Joint Tests of Financial Market Contagion with Applications*

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Abstract

A new class of multiple-channel tests of financial market contagion is introduced in which the transmission channels of financial market crises are identified jointly through the correlation, co-skewness and co-kurtosis of the distribution of returns. The proposed tests have the advantage over the existing single-channel tests of contagion in the literature in that they yield the correct size in small samples which is typical of crisis periods. Regarding the power of the tests, the multiple-channel tests display the second highest power following the single-channel tests if the data generating process for an experiment contains the transmission channel of contagion consistent with the single-channel test. The proposed tests are applied to test for financial market contagion in equity markets during the three financial crises of 2007-12. The results show that the joint tests identify various combinations of transmission channels during the three crises.

Key words: Co-skewness, Co-kurtosis, Co-volatility, Contagion testing, European debt crisis, Financial crisis, Lagrangian multiplier tests.

JEL Classification: C12, F30

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1 Introduction

Recent financial crises, particularly the sovereign debt crisis in Europe, have aroused researchers' interest in studying international equity market co-movement, around periods of crisis that is characterized by financial contagion. An extensive literature on market contagion has investigated the nature of market co-movement and explored the impact of financial shocks from one region on the international financial markets in other regions. The earliest articles by King and Wadhwani (1990) focus on a comparison of the linear cross-market co-moment (i.e. correlation) measured in stable and volatile market periods. Forbes and Rigobon (2002) correct the constant correlation test by introducing a heteroskedasticity correlation in which the change in cross-market co-movement during crisis periods is conditional on an increase in the returns volatility of the source country.

One of the critical issues faced by the literature on contagion is that correlation analysis delivers partial information on measuring cross-market linkages as asset returns are characterized statistically by skewness and fat tails. A range of studies have taken into account this issue by investigating alternative statistics such as high order co-movements instead of using the standard methodology (the correlation test) for measuring financial contagion. Favero and Giavazzi (2002) explore the outlier tests of contagion, Bae et al. (2003) develop the co-exceedance approach based on extreme value theory, Pesaran and Pick (2007) propose the threshold tests of contagion, Rodriguez (2007) studies tail dependence using copulas, Fry et al. (2010) introduce the co-skewness test of contagion, and Fry-McKibbin and Hsiao (2014) develops the co-kurtosis and co-volatility tests of contagion.

The previous literature, however, examines contagion test either through the correlation channel (Forbes and Rigobon, 2002) or through the higher order co-moments channel such as the asymmetric dependence test (Fry et al., 2010) or extremal dependence test (Fry-McKibbin and Hsiao, 2014). Chan et al. (2013) is one example, but they only focus on joint tests of contagion based on changes in the second and third order co-moments. Ignoring either the correlation test or the higher order test of contagion could result in omitting one of possible transmission channels such as cross-market mean and mean spillover, cross-market mean and volatility spillover, and cross-market mean and skewness spillover.

This paper introduces a new class of multiple-channel contagion tests in which the transmission channels of financial market crises can be identified jointly through both the second order co-moments and the higher order co-moments of the distribution of returns. This framework is set up to measure five different combinations of channels through the correlation, co-skewness, and co-kurtosis channels of financial market contagion.

The proposed approaches are based on the work of Fry et al. (2010), by adopting the bivariate generalized normal distribution through the second to the fourth order co-moments to derive five forms of test statistics for the joint co-moments such as correlation, co-skewness and co-kurtosis using the Lagrange multiplier test. More importantly, this framework captures a range of possible transmission channels of contagion, not only linear cross-market channels between the mean of market i and the mean of market j, but also non-linear cross-market channels between the mean of market i and the return volatility of market j as well as between the mean of market i and return skewness of market j. This is the first contribution of this paper.

The second contribution of this paper is that Monte Carlo simulation studies are conducted to investigate the finite sampling properties of multiple-channel tests for financial contagion. In practice, the multiple-channel tests provide a much more reasonable size than the signal-channel tests existing in the literature such as the co-skewness contagion tests (Fry et al., 2010) and the co-kurtosis and co-volatility contagion tests (Fry-McKibbin and Hsiao, 2014), with the size of multiple-channel tests being close to the finite sample distribution with the nominal size of 5%.

In terms of the power test for each multiple-channel test, the simulation studies demonstrate that the proposed joint tests of contagion display the second highest power following the single-channel tests investigated if the data generating process (DGP) contains the transmission channel of contagion consistent with the single channel test. For instance, if the DGP for an experiment contains only the correlation channel, then there is no doubt that the correlation change test of contagion is the most powerful test, followed by the joint test of contagion through the correlation and co-skewness channels.

The final contribution is the empirical application of the five types of multiple-channel contagion tests to the three financial crises of 2007-12 of the subprime mortgage crisis, the great recession and the European debt crisis. The empirical results show that the great recession is the worst amongst the three crises since the five types of multiple-channel of contagion between the US equity market and the European equity markets are operating for all cases. This result reflects that the great recession is a true global financial crisis (Fry et al., 2012). The European debt crisis is the second worst crisis as the results of the joint correlation, co-skewness and co-kurtosis channels of contagion tests provide significant evidence of contagion from the Greek equity market to the European equity market for almost all cases. Among the five types of multiple-channel contagion tests, the results of the joint correlation, co-skewness and co-kurtosis tests provide more evidence of contagion than those of the joint correlation and co-skewness tests among the three financial crises, indicating the importance of changes in multiple non-linear cross-market dependence channels during crises.

The rest of the chapter proceeds as follows. Section 2 provides the test statistics of joint co-moments, which gives a framework for developing five types of multiple-channel tests of contagion in Section 3. Sampling properties of these tests are compared with the single-channel contagion tests existing in the contagion literature using a range of Monte Carlo experiments in Section 4. Section 5 provides an application of the various multiple-channel contagion tests to the three financial crises: the subprime mortgage crisis of 2007-08, the great recession crisis of 2008-09, and the European debt crisis of 2010-12. Conclusions are presented in Section 6 with details of the derivations given in Appendix B.

