A Macroeconomic Model of Imperfect Competition with Patent Licensing

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Abstract

This paper sets up an imperfectly competitive macroeconomic model that features the strategic interaction between the patent-holding firm and licensees, and uses it to analyze the relevant macro variables under various licensing arrangements. Some main findings emerge from the analysis. First, the equilibrium aggregate output and aggregate consumption under fixed-fee and royalty licensing regimes are always greater than those under the no licensing regime. Moreover, the equilibrium aggregate output and consumption under the fixed-fee licensing regime are always greater than those under the royalty licensing regime. Second, with the higher (lower) technology level the patent-holder prefers the fixed-fee (royalty) contract. Third, welfare could be improved through technology transfer, and the level of welfare under the fixed-fee licensing regime is higher than that under the royalty licensing regime. Lastly, this paper discusses some extensions of the baseline model.

Keywords: Imperfect competition, Macroeconomic model, Fixed-fee licensing, Royalty licensing

JEL classifications: D45, E10, L16

1. Introduction

Over the past few decades, the volume of transactions in patent licensing has increased dramatically in many countries.[1] Patent-holding firms earn huge profits through patent licensing to incumbent firms, thereby causing the promotion of technology diffusion. Based on empirical observations that patent licensing has become more important, the issue concerning patent licensing has been studied extensively in the field of industrial economics. In general, the existing studies on patent licensing focus on the following two subjects: (i) what the optimal licensing contract is for a profit-maximizing patent-holder; (ii) how the level of social welfare would change under different types of licensing contracts and industrial structures. With regard to the first subject, Wang (1998, 2002) and Kamien and Tauman (2002) find that the patent-holder prefers the royalty licensing contract to other licensing agreements in most instances. With regard to the second subject, existing studies conduct the welfare analysis from a variety of perspectives, such as Cournot versus Bertrand competition (Faulí-Oller and Sandonís, 2002; Mukherjee, 2010), spatial competition (Poddar and Sinha, 2004), the Stackelberg structure (Kabiraj, 2005), the per-unit royalty versus ad valorem royalty (San Martin and Saracho, 2010), horizontal mergers versus patent licensing (Faulí-Oller and Sandonís, 2003), and the insider

[1] The empirical evidence reveals that receipts and payments in relation to patent licensing have increased significantly in Japan, the U.K. and the U.S. in the past few decades. See, for instance, Nadiri (1993, Table 3) and the statistical data reported in Jovanovic (2009).

versus outsider patent-holder (Sen and Tauman, 2007).[2] A common feature in these studies is that they only focus on the partial equilibrium framework, and hence neglect the mutual interaction between the product market and the other markets. While simple, such an implementation may not only fail to reflect reality, but may also mislead us in terms of our understanding the efficacy of patent licensing.[3]

Ever since Dixit and Stiglitz (1977) published their pioneering paper, a vast number of recent studies have focused on macroeconomic policies in the presence of the imperfectly competitive product market; e.g., Dixon (1987), Startz (1989), Molana and Moutos (1992), Chen et al. (2005), and Heijdra (2009). The main purpose of these studies is to explore how the macroeconomic policies affect the relevant macro variables when the product market is imperfectly competitive. More recently, in departing from these existing studies, Lai et al. (2010) focus on the issue from the perspective of the industrial structure. To be more specific, they set up an imperfectly competitive macroeconomic model that is able to deal with both vertical separation and vertical integration, and make an effort to examine the relative macroeconomic performance between these two industrial regimes.

In a departure from the literature on patent licensing, this paper extends the framework developed by Heijdra (2009), and sets up an imperfectly competitive macroeconomic model featured by the strategic interaction between the patent-holding firm and licensees. To highlight the importance arising from different types of licensing agreements, this paper considers three regimes: no licensing, fixed-fee licensing, and royalty licensing. The paper then examines how the patent licensing behavior of firms governs the determination of the relevant macro variables under the three regimes.

To the best of my knowledge, this paper is the first to deal with patent licensing in terms of general equilibrium analysis. Compared with the existing literature on patent licensing characterized by partial equilibrium analysis, two additional channels emerge from our macroeconomic model featured by general equilibrium analysis. The first is that the partial equilibrium analysis ignores the role of demand and supply

[2] In the existing literature, the patent-holder is separated into the insider patent-holder and the outsider patent-holder. The insider patent-holder is a producer, and its total profits contain profit from its own output and patent licensing revenues; see, for instance, Wang (1998, 2002), Mukerjee and Balasubramanian (2001), and Kamien and Tauman (2002). The outsider patent-holder is an independent laboratory and non-producer, and its profit is only patent licensing revenues; see, for instance, Kamien and Tauman (1986), Katz and Shapiro (1986), and Kamien et al. (1992). This paper only focuses on the behavior of the insider patent-holder.

[3] A notable exception is Beladi and Chao (1993), which compares the welfare effects of technology transfer between royalty licensing and direct foreign investment in a general equilibrium model. This paper instead focuses on comparing the relative performance of macroeconomic variables under different patent licensing contracts.

forces in the labor market and, as a result, treats the real wage rate as given. Our general equilibrium analysis explicitly brings the mutual interaction between the goods market and the labor market into the picture, and hence endogenizes the real wage. The second is that the partial equilibrium analysis treats the household's income as given; our general equilibrium analysis instead not only deals with the household's optimizing behavior, but also endogenizes the household's income. With these two additional channels, our analysis finds some conflicting results compared to the existing studies on patent licensing.

Apart from providing a positive analysis on the performance of patent licensing, this paper also engages in a normative analysis concerning the relative welfare level among various licensing agreements. Within the existing literature, Poddar and Sinha (2004) and San Martin and Saracho (2010) find that the welfare level under the royalty licensing regime is greater than that under the no licensing regime. Kabiraj (2005) shows that, with a high (low) degree of technical innovation, the welfare level under the fixed-fee licensing regime is greater (less) than that under the royalty licensing regime. In view of the fact that the findings proposed by Poddar and Sinha (2004), Kabiraj (2005), and San Martin and Saracho (2010) are characterized by a partial equilibrium analysis, our paper can thus serve as a vehicle to examine whether their findings are tenable in a general equilibrium analysis.

The remainder of this paper proceeds as follows. Section 2 sets up a monopolistic competition macroeconomic model. Section 3 deals with the determination of the relevant macro variables under the three licensing regimes. A comparison of the relative sizes of the macro variables among the three licensing regimes is reported in Section 4. Section 5 compares the relative welfare level among the three licensing regimes. Section 6 discusses some extensions of the baseline model. Finally, the main findings of the analysis are summarized in Section 7.

2. The baseline model

The baseline model we set up can be treated as an extended version of Heijdra (2009, p. 358) in which a monopolistically competitive product market is present. The economy under consideration consists of two types of agents: households and firms. We then in turn describe the behavior of each of these agents.

2.1 Households

The economy is populated by a unit of infinitely-lived identical households. The representative household derives utility from the consumption of composite goods C and leisure l. The household's utility function U can be expressed as:

$$Max \quad U = \alpha \ln C + (1-\alpha)\ln l \,; \quad 0 < \alpha < 1, \tag{1}$$

where the parameters α and $1-\alpha$ refer to the weights in terms of the utility which the representative household attaches to consumption and leisure, respectively.

Assume that there are *m* types of consumption goods. In line with Dixit and Stiglitz (1977), the composite consumption is defined as:

$$C = m\left[\frac{1}{m}\sum_{i=1}^{m} c_i^{1-\mu}\right]^{1/(1-\mu)} \,; \quad 0 \le \mu < 1, \tag{2}$$

where c_i is a consumption good of variety *i*, and μ is the inverse of the elasticity of substitution between any two consumption goods. We will show later that μ measures the degree of monopoly power of firms.

The representative household is bound by a flow constraint linking its income and its expenditure. In addition to wage income, as the owner of all firms, the representative household receives aggregate profits of all firms in the form of dividends. Accordingly, the household's budget constraint can be expressed as:

$$PC = \sum_{i=1}^{m} p_i c_i = wN_s + \Pi \,; \quad N_s = T - l, \tag{3}$$

where P is the price index of composite consumption, p_i is the price of the *i*th consumption good, w is the wage rate, T is the time endowment, N_s is labor supply, and Π is aggregate profits of all firms. The price index of composite consumption P can be written as:

$$P = \left[\frac{1}{m}\sum_{i=1}^{m} p_i^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}. \tag{4}$$

It should be noted that in this paper the bundle of composite final goods is treated as the *numéraire*, and hence in what follows the price index of composite consumption P is normalized to unity (i.e., $P=1$).[4] As a result, the price of the *i*th consumption good p_i ($i = 1, 2, ..., m$) can be treated as the relative price in terms of P.

The household's optimization problem can be solved by applying a two-stage

[4] To simplify our analysis, the existing studies on imperfect competition, e.g., Benhabib and Farmer (1994), Devereux et al. (1996, 2000), and Wang and Wen (2008) assume that money is absent in the economy. Accordingly, these studies either explicitly or implicitly assume that the final good is *numeraire*. In line with these studies, in this paper the bundle of composite final goods acts as the *numeraire*, and hence the price index of composite consumption *P* is normalized to unity.

budgeting decision. In the first stage, the household chooses the composite consumption C and leisure l to maximize its utility reported in equation (1) subject to the budget constraint stated in equation (3). In the second stage, the household chooses c_i ($i = 1, 2, ..., m$) to maximize the composite consumption C subject to $PC = \sum_{i=1}^{m} p_i c_i$. Given $P = 1$, the solution to the household's optimization problem yields the following expressions:

$$C = \alpha(wT + \Pi), \tag{5a}$$

$$l = (1-\alpha)\frac{wT + \Pi}{w}, \tag{5b}$$

$$c_i = \left[\frac{P}{p_i}\right]^{1/\mu} \frac{C}{m} = \left[\frac{1}{p_i}\right]^{1/\mu} \frac{C}{m}. \tag{5c}$$

Equation (5c) is the demand function for the *i*th consumption good ($i = 1, 2, ..., m$), and indicates that (the absolute value of) the price elasticity of demand for the *i*th consumption good is $1/\mu$.

2.2 Firms

The production sector that we consider is composed of *m* firms, each of which produces a specific good y_i that is an imperfect substitute for the other goods. We utilize the usual "large numbers" assumption that each firm treats the prices of other firms and aggregate demand as given with respect to its own actions.

Among m firms, one patent-holding firm, denoted as "firm 1", owns an advanced technology that enables it to enjoy a lower marginal cost than the other $m-1$ firms. Each of the other $m-1$ firms produces a differentiated product and incurs an identical and constant marginal cost. To describe such a circumstance, firm 1's production function is specified as follows:

$$y_1 = An_1; \quad A > 1, \tag{6a}$$

where y_1 is firm 1's output, A is the exogenous technology level (the size of the advanced technology), and n_1 is firm 1's employment. To reflect the fact that with advanced technology firm 1 has a lower marginal cost (i.e., a higher labor productivity), we impose the restriction $A > 1$.

