A Macroeconomic Model of Imperfect Competition with Patent Licensing

Hui-ting Hsieh

Advisor: Ching-chong Lai, Ph.D

Li-kung Ferng, Ph.D

Literature (I)

- The existing studies on patent licensing focus on two subjects:
  - The optimal contract; e.g., Wang (1998, 2002), and Kamien and Tauman (2002).
  - The social welfare; e.g., Faulí-Oller and Sandonís (2002), Poddar and Sinha (2004), Kabiraj (2005), and Sen and Tauman (2007).

These studies only focus on the partial equilibrium framework.

Literature (II)

- The recent studies have focused on macroeconomic policies in the presence of the imperfectly competitive product market; e.g., Dixon (1987), Startz (1989), Molana and Moutos (1992), Chen et al. (2005), and Heijdra (2009).
- Departing from these existing studies, Lai et al. (2010) set up an imperfectly competitive macroeconomic model that is able to deal with both vertical separation and vertical integration.

Motivation

- Comparing the partial equilibrium analysis with our model:

	Partial equilibrium analysis	General equilibrium analysis
Real wage	given	*endogenize*
Household's income	given	*endogenize*

- With these two additional channels, our analysis finds some conflicting results compared to the existing studies on patent licensing.

Baseline Model

- We set up can be treated as an extended version of Heijdra (2009) in which a monopolistically competitive product market is present.
- Two types of agents:
  - Households
  - Firms

Baseline Model –Households (I)

- Household's optimal decision:

$$Max \quad U = \alpha \ln C + (1-\alpha) \ln l \quad ; \quad 0 < \alpha < 1$$

$$s.t. \quad C = m\left[\frac{1}{m}\sum_{i=1}^{m} c_i^{1-\mu}\right]^{1/(1-\mu)} \quad ; \quad 0 \le \mu < 1$$

$$PC = \sum_{i=1}^{m} p_i c_i = wN_s + \Pi \quad ; \quad N_s = T - l$$

$$P = \left[\frac{1}{m}\sum_{i=1}^{m} p_i^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)} \quad .$$

Baseline Model --Households (II)

The composite consumption is the *numeraire*, and the price index of composite consumption is normalized to unity, the household's optimization problems:

$$C = \alpha(wT + \Pi)$$

$$l = (1-\alpha)\frac{wT + \Pi}{w}$$

$$c_i = \left[\frac{P}{p_i}\right]^{1/\mu} \frac{C}{m} = \left[\frac{1}{p_i}\right]^{1/\mu} \frac{C}{m}$$

Baseline Model --Firms

- Firm: we consider is composed of *m* firms, each of which produces a specific good y_i that is an imperfect substitute for the other goods.
  - Firm 1's (patent-holding firm) production function:

    $$y_1 = An_1 \quad ; \quad A > 1 \ .$$

  - Firm *i*'s production function:

    $$y_i = n_i \quad ; \quad i = 2, \ldots, m \ .$$

Baseline Model
---No Licensing Regime (I)

- Firm 1:

$$Max\ \pi_1^{NL} = p_1^{NL} y_1^{NL} - w^{NL} n_1^{NL}$$

$$s.t.\quad c_1 = p_1^{-\frac{1}{\mu}} \frac{C}{m}$$

$$y_1 = A n_1$$

- Firm *i*:

$$Max\ \pi_i^{NL} = p_i^{NL} y_i^{NL} - w^{NL} n_i^{NL};\quad i = 2, \ldots, m.$$

$$s.t.\quad c_i = p_i^{-\frac{1}{\mu}} \frac{C}{m}$$

$$y_i = n_i$$

Baseline Model
--- No Licensing Regime (II)

- The firm's optimal production decision and its corresponding price:

$$y_1^{NL} = \left[\frac{(1-\mu)A}{w^{NL}}\right]^{1/\mu} \frac{C^{NL}}{m}$$

$$p_1^{NL} = \frac{w^{NL}}{(1-\mu)A}$$

$$y_i^{NL} = \left[\frac{(1-\mu)}{w^{NL}}\right]^{1/\mu} \frac{C^{NL}}{m}$$

$$p_i^{NL} = \frac{w^{NL}}{1-\mu}$$

Baseline Model
--- No Licensing Regime (III)

