families are wealth-constrained, however, if $\phi > \phi^{***}$ type-u families are still wealth-constrained even though a student loan can fully cover the college tuition and has no default risk. Student loans can solve the insufficient wealth problem of the poorest families *after* their children go to college. However, the poorest families might encounter insufficient wealth problem *before* their children go to college, which cannot be solved by student loans. Similarly, scholarships granted to students after they register in colleges cannot fully amend the inequity problem.

4 Concluding Remarks

In this paper, we are particularly interested in the relation between the wealth constraint and the opportunity to acquire more skills through attending higher education. When a poor child cannot go to college but a rich child with a lower innate ability can, an inequity issue arises. This inequity raises concerns if it is recognized that a child's opportunity of receiving higher education should not be directly tied to his parent's wealth. The inequity issue matters also because inequity implies inefficiency in human capital investment: training a child of low innate ability into a skilled worker is more expensive than training a child of high innate ability.

In this paper we study the possible effects of PT on the social inequity issue, which is rarely discussed theoretically in the literature. The main contribution of this paper is to incorporate PT into a two-period OG model to study under what circumstances the inequity issue arises. A key step in the model is to construct a PT-effectiveness threshold which is closely related to the college tuition and the wealth gap between the poor and the rich families. We then show that when the effectiveness of PT exceeds the PT-effectiveness threshold, poor families become wealth-constrained and relatively disadvantaged in sending their children to college. At the same time, the average spending on PT of the poor families is smaller than that of the rich families. Under these circumstances, the existence of PT causes educational inequity. Thus, the PT-effectiveness threshold and the disparity of average spending on PT between different wealth groups may serve empirical researches as indicators for uncovering whether the poor families are wealth-constrained or not.

Since a country with higher wealth inequality between different wealth groups has a smaller PT-effectiveness threshold, the effectiveness of PT is more likely to surpass the threshold, thereby leading the poor to be wealth-constrained in investing on PT. This implies that a country with higher wealth inequality has to pay more attention to the potential consequence of educational inequity that PT has caused. Taking account of PT allows we to examine its effects on social inequity which have not been discussed in the theoretical literature.

Another innovation of our model is to take account of the number of college admissions and, therefore, to understand how the size of higher education affects the activity of PT industry. We show that an increase in the number of college admissions results in more skilled workers, but it is unable to narrow down the gap in the probabilities between poor and rich families for having a child to attend college. Moreover, an increase in the number of college admissions may even encourage parents to invest more on PT. This finding contrasts to the much-discussed policy that more placements in colleges for students can alleviate the competition of college entrance exam so as to diminish the PT activities.

Once we understand how PT causes inequity, we are ready to study the policy implications of this model. Our analysis shows that policies of tax and transfer are able to amend the social inequity by affecting the PT-effectiveness threshold. We also study the role of student loans, which indicates that extending more loans for college tuition without considering a reduction in the financial burden from PT may not amend the inequity.

As we have shown, extending more student loans for college tuition and establishing more colleges may not help to achieve the goal of equity in education. Instead, the government should strengthen the connection between formal education in schools and the sorting mechanism of the college entrance exam, so that the effectiveness of PT can be reduced and students do not have to rely on private tutoring to pass the entrance exam.

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Table 1

The Steady-State Equilibrium given $\phi^* < 1$.

Case z	parameters' values	$\overline{h}_z$	$\overline{e}_z$
A	$\phi \leq \phi^*$, $y < \phi$, $N \in \left(y, 1 - y\left(\frac{1}{\phi} - 1\right)\right]$	$1 + y - N$	$\frac{y^2}{2\phi^2}$
B	$\phi \leq \phi^*$, $y \leq \phi$, $N \in (0, y]$	$1 + \frac{y-N}{\phi}$	$\frac{y^2-(y-N)^2}{2\phi^2}$
C	$\phi > \phi^*, 1 + x > y$, $y < \phi$, $N \in \left(\frac{x}{1+x-y}, 1 - \frac{y}{1+x-y}\left(\frac{1}{\phi} - 1\right)\right]$	$1 + x - N(1 + x - y)$	$\frac{Ny^2+(1-N)x^2}{2\phi^2}$
D	$\phi > \phi^*$, $y \leq \phi$, $1 + x > y$, $N \in \left(0, \frac{x}{1+x-y}\right]$	$1 + \frac{x-N(1+x-y)}{\phi}$	$\frac{Ny^2+(1-N)x^2}{2\phi^2}$ $-\frac{[x-N(1+x-y)]^2}{2\phi^2}$