The other $m-1$ firms do not process advanced technology, and hence their production function can be described by:

$$y_i = n_i; \quad i = 2, ..., m. \tag{6b}$$

As is evident in equation (6b), other firms' marginal labor productivity is less that that of firm 1.

3. Macroeconomic Equilibrium

We are now in a position to deal with the macroeconomic equilibrium. In this section, three regimes will be discussed in turn. The first regime is the no licensing regime. In this regime, the patent-holding firm does not license its technology to the other $m-1$ firms and keeps the advanced technology for its own use. The second regime is the fixed-fee licensing regime. In the context of this regime, the patent-holding firm licenses its advanced technology to the other $m-1$ firms and charges an upfront fixed fee. The third regime is the royalty licensing regime. In this regime, the patent-holding firm licenses its advanced technology to the other $m-1$ firms and charges a royalty fee on a per unit of production basis.[5]

To describe the sequence of decisions of both the patent-holding firm and the other $m-1$ firms, we specify the game structure under the three regimes in the following way. Under both the fixed-fee licensing and royalty licensing regimes, the game structure includes three stages. In the first stage, the patent-holding firm offers a fixed-fee contract or royalty contract. In the second stage, the other $m-1$ firms decide whether or not to accept the contract. In the last stage, all firms engage in a quantity competition. Under the regime of no licensing, the game structure only includes a single stage, i.e., all firms engage in a quantity competition. In what follows, we will in turn discuss the macroeconomic equilibrium under these three regimes.

3.1 No licensing

We first deal with the no licensing regime. Under such a regime, firm 1 does not license its advanced technology to the other $m-1$ firms and keeps the superior technology for its own use. As is evident, licensing behavior does not occur under this regime. Let the superscript "NL" denote the regime of no licensing. Then, the real profit of the ith firm can be expressed as:

$$\pi_i^{NL} = p_i^{NL} y_i^{NL} - w^{NL} n_i^{NL}; \quad i = 1, 2, ..., m. \tag{7}$$

Maximizing equation (7) subject to equations (5c), (6a) and (6b) yields the following firm's optimal production decision and its corresponding price:

[5] We can follow Fauli-Oller et al. (2012) to deal with the optimal number of licensees. This issue will be discussed in a later section.

$$y_1^{NL} = \left[\frac{(1-\mu)A}{w^{NL}}\right]^{1/\mu} \frac{C^{NL}}{m}, \tag{8a}$$

$$p_1^{NL} = \frac{w^{NL}}{(1-\mu)A}, \tag{8b}$$

$$y_i^{NL} = \left[\frac{(1-\mu)}{w^{NL}}\right]^{1/\mu} \frac{C^{NL}}{m}; \quad i = 2,\ldots,m, \tag{8c}$$

$$p_i^{NL} = \frac{w^{NL}}{1-\mu}; \quad i = 2,\ldots,m. \tag{8d}$$

As a result, the real profit of the *i*th firm is given by:

$$\pi_i^{NL} = \mu p_i^{NL} y_i^{NL}; \quad i = 1, 2,\ldots,m. \tag{8e}$$

Equations (8b) and (8d) reveal the fact that the parameter μ can be treated as the degree of monopoly power. To be more specific, if the goods market is featured with imperfect competition ($1 > \mu > 0$), the firm will set the market price greater than the marginal cost. By contrast, the firm will set the market price equal to the marginal cost when the goods market is perfectly competitive ($\mu = 0$).[6] Moreover, by comparing equations (8a) with (8c) and (8b) with (8d), it is quite easy to find that the cost-reducing invention firm (i.e., firm 1) can benefit from setting a lower price and producing a higher output than the non-invention firms (i.e., the other $m-1$ firms). Equation (8e) states that each firm's profits are positively related to the degree of monopoly power.

Let a variable with "~" denote its equilibrium value. We can derive the equilibrium wage rate $\tilde{w}^{NL}$, aggregate output $\tilde{Y}^{NL}$, composite consumption $\tilde{C}^{NL}$, leisure $\tilde{l}^{NL}$, firm 1's real profit $\tilde{\pi}_1^{NL}$, firm i's real profit $\tilde{\pi}_i^{NL}$, and real aggregate profits $\tilde{\Pi}^{NL}$ under the no licensing regime as follows:[7]

$$\tilde{w}^{NL} = \frac{(1-\mu)A}{\{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{\mu/(\mu-1)}}. \tag{9a}$$

$$\tilde{Y}^{NL} = \tilde{C}^{NL} = \frac{\alpha(1-\mu)AT}{(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{\mu/(\mu-1)}}, \tag{9b}$$

6 For a detailed discussion concerning market power, see Torregrosa (1998).
7 The detailed derivations of the equilibrium macro variables are relegated to Appendix A.

$$\tilde{l}^{NL} = \frac{(1-\alpha)T}{1-\alpha\mu}. \tag{9c}$$

$$\tilde{\pi}_1^{NL} = \frac{\alpha\mu(1-\mu)AT}{m(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}}, \tag{9d}$$

$$\tilde{\pi}_i^{NL} = \frac{\alpha\mu(1-\mu)A^{(2\mu-1)/\mu}T}{m(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}};\ i = 2,3,\ldots,m, \tag{9e}$$

$$\tilde{\Pi}^{NL} = \frac{\alpha\mu(1-\mu)AT}{(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{\mu/(\mu-1)}}. \tag{9f}$$

Equations (9d) and (9e) lead us to derive the result $\tilde{\pi}_i^{NL} = A^{(\mu-1)/\mu}\tilde{\pi}_1^{NL}$. Since firm 1 can produce its output at a lower marginal cost due to advanced technology, the real profit of firm 1 is greater than that of the other $m-1$ firms.

By inserting equations (9b) and (9c) into the household's utility function (1), the household's welfare level (i.e., the level of indirect utility) under the no licensing regime $\tilde{U}^{NL}$ is given by:

$$\tilde{U}^{NL} = \alpha \ln \frac{\alpha(1-\mu)AT}{(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{\mu/(\mu-1)}} + (1-\alpha)\ln\frac{(1-\alpha)T}{1-\alpha\mu}. \tag{10}$$

3.2 Fixed-fee licensing

We then turn to discuss the fixed-fee licensing regime. In this regime, firm 1 licenses its technology to the other $m-1$ firms by charging a fixed fee F, which is invariant of the licensee's production decision.

As mentioned previously in this section, the game structure under the fixed-fee licensing regime is composed of three stages. The model is solved backwards by starting with the third stage. Let the superscript "F" denote the fixed-fee licensing regime. In the third stage, provided that each of the licensees accepts a licensing contract, firm 1's real total profit (π_1^F) can be expressed as follows:

$$\pi_1^F = p_1^F y_1^F - w^F n_1^F + (m-1)F, \tag{11}$$

where F denotes the fixed fee charged for each licensee. Equation (11) indicates that firm 1's real total profit includes the profit from its own output ($p_1^F y_1^F - w^F n_1^F$) and licensing revenues ($(m-1)F$).

Maximizing equation (11) subject to equations (5c) and (6a) leads to the following firm 1's optimal production decision and its corresponding price:

$$y_1^F = \left[\frac{(1-\mu)A}{w^F}\right]^{1/\mu} \frac{C^F}{m}, \tag{12a}$$

$$p_1^F = \frac{w^F}{(1-\mu)A}. \tag{12b}$$

As a result, firm 1's real total profit is given by:

$$\pi_1^F = \mu p_1^F y_1^F + (m-1)F. \tag{12c}$$

Let π_i^F ($i = 2,...,m$) be the real total profit of firm i. Then, π_i^F can be written as:

$$\pi_i^F = p_i^F y_i^F - w^F n_i^F - F. \tag{13}$$

Equation (13) states that firm i's real total profit is equal to the difference between the profit from its own output ($p_i^F y_i^F - w^F n_i^F$) and licensing fee (F). It should be noted that the production function for each of the other $m-1$ firms changes to $y_i = An_i$ when the other $m-1$ firms receive the new technology. The optimization problem for the each of licensees is to maximize π_i^F in equation (13) subject to equations (5c) and (6a) by choosing y_i^F. This leads to the following optimal output production and corresponding price level:

$$y_i^F = \left[\frac{(1-\mu)A}{w^F}\right]^{1/\mu} \frac{C^F}{m}; \quad i = 2,...,m, \tag{14a}$$

$$p_i^F = \frac{w^F}{(1-\mu)A}; \quad i = 2,...,m. \tag{14b}$$

As a result, firm i's real total profit is given by:

$$\pi_i^F = \mu p_i^F y_i^F - F; \quad i = 2,...,m. \tag{14c}$$

As is evident in equations (12a) and (14a), because all the other $m-1$ firms possess the same technology as firm 1, all firms (including firm 1) have the same optimal output decision under this regime.

In the second stage, when firm i's real total profit under the fixed-fee licensing regime is at least as great as that under the no licensing regime, firm i is inclined to accept the licensing contract. Hence, firm i's incentive constraint condition is:

$$\pi_i^F - \pi_i^{NL} \geq 0. \tag{15}$$

In the first stage, to maximize the real total profit firm 1 can charge firm i a fixed fee F that will make firm i indifferent between fixed-fee licensing and no licensing (i.e., $\pi_i^F - \pi_i^{NL} = 0$). To be more precise, the fixed fee F that firm 1 charges is equal to the maximum value that the licensee is willing to pay. As a consequence, substituting equations (7) and (13) into the constraint $\pi_i^F - \pi_i^{NL} = 0$ yields:

$$F = (p_i^F y_i^F - w^F n_i^F) - (p_i^{NL} y_i^{NL} - w^{NL} n_i^{NL}) \tag{16}$$

Equipped with the above relationships, we can derive the following equilibrium macro variables: the wage rate $\tilde{w}^F$, aggregate output $\tilde{Y}^F$, composite consumption $\tilde{C}^F$, leisure $\tilde{l}^F$, firm 1's real total profit $\tilde{\pi}_1^F$, firm i's real total profit $\tilde{\pi}_i^F$, aggregate real profits $\tilde{\Pi}^F$, and the fixed fee $\tilde{F}$:[8]

$$\tilde{w}^F = (1-\mu)A. \tag{17a}$$

$$\tilde{Y}^F = \tilde{C}^F = \frac{\alpha(1-\mu)AT}{1-\alpha\mu}, \tag{17b}$$

$$\tilde{l}^F = \frac{(1-\alpha)T}{1-\alpha\mu}, \tag{17c}$$

$$\tilde{\pi}_1^F = \frac{\alpha\mu(1-\mu)AT}{(1-\alpha\mu)}\left\{1 - \frac{(m-1)A^{(\mu-1)/\mu}}{m\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}}\right\}, \tag{17d}$$

$$\tilde{\pi}_i^F = \frac{\alpha\mu(1-\mu)A^{(2\mu-1)/\mu}T}{m(1-\alpha\mu)\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}}, \tag{17e}$$

$$\tilde{\Pi}^F = \frac{\alpha\mu(1-\mu)AT}{1-\alpha\mu}. \tag{17f}$$

$$\tilde{F} = \frac{\alpha\mu(1-\mu)AT}{m(1-\alpha\mu)}\left\{1 - \frac{mA^{(\mu-1)/\mu}\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{\mu/(1-\mu)}}{[1+(m-1)A^{(\mu-1)/\mu}]}\right\}. \tag{17g}$$

By making a comparison between (17d) and (17e), it is quite straightforward to show

[8] The detailed derivations of the equilibrium macro variables under the fixed-fee licensing regime are provided in Appendix A.

that the real total profit of firm 1 is also greater than that of firm i (i.e., $\tilde{\pi}_i^F < \tilde{\pi}_1^F$) under the fixed-fee regime.[9]

Finally, by inserting equations (17b) and (17c) into the household's utility function, we obtain the household's welfare under the fixed-fee licensing regime:

$$\tilde{U}^F = \alpha \ln \frac{\alpha(1-\mu)AT}{1-\alpha\mu} + (1-\alpha)\ln\frac{(1-\alpha)T}{1-\alpha\mu}. \tag{18}$$

3.3 Royalty licensing

This subsection discusses the royalty licensing regime. Under this regime, firm 1 licenses its technology to the other $m-1$ firms at a flat royalty rate s. Each of the other $m-1$ firms pays the license fee according to its level of output.