- Macroeconomic Equilibrium
  - By substituting $\Pi^{NL} = Y^{NL} - w^{NL} N_d^{NL}$ into the individual household's budget constraint, we obtain:

$$C^{NL} - Y^{NL} + w^{NL}(N_d^{NL} - N_s^{NL}) = 0$$

  - The goods market equilibrium condition:

$$Y^{NL} = C^{NL}$$

  - The labor market equilibrium condition:

$$N_s^{NL} = N_d^{NL} \Rightarrow \tilde{w}^{NL}$$

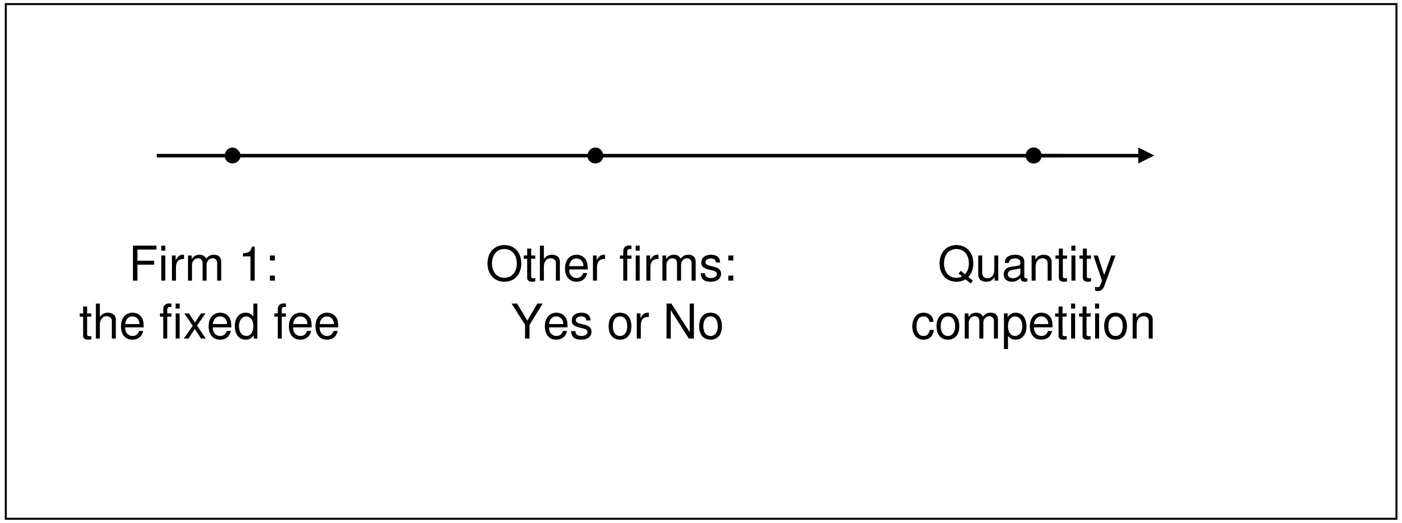
(Let a variable with "~" denote its equilibrium value.)

Baseline Model
--- Fixed Fee Licensing and Royalty Licensing

- Fixed fee licensing: the lump-sum payment that is paid up front when the licensing contract is made.
- Royalty licensing: the per-unit royalty that is paid according to the number of units that are produced.

Baseline Model
--- Fixed Fee Licensing Regime (I)

- Game stage:



- Use the backward induction.

Baseline Model
--- Fixed Fee Licensing Regime (II)

- In the third stage:
  - Firm 1's real total profit:

$$\pi_1^F = p_1^F y_1^F - w^F n_1^F + (m-1)F$$

  - Firm *i*'s real total profit:

$$\pi_i^F = p_i^F y_i^F - w^F n_i^F - F$$

- The firm's optimal decisions:

$$y_i^F = \left[\frac{(1-\mu)A}{w^F}\right]^{1/\mu} \frac{C^F}{m};\ i = 1,\ldots,m.$$

$$p_i^F = \frac{w^F}{(1-\mu)A}$$

Baseline Model
--- Fixed Fee Licensing Regime (III)