2 Statistics of Joint Co-moments

2.1 A Bivariate Generalized Exponential Family

To develop the eight types of multiple-channel tests of contagion, test statistics of joint co-moments need to be developed first. A non-normal multivariate returns distribution is specified in order to model the distribution with asymmetry and leptokurtosis. Following the work of Fry et al. (2010), the bivariate generalized exponential family of the distribution through the first to the fourth order moments and co-moments is used for developing the statistics of co-moments and it is represented as[1]

$$\begin{aligned} f(r) &= \exp(h-\eta) \qquad (1)\\ &= \exp\left(\sum_{i=1}^{12}\theta_i g_i(r)-\eta\right)\\ &= \exp\big(\theta_1 r_1^2+\theta_2 r_2^2+\theta_3 r_1 r_2+\theta_4 r_1 r_2^2+\theta_5 r_1^2 r_2+\theta_6 r_1 r_2^3\\ &\quad +\theta_7 r_1^3 r_2+\theta_8 r_1^2 r_2^2+\theta_9 r_1^3+\theta_{10} r_2^3+\theta_{11} r_1^4+\theta_{12} r_2^4-\eta\big), \end{aligned}$$

where h is a function of parameters θ_i and $g_i(r)$. The choices for $g_i(r)$ are polynomials and cross-products in the elements of r which are assumed to follow an independent bivariate

[1] The univariate class of generalized exponential family of distributions is developed by Cobb et al. (1983) and Lye and Martin (1993).

normal distribution, and η is a normalising constant denoted as

$$\eta = \ln \int\int \exp\left(\theta_1 r_1^2 + \theta_2 r_2^2 + \theta_3 r_1 r_2 + \theta_4 r_1 r_2^2 + \theta_5 r_1^2 r_2 + \theta_6 r_1 r_2^3 \right. \\ \left. + \theta_7 r_1^3 r_2 + \theta_8 r_1^2 r_2^2 + \theta_9 r_1^3 + \theta_{10} r_2^3 + \theta_{11} r_1^4 + \theta_{12} r_2^4\right) dr_1 dr_2. \quad (2)$$

The parameters θ_1 and θ_2 control the variances of assets 1 (r_1^2) and 2 (r_2^2) respectively. The parameter θ_3 controls the degree of linear dependence in the relationship between assets 1 and 2 (r_1 and r_2), referring to a correlation coefficient. The parameters θ_4 and θ_5 measure the asymmetry of the probability distribution of the two assets, capturing dependent links between the first moment of asset 1 and the second moment of asset 2 (r_1 and r_2^2), and between the second moment of asset 1 and the first moment of asset 2 (r_1^2 and r_2). These parameters represent the co-skewness coefficients. The parameters θ_6, θ_7 and θ_8 measure the tailedness and peakedness of the probability distribution of the two assets (i.e. co-kurtosis and co-volatility), capturing the dependent links between the first moment of asset 1 and the third moment of asset 2 (r_1 and r_2^3), between the third moment of asset 1 and the first moment of asset 2 (r_1^3 and r_2), and between the second moment of two assets 1 and 2 (r_1^2 and r_2^2). The parameters θ_9 and θ_{10} as well as θ_{11} and θ_{12} control the skewness (r_1^3 and r_2^3) and kurtosis (r_1^4 and r_2^4) for assets 1 and 2 respectively.

2.2 Test Statistics of Joint Co-moments

In this paper, the Lagrange multiplier test is used to develop the test statistics of joint co-moments as the bivariate generalized exponential family of the distribution in equation (1) is nested in the bivariate normal distribution by setting the restrictions $\theta_4 = ... = \theta_{12} = 0$.

Consider a sample of size T from the bivariate generalized exponential family of the distribution with a finite number K of unknown parameters $\theta = (\theta_1, ..., \theta_K)'$ summarizing the moments of a log likelihood function $\ln L_t(\theta) = h - \eta$ in equation (1) where $h = \sum_{i=1}^{K} \theta_i g_i(r)$ and η is the normalising constant respectively. The hypothesis to be tested is specified as

$$H_0 : \theta_1, ..., \theta_p = 0; p \leq K. \quad (3)$$

Let $\widehat{\theta}$ be the maximum likelihood estimator of θ. The Lagrange multiplier test statistic is given by

$$LM = q\left(\widehat{\theta}\right)' I\left(\widehat{\theta}\right)^{-1} q\left(\widehat{\theta}\right), \tag{4}$$

which is asymptotically distributed as χ_p^2. Here, $q\left(\widehat{\theta}\right)$ is the score function evaluated at $\widehat{\theta}$ given by

$$q\left(\widehat{\theta}\right) = \left(\frac{\partial \ln L_t(\theta)}{\partial \theta}\right)_{\theta=\widehat{\theta}}, \tag{5}$$

and $I\left(\widehat{\theta}\right)$ is the asymptotic information matrix evaluated at $\widehat{\theta}$, that is

$$I\left(\widehat{\theta}\right) = T\left(E\left[\frac{\partial h}{\partial \theta}\frac{\partial h}{\partial \theta'}\right]_{\theta=\widehat{\theta}} - E\left[\frac{\partial h}{\partial \theta}\right]_{\theta=\widehat{\theta}} E\left[\frac{\partial h}{\partial \theta'}\right]_{\theta=\widehat{\theta}}\right). \tag{6}$$

2.2.1 Test Statistic of Correlation and Co-skewness

Consider the restricted model, the bivariate normal distribution with the third order co-moments, that is

$$\begin{aligned} f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_5\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right) - \eta_2\right], \end{aligned} \tag{7}$$

where $\eta_2 = \ln \int\int \exp\left[h_2\right] dr_1 dr_2$, and

$$\begin{aligned} h_2 &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\ &\quad +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_5\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\Bigg]. \end{aligned}$$

A test of the restriction for the hypothesis of independent bivariate normality is set up as

$$H_0 : \rho = \theta_4 = \theta_5 = 0,$$

which constitutes a joint test of correlation and co-skewness. Under the null hypothesis, the distribution is an independent bivariate normal where the maximum likelihood estimators of the unknown parameters are

$$\widehat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}; \widehat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T} \left(r_{i,t} - \widehat{\mu}_i\right)^2; \widehat{\rho} = \frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{1,t} - \widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t} - \widehat{\mu}_2}{\widehat{\sigma}_2}\right); \forall i = 1, 2, \quad (8)$$

The Lagrange multiplier statistics for the joint test of correlation and co-skewness is (see Appendix B.2 for details)

$$\begin{aligned} LM_2 &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1}{T}}}\right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2}{T}}}\right)^2 \quad (9) \\ &+ \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^1}{\sqrt{\frac{2}{T}}}\right)^2. \end{aligned}$$

Under the null hypothesis, LM_2 is distributed asymptotically as χ_3^2.