Similar to the fixed-fee licensing regime, as stated in Section 3, the game structure under the royalty licensing regime is also composed of three stages. The model is solved backwards by starting with the third stage. We first deal with the third stage in which all firms engage in a quantity competition.

Let the superscript "R" denote the regime of royalty licensing. Given the fact that firm 1 licenses its technology to the other $m-1$ firms and charges a flat royalty fee s according to the output level, firm 1's real total profit can then be expressed as:

$$\pi_1^R = p_1^R y_1^R - w^R n_1^R + (m-1)s y_i^R. \tag{19}$$

Firm 1's optimization problem is to maximize π_1^R as reported in (19), subject to (5c) and (6a), by choosing its output level. The first-order condition for this problem leads to firm 1's optimal output production and its corresponding price level as follows:

$$y_1^R = \left[\frac{(1-\mu)A}{w^R}\right]^{1/\mu} \frac{C^R}{m}, \tag{20a}$$

$$p_1^R = \frac{w^R}{(1-\mu)A}. \tag{20b}$$

As a result, firm 1's real total profit is given by:

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[9] From equations (17d) and (17e) we can obtain the following expression:

$$\pi_i^F = \{[A^{(1-\mu)/\mu} + (m-1)]\{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{\mu/(\mu-1)} - (m-1)\}^{-1}\pi_1^F.$$

Given $\{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{\mu/(\mu-1)} > 1$ and $[A^{(1-\mu)/\mu} + (m-1)] > (m-1) \geq 1$, it is easy to infer that $\pi_i^F < \pi_1^F$.

$$\pi_1^R = \mu p_1^R y_1^R + (m-1)sy_i^R . \tag{20c}$$

Let π_i^R ($i = 2,...,m$) denote the licensee's real total profit. Then, π_i^R can be expressed as:

$$\pi_i^R = p_i^R y_i^R - w^R n_i^R - sy_i^R . \tag{21}$$

By choosing y_i^R to maximize π_i^R as stated in equation (21) subject to (5c) and (6a), we can derive firm i's optimal output production and its corresponding price level:

$$y_i^R = \left[\frac{(1-\mu)}{w^R / A + s} \right]^{1/\mu} \frac{C^R}{m} , \tag{22a}$$

$$p_i^R = \frac{w^R}{(1-\mu)A} + \frac{s}{(1-\mu)} . \tag{22b}$$

As is clear from (22a) and (22b), unlike the fixed-fee licensing regime, under the royalty licensing regime firm i's optimal decision is affected by the royalty rate s. Moreover, equations (22a) and (22b) indicate that, in response to a higher royalty rate, the other $m-1$ firms tend to reduce output production and raise the output price under the royalty licensing regime.

In the second stage, similar to the fixed-fee licensing, firm i ($i = 2,...,m$) will accept the licensing contract if the following incentive constraint is satisfied:

$$\pi_i^R - \pi_i^{NL} \geq 0 . \tag{23a}$$

To simplify the analysis, in what follows we focus on the situation where firm i accepts the licensing contract, i.e., we impose the restriction $\pi_i^R - \pi_i^{NL} \geq 0$.[10]

In the first stage, firm 1 chooses s to maximize its real total profit. In addition, the following incentive constraint for firm 1 must be satisfied:

$$\pi_1^R - \pi_1^{NL} \geq 0 . \tag{23b}$$

Then, by equation (22a) together with (19), we obtain the optimal royalty rate:

[10] We have also considered the situation in association with $\pi_i^R - \pi_i^{NL} < 0$. Under such a situation, firm i ($i = 2,...,m$) tends not to accept the licensing contract. As a result, the best way for the patent-holding firm to motivate firm i to accept the licensing contract is to set the royalty rate so as to satisfy $\pi_i^R - \pi_i^{NL} = 0$.

$$s = \frac{\mu}{1-\mu}\frac{w^R}{A}. \tag{24}$$

Substituting equation (24) into equations (22a) and (22b), firm i's output production and price level are given by:

$$y_i^R = \left[\frac{(1-\mu)^2 A}{w^R}\right]^{1/\mu} \frac{C^R}{m}, \tag{25a}$$

$$p_i^R = \frac{w^R}{(1-\mu)^2 A}. \tag{25b}$$

As a result, firm i's real total profit is given by:

$$\pi_i^R = \mu p_i^R y_i^R; \quad i = 2,...,m. \tag{25c}$$

Equation (25b) reveals an important result: Royalty licensing leads to double marginalization since both the licenser (firm 1) and the licensee (firm i) add their own price-cost margin at each stage of production.

Similar to the derivations under the no licensing regime, we can infer the following equilibrium macro variables under the royalty licensing regime: aggregate output $\tilde{Y}^R$, composite consumption $\tilde{C}^R$, leisure $\tilde{l}^R$, firm 1's real total profit $\tilde{\pi}_1^R$, firm i's real total profit $\tilde{\pi}_i^R$, aggregate real profits $\tilde{\Pi}^R$, and royalty rate $\tilde{s}$:[11]

$$\tilde{w}^R = \frac{(1-\mu)A}{\{(1/m)[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]\}^{\mu/(\mu-1)}}. \tag{26a}$$

$$\tilde{Y}^R = \tilde{C}^R = \frac{\alpha(1-\mu)AT}{(1-\alpha\mu\Phi)\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{\mu/(\mu-1)}}, \tag{26b}$$

$$\tilde{l}^R = \frac{(1-\alpha)T}{1-\alpha\mu\Phi}, \tag{26c}$$

$$\tilde{\pi}_1^R = \frac{\alpha\mu(1-\mu)AT[1+(m-1)(1-\mu)^{1/\mu}]}{m(1-\alpha\mu\Phi)\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}}, \tag{26d}$$

$$\tilde{\pi}_i^R = \frac{\alpha\mu(1-\mu)^{1/\mu}AT}{m(1-\alpha\mu\Phi)\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{(2\mu-1)/(\mu-1)}}, \tag{26e}$$

[11] The detailed proofs regarding the equilibrium macro variables under the royalty licensing regime are relegated to Appendix A.

$$\tilde{\Pi}^R = \frac{\alpha\mu(1-\mu)AT\Phi}{(1-\alpha\mu\Phi)\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{\mu/(\mu-1)}}, \tag{26f}$$

$$\tilde{s} = \mu\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{\mu/(1-\mu)}, \tag{26g}$$

where $\Phi = \dfrac{1+(m-1)(2-\mu)(1-\mu)^{(1-\mu)/\mu}}{1+(m-1)(1-\mu)^{(1-\mu)/\mu}} > 1$.

By comparing equation (26d) with (26e), it is easy to find that the real total profit of firm 1 is also greater than that of firm i (i.e., $\tilde{\pi}_i^R < \tilde{\pi}_1^R$) under the royalty licensing regime.[12] In addition, by inserting equations (9e) and (26e) into (23a), we can infer that the following lower bound on the productivity of labor of the licensee (firm i) should be imposed to ensure that $\pi_i^R - \pi_i^{NL} \geq 0$:

Condition P (Productivity of Labor of the Licensee):

$$A(1-\mu) \geq \Gamma(\alpha), \text{ where } \Gamma(\alpha) = \frac{1-\alpha\mu\Phi}{1-\alpha\mu} < 1.^{13} \tag{27}$$

The proper economic interpretation of Condition P is that if A is too small (i.e., if Condition P is violated), then firms $i = 2,...,m$ have insufficient incentives to engage in licensing at all. Condition P can be explained alternatively. From equations (8e) and (25c), we can infer that Condition P holds if $p_i^R y_i^R \geq p_i^{NL} y_i^{NL}$. It is clear that $p_i^R y_i^R \geq p_i^{NL} y_i^{NL}$ can be rewritten as $p_i^R / p_i^{NL} \geq y_i^{NL} / y_i^R$. With $p_i^{NL} = w^{NL}/(1-\mu)$ in equation (8d) and $p_i^R = w^R/(1-\mu)^2 A$ in equation (25b), the price ratio between the royalty licensing regime and the no licensing regime (p_i^R / p_i^{NL}) is closely related to $A(1-\mu)$. This term is reflected as the left-hand side of Condition P reported in equation (27). Moreover, the quantity ratio between the no licensing regime and the royalty licensing regime (y_i^{NL} / y_i^R) is closely related to $\Gamma(\alpha)$. This term is reflected as the right-hand side of Condition P reported in equation (27).

Finally, by substituting equations (26b)-(26c) into the household's utility function, we then obtain the household's welfare level under the royalty licensing regime:

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[12] From equations (26d) and (26e) we have $\tilde{\pi}_i^R = (1-\mu)^{(1-\mu)/\mu}/[1+(m-1)(1-\mu)^{1/\mu}]\tilde{\pi}_1^R$. Given $(1-\mu)^{(1-\mu)/\mu} < 1$ and $[1+(m-1)(1-\mu)^{1/\mu}] > 1$, it is quite easy to obtain $\tilde{\pi}_i^R < \tilde{\pi}_1^R$.

[13] Given the definition $\Phi = \dfrac{1+(m-1)(2-\mu)(1-\mu)^{(1-\mu)/\mu}}{1+(m-1)(1-\mu)^{(1-\mu)/\mu}} > 1$ reported in equation (26g), it is quite easy to infer $\Gamma(\alpha) = \dfrac{1-\alpha\mu\Phi}{1-\alpha\mu} < 1$.

$$\tilde{U}^R = \alpha \ln \frac{\alpha(1-\mu)AT}{(1-\alpha\mu\Phi)\{[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]/m\}^{\mu/(\mu-1)}} + (1-\alpha)\ln\frac{(1-\alpha)T}{1-\alpha\mu\Phi}. \tag{28}$$

4. Comparison of the three regimes: macroeconomic variables

In this section, we will compare the relative sizes of the relevant macroeconomic variables among the three regimes. We will first compare the no licensing regime with the fixed-fee licensing regime, followed by the no licensing regime with the royalty licensing regime, and finally compare the fixed-fee licensing regime with the royalty licensing regime.