Figure 1

Values of Parameters in Cases A to D.

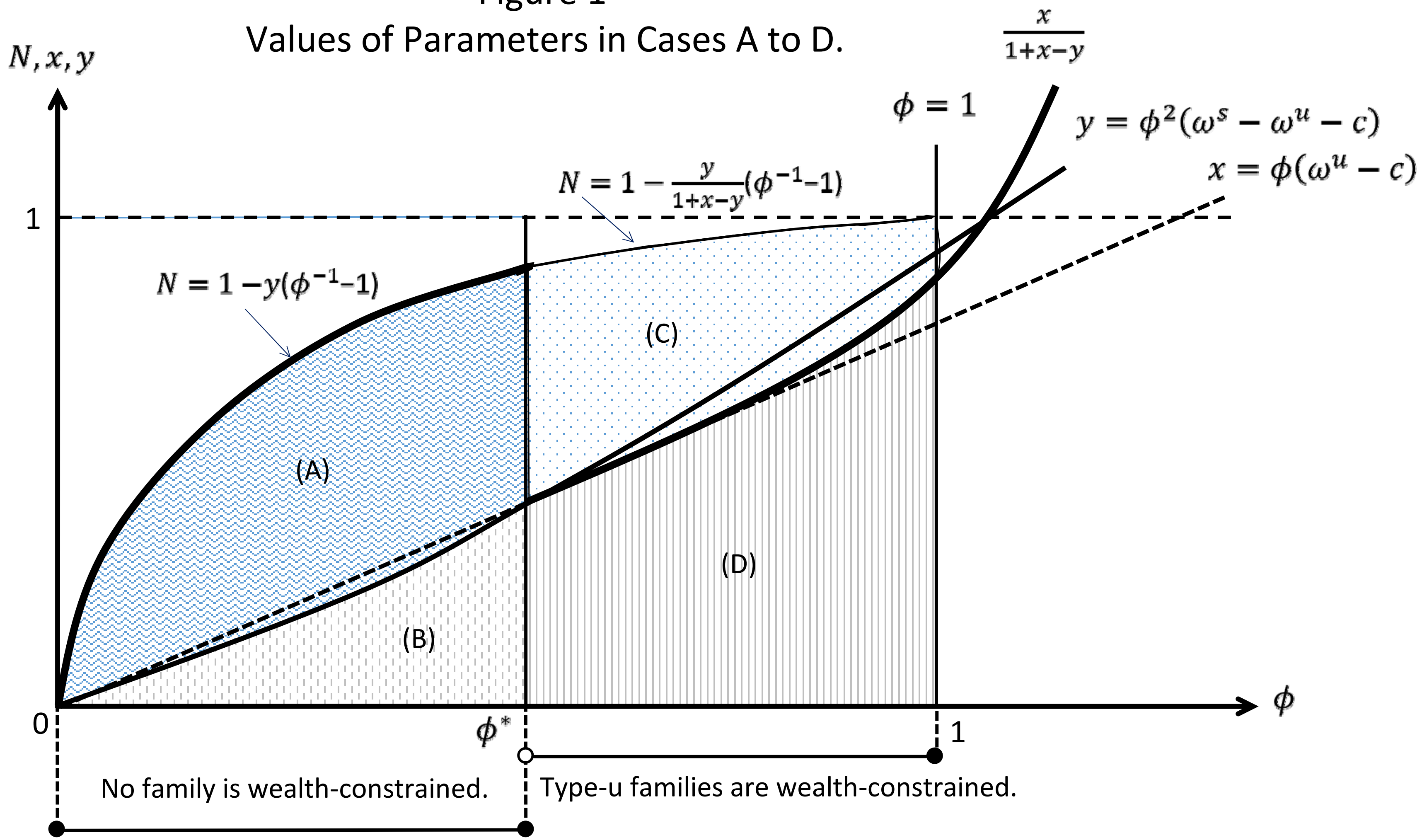

Figure 2

PT-Effectiveness Threshold Increases from ϕ^* to $\tilde{\phi}$ or to $\tilde{\tilde{\phi}}$ by Tax and Transfer.