2.2.2 Test Statistic of Correlation, Co-skewness and Co-kurtosis

Consider the restricted model, the bivariate normal distribution with the third and fourth order co-moments, that is

$$\begin{aligned} f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\left. +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 - \eta_3\right], \quad (10) \end{aligned}$$

where $\eta_3 = \ln\int\int \exp\left[h_3\right] dr_1 dr_2$, and

$$\begin{aligned} h_3 &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\ &+\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3\Bigg]. \end{aligned}$$

A test of the restriction for the hypothesis of independent bivariate normality is set up as

$$H_0 : \rho = \theta_4 = \theta_6 = 0.$$

The Lagrange multiplier statistic for the joint test of correlation, co-skewness and co-kurtosis is (see Appendix B.3 for details)

$$\begin{aligned} LM_3 &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{2}{5T}}} \right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2}{T}}} \right)^2 \\ &+ \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3}{\sqrt{\frac{6}{T}}} \right)^2 \\ &- \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1}{T}}} \right)\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3}{\sqrt{\frac{1}{T}}} \right). \end{aligned} \tag{11}$$

Under the null hypothesis, LM_3 is distributed asymptotically as χ^2_3.

3 Multiple-channel Tests of Contagion

This section presents the five multiple-channel tests of contagion. The first three types of joint tests of contagion are examined based on significant changes in the possible combinations of correlation and co-skewness during the crisis period compared to the non-crisis period. The remaining tests are to test for contagion through the correlation, co-skewness and co-kurtosis channels.

In deriving multiple-channel tests of contagion, the following notation is used. The non-crisis period is denoted as x, and the crisis period as y. The correlation between the two asset returns is denoted as ρ_x (non-crisis) and ρ_y (crisis). The sample sizes of the non-crisis and crisis periods are respectively T_x and T_y. Let i denote the source crisis asset market and j denote the recipient market of contagion. $\widehat{\mu}_{xi}$, $\widehat{\mu}_{xj}$, $\widehat{\mu}_{yi}$ and $\widehat{\mu}_{yj}$ are the sample means of the asset returns for markets i and j during the non-crisis and crisis periods, and $\widehat{\sigma}_{xi}$, $\widehat{\sigma}_{xj}$, $\widehat{\sigma}_{yi}$ and $\widehat{\sigma}_{yj}$ are the sample standard deviations of the asset returns for markets i and j during the non-crisis and crisis period, respectively.

3.1 Linear and Asymmetric Dependence Joint Tests

The linear and asymmetric dependence joint tests of contagion are based on identifying significant changes in the second and third order co-moments of returns (correlation and co-skewness) for asset markets 1 and 2 during the crisis period compared to the non-crisis period. Three types of linear and asymmetric dependence joint tests are considered depending on whether correlation is joint with either one form of co-skewness ($JLAD_1$ and $JLAD_2$) or both forms of co-skewness ($JLAD_3$).

The first test ($JLAD_1$) for contagion identifies the transmission channels of financial shocks from the mean returns of source market i to both mean returns and return volatility of the recipient market j. The second type ($JLAD_2$) is to test for financial contagion which identifies transmission channels from both mean returns and return volatility in the source market i to the mean returns in the recipient market j. The last type ($JLAD_3$) is to test for three transmission channels of contagion through i) the mean returns between two asset markets i and j; ii) the mean returns in the source market i to the return volatility of the recipient market j; and iii) the return volatility in the source market i to the mean returns of the recipient markets j. Three types of statistics to test for contagion from the source market i to the recipient market j are

$$JLAD_1 = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}} \right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^2\right) - \widehat{\psi}_x\left(r_i^1, r_j^2\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}} \right)^2, \quad (12)$$

$$JLAD_2 = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}} \right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^1\right) - \widehat{\psi}_x\left(r_i^2, r_j^1\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}} \right)^2, \quad (13)$$

and

$$JLAD_3 = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}} \right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^2\right) - \widehat{\psi}_x\left(r_i^1, r_j^2\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}} \right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^1\right) - \widehat{\psi}_x\left(r_i^2, r_j^1\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}} \right)^2, \quad (14)$$

where

$$\widehat{\psi}_y(r_i^m, r_j^n) = \frac{1}{T_y}\sum_{t=1}^{T_y}\left(\frac{y_{i,t}-\widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}}\right)^m\left(\frac{y_{j,t}-\widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}}\right)^n, \tag{15}$$

$$\widehat{\psi}_x(r_i^m, r_j^n) = \frac{1}{T_x}\sum_{t=1}^{T_x}\left(\frac{x_{i,t}-\widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}}\right)^m\left(\frac{x_{j,t}-\widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}}\right)^n. \tag{16}$$

$\widehat{\psi}_y(r_i^m, r_j^n)$ and $\widehat{\psi}_x(r_i^m, r_j^n)$ are the estimated correlation ($m = n = 1$) and co-skewness ($m = 1, n = 2$) coefficients during the crisis and non-crisis periods. Under the null hypothesis of no contagion, the test statistics are asymptotically distributed as

$$JLAD_1, JLAD_2 \xrightarrow{d} \chi_2^2, \tag{17}$$
$$JLAD_3 \xrightarrow{d} \chi_3^2.$$

3.2 Linear, Asymmetric and Extremal Dependence Joint Tests

The linear, asymmetric and extremal dependence joint tests of contagion are determined by identifying significant changes jointly in the second to fourth order co-moments of returns (correlation, co-skewness and co-kurtosis) for asset markets i and j during the crisis period compared to the non-crisis period.

The linear, asymmetric and extremal dependence joint tests have two forms. The first is to test for contagion where the financial shocks transmit from the mean returns of a source market i to the mean returns, return volatility and skewness of a recipient market j. The test statistic ($JLAED_1$) to test contagion from the source market i to the recipient market j is

$$JLAED_1 = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{2}{5T_y} + \frac{2}{5T_x}}}\right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^2\right) - \widehat{\psi}_x\left(r_i^1, r_j^2\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}}\right)^2 \tag{18}$$
$$+ \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^3\right) - \widehat{\psi}_x\left(r_i^1, r_j^3\right)}{\sqrt{\frac{6}{T_x} + \frac{6}{T_y}}}\right)^2 - \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{1}{T_y} + \frac{1}{T_x}}}\right)\left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^3\right) - \widehat{\psi}_x\left(r_i^1, r_j^3\right)}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}}\right).$$