4.1 No licensing versus fixed-fee licensing

By making a comparison between equations (9b)-(9f) and (17b)-(17f), we obtain the following results: $\tilde{Y}^F > \tilde{Y}^{NL}$, $\tilde{C}^F > \tilde{C}^{NL}$, $\tilde{w}^F > \tilde{w}^{NL}$, $\tilde{l}^F = \tilde{l}^{NL}$, $\tilde{N}^F = \tilde{N}^{NL}$, $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$, $\tilde{\pi}_i^F = \tilde{\pi}_i^{NL}$, and $\tilde{\Pi}^F > \tilde{\Pi}^{NL}$. The economic intuition behind these results is as shown below.

We first discuss the result $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$. As defined previously in equation (11), firm 1's real total profit is composed of the profit from its own output ($p_1^F y_1^F - w^F n_1^F$) and licensing revenues ($(m-1)F$). Given that firm 1 does not obtain licensing revenues under the fixed-fee licensing regime (i.e., $\pi_1^F = p_1^F y_1^F - w^F n_1^F$), to explain the result $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$ it is better to define firm 1's real profit that excludes licensing revenues under the fixed-fee licensing $(\pi_1^F)_O$ as follows:

$$(\pi_1^F)_O = p_1^F y_1^F - w^F n_1^F. \tag{29a}$$

Substituting $\tilde{p}_1^F$, $\tilde{y}_1^F$, $\tilde{w}^F$, and the relation $\tilde{y}_1^F = A\tilde{n}_1^F$ into equation (29a) yields:

$$(\tilde{\pi}_1^F)_O = \frac{\alpha\mu(1-\mu)AT}{m(1-\alpha\mu)}. \tag{29b}$$

Subtracting (29b) from (9d), we obtain:

$$(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL} = \frac{\alpha\mu(1-\mu)AT}{m(1-\alpha\mu)}\left\{1-\{[1+(m-1)A^{(\mu-1)/\mu}]/m\}^{\frac{2\mu-1}{1-\mu}}\right\} \gtrless 0 \; ; \;\; \text{if } \mu \gtrless 0.5 . \tag{30a}$$

Equation (30a) reveals that firm 1's real profit from its own output under the fixed-fee licensing regime is greater or lower than that under the no licensing regime depending upon whether $\mu > 0.5$ or $\mu < 0.5$. Why does firm 1's real profit from its own output increase with the higher degree of product differentiation when it freely transfers advanced technology to firm i? When firm 1 transfers its superior technology to firm i, an improvement in firm i's technology leads it to increase its labor demand, and hence results in a higher real wage. Two effects emerge in response to a rise in the real wage. First, a rise in the real wage causes an increase in firm 1's marginal cost of production. Second, a rise in the real wage leads the households to hold more real wealth, and further causes an increase in the demand for firm 1's goods (hence the marginal revenue of firm 1's goods also rises in response). However, since the upward shift in firm 1's marginal cost of production is greater than that for firm 1's marginal revenue, the equilibrium output of good 1 then falls and its corresponding equilibrium price rises in response. When μ is relatively high (low), the strength stemming from a rise in the equilibrium price dominates (falls short of) that stemming from a fall in the equilibrium output. Accordingly, firm 1's equilibrium real profit from its own output (i.e., $(\tilde{\pi}_1^F)_O$) will rise (fall) in response.

However, if licensing revenues are brought into the picture, then from equations (11) and (29a) we have the expression: $\tilde{\pi}_1^F = (\tilde{\pi}_1^F)_O + (m-1)\tilde{F}$. We can thus further infer the following result:

$$\tilde{\pi}_1^F - \tilde{\pi}_1^{NL} = [(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL}] + (m-1)\tilde{F} \tag{30b}$$

As shown in equation (30a), $(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL} \gtrless 0$, if $\mu \gtrless 0.5$. With an additional positive term $(m-1)\tilde{F}$, two points should be noted. First, it is clear that under the situation $\mu > 0.5$, $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$ always holds since $[(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL}] > 0$ and $(m-1)\tilde{F} > 0$ are true. Second, under the situation $\mu < 0.5$, equation (30a) indicates that $[(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL}] < 0$. However, the negative effect in $[(\tilde{\pi}_1^F)_O - \tilde{\pi}_1^{NL}]$ is smaller than the positive effect in $(m-1)\tilde{F}$. As a consequence, we can infer from equation (30b) that the result $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$ always holds.

Based on the above discussion, we have the following proposition:

Proposition 1. *The patent holder's real profit from its own output under the fixed-fee regime may be greater or less than that under the no licensing regime depending upon whether the degree of product differentiation* μ *is higher (lower) than* 0.5. *Moreover, the patent holder's real total profit under the fixed-fee regime is always greater than that under the no licensing regime regardless of the degree of product differentiation* μ.

In their partial equilibrium analysis, Wang (2002), Fauli-Oller and Sandonis (2003) and Mukherjee (2010) find that the patent-holding firm is inclined not to license its technology as the products come close to being perfect substitutes and the innovation is sufficiently large. The economic intuition underlying their findings can be grasped as follows. When the licensees acquire the new technology, an expansion in their output production will crowd out firm 1's profit. These studies further show that, for the patent-holding firm, licensing revenues from fixed-fee licensing cannot cover the loss of technology transfer if the differentiated products are highly substitutable and the innovation is sufficiently large. As is evident, our analytical result stated in Proposition 1 does not support their finding. The difference between these studies and our paper is that their analysis is derived within the goods market only. More specifically, their analysis ignores not only the mutual interaction between the goods market and the labor market, but also the change in the household's income.

We then discuss the results: $\tilde{Y}^F > \tilde{Y}^{NL}$ and $\tilde{C}^F > \tilde{C}^{NL}$. In the fixed-fee licensing regime, the licensees (the other $m-1$ firms) benefit from the lower marginal cost of production (the higher marginal productivity of labor) through buying the advanced technology. This leads them to boost their output production, thereby resulting in higher aggregate output production compared to the level under the no licensing regime (i.e., $\tilde{Y}^F > \tilde{Y}^{NL}$). Moreover, given that the equilibrium condition for the goods market requires aggregate output production to be equal to aggregate demand under the both regimes (i.e., $\tilde{Y}^F = \tilde{C}^F$ and $\tilde{Y}^{NL} = \tilde{C}^{NL}$), we can thus obtain the result $\tilde{C}^F > \tilde{C}^{NL}$.

We finally discuss the result $\tilde{l}^F = \tilde{l}^{NL}$. By observing equation (5b), compared with the no licensing regime, the fixed-fee licensing regime leads to higher real wages and more aggregate profits, and hence induces two effects. The first effect is the substitution effect. With this effect, a higher wage rate motivates the representative household to cut its leisure time. The second effect is the income effect. With this effect, the representative household tends to boost the leisure time since leisure is a normal good. It happens that both conflicting effects on the leisure time are equal, and accordingly the leisure time under the no licensing regime is the same as that under the fixed-fee licensing regime.

Summing up the above discussion, we can establish the following proposition:

Proposition 2. *The equilibrium aggregate output and consumption under the fixed-fee licensing regime are always greater than those under the no licensing regime. However, the equilibrium leisure under the fixed-fee licensing regime is equal to that under the no licensing regime.*

4.2 No licensing versus royalty licensing

By comparing equations (9b)-(9f) with (26b)-(26f), the following results are established: $\tilde{Y}^R > \tilde{Y}^{NL}$, $\tilde{C}^R > \tilde{C}^{NL}$, $\tilde{w}^R \gtrless \tilde{w}^{NL}$, $\tilde{l}^R > \tilde{l}^{NL}$, $\tilde{N}_s^R < \tilde{N}_s^{NL}$, $\tilde{\pi}_1^R > \tilde{\pi}_1^{NL}$, $\tilde{\pi}_i^R \geq \tilde{\pi}_i^{NL}$, and $\tilde{\Pi}^R > \tilde{\Pi}^{NL}$.[14] We then discuss the economic intuition behind these results.

In the royalty licensing regime, Condition P ensures that firm i accepts the licensing contract with a proportional royalty rate. Then, firm i has an incentive to boost its output production since firm i benefits from a decline in the marginal cost of production, thereby causing a rise in aggregate output production compared to the level under the no licensing regime (i.e., $\tilde{Y}^R > \tilde{Y}^{NL}$). Moreover, since the equilibrium condition for the goods market under the two regimes requires that $\tilde{Y}^R = \tilde{C}^R$ and $\tilde{Y}^{NL} = \tilde{C}^{NL}$, it is quite straightforward from the relation $\tilde{Y}^R > \tilde{Y}^{NL}$ to infer the result $\tilde{C}^R > \tilde{C}^{NL}$.

The intuition behind the result $\tilde{\pi}_1^R > \tilde{\pi}_1^{NL}$ is the same as for $\tilde{\pi}_1^F > \tilde{\pi}_1^{NL}$ in Subsection 4.1. To save space, we do not repeat it again. The result $\tilde{\pi}_i^R \geq \tilde{\pi}_i^{NL}$ is the incentive-compatibility constraint for firm i. Equipped with $\tilde{\pi}_1^R > \tilde{\pi}_1^{NL}$ and $\tilde{\pi}_i^R \geq \tilde{\pi}_i^{NL}$, we can then infer the result $\tilde{\Pi}^R > \tilde{\Pi}^{NL}$.

By comparing the equilibrium leisure time under royalty licensing with that under the no licensing regime, we obtain the result $\tilde{l}^R > \tilde{l}^{NL}$. We will discuss the economic intuition behind this result in Subsection 4.3 below.

To summarize the above results, we have the following proposition:

Proposition 3. *The equilibrium aggregate output, consumption, and leisure under the royalty licensing regime are always higher than those under the no licensing regime.*

4.3 Fixed-fee licensing versus royalty licensing

This subsection turns to discuss the relative performance of the macro variables between fixed-fee licensing and royalty licensing. Comparing (17b)-(17f) with (26b)-(26f) yields the following results: $\tilde{Y}^F > \tilde{Y}^R$, $\tilde{C}^F > \tilde{C}^R$, $\tilde{w}^F > \tilde{w}^R$, $\tilde{l}^F < \tilde{l}^R$, $\tilde{N}_s^F > \tilde{N}_s^R$, $\tilde{\pi}_1^F \gtrless \tilde{\pi}_1^R$, $\tilde{\pi}_i^F \leq \pi_i^R$, and $\tilde{\Pi}^F \gtrless \tilde{\Pi}^R$. We then provide economic reasoning for these results.