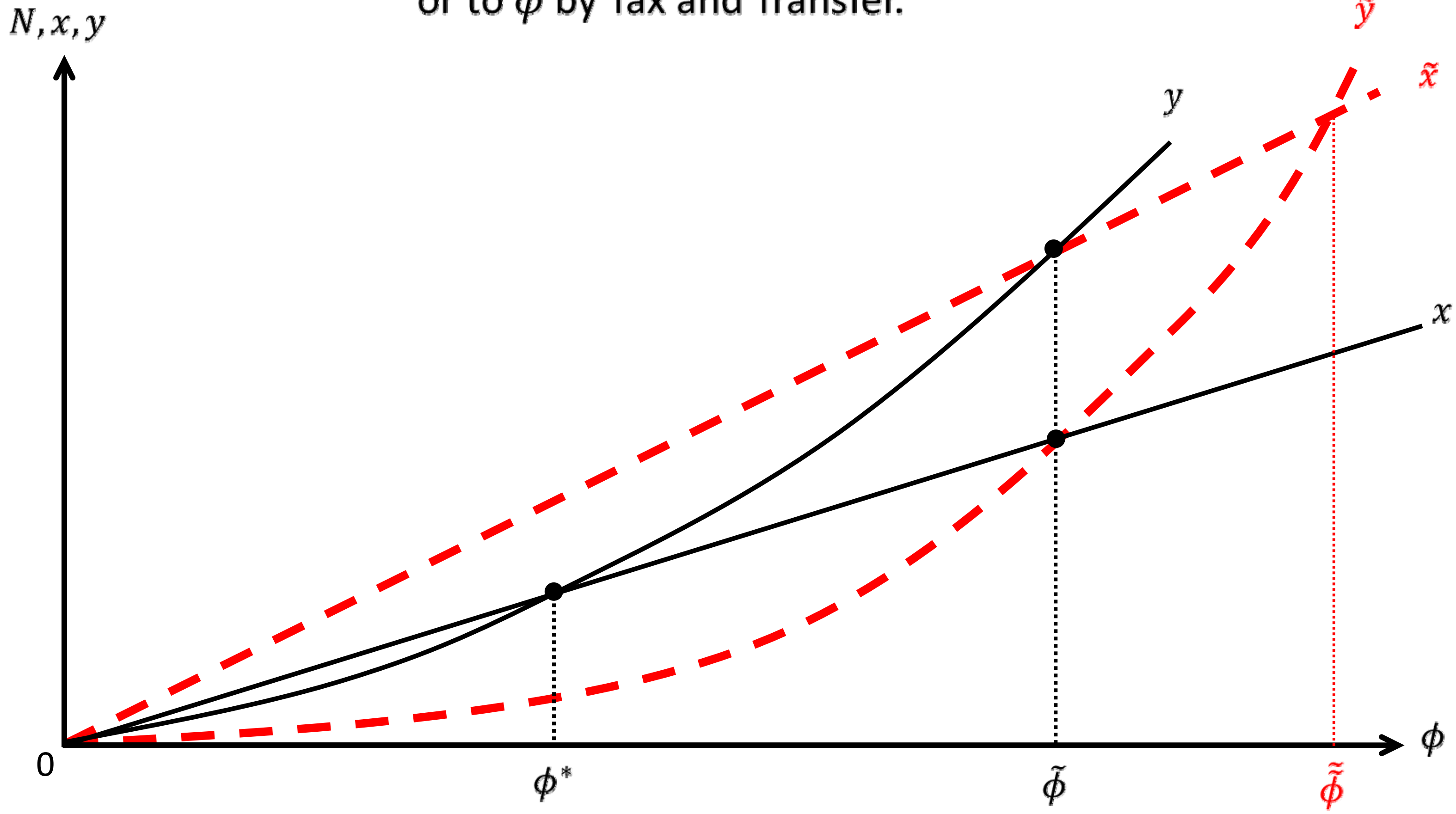

Figure 3

PT-Effectiveness Thresholds Created by Student Loans.