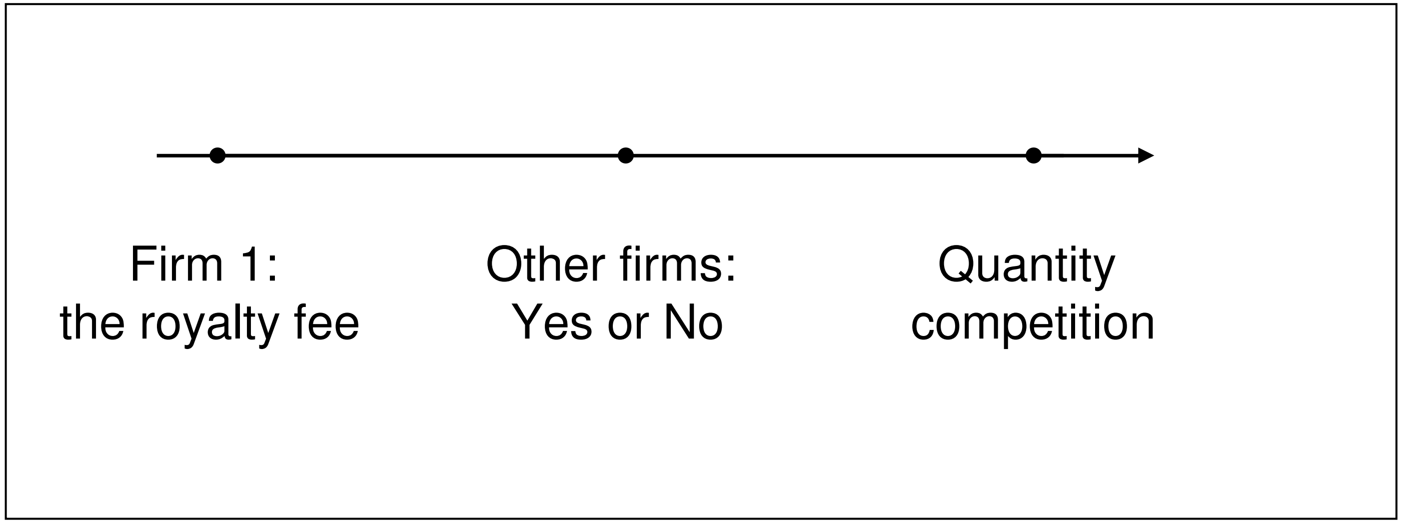
- In the second stage: $\pi_i^F - \pi_i^{NL} \geq 0$
- In the first stage:
  - The fixed fee *F* that firm 1 charges is equal to the maximum value that licensee is willing to pay, $\pi_i^F - \pi_i^{NL} = 0$. We obtain:

$$F = (p_i^F y_i^F - w^F n_i^F) - (p_i^{NL} y_i^{NL} - w^{NL} n_i^{NL})$$

- By the equilibrium condition for the labor market, we can obtain the equilibrium macro variables and household's welfare level.

Baseline Model
--- Royalty Licensing Regime (I)

- Game stage:



- Use the backward induction.

Baseline Model
--- Royalty Licensing Regime (II)

- In the third stage:
  - Firm 1's real total profit can be expressed as follow:

$$\pi_1^R = p_1^R y_1^R - w^R n_1^R + (m-1) s y_i^R$$

  - Firm *i*'s real total profit can be written as:

$$\pi_i^R = p_i^R y_i^R - w^R n_i^R - s y_i^R$$

- In the second stage: $\pi_i^R - \pi_i^{NL} \geq 0$
- In the first stage: $s = \mu w^R / [(1-\mu) A]$

Royalty Licensing Regime (III)

- The firm's optimal production decision and its corresponding price:

$$y_1^R = \left[\frac{(1-\mu)A}{w^R}\right]^{1/\mu} \frac{C^R}{m}$$

$$p_1^R = \frac{w^R}{(1-\mu)A}$$

$$y_i^R = \left[\frac{(1-\mu)^2 A}{w^R}\right]^{1/\mu} \frac{C^R}{m}$$

$$p_i^R = \frac{w^R}{(1-\mu)^2 A}$$

Royalty licensing leads to the **double marginalization** since both licenser (firm 1) and licensee (firm) add their own price-cost margin at each stage of production.

Comparison of Three Regimes

- The equilibrium condition for labor under three regimes:
  - No licensing:

$$\frac{\alpha(1-\mu)T}{1-\alpha\mu} = [1+(m-1)A^{(\mu-1)/\mu}](1-\mu)^{1/\mu} A^{(1-\mu)/\mu} \frac{\alpha T (w^{NL})^{(\mu-1)/\mu}}{m(1-\alpha\mu)}$$

  - Fixed fee licensing:

$$\frac{\alpha(1-\mu)T}{1-\alpha\mu} = (1-\mu)^{1/\mu} A^{(1-\mu)/\mu} \frac{\alpha T (w^{F})^{(\mu-1)/\mu}}{1-\alpha\mu}$$