The second form of the linear, asymmetric and extremal dependence joint test is to test for contagion where the financial shocks transmit from the mean returns, return volatility and skewness of a source market i to the mean returns of a recipient market j. This type of test statistic is given by

$$JLAED_2 = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{2}{5T_y} + \frac{2}{5T_x}}} \right)^2 + \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^1\right) - \widehat{\psi}_x\left(r_i^2, r_j^1\right)}{\sqrt{\frac{2}{T_x} + \frac{2}{T_y}}} \right)^2 \tag{19}$$

$$+ \left(\frac{\widehat{\psi}_y\left(r_i^3, r_j^1\right) - \widehat{\psi}_x\left(r_i^3, r_j^1\right)}{\sqrt{\frac{6}{T_x} + \frac{6}{T_y}}} \right)^2 - \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^1\right) - \widehat{\psi}_x\left(r_i^1, r_j^1\right)}{\sqrt{\frac{1}{T_y} + \frac{1}{T_x}}} \right) \left(\frac{\widehat{\psi}_y\left(r_i^3, r_j^1\right) - \widehat{\psi}_x\left(r_i^3, r_j^1\right)}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}} \right),$$

where $\widehat{\psi}_y(r_i^m, r_j^n)$ and $\widehat{\psi}_x(r_i^m, r_j^n)$ are the estimated correlation ($m = n = 1$), co-skewness ($m = 1, n = 2$) and co-kurtosis ($m = 1, n = 3$) coefficients during the crisis and non-crisis periods in equations (15) and (16).

Under the null hypothesis of no contagion, the test statistics are asymptotically distributed as

$$JLAED_1, JLAED_2 \xrightarrow{d} \chi_3^2. \tag{20}$$

4 Sample Properties

This section presents the sample properties of the four types of single-channel contagion tests and three types of multiple-channel tests using a range of Monte Carlo experiments. The experiments are conducted to study the size and power properties of the tests. The four types of single-channel tests of contagion considered are i) the FR test in equation (57) in Appendix C; ii) the CS_{12} test in equation (59); iii) the CK_{13} test in equation (61); and iv) the CV_{22} tests in equation (65).[2] The three types of multiple-channel tests of contagion considered are i) the $JLAD_1$ test in equation (12); ii) the $JLAD_3$ test in equation (14); and iii) the $JLAED_1$ test in equation (18). The descriptions of single- and multiple-channel tests are summarized in Table 1 and the test statistic of the adjusted correlation change test is shown in Appendix C.

[2] The four types of single-channel tests of contagion are summarized in appendix C.

Table 1:
Summary of single- and multiple-channel tests of contagion.

Test	Equation	Description	Transmission channel from
Single-channel tests of contagion			
FR	57	Correlation change test	mean returns to mean returns
CS_{12}	59	Co-skewness change test	mean returns to return volatility
CK_{13}	61	Co-kurtosis change test	mean returns to return skewness
CV_{22}	65	Co-volatility change test	return volatility to return volatility
Multiple-channel tests of contagion			
$JLAD_1$	12	A joint test of FR and CS_{12}	i) mean returns to mean returns ii) mean returns to return volatility
$JLAD_3$	14	A joint test of FR, CS_{12} and CS_{21}	i) mean returns to mean returns ii) mean returns to return volatility iii) return volatility to mean returns
$JLAED_1$	18	A joint test of FR, CS_{12} and CK_{13}	i) mean returns to mean returns ii) mean returns to return volatility iii) mean returns to return skewness

4.1 Data Generating Process

To investigate transmission mechanisms of market linkings between asset returns of two markets, the DGP follows the generalized bivariate normal distribution with higher order moments and co-moments used to capture the linkages between two asset markets 1 and 2 with linear dependence ($\theta_{x,3}$), asymmetric dependence ($\theta_{x,4}$ and $\theta_{x,5}$) and extremal dependence ($\theta_{x,6}$, $\theta_{x,7}$ and $\theta_{x,8}$) in a non-crisis period (x)

$$\begin{aligned} f(x_1,x_2) &= \exp\left[\theta_{x,1}x_1^2+\theta_{x,2}x_2^2+\theta_{x,3}x_1x_2+\theta_{x,4}x_1x_2^2+\theta_{x,5}x_1^2x_2+\theta_{x,6}x_1x_2^3\right. \\ &\quad \left.+\theta_{x,7}x_1^3x_2+\theta_{x,8}x_1^2x_2^2+\theta_{x,9}x_1^3+\theta_{x,10}x_2^3+\theta_{x,11}x_1^4+\theta_{x,12}x_2^4-\eta_x\right], \end{aligned} \tag{21}$$

with

$$\begin{aligned} \theta_{x,1} &= \theta_{x,2} = -\frac{0.5}{(1-\rho_x^2)}, \\ \theta_{x,3} &= \frac{\rho_x}{(1-\rho_x^2)}, \end{aligned}$$

and in the crisis period (y)

$$\begin{aligned} f(y_1,y_2) &= \exp\left[\theta_{y,1}y_1^2+\theta_{y,2}y_2^2+\theta_{y,3}y_1y_2+\theta_{y,4}y_1y_2^2+\theta_{y,5}y_1^2y_2+\theta_{y,6}y_1y_2^3\right. \\ &\quad \left.+\theta_{y,7}y_1^3y_2+\theta_{y,8}y_1^2y_2^2+\theta_{y,9}y_1^3+\theta_{y,10}y_2^3+\theta_{y,11}y_1^4+\theta_{y,12}y_2^4-\eta_y\right], \end{aligned} \tag{22}$$

with

$$\begin{aligned} \theta_{y,1} &= \theta_{y,2} = -\frac{0.5}{(1-\rho_y^2)}, \\ \theta_{y,3} &= \frac{\rho_y}{(1-\rho_y^2)}, \end{aligned}$$

η_x and η_y are the normalising constants such that $\int\int f(x_1,x_2)dx_1dx_2 = 1$ and $\int\int f(y_1,y_2)dy_1dy_2 = 1$. The non-crisis data for asset markets 1 and 2 $\{x_{1,t}, x_{2,t}\}$ and crisis data for asset markets 1 and 2 $\{y_{1,t}, y_{2,t}\}$ can be generated from the above generalized bivariate normal distribution with higher order moments and co-moments in equations (21) and (22) given its cumulative distribution function based on the inverse-transform method.

To perform the size properties of the contagion tests, the Monte Carlo experiment is conducted under the null hypothesis of no contagion by setting

$$\rho_x = \rho_y = 0, \;\; \theta_{x,i} = \theta_{y,i} = 0, \;\; \forall_i = 4, \ldots, 8, \tag{23}$$

in equations (21) and (22). Given that non-crisis sample periods tend to be relatively large, while crisis sample periods tend to be relatively short, six Monte Carlo experiments are conducted for the size tests with the non-crisis period being set at 500 days ($T_x = 500$) and the crisis period varying between 50 and 500 days ($T_y = 50, 100, 200, 300, 400, 500$).