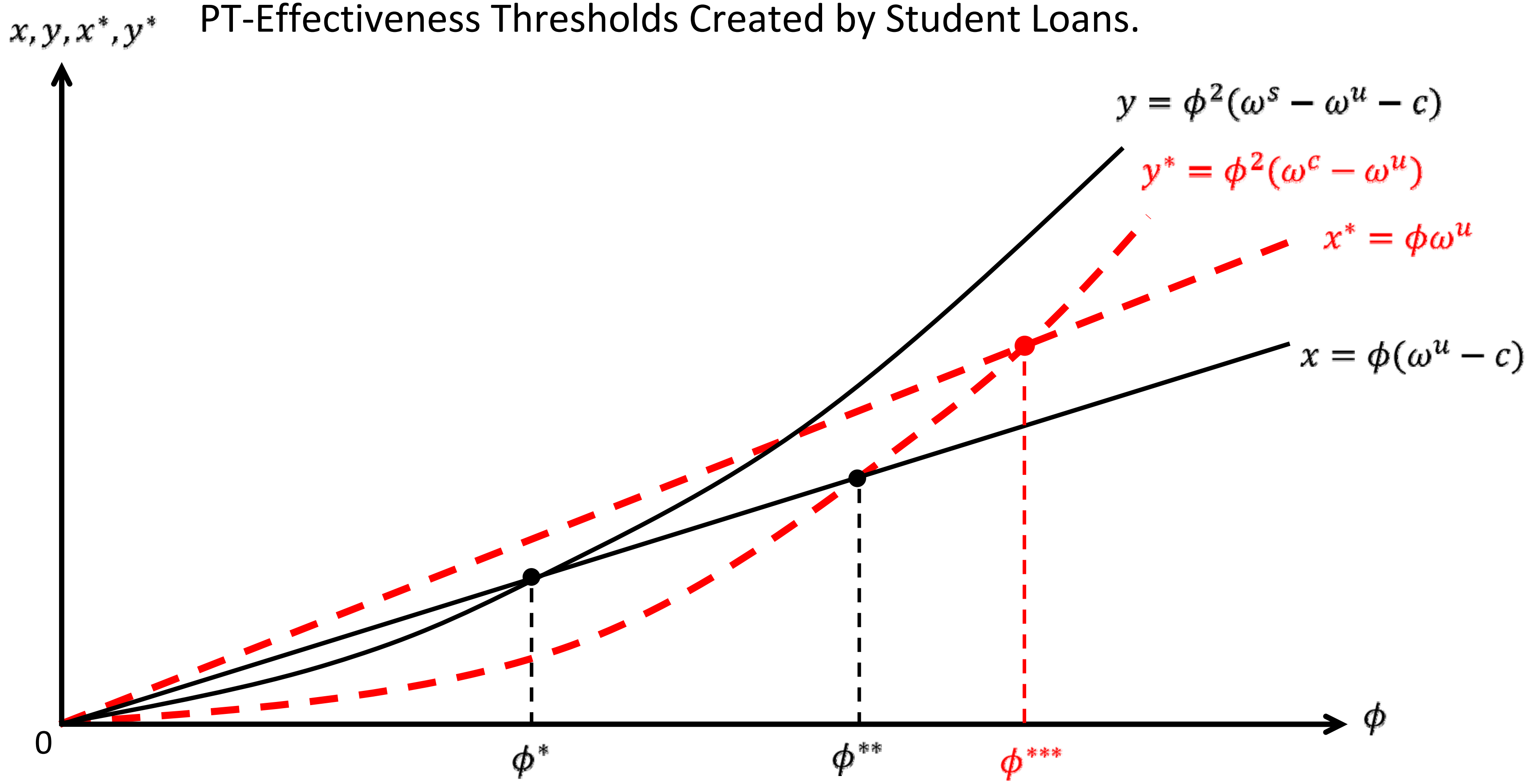

Figure 4
Incentive and Wealth Constraints of Type-u Old Agents
When $\phi^{**} < \phi < \phi^{***}$.