In comparison with the fixed-fee licensing regime, under the royalty licensing

[14] As stated in footnote 9, firm i will accept the royalty fee so as to satisfy $\pi_i^R - \pi_i^{NL} = 0$ when Condition P does not hold. Under such a situation, due to mathematical complexity, the relative sizes of the macro variables between the royalty licensing regime and the no licensing regime require that we resort to numerical simulations. The numerical results are available upon request from the authors.

regime firm i incurs an additional marginal cost since it pays the licensing fee according to the output level. This leads firm i to shrink its output production. As a result, aggregate output under the royalty licensing regime is lower than that under the fixed-fee licensing regime (i.e., $\tilde{Y}^F > \tilde{Y}^R$). Moreover, with the equilibrium condition for the goods market under the two regimes $\tilde{Y}^F = \tilde{C}^F$ and $\tilde{Y}^R = \tilde{C}^R$, we thus have the result $\tilde{C}^F > \tilde{C}^R$.

We then discuss the result $\tilde{l}^F < \tilde{l}^R$. In a way that is similar to the inference in Subsection 4.1, from equation (5b) and compared with the royalty licensing regime, the fixed-fee licensing regime leads to a higher real wage, and hence the substitution effect under the fixed-fee licensing regime is greater than that under the royalty licensing regime. The representative household tends to have less leisure time under the fixed-fee licensing regime compared to under the royalty licensing regime. Accordingly, the leisure time under the fixed-fee licensing regime is lower than that under the royalty licensing regime.

We are now in a position to compare the relative sizes between $\tilde{l}^R$ and $\tilde{l}^{NL}$ in Subsection 4.2. Using the results $\tilde{l}^F = \tilde{l}^{NL}$ in Subsection 4.1 and $\tilde{l}^F < \tilde{l}^R$ in this subsection, it is quite straightforward to infer the result $\tilde{l}^R > \tilde{l}^{NL}$.

The above discussion leads us to the following proposition:

Proposition 4. *The equilibrium aggregate output and consumption under the fixed-fee licensing regime are always greater than those under the royalty licensing regime. Nevertheless, the equilibrium leisure under the fixed-fee licensing regime is lower than that under the royalty licensing regime.*

We then discuss the result $\tilde{\pi}_1^F \gtrless \tilde{\pi}_1^R$. Due to the complexity of firm 1's equilibrium total profit stated in equations (17d) and (26d), we resort to the numerical solution to illustrate the relative size of firm 1's equilibrium real total profit between royalty licensing and fixed-fee licensing. The parameters we set are adapted from the manufacturing industries data in Taiwan and the value in Chang et al. (2011), where the number of firms $m = 25$ and the market power $\mu = 0.29$.[15,16] The weight of consumption $\alpha = 0.42$ is chosen to generate $N = (1/3)T$, as in Costa and Dixon (2011).

With the structural parameters $m = 25$, $\mu = 0.29$, $T = 1$, and $\alpha = 0.42$, we

[15] Fauli-Oller et al. (2012) use the manufacturing industries data in Spain, and find that the number of firms is 27. Based on the manufacturing industries data in Taiwan, we find that the number of firms is 25.

[16] Chang et al. (2011) state that many empirical studies support the view that the markup ratio $1/(1-\mu)$ is around 1.4, implying that the market power μ is equal to 0.29. See Chang et al. (2011) for a detailed explanation.

should impose the restriction $A \geq 1.29$ to satisfy Condition P. Moreover, as exhibited in Figure 1, we find a threshold value of A, namely $A_T = 2.08$, such that $\tilde{\pi}_1^F > \tilde{\pi}_1^R$, if $A > A_T = 2.08$, while $\tilde{\pi}_1^F < \tilde{\pi}_1^R$, if $1.29 \leq A < A_T = 2.08$. This simulated result indicates that a maximizing-profit patent-holding firm is inclined to select fixed-fee licensing when it has a relatively high technology level. [17,18]

Our simulated result stands in sharp contrast to the finding proposed by Kamien and Tauman (2002), Wang (2002) and Kabiraj (2005),[19] which purport that the royalty licensing contract is superior to fixed-fee licensing with a sufficiently large innovation. The major difference between these previous studies and our paper can be reported as follows. In their partial equilibrium analysis, Kamien and Tauman (2002), Wang (2002) and Kabiraj (2005) focus on the goods market only. Under such a situation, for the patent-holding firm licensing revenues from fixed-fee licensing cannot cover the loss of technology transfer when the degree of product differentiation is lower and the level of technology is higher. As a consequence, to maximize its profits the patent-holding firm is inclined to choose the royalty licensing contract under the situation where the degree of product differentiation is lower and the level of technology is higher.

However, in our model the mutual interdependence between the goods market and the labor market is explicitly taken into consideration. Faced with such a situation, one additional channel emerging from our general equilibrium analysis should be noted. Compared with the royalty licensing regime, under the fixed-fee licensing regime technology transfer leads to higher labor demand, and hence results in higher real wages. The higher real wages tend to bring forth higher household income, and hence in turn cause a higher demand for the patent holder's goods. As a consequence, the introduction of the interaction between the goods market and the labor market creates a new channel through which the patent-holding firm earns more total profit under the fixed-fee licensing regime than under the royalty licensing regime. Moreover, this beneficial effect for the fixed-fee licensing regime will be reinforced with a higher level of technology. As is evident, when the level of

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[17] The simulated result is robust if the number of the firms is within the range $m \geq 2$ and the level of market power is within the range $1 > \mu > 0$. The simulated results are available from the authors upon request.

[18] In line with Fauli-Oller et al. (2012), we can consider the situation where the patent-holding firm (firm 1) can choose the optimal number of licensees. Due to mathematical complexity, we resort to numerical simulations to deal with this situation. Our simulated result shows that the patent-holding firm will license to $m-1$ firms (all firms are licensed) regardless of whether it adopts the fixed-fee or royalty contract. This simulated result is consistent with the Fauli-Oller et al. (2012) finding, although their analysis assumes that the patent-holding firm adopts the two-part tariff licensing contract. This is the reason why our analysis makes an assumption that firm 1 licenses to all other $m-1$ firms.

[19] Kabiraj (2005) points out that for the profit-maximizing patent holding firm the royalty licensing contract is always preferable to the fixed-fee licensing contract in a Stackelberg structure.

technology is higher, the force of this new channel will dominate that of the partial equilibrium analysis, and the patent holder will instead choose the fixed-fee licensing contract.

5. Comparison of the three regimes: the welfare level

We now proceed to compare the household's welfare level among the three regimes. First of all, we compare the household's welfare level between the fixed-fee licensing and no licensing regimes. Comparing equation (10) with (18) yields:

$$\tilde{U}^{F} - \tilde{U}^{NL} = \frac{\mu}{\mu - 1} \ln\{[1 + (m-1)A^{(\mu-1)/\mu}]/m\} > 0 . \tag{31a}$$

The reason for equation (31a) is quite clear. As indicated in Proposition 2, in comparison with the no licensing regime, fixed-fee licensing generates higher equilibrium aggregate consumption and the same leisure time. Therefore, the welfare level under the fixed-fee licensing regime is always greater than that under the no licensing regime.

We then compare the household's welfare level between the royalty licensing and no licensing regimes. By comparing equation (10) with (28), we have:

$$\tilde{U}^{R} - \tilde{U}^{NL} = \alpha \ln \frac{(1-\alpha\mu)[1+(m-1)A^{(\mu-1)/\mu}]^{\mu/(\mu-1)}}{(1-\alpha\mu\Phi)[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]^{\mu/(\mu-1)}} + (1-\alpha)\ln\frac{1-\alpha\mu}{1-\alpha\mu\Phi} > 0 . \tag{31b}$$

Proposition 3 reveals that, compared with the no licensing regime, royalty licensing leads to more aggregate consumption and more leisure time, thereby generating a higher welfare level. As a result, the welfare level under the royalty licensing regime is always greater than that under the no licensing regime.

From equations (31a) and (31b) we can establish the following proposition:

Proposition 5. *The welfare level under the two licensing regimes is always greater than that under the no licensing regime.*

The result of Proposition 5 is consistent with the finding put forth by Fauli-Oller and Sandonis (2002), and San Martin and Saracho (2010).[20]

We finally turn to compare the relative welfare level between fixed-fee licensing and royalty licensing. Due to the fact that the functional forms under these two

[20] Fauli-Oller and Sandonis (2002) and San Martin and Saracho (2010) respectively show that welfare is improving under the two-part tariff licensing and royalty licensing.

regimes, as reported in equations (18) and (28), are too complex, we must resort to numerical simulations.

The numerical parameters we utilize are the same as those in Section 4, and hence we impose the same restriction $A \geq 1.29$ to satisfy Condition P. In Figure 2, we plot the welfare level under both the fixed-fee licensing regime and the royalty licensing regime. As displayed in Figure 2, it is quite clear that the welfare level under the fixed-fee licensing regime is always larger than that under the royalty licensing regime ($\tilde{U}^F > \tilde{U}^R$). Proposition 5 indicates that, in comparison with the royalty licensing regime, fixed-fee licensing leads to more aggregate consumption and less leisure time. The greater aggregate consumption contributes to a positive effect on the welfare level, while the reduction in leisure time leads to a negative effect on the welfare level. As is obvious, with our plausible numerical parameters, the positive effect dominates the negative effect, thereby resulting in the welfare level under the fixed-fee licensing regime being higher than that under the royalty licensing regime.

6. Extensions and Discussion

This section discusses four extensions of the baseline model developed in Section 2. To save space, in this section we only focus on how the patent-holding firm makes an alternative choice between the fixed-fee licensing contract and the royalty licensing contract. More specifically, we respectively provide a brief discussion on whether our finding regarding the relative superiority between the fixed-fee contract and the royalty contract in Section 4.3 is tenable with each extension. Before preceding this section, one point should be made here. To make a distinction of the equilibrium macro variables between the extended models and the baseline model, a variable with "^" denotes its equilibrium value in the extended models.

6.1 R&D Investment

For simplicity, the baseline model in Section 2 assumes that the patent-holding firm (firm 1) has access to its superior technology in an exogenous and costless manner. It would be interesting to extend our model to consider a more general case where firm 1 undertakes a costly innovation and licenses its invented technology to the other $m-1$ firms. We assume that firm 1 uses R&D costs C_r (in terms of aggregate output) to promote the technology level. When firm 1 does not undertake R&D, the technology level is equal to 1, and firm 1's R&D costs can then be expressed as:[21]

[21] In line with D'Aspremont and Jacquemin (1988), the cost of R&D is assumed to be in quadratic

$$C_r = \psi \frac{(A-1)^2}{2}; \ \ \psi > 0, \tag{32a}$$

where ψ is a constant parameter that reflects the extent of the R&D cost.

Accordingly, the real total profit of firm 1 under the fixed-fee licensing and royalty licensing regimes reported in equations (11) and (19) is modified as follows:

$$\pi_1 = \begin{cases} p_1 y_1 - wn_1 + (m-1)F - C_r; & \text{under the fixed - fee licensing regime,} \\ p_1 y_1 - wn_1 + (m-1)sy_i - C_r; & \text{under the royalty licensing regime.} \end{cases} \tag{32b}$$

In the presence of endogenous innovation, the game structure under the fixed-fee licensing and royalty licensing regimes is modified as follows. In the first stage, the patent-holding firm decides its technology level. In the second stage, the patent-holding firm offers a fixed-fee contract or royalty contract. In the third stage, the other $m-1$ firms determine whether or not to accept the contract proposed by the patent-holding firm. In the last stage, all firms engage in a quantity competition. In comparison with the game structure of the baseline model, the first stage regarding the optimal choice for the technology level is added.