To compute the power properties of contagion tests, three Monte Carlo experiments are conducted under the alternative hypothesis of contagion

$$\begin{aligned} &\text{Experiment I} &&: \rho_y > 0, \\ &\text{Experiment II} &&: \theta_{y,8} < 0, \\ &\text{Experiment III} &&: -0.9 < \theta_{y,4} < 0.9, \; \theta_{y,8} = -0.5, \end{aligned} \tag{24}$$

in equations (21) and (22). Each experiment provides additional transmission channels of contagion between asset markets 1 and 2. For example, the purely correlation channel of contagion of experiment I is conducted by increasing values of the parameter ρ_y. The co-volatility channel of contagion can be found by decreasing values of the parameters $\theta_{y,8}$ in experiment II. The co-skewness channel of contagion can be identified by either decreasing or increasing values of the parameter $\theta_{y,4}$ in experiment III but the co-volatility parameter is set up to be negative ($\theta_{y,8} = -0.5$) to keep the distribution bounded. The range of $\theta_{y,4}$ is between -0.9 to 0.9. The strength of contagion is controlled by the contagion parameters ρ_y, $\theta_{y,4}$ and $\theta_{y,8}$. By performing Monte Carlo experiments I to III for changing the value of each parameter, the power function for each test is computed. Table 2 summarizes the restrictions on the parameters in equations (21) and (22) for the size and power tests.

4.2 Size

Table 3 gives the results of the size properties from the Monte Carlo experiments for the four single-channel tests and five multiple-channel tests of contagion, with attention to increasing the duration of the crisis period T_y from 50 to 500 days while keep the duration of the non-crisis period set of $T_x = 500$ days. The number of replications is set at 10,000 for all simulations. To calculate the size values for each test statistic, the simulated models following equations (21) and (22) are built based on the restriction of no contagion in equation (23) and the assumption of no skewness and kurtosis for asset markets 1 and 2. The sizes for each test are based on the 5% asymptotic χ^2_1 critical values for the four types of single-channel tests of contagion (i.e. the FR, CS_{12}, CK_{13} and CV_{22} tests). The critical values of the three types of multiple-channel tests of contagion are also based on 5% asymptotic χ^2_2 critical values for the $JLAD_1$ test and χ^2_3 for the $JLAD_3$ and $JLAED_1$ tests.

The results in Table 3 show that the multiple-channel tests present superior size properties relative to the single-channel tests, with the simulated size being close to the nominal size of 5% for all sample sizes. In particular, in the case when the non-crisis sample period is relatively large ($T_x = 500$), while the crisis sample period is relatively short ($T_y = 50$), the three types of multiple-channel test statistics ($JLAD_1$, $JLAD_3$, $JLAED_1$) present a good approximation to the finite sample distribution with the nominal size of 5%. In contrast, the four types of single-channel tests do not exhibit reasonable size as the sample size of the crisis period decreases. Moreover, in the case of the on-crisis period being 500 days and the crisis period being 50 days, the FR test is an oversized 5.7%, while the CS_{12}, CK_{13} and CV_{22} tests are all undersized with 4.3%, 3.5% and 2.7%, respectively.

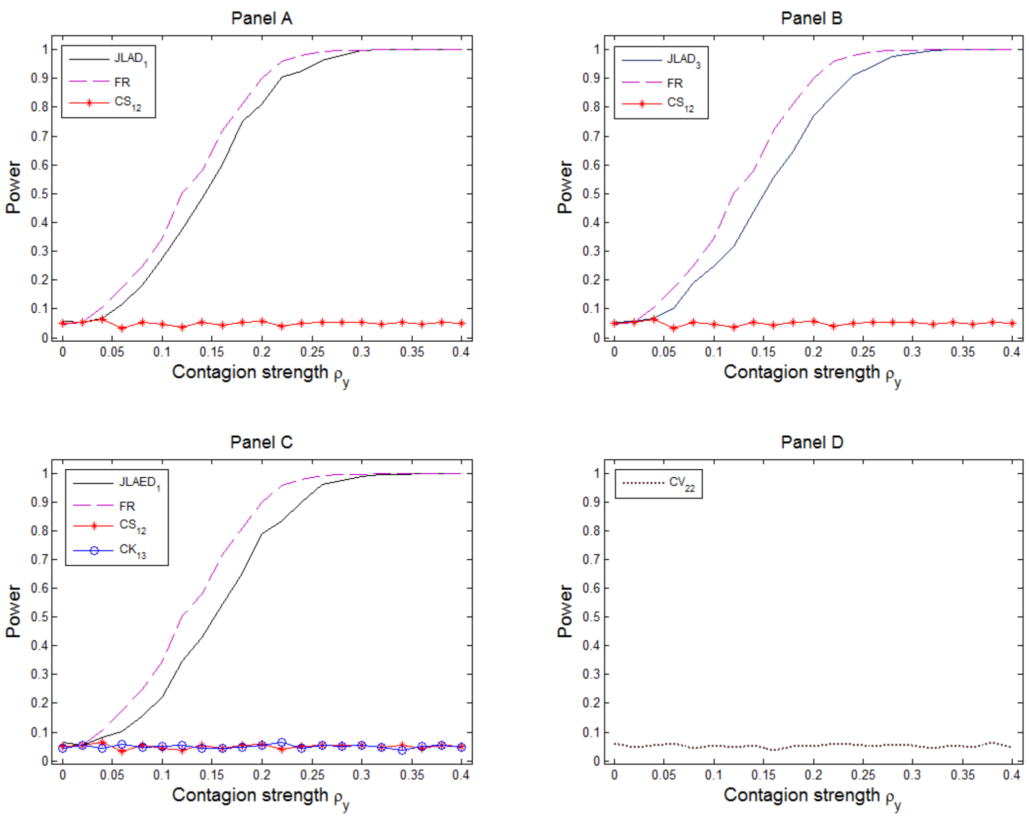
4.3 Power

Figures 1 to 3 show the power functions of the four types of single-channel and three types of multiple-channel tests across experiments I to III with the sample sizes of non-crisis and crisis being 500 days ($T_x = T_y = 500$). The DGP in each experiment contains additional transmission channels of contagion. The power functions are determined by increasing the intensity level of contagion. For instance, in experiment I in Figure 1, the power functions for each test are simulated based on the alternative hypothesis of contagion through the correlation channel for increasing values of ρ_y ranging from 0 to 0.4. Given the restriction of no contagion $\rho_y = \rho_x = 0$, the simulated power functions are size adjusted with a power of 5%.