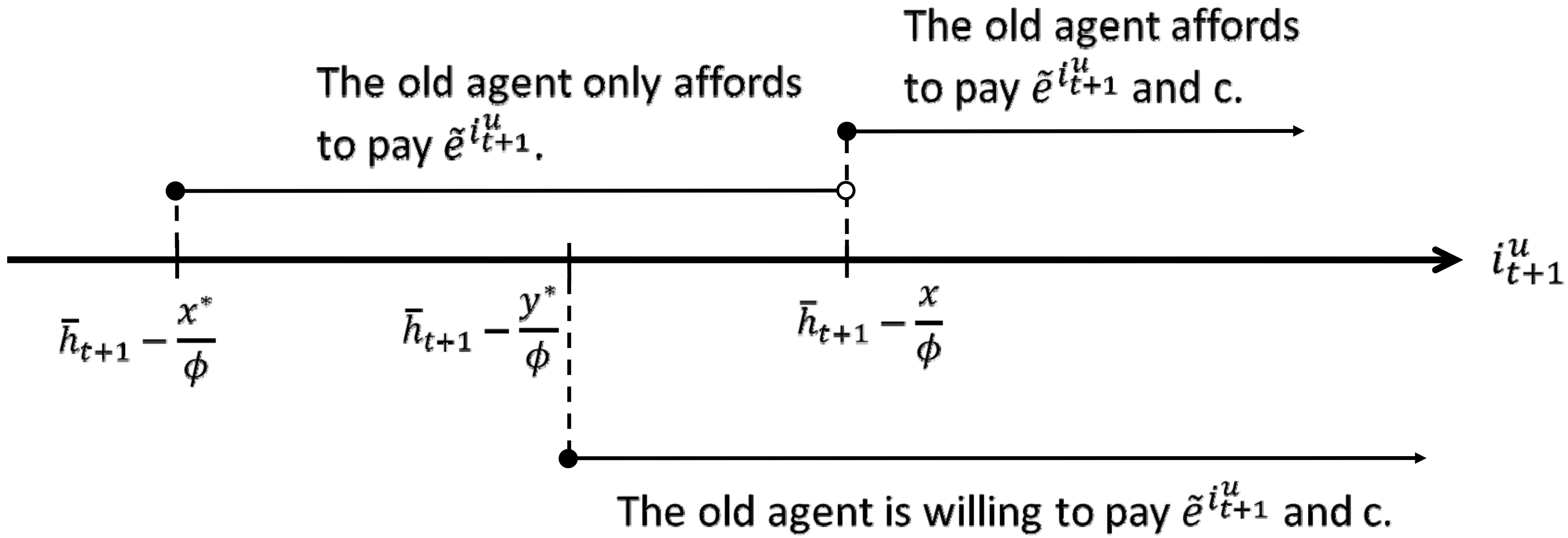

Figure 5

Incentive and Wealth Constraints of Type-u Old Agents When $\phi > \phi^{***}$.