  - Royalty licensing:

$$\frac{\alpha(1-\mu\Phi)T}{1-\alpha\mu\Phi} = [1+(m-1)(1-\mu)^{1/\mu}](1-\mu)^{1/\mu} A^{(1-\mu)/\mu} \frac{\alpha T (w^{R})^{(\mu-1)/\mu}}{m(1-\alpha\mu\Phi)}$$

Baseline Model --- Main Findings (I)

- The patent holder's real profit:

$$\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}, \text{ and } \tilde{\pi}_1^R > \tilde{\pi}_1^{NL}.$$

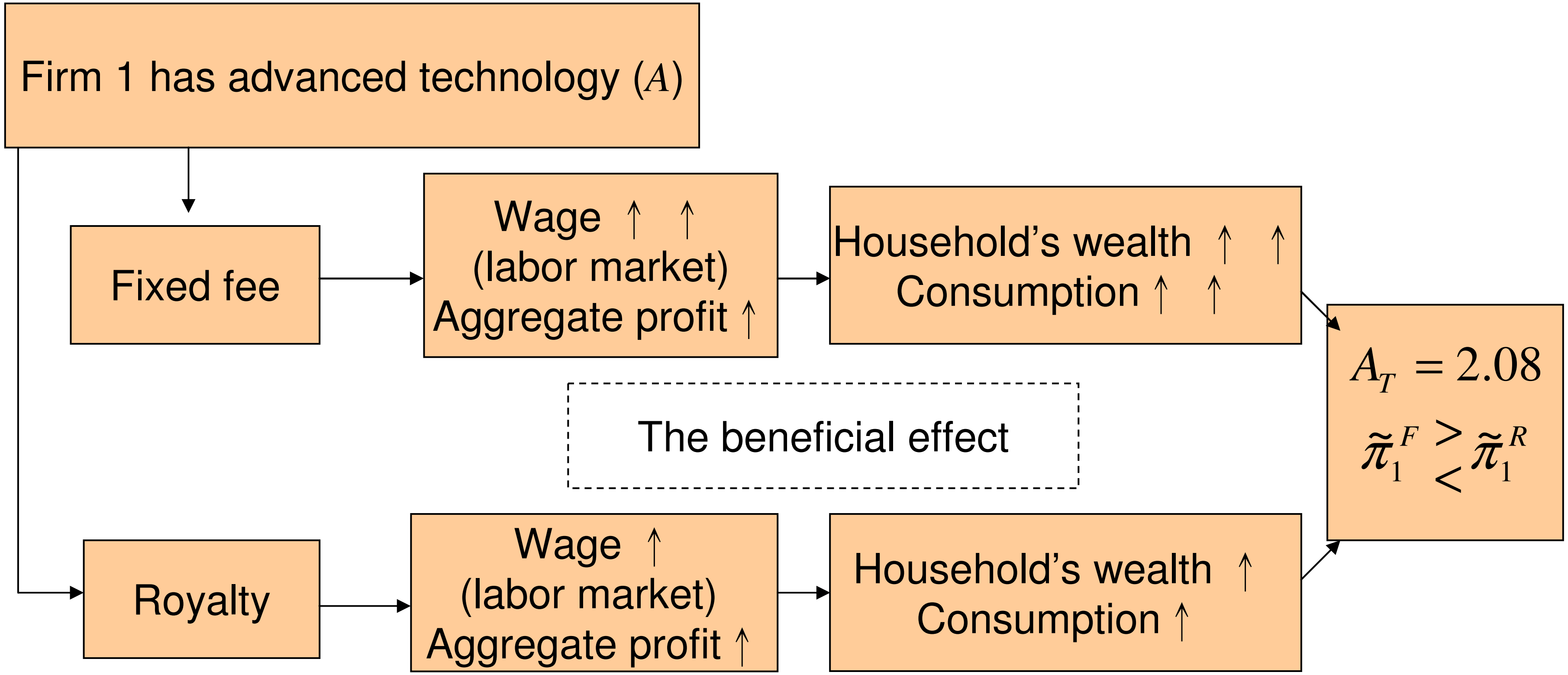
- The problem of choosing the optimal contract: With the higher (lower) technology level the patent-holder prefers the fixed fee (royalty) contract. $\tilde{\pi}_1^F \gtrless \tilde{\pi}_1^R$.