In a way that is similar to the procedure in Section 3, these games are solved by backward induction. We obtain that the optimal decisions of firms in the second, third, and fourth stages are the same as those in Section 3. In the first stage, firm 1 determines the technology level to maximize π_1 as stated in equation (32b) subject to the firms' optimal decisions in the second, third, and fourth stages. The first-order conditions under the two regimes are respectively given by:

$$(1-\mu)\left(\frac{w^F}{1-\mu}\right)^{\frac{\mu-1}{\mu}} C^F A^{F^{\frac{1-2\mu}{\mu}}} = \psi(A^F - 1), \tag{33a}$$

$$[1+(m-1)(1-\mu)^{\frac{1}{\mu}}]\frac{1-\mu}{m}\left(\frac{w^R}{1-\mu}\right)^{\frac{\mu-1}{\mu}} C^R A^{R^{\frac{1-2\mu}{\mu}}} = \psi(A^R - 1). \tag{33b}$$

In equations (33a) and (33b), firm 1 equates the marginal benefit (on the left-hand side) to the marginal cost (on the right-hand side) of innovation under the fixed fee licensing and the royalty licensing regimes, respectively.

As the extended model is too complicated to solve analytically, we thus resort to

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form.

numerical simulations. The benchmark parameters we choose are the same as those in Subsection 4.3 of the baseline model: $m=25$, $\mu=0.29$, $T=1$, and $\alpha=0.42$. We obtain the following results: (i) the relative size of the technology innovation between the two regimes is $\hat{A}^F > \hat{A}^R$; and (ii) the patent holder's real total profit under the fixed-fee regime is lower than that under the royalty licensing regime ($\hat{\pi}_1^F < \hat{\pi}_1^R$).[22]

We first discuss the result: $\hat{A}^F > \hat{A}^R$. By comparing (33a) and (33b), we see that the marginal benefit of innovation under the royalty licensing regime is less than that under the fixed-fee licensing regime. This result stems from the fact that, as described by equation (25a), the licensing revenue under the royalty licensing regime is featured with the double marginalization effect. Thus, we have the result $\hat{A}^F > \hat{A}^R$.

Then, we discuss $\hat{\pi}_1^F < \hat{\pi}_1^R$. Due to the presence of endogenous innovation, two additional effects with a higher technology level emerge in response. First, similar to the analysis in Subsection 4.3, the mutual interdependence between the goods market and the labor market makes for a greater beneficial effect under the fixed-fee licensing regime compared to the royalty licensing regime. Second, a higher technology level leads to a higher R&D cost. The R&D cost under the fixed-fee licensing regime is greater than that under the royalty licensing regime due to $\hat{A}^F > \hat{A}^R$. If the first positive effect falls short of the second negative effect, we obtain the conclusion: $\hat{\pi}_1^F < \hat{\pi}_1^R$.

However, if the first positive effect dominates the second negative effect, our result $\hat{\pi}_1^F > \hat{\pi}_1^R$ in Subsection 4.3 holds even though endogenous innovation is brought into the picture. To highlight this point, we increase the degree of product differentiation μ from $\mu=0.29$ to $\mu=0.31$. Under such a situation, the first positive effect will be strengthened, and hence is greater than the second negative effect. As a result, our result $\hat{\pi}_1^F > \hat{\pi}_1^R$ in Subsection 4.3 under the situation where the superior technology of the patent-holding firm is assumed to be exogenous and costless remains true.

6.2 Mixed Industrial Structure

The baseline model in Section 2 specifies that there is only a single product, albeit a differentiated one. As pointed out by Dixon and Hansen (1999), and Censolo and Colombo (2008), it is more realistic for the structure of industries to be diverse, i.e.,

[22] The numerical results are available upon request from the authors.

the goods market is composed of both monopolistically competitive goods and perfectly competitive goods. As such, it would be worthwhile to discuss whether our finding in Subsection 4.3 is robust if there is a second goods market, namely, that of perfectly competitive goods, to absorb part of the available labor.

This subsection deals with the mixed industrial structure of an economy in which some specific proportions of goods are perfectly competitive and other proportions of goods are monopolistically competitive. In the presence of the mixed industrial structure, the household's consumption includes two types of final goods. One is consumption in the perfectly competitive sector C_C, and the other is consumption in the monopolistically competitive sector C_M. As a result, in line with Dixon and Hansen (1999), the household's aggregate consumption C can be expressed as:

$$C=\frac{C_M^{\beta}C_C^{1-\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}}. \tag{34a}$$

The parameter β denotes the share of C_M in aggregate consumption. It is clear that $\beta=1$ implies that perfectly competitive goods are absent from the analysis, and the extended model in this subsection thus degenerates to the baseline model in Section 2.

The household's utility function can then be modified as follows:

$$Max \quad U=\alpha\ln C+(1-\alpha)\ln l\,; \quad C_M=m[(\textstyle\sum_{i=1}^{m}c_i^{1-\mu})/m]^{1/(1-\mu)}, \tag{34b}$$

where the definition of the consumption in the monopolistically competitive sector C_M is the same as that in equation (2).

The budget constraint of the household can be written as:

$$PC=P_M C_M+P_C C_C=wN_s+\Pi\,; \quad P_M C_M=\textstyle\sum_{i=1}^{m}p_i c_i. \tag{34c}$$

The corresponding consumer price index P is given by:

$$P=P_M^{\beta}P_C^{1-\beta}=1\,; \quad P_M=\left[\frac{1}{m}\sum_{i=1}^{m}p_i^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}. \tag{34d}$$

Similar to Subsection 2.1, the bundle of two types of final goods is treated as the *numéraire*, and hence $P=1$. The solution to the household's optimization problem yields the following expressions:

$$C_C=\frac{\alpha(1-\beta)(wT+\Pi)}{P_C}, \tag{35a}$$

$$C_M = \frac{\alpha\beta(wT+\Pi)}{P_M}, \tag{35b}$$

$$l = (1-\alpha)\frac{wT+\Pi}{w}, \tag{35c}$$

$$c_i = \left[\frac{P_M}{p_i}\right]^{1/\mu}\frac{C_M}{m} = \left[\frac{P_M}{p_i}\right]^{1/\mu}\frac{\alpha\beta(wT+\Pi)}{mP_M}. \tag{35d}$$

The game structure is the same as that in Section 3. In addition, each firm produces either the monopolistically competitive good or the perfectly competitive good. Moreover, we assume that each firm in both sectors employs labor to produce its product. In the monopolistically competitive sector, we assume that the production technology of each firm is the same as that in Subsection 2.2. Accordingly, the optimal decisions on the product pricing and production of each firm remain intact as those in Subsection 2.2. The representative firm's production function in the perfectly competitive sector is given by:

$$y_C = n_C. \tag{36a}$$

The real profit of the representative firm is:

$$\pi_C = P_C y_C - wn_C. \tag{36b}$$

Based on equations (36a) and (36b), the representative firm's optimal decision is to produce output until price is equal to the marginal cost, i.e., $P_C = w$. As a result, we can infer the result $\pi_C = 0$.

With the same structural parameters in Subsection 4.3, we use numerical simulations to compute the macro variables under the two regimes. We find that the presence of a mixed industrial structure will affect the threshold value of A (i.e., A_T) for determining the relative size between $\hat{\pi}_1^F$ and $\hat{\pi}_1^R$.[23] To be more specific, we find that the threshold value of the technology level A_T is crucially related to the share of monopolistically competitive goods in aggregate consumption β. The result in Table 1 indicates how β will affect A_T.

Two results displayed in Table 1 should be noted. First, as exhibited in Figure 3, when perfectly competitive products are absent from the goods market ($\beta = 1$), both the $\hat{\pi}_1^F(\beta = 1)$ line and the $\hat{\pi}_1^R(\beta = 1)$ line intersect at $A = 2.08$, implying that the value of the threshold value of the technology level is equal to 2.08. This threshold

[23] The detailed numerical results are available upon request from the authors.

value is the same as that in Section 4.3 of the baseline model, and is also exhibited in Figure 1. Second, a significantly negative relationship between A_T and β is established. This result implies that the patent-holding firm is more inclined to choose the royalty contract, and hence it needs a higher technology level to select the fixed-fee contract. As depicted in Figure 3, when β falls from $\beta = 1$ to $\beta = 0.5$, both $\hat{\pi}_1^F(\beta = 1)$ and $\hat{\pi}_1^R(\beta = 1)$ shift downward to $\hat{\pi}_1^F(\beta = 0.5)$ and $\hat{\pi}_1^R(\beta = 0.5)$, respectively. Both $\hat{\pi}_1^F(\beta = 0.5)$ and $\hat{\pi}_1^R(\beta = 0.5)$ intersect at the technology level in association with $A = 3.30$. It is clear from Figure 3 that $\hat{\pi}_1^F(\beta = 0.5) >$ $\hat{\pi}_1^R(\beta = 0.5)$, if $A > 3.30$, while $\hat{\pi}_1^F(\beta = 0.5) < \hat{\pi}_1^R(\beta = 0.5)$, if $1.32 < A < 3.30$.[24] The intuition regarding why the patent-holding firm gives more preference to the royalty licensing contract in association with a smaller value of β can be explained as follows.

In the presence of a mixed industrial structure, as indicated in (35d), the patent-holding firm (firm 1) faces a lower demand in response to a smaller value of β. The extent of the reduction in demand under the royalty licensing regime is smaller than that under the fixed-fee licensing regime because firm 1 possesses the advantage of lower marginal costs under the royalty licensing contract compared to the fixed-fee licensing contract.[25] The lower demand for the patent holder leads it to have lower total profit, and thereby the beneficial effect of the fixed-fee licensing (compared to the royalty licensing) mentioned in Subsection 4.3 will be weakened. Consequently, the patent-holding firm is more inclined to choose the royalty contract with a smaller value of β.

6.3 Free Entry

The baseline model assumes that the number of firms is constant. However, in our baseline model, firms in the monopolistically competitive market earn positive profits. This provides a motivation for the firm to enter the market. Accordingly, it would be interesting to extend our model to consider the free entry case.

In this subsection, we extend the baseline model with free entry. Assume that each firm bears a fixed cost of entry ϕ (in terms of aggregate output), and firms which have non-advanced technology continue to enter the market until profits fall to

[24] It should be noted that, in association with $\beta = 0.5$, Condition P as reported in equation (27) requires that $1.32 < A$.