4.3.1 Experiment I: Vary the Correlation Channel

The results in Figure 1 show that the FR test (dashed line) is decisively more powerful than the remaining single-channel and all multiple-channel tests as the DGP for experiment I only contains the additional transmission channel of contagion through the correlation channel and does not include changes in the higher order co-moments channels. As expected, the

Figure 1: Simulated power functions of contagion statistics in experiment I.



Note: Simulated power functions of contagion statistics in experiment I where the DGP only contains the correlation channel of contagion. The sample sizes of non-crisis and crisis periods are set at $T_x = T_y = 500$. The contagion strength is controlled by ρ_y with a range of values of 0 to 0.4.

CS_{12}, CK_{13} and CV_{22} tests exhibit no power in detecting this channel of contagion with power of around 5% over the range of values of the correlation contagion parameter ρ_y.

The three types of multiple-channel tests (i.e. $JLAD_1$, $JLAD_3$ and $JLAED_1$) are the less powerful tests than the FR test. However, the results are quite reasonable since three of these tests capture not only the correlation channel but also one of the higher order co-moments channels. The information on capturing the transmission channels of the higher order co-moments is not contained in experiment I for the DGP, which is why the power of the three types of multiple-channel tests are smaller than that of the FR test which tests for contagion only through the correlation channel. The power functions of both the FR test and the multiple-channel tests monotonically increase over the range of ρ_y. More specifically, the speed of approaching 100% power for the FR test is faster than that for either of the joint tests. Inspection of Panel A of Figure 1 shows that the power of the FR test is close to 100% as the contagion strength reaches $\rho_y = 0.25$, while for the $JLAD_1$ test, the power reaches to 100% as ρ_y becomes close to 0.30.

4.3.2 Experiment II: Vary the Co-volatility Channel

Figure 2 shows the power functions of the four singe-channel and three multiple-channel tests for experiment II which is based on the alternative hypothesis of contagion through the co-volatility channel for the range of contagion strength $\theta_{y,8}$ between -0.9 and 0. Given the contagion strength being zero, all the tests yield a probability of 5% for finding contagion as the power functions are size adjusted ($\theta_{y,8} = \theta_{x,8} = 0$).

Panel D of Figure 2 shows that the CV_{22} test monotonically increases over the range of $\theta_{y,8}$ in experiment II as it captures the change in co-volatility. As expected, the remaining single-channel tests (FR, CS_{12} and CK_{13}) and three multiple-channel tests ($JLAD_1$, $JLAD_3$ and $JLAED_1$) present a constant power function of around 5% as the level of contagion $\theta_{y,8}$ decreases from 0 to -0.9.

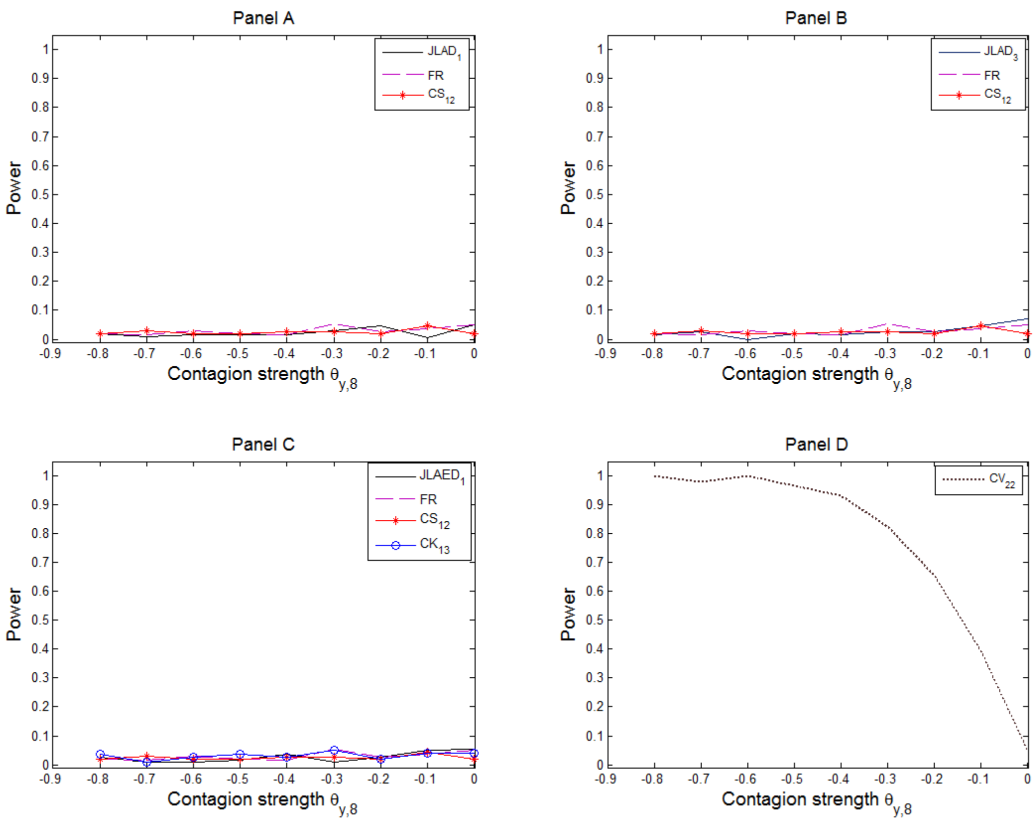
4.3.3 Experiment III: Vary Co-skewness and Co-volatility Channels

Figure 3 shows the simulated power functions for each test conducted using the DGP of contagion through the first form of co-skewness (co-skewness$_{12}$) and co-volatility channels in experiment III. The parameter $\theta_{y,4}$ represents the contagion strength through the co-skewness channel ranging from -0.9 to 0.9. To ensure that the generalized bivariate normal distribution with higher order moments and co-moments in the DGP is bounded, the parameter $\theta_{y,8}$ is set up to -0.5 in experiment III in which the transmission channel of contagion through the co-volatility channel can be detected.

The results of the contagion tests using the DGP of Experiment III containing one of transmission channels of contagion through the first form of co-skewness reveals that the CS_{12} test is indeed the best power in detecting this type of contagion channels among three types of multiple-channel tests since this test is the only test purely detecting a specific channel of contagion based on the first form of co-skewness. As shown in Panel A of Figure 3, the power function of the CS_{12} test monotonically increases if the contagion strength $\theta_{y,4}$ increases from 0 to 0.9. When the contagion strength $\theta_{y,4}$ reaches 0.9, the power of the CS_{12} test is close to 100%.

As shown in Panels A to C of Figure 3, the joint tests, i.e. $JLAD_1$, $JLAD_3$ and $JLAED_1$,

Figure 2: Simulated power functions of contagion statistics in experiment II.