- Comparing the level of social welfare:

$$\tilde{U}^F > \tilde{U}^R > \tilde{U}^{NL}$$

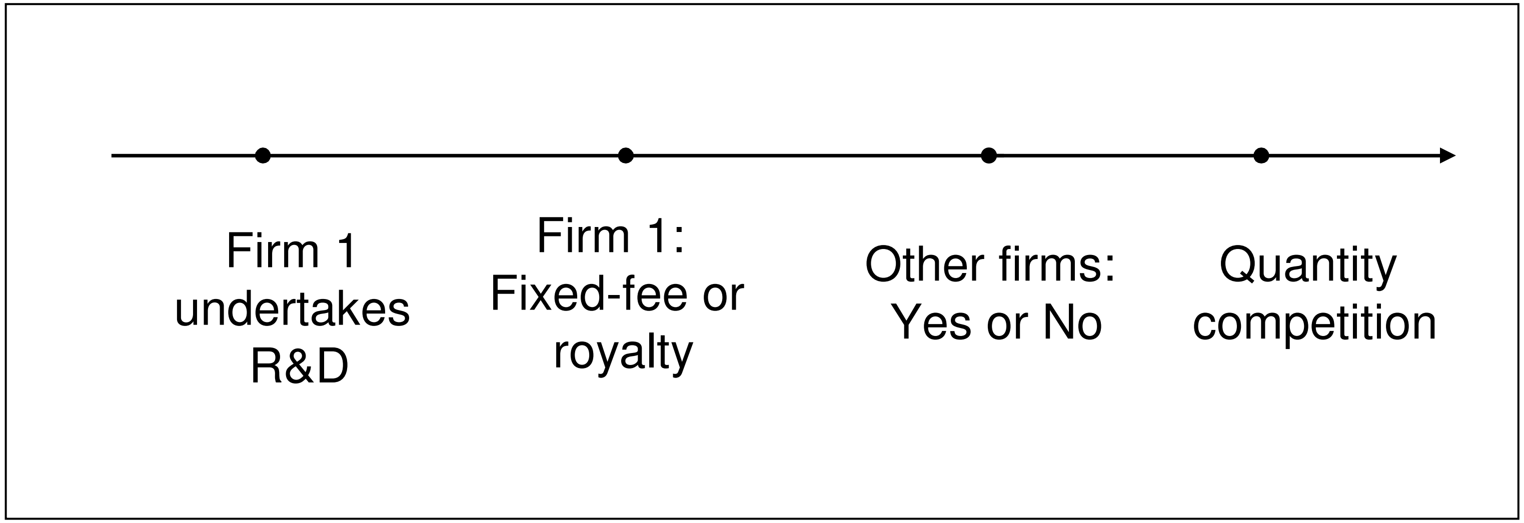
Baseline Model --- Main Findings (II)

- Intuition:



Extensions and Discussion ---R&D Investment (I)

- Game stage:



Extensions and Discussion ---R&D Investment (II)

- Firm 1's R&D costs: $C_r = \psi (A-1)^2 / 2$
- The real total profit of firm 1 is modified as follows:

$$\pi_1 = \begin{cases} p_1 y_1 - w n_1 + (m-1)F - C_r; \\ p_1 y_1 - w n_1 + (m-1) s y_i - C_r. \end{cases}$$

Extensions and Discussion ---R&D Investment (III)

- The first-order conditions under the two regimes are respectively given by:

$$(1-\mu)\left(\frac{w^F}{1-\mu}\right)^{\frac{\mu-1}{\mu}} C^F A^{F^{\frac{1-2\mu}{\mu}}} = \psi(A^F - 1)$$

$$\frac{[1+(m-1)(1-\mu)^{\frac{1}{\mu}}]}{m}(1-\mu)\left(\frac{w^R}{1-\mu}\right)^{\frac{\mu-1}{\mu}} C^R A^{R^{\frac{1-2\mu}{\mu}}} = \psi(A^R - 1)$$

- The results:

$\hat{A}^F > \hat{A}^R$ and $\hat{\pi}_1^F < \hat{\pi}_1^R$.

Extensions and Discussion ---R&D Investment (IV)

- Intuition:
  - With the double marginalization effect, the $MB^R < MB^F$, and then $\hat{A}^F > \hat{A}^R$.
  - There are two additional effect with a higher A:

    (i) the greater beneficial effect.

    (ii) the higher R&D cost.

    When first effect falls short of the second effect, we obtain $\hat{\pi}_1^F < \hat{\pi}_1^R$.