[25] Under fixed-fee licensing, each firm including firm 1 uses the same technology to produce its good. Accordingly, firm 1 does not have an advantage in the production with lower marginal costs compared with firm *i*. However, under royalty licensing, firm 1 instead possesses lower marginal costs than firm *i*. This is the reason why firm 1 has the advantage of lower marginal costs under royalty licensing compared to fixed-fee licensing.

zero. Nevertheless, the patent-holding firm still has positive profits. We modify the real total profit of firm 1 under the two regimes as follows:

$$\pi_1 = \begin{cases} p_1 y_1 - wn_1 + (m-1)F - \phi; & \text{under the fixed-fee licensing regime,} \\ p_1 y_1 - wn_1 + (m-1)sy_i - \phi; & \text{under the royalty licensing regime.} \end{cases} \quad (37a)$$

Moreover, the real total profit of firm i under the two regimes described in equations (13) and (21) is modified as follows:

$$\pi_i = \begin{cases} p_i y_i - wn_i - F - \phi; & \text{under the fixed-fee licensing regime,} \\ p_i y_i - wn_i - sy_i - \phi; & \text{under the royalty licensing regime.} \end{cases} \quad (37b)$$

The game structure is the same as that in Section 3. In a way that is similar to the procedure in Section 3, we can derive the optimal decision of firms under the two regimes. In addition, we utilize the zero-profit condition to infer the optimal number of firms as follows:

$$m = \begin{cases} \mu Y^F / (\phi + F); & \text{under the fixed-fee licensing regime,} \\ \mu Y^R / \phi - (1-\mu)^{(\mu-1)/\mu} + 1; & \text{under the royalty licensing regime.} \end{cases} \quad (37c)$$

It should be noted that aggregate profits only arise from the patent-holding firm's total profit due to free entry. Therefore, aggregate profits in this extended model are lower than those in the baseline model.

Due to the free entry of firms, the number of firms m is an endogenous variable and cannot be treated as a fixed parameter. As a result, in our numerical analysis we see that all parameters except for the number of firms m are the same as those in Subsection 4.3, i.e., $\mu = 0.29$, $T = 1$, and $\alpha = 0.42$.

Three interesting results emerge from the numerical analysis. First, as shown in Table 2, the number of firms is inversely related to the size of the fixed cost. The intuition for this result is quite obvious. A higher fixed cost would constitute a more severe impediment to potential entrants, thereby causing fewer firms to enter the market.

Second, the threshold value of the technology level is negatively related to the size of the fixed cost, implying that the patent holder gives greater preference to choosing the fixed-fee licensing contract with a higher fixed cost. The intuition behind this result is as follows. A higher fixed cost results in fewer firms in the goods market. Equipped with this, each firm, including the patent holder, faces a greater demand for its goods. Moreover, the beneficial effect of the fixed-fee licensing (compared to the royalty licensing) mentioned in Subsection 4.3 will be reinforced due to the greater demand for the patent holder. As a result, the patent holder is more inclined to choose the fixed-fee contract with a higher fixed cost.

Third, as shown in Table 2, the number of firms under the royalty licensing regime is greater than that under the fixed-fee licensing regime (i.e., $\hat{m}^R > \hat{m}^F$). The intuition underlying this result is straightforward. With no entry for the firm, as indicated in Subsection 3.2, the patent holding firm charges the fixed fee F such that firm i ($i = 2,...,m$) is indifferent between fixed-fee licensing and no licensing, i.e., $\pi_i^F = \pi_i^{NL}$. In addition, the incentive constraint that firm i will accept the royalty licensing contract is $\pi_i^R - \pi_i^{NL} \geq 0$. These two constraints impose the following result: $\pi_i^R \geq \pi_i^F$. The result $\pi_i^R \geq \pi_i^F$ implies that, except for the extreme case of $\pi_i^R = \pi_i^F$, in comparison with the fixed-fee licensing contract, the royalty licensing contract provides more incentives for potential entrants to join the market. As a result, we can infer the outcome $\hat{m}^R > \hat{m}^F$.

6.4 Alternative Markup Ratio

Yang and Heijdra (1993) propose that, if each monopolistically competitive firm considers its effect on the aggregate price level, the price elasticity of demand is different from that in Dixit and Stiglitz (1977). As is obvious, the change in the price elasticity of demand will further affect the markup ratio of monopolistically competitive firms. With this feature, it would be interesting to extend our model to consider the Yang and Heijdra (1993) insight. Based on the demand function for the *i*th consumption good and the price index of composite consumption reported in equations (4) and (5c) in the baseline model, we have the following expression:

$$c_i = \left[\frac{P}{p_i}\right]^{1/\mu} \frac{C}{m}; \quad P = \left[\frac{1}{m}\sum_{i=1}^{m} p_i^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}; \quad i = 1,2,...,m. \tag{38a}$$

In the baseline model, the *i*th firm determines its optimal price p_i by treating the index of composite consumption P as given. Under such a simplified assumption, equation (5c) indicates that the price elasticity of demand for the *i*th consumption good is $\varepsilon_d = 1/\mu$, and hence the markup ratio $\eta_i = p_i / MC_i$ is:

$$\eta = \eta_i = \frac{1}{1-\mu}; \quad i = 1,2,...,m, \tag{38b}$$

where η is the common markup ratio of all firms. It is quite obvious from equation (38b) that each firm has the same markup ratio. Moreover, the markup ratio depends upon the degree of product differentiation.

We now deal with the situation where the firm takes into account the effect of its price on the aggregate price level when it decides optimal pricing. Under such an extended model, the price elasticity of demand for the ith firm can be expressed as follows:

$$\varepsilon_{i,d}(m) = \frac{1}{\mu} - \frac{1}{\mu m}(P/p_i)^{\frac{1-\mu}{\mu}}; \quad i = 1,2,...,m. \tag{38c}$$

Equation (38c) indicates that the price elasticity of demand in this extended model is smaller than that in the baseline model, $\varepsilon_{i,d}(m) < \varepsilon_d$, since $(P/p_i)^{(1-\mu)/\mu}/(\mu m) > 0$. It is clear from equation (38c) that the price elasticity of demand is affected by the number of firms and the relative price between the ith consumption good and the composite goods. Then, the markup ratio for the ith firm is:

$$\eta_i(m) = \frac{\varepsilon_{i,d}(m)}{\varepsilon_{i,d}(m) - 1} = \frac{1 - (P/p_i)^{(1-\mu)/\mu}/m}{(1-\mu) - (P/p_i)^{(1-\mu)/\mu}/m}; \quad i = 1,2,...,m. \tag{38d}$$

By comparing equation (38b) with equation (38d), we can infer that the markup ratio under the situation where the impact of p_i on P is brought into the picture is greater than that under the situation where the impact of p_i on P is ignored, i.e., $\eta_i(m) > \eta$. Moreover, the markup ratio in this extended model is closely related to the degree of product differentiation, the number of firms, and the relative price between the ith consumption good and composite goods.

Since the extended model is too complicated to obtain a closed-form solution, just as in the case of the baseline model, we present our results via numerical analysis. With the same structural parameters in Subsection 4.3, we find that, once the impact of p_i on P is brought into the picture, the threshold value of the technology level declines from $A_T = 2.08$ in the baseline model to $A_T = 2.06$ in the extended model.[26] This implies that the patent holder is more inclined to choose the fixed-fee contract (compared to the royalty contract) when each firm considers the effect of its price on the aggregate price level. The economic reasoning for this result can be explained intuitively. When each firm contemplates the impact of p_i on P, its markup ratio increases in response. Then, each firm (including the patent holder) faces a rise in its total profit. As a consequence, the beneficial effect (mentioned in Subsection 4.3) for the fixed-fee licensing stemming from the greater total profits of the patent holder will be strengthened. The patent holder is thus more inclined to

[26] The detailed numerical results and a graphical apparatus similar to Figure 3 are available upon request from the authors.

choose the fixed-fee contract when the impact of p_i on P is taken into consideration by each of the monopolistic firms.

7. Conclusions

This paper sets up an imperfectly competitive macroeconomic model that is characterized by the strategic interaction between the patent-holding firm and the licensees. We then use the model to examine the relative performance of macroeconomic variables under the three different regimes. Several interesting results emerge from our analysis. First, the equilibrium aggregate output and aggregate consumption under the two licensing regimes are always greater than those under the no licensing regime. Moreover, the equilibrium aggregate output and consumption under the fixed-fee licensing regime are always greater than those under the royalty licensing regime. Second, the patent holder’s real profit from its own output under the fixed-fee regime may be greater or less than that under the no licensing regime depending upon the relative degree of product differentiation. In addition, the patent-holder’s real total profit and aggregate profits under the two licensing regimes are always greater than those under the no licensing regime regardless of the degree of product differentiation and the extent of the advanced technology. Third, with the higher (lower) technology level the patent-holder prefers the fixed-fee (royalty) contract. Fourth, the welfare level under the two licensing regimes is always higher than that under the no licensing regime. Moreover, with plausible numerical parameters, the welfare level under the fixed-fee licensing regime is higher than that under the royalty licensing regime.

This paper also discusses four extensions of the baseline model. With plausible numerical parameters, the patent-holding firm is more inclined to choose the royalty licensing contract compared to the fixed-fee licensing contract with the following extensions: (i) the patent holding firm undertakes endogenous innovation; and (ii) the mixed industrial structure of an economy (the goods market is composed of both monopolistically competitive goods and perfectly competitive goods) is present. However, the patent-holding firm is instead more likely to choose the fixed-fee licensing contract with the following two extensions: (i) the firms can freely enter the market if existing firms earn positive profits; and (ii) each firm in the monopolistically competitive market considers the impact of its price on the aggregate general price level.

Appendix A

This appendix provides a derivation procedure of the equilibrium macro variables.

The inferences under the three regimes are as the followings.

Under the no licensing regime, using equations (2), (4), (6a), (6b), and (8a)-(8d), we can infer aggregate output of the final good, aggregate labor hired by the firms, the composite consumption, the relative price between each good and the composite consumption good, and real aggregate profits as follows:

$$Y^{NL} = p_1^{NL} y_1^{NL} + (m-1) p_i^{NL} y_i^{NL} = m\{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{1/(1-\mu)} y_1^{NL}, \quad \text{(A1)}$$

$$N_d^{NL} = n_1^{NL} + (m-1) n_i^{NL}. \quad \text{(A2)}$$

$$C^{NL} = m\{(1/m)[(c_1^{NL})^{1-\mu} + (m-1)(c_i^{NL})^{1-\mu}]\}^{1/(1-\mu)}; \ \ i = 2,\ldots,m, \quad \text{(A3)}$$

$$p_1^{NL} = \{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{\mu/(1-\mu)}, \quad \text{(A4)}$$

$$p_i^{NL} = A\{(1/m)[1+(m-1)A^{(\mu-1)/\mu}]\}^{\mu/(1-\mu)}; \ \ i = 2,\ldots,m, \quad \text{(A5)}$$

$$\Pi^{NL} = \mu Y^{NL}. \quad \text{(A6)}$$

By substituting real aggregate profits ($\Pi^{NL} = Y^{NL} - w^{NL} N_d^{NL}$) into the individual household's budget constraint reported in (3), we obtain the equilibrium condition for the labor market $N_s^{NL} = N_d^{NL}$. It should be noted that, by virtue of Walras' Law, the equilibrium condition for the goods market can be abstracted from the analysis.[27]

Substituting equations (5a), (5b), and (A2) into the equilibrium condition for the labor market $N_s^{NL} = N_d^{NL}$ yields:

$$\frac{\alpha(1-\mu)T}{1-\alpha\mu} = [1+(m-1)A^{(\mu-1)/\mu}](1-\mu)^{1/\mu} A^{(1-\mu)/\mu} \frac{\alpha T (w^{NL})^{(\mu-1)/\mu}}{m(1-\alpha\mu)}. \quad \text{(A7)}$$

Then, from the equilibrium condition for the labor market reported in equation (A7), we drive the equilibrium wage rate. By inserting the equilibrium wage rate into (5a) and (5b), we can infer the equilibrium aggregate output $\tilde{Y}^{NL}$, composite consumption

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[27] Substituting the firm's profit function $\Pi^{NL} = Y^{NL} - w^{NL} N_d^{NL}$ into the individual household's budget constraint (3) yields:

$$C^{NL} - Y^{NL} + w^{NL}(N_d^{NL} - N_s^{NL}) = 0.$$

Let $ED^G = C^{NL} - Y^{NL}$ and $ED^N = w^{NL}(N_d^{NL} - N_s^{NL})$ denote the excess demand in the goods market and the labor market. The above equation can be rewritten as $ED^G + ED^N = 0$, indicating that one of the two markets is redundant. Accordingly, in what follows we do not deal with the goods market equilibrium condition. See Lai et al. (2010) for a detailed discussion regarding the Walras Law in the monopolistic competition macroeconomic model.