Note: Simulated power functions of contagion statistics in experiment II where the DGP contains only the co-volatility channel of contagion. The sample sizes of non-crisis and crisis periods are set at $T_x = T_y = 500$. The contagion strength is controlled by $\theta_{y,8}$ with a range of values of -0.9 to 0. All the tests yield a probability of 5% for finding contagion as the power functions are size adjusted ($\theta_{y,8} = 0$).

are the next most powerful tests following three single-channel tests (FR, CS_{12} and CK_{13}). These three types of joint tests also reveal monotonically increasing power functions over the range of contagion strength $\theta_{y,4}$ between 0 and 0.9. The remaining single-channel tests (FR and CK_{13}) shown in Panels A to C of Figure 3 all have very low power with around 5% as the contagion strength $\theta_{y,4}$ is between -1 and 1. Not surprisingly, the CV_{22} test shown in Panel D presents 100% of the power given that the DGP for experiment III contains the transmission channels of contagion through both co-skewness and co-volatility channels.

Overall, the three simulation experiments show that the proposed joint tests of contagion exhibit the second most powerful tests following the single-channel tests if the DGP contains only the transmission channel of contagion consistent with the single channel test.

5 Empirical Applications

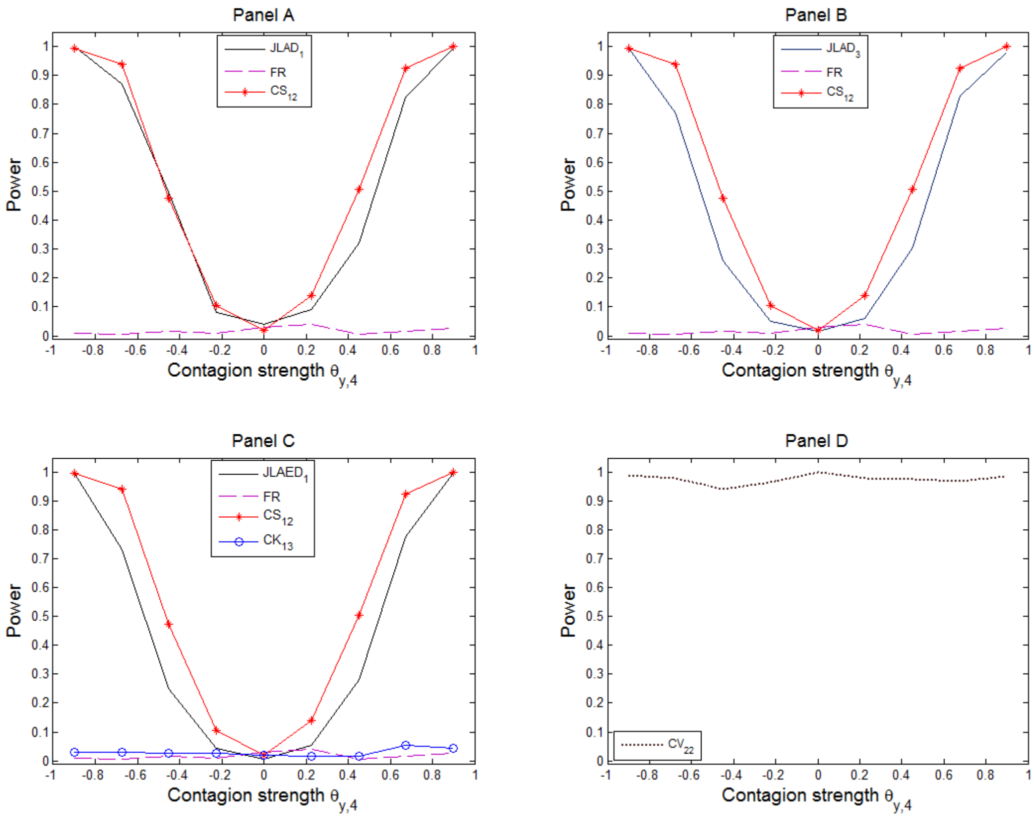
The five types of multiple-channel tests of contagion are applied to test for contagion during the three financial crises: the subprime mortgage crisis of 2007-08, the great recession of 2008-09, and the European sovereign-debt crisis of 2010-12.

5.1 The Data

The data consists of the daily percentage returns of equity market indices of fifteen European countries (Austria, Belgium, Denmark, France, Germany, Greece, Italy, Ireland, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the UK) and one north American country (the US). The daily percentage returns are calculated as 100 times the log first difference of the value of the equity indices. All data series are denominated in US dollars.[3] The non-crisis period begins January 3, 2005 and ends July 25, 2007. The crisis period begins July 26, 2007 and ends August 30, 2012. It can be divided into three sub-periods of crisis, namely the subprime mortgage crisis (July 26, 2007 to September 14, 2008), the great recession crisis