Extensions and Discussion
---Mixed Industrial Structure (I)

- Two goods: the monopolistically competitive goods and the perfectly competitive goods.
- The household's utility :

$$Max \quad U = \alpha \ln C + (1-\alpha)\ln l$$

$$C = \frac{C_M^{\beta} C_C^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}}$$

Extensions and Discussion
---Mixed Industrial Structure (II)

- The household's optimization problem:

$$C_C = \frac{\alpha(1-\beta)(wT+\Pi)}{P_C}$$

$$C_M = \frac{\alpha\beta(wT+\Pi)}{P_M}$$

$$l = (1-\alpha)\frac{wT+\Pi}{w}$$

$$c_i = \left[\frac{P_M}{p_i}\right]^{1/\mu}\frac{C_M}{m} = \left[\frac{P_M}{p_i}\right]^{1/\mu}\frac{\alpha\beta(wT+\Pi)}{mP_M}$$

Extensions and Discussion
---Mixed Industrial Structure (III)

- Results: the patent-holding firm is more inclined to choose the royalty contract when the β decreases.
- Intuition: the smaller β causes the lower demand, and further leads to the lower total profit. The beneficial effect of fixed-fee licensing will be weakened.

Extensions and Discussion ---Free Entry (I)

- Assumption:
  - Each firm bears a fixed cost of entry ϕ.
  - Non-advanced technology firms has zero profit. Firm 1 still has positive profits.
- The profit of firms:

$$\pi_1 = \begin{cases} p_1 y_1 - w n_1 + (m-1)F - \phi, \\ p_1 y_1 - w n_1 + (m-1) s y_i - \phi. \end{cases}$$

$$\pi_i = \begin{cases} p_i y_i - w n_i - F - \phi, \\ p_i y_i - w n_i - s y_i - \phi. \end{cases}$$

Extensions and Discussion ---Free Entry (II)

- the optimal number of firms:

$$m = \begin{cases} \mu Y^F / (\phi + F), \\ \mu Y^R / \phi - (1-\mu)^{(\mu-1)/\mu} + 1. \end{cases}$$

- Results:
  - The greater ϕ, the fixed-fee contract is better.
  - $\hat{m}^R > \hat{m}^F$

Extensions and Discussion ---Free Entry (III)

- Intuition:

The higher ϕ results in fewer firms. Each firm faces a greater demand for its goods. The beneficial effect of the fixed-fee licensing will be reinforced.

Extensions and Discussion ---Alternative Markup Ratio (I)

- We follow Yang and Heijdra (1993) by taking into account both the direct and indirect effects.
- The price elasticity of demand:
  - Baseline model: $\varepsilon_d = 1/\mu$
  - This model: $\varepsilon_{i,d}(m) = \frac{1}{\mu} - \frac{1}{\mu m}(P/p_i)^{\frac{1-\mu}{\mu}}$

Extensions and Discussion
---Alternative Markup Ratio (II)

- The markup ratio $\eta_i = p_i / MC_i$:
  - Baseline model: $$\eta = \eta_i = \frac{1}{1-\mu}$$
  - This model: $$\eta_i(m) = \frac{\varepsilon_{i,d}(m)}{\varepsilon_{i,d}(m) - 1} = \frac{1-(P/p_i)^{(1-\mu)/\mu}/m}{(1-\mu)-(P/p_i)^{(1-\mu)/\mu}/m}$$

- Result: the patent holder is more inclined to choose the fixed-fee contract when each firm considers the effect of its price on the aggregate price level.

Extensions and Discussion ---Alternative Markup Ratio (III)

- Intuition:

  When each firm considers the effect of its price on *P*, its markup ratio increases, and then each firm faces a rise in its total profit. The beneficial effect for the fixed-fee licensing well be strengthened.

Conclusions (I)

- In the baseline model:
  - The patent-holder's real total profit and aggregate profits under the two licensing regimes are always greater than those under the no licensing regime.
  - With the higher (lower) technology level the patent-holder prefers the fixed-fee (royalty) contract.
  - The welfare level under the fixed-fee licensing regime is higher than that under the royalty licensing regime.

Conclusions (II)

- In the extension models:
  - The patent-holding firm is more inclined to choose the royalty licensing contract: (i) R&D investment; and (ii) the mixed industrial structure.
  - The patent-holding firm is instead more likely to choose the fixed-fee licensing contract: (i) free entry; and (ii) alternative Markup Ratio.

The End