$\tilde{C}^{NL}$, and leisure $\tilde{l}^{NL}$. Moreover, substituting the equilibrium wage rate into equations (7) and (A6), we obtain firm 1's real profit $\tilde{\pi}_1^{NL}$, firm i's real profit $\tilde{\pi}_i^{NL}$, and real aggregate profits $\tilde{\Pi}^{NL}$.

Under the fixed-fee licensing regime, from equations (12a), (12b), (14a), and (14b) we can infer that a symmetric equilibrium under which $p_i = p$, $y_i = y$, $n_i = n$, and $c_i = c$ ($i = 1,...m$) is present in this regime. Based on this feature, we obtain aggregate output of the final good, aggregate labor hired by all firms, the composite consumption, the relative price of goods, and real aggregate profits as follows:

$$Y^F = my^F, \tag{A8}$$

$$N_d^F = mn^F, \tag{A9}$$

$$C^F = mc^F, \tag{A10}$$

$$p^F = \frac{w^F}{(1-\mu)A} = 1, \tag{A11}$$

$$\Pi^F = m\mu y^F = \mu Y^F. \tag{A12}$$

Based on the results reported in equations (A8)-(A12), we can infer the equilibrium condition for the labor market. Substituting equations (5a), (5b), (6a), (6b), (12a), (14a), and (A9) into the equilibrium condition for the labor market $N_s^F = N_d^F$ yields:

$$\frac{\alpha(1-\mu)T}{1-\alpha\mu} = (1-\mu)^{1/\mu} A^{(1-\mu)/\mu} \frac{\alpha T (w^F)^{(\mu-1)/\mu}}{1-\alpha\mu}. \tag{A13}$$

From (A13) we can solve the equilibrium wage rate under the fixed-fee licensing regime. Moreover, substituting the equilibrium wage rate into equations (5a)-(5b), (11), (13), and (A12), the equilibrium aggregate output $\tilde{Y}^F$, composite consumption $\tilde{C}^F$, leisure $\tilde{l}^F$, firm 1's real total profit $\tilde{\pi}_1^F$, firm i's real total profit $\tilde{\pi}_i^F$, aggregate real profits $\tilde{\Pi}^F$, and the fixed fee $\tilde{F}$.

Under the royalty licensing regime, in a way that is similar to the inference under the no licensing regime, we can express the following macroeconomic variables under the royalty licensing regime:

$$Y^R = m\{(1/m)[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]\}^{1/(1-\mu)} y_1^R, \tag{A14}$$

$$N_d^R = n_1^R + (m-1)n_i^R; \quad i = 2,...,m, \tag{A15}$$

$$C^R = m\{(1/m)[(c_1^R)^{1-\mu} + (m-1)(c_i^R)^{1-\mu}]\}^{1/(1-\mu)}\,; \;\; i = 2,...,m\,, \tag{A16}$$

$$p_1^R = \{(1/m)[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]\}^{\mu/(1-\mu)}\,, \tag{A17}$$

$$p_i^R = (1-\mu)^{-1}\{(1/m)[1+(m-1)(1-\mu)^{(1-\mu)/\mu}]\}^{\mu/(1-\mu)}\,; \;\; i = 2,...,m\,, \tag{A18}$$

$$\Pi^R = \mu\Phi Y^R\,. \tag{A19}$$

where $\Phi = \dfrac{1+(m-1)(2-\mu)(1-\mu)^{(1-\mu)/\mu}}{1+(m-1)(1-\mu)^{(1-\mu)/\mu}} > 1$.

By means of a similar procedure under the no licensing regime, the equilibrium condition for the labor market $N_s^R = N_d^R$ can be expressed as:

$$\frac{\alpha(1-\mu\Phi)T}{1-\alpha\mu\Phi} = [1+(m-1)(1-\mu)^{1/\mu}](1-\mu)^{1/\mu}A^{(1-\mu)/\mu}\frac{\alpha T(w^R)^{(\mu-1)/\mu}}{m(1-\alpha\mu\Phi)}\,. \tag{A20}$$

Accordingly, from equation (A20), we can infer the equilibrium wage rate $\tilde{w}^R$. Then, we can infer the equilibrium aggregate output $\tilde{Y}^R$, composite consumption $\tilde{C}^R$, leisure $\tilde{l}^R$, firm 1's real total profit $\tilde{\pi}_1^R$, firm i's real total profit $\tilde{\pi}_i^R$, aggregate real profits $\tilde{\Pi}^R$, and royalty rate $\tilde{s}$.

Figure 1. Firm 1's total profit under the two regimes

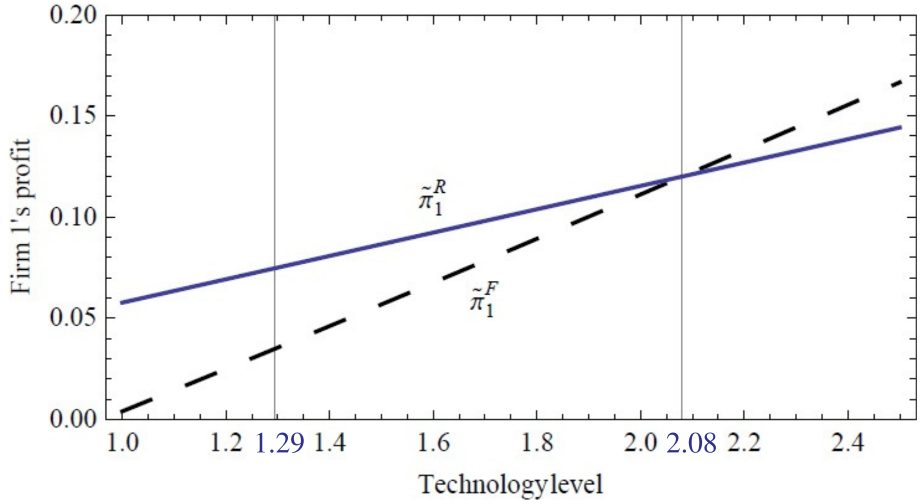

Figure 2. The welfare level under the two regimes

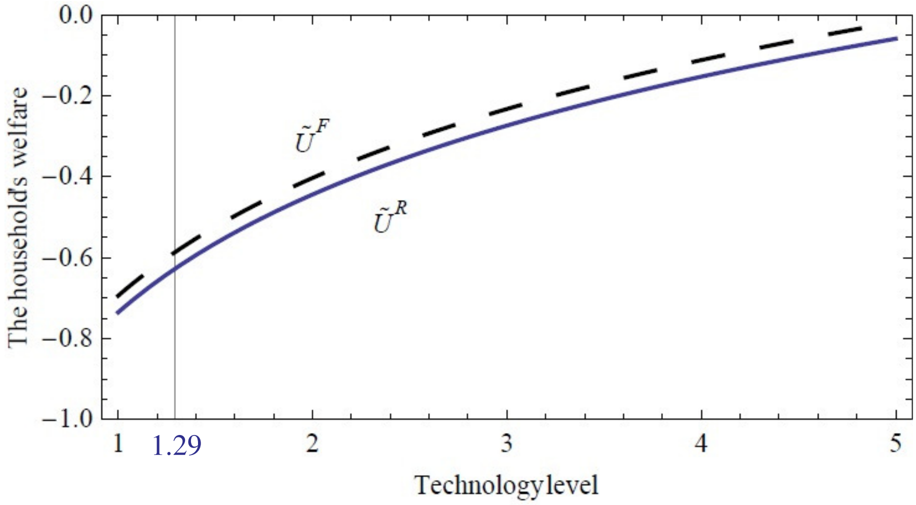

Figure 3. Firm 1's total profit under the two regimes: A change in β

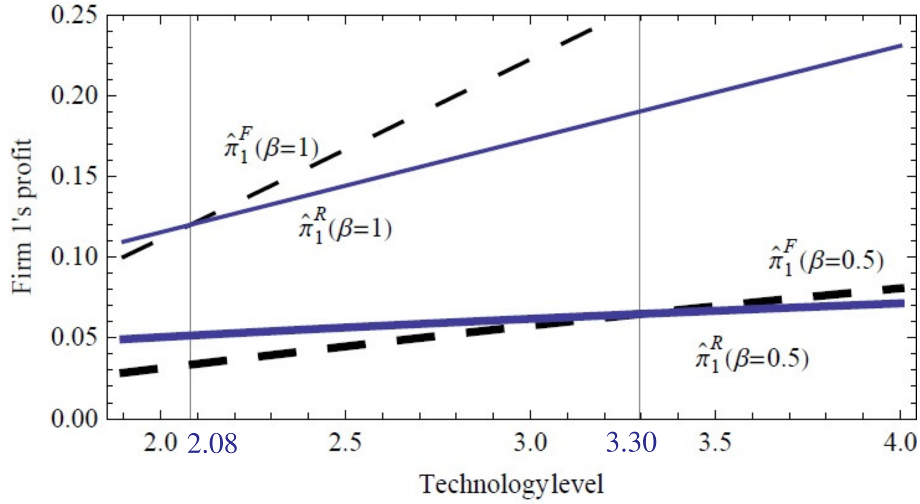

Table 1. The relationship between A_T and β

β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
A_T	5.01	4.52	4.07	3.66	3.30	2.97	2.69	2.45	2.25	2.08

Table 2. The relationship among ϕ, A_T, and $\hat{m}$

ϕ	0.001	0.003	0.005	0.007	0.009	0.011	0.013	0.015
A_T	3.10	2.72	2.49	2.33	2.21	2.12	2.04	1.97
$\hat{m}^F$	110	33	19	13	10	8	7	6
$\hat{m}^R$	304	88	48	32	23	18	15	12

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