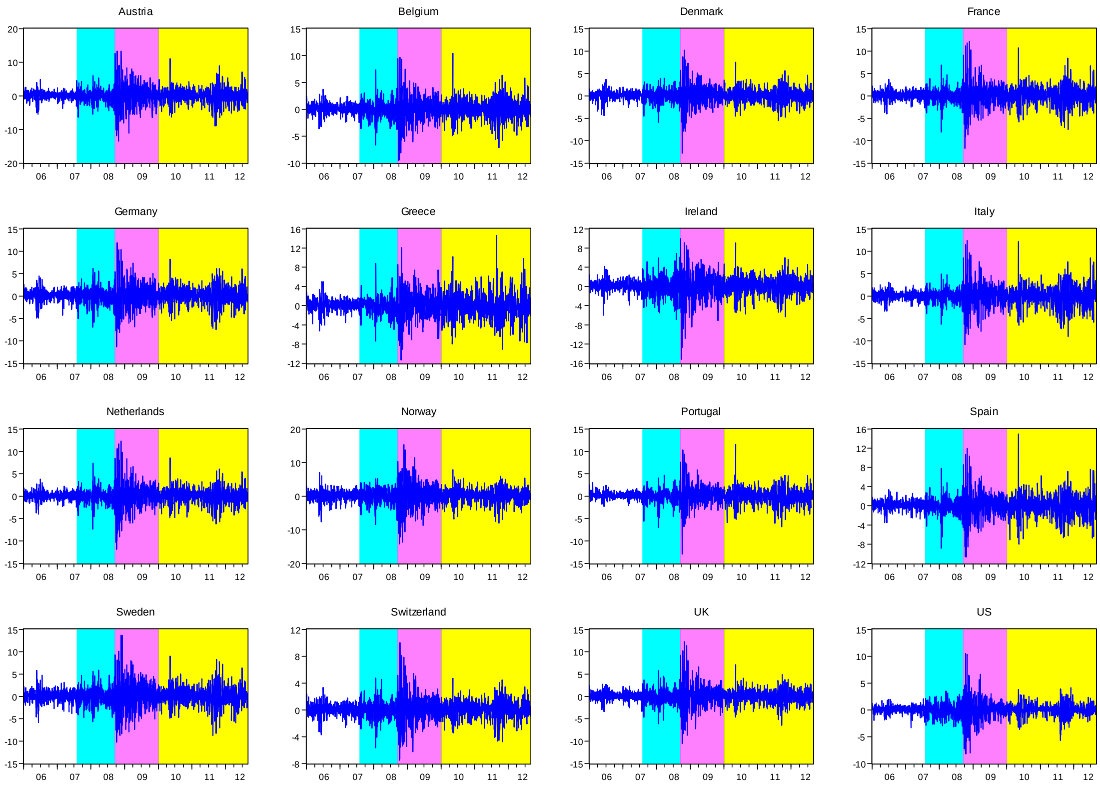
[3] All data are collected from Datastream. The mnemonics are: Austria - Austrian Traded index (ATXINDEX); Belgium - BEL 20 price index (BGBEL20); Denmark - OMX Copenhagen price index (COSEASH); France - France CAC 40 price index (FRCAC40); Germany - MDAX Frankfurt price index (MDAXIDX); Greece - Athex Composite price index (GRAGENL); Italy - FTSE MIB price index (FTSEMIB); Ireland - Ireland Se Overall price index (ISEQUIT); Netherlands - Aex price index (AMSTEOE); Norway - MSCI Norway price index (MSNWAYL); Portugal - Portual PSI All Share price index (POPSIGN); Spain - IBEX 35 price index (IBEX351); Sweden - OMX Stockholm 30 price index (SWEDOMX); Switzerland - Swiss Market price index (SWISSMI); UK - FTSE 100 price index (FTSE100); US - Dow Jones Industrials price index (DJINDUS).

Figure 3: Simulated power functions of contagion statistics in experiment III.



Note: Simulated power functions of contagion statistics in experiment III where the DGP contains two transmission channels of contagion through the co-skewness and co-volatility channels. The sample sizes of non-crisis and crisis periods are set at $T_x = T_y = 500$. The contagion strength of the co-skewness channel is controlled by $\theta_{y,4}$ ranging between -0.9 and 0.9. The co-volatility contagion parameter $\theta_{y,8}$ is set at -0.5.

Figure 4: Daily equity returns from 2005 to 2012.



Note: The shaded areas refer to episodes of crisis in international equity markets. These are (i) the subprime mortgage crisis (July 26, 2007 to September 14, 2008); (ii) the great recession (September 15, 2008 to December 31, 2009); and (iii) the European debt crisis (January 1, 2010 to August 30, 2012).

(September 15, 2008 to December 31, 2009), and the European debt crisis (January 2, 2010 to August 31, 2012). The choice of dates is based on Fry et al. (2011).

The returns series are plotted in Figure 4. The figure highlights the most extreme changes in volatility in most equity markets during the three periods of financial crises extending from July 2007 to August 2012. Before implementing the joint tests of contagion, the market fundamentals are formally modeled by specifying a vector autoregressive model (VAR). To deal with the problems of the equity markets open in different time zones, two-days rolling average returns are used to fit into a VAR with two lags containing the countries listed in each crisis period.[4] The residuals of the VAR are used to compute the test statistics.

[4] Two lags are set in the VAR based on the criteria of Akaike information (AI) and Akaike final prediction error (FPE).

5.2 Evidence of Contagion

This section illustrates the performance of the joint tests of contagion. The results of the joint tests of contagion are reported in Tables 4 and 5 for the three financial crises occurring during 2007-12. The source crisis country is selected to be the US during the subprime mortgage crisis and great recession, and to be Greece during the European debt crisis. Table 4 presents the results of contagion tests through three combinations of the linear and asymmetric dependence channels, while Table 5 presents results of contagion tests that consider two combinations of channels operating through the linear, asymmetric and extremal dependence channels.

5.2.1 Linear and Asymmetric Dependence Joint Tests of Contagion

Table 4 shows the results of contagion tests based on changes in three combinations of the linear and asymmetric dependence channels, depending on whether the correlation is joint with either one form of co-skewness ($JLAD_1$ and $JLAD_2$) or both forms of co-skewness ($JLAD_3$) during the three financial crises.

Table 2: Summary of restrictions on the parameters in equations (21) and (22) for size and power tests.

	Size	Power		
Parameters		Exp I	Exp II	Exp III
ρ_x	0	0	0	0
ρ_y	0	$[0.01, 0.40]$	0	0
$\theta_{x,4}$	0	0	0	0
$\theta_{y,4}$	0	0	0	$[-0.90, 0.90]$
$\theta_{x,5}$	0	0	0	0
$\theta_{y,5}$	0	0	0	0
$\theta_{x,6}$	0	0	0	0
$\theta_{y,6}$	0	0	0	0
$\theta_{x,7}$	0	0	0	0
$\theta_{y,7}$	0	0	0	0
$\theta_{x,8}$	0	0	0	0
$\theta_{y,8}$	0	0	$[-0.9, 0]$	-0.50
$\theta_{x,9}$	0	0	0	0
$\theta_{y,9}$	0	0	0	0
$\theta_{x,10}$	0	0	0	0
$\theta_{y,10}$	0	0	0	0
$\theta_{x,11}$	0	0	0	0
$\theta_{y,11}$	0	0	0	0
$\theta_{x,12}$	0	0	0	0
$\theta_{y,12}$	0	0	0	0
T_x	500	500	500	500
T_y	$[50, 500]$	500	500	500

Note: The correlation channel of contagion of Exp I is conducted by increasing values of the parameter ρ_y. The co-volatility channel of contagion in Exp II is conducted by decreasing values of the parameter $\theta_{y,8}$. The co-skewness channel of contagion in Exp III is conducted by either decreasing or increasing values of the parameter $\theta_{y,4}$ ranging between -0.9 and 0.9. In Exp III, the parameter of co-volatility is set to be negative ($\theta_{y,8} = -0.5$) to ensure the distribution is bounded.

Table 3:
Size properties of single-channel and multiple-channel tests of contagion based on the different sizes of the crisis sample period, for non-crisis samples of size $T_x = 500$. The results are based on $10,000$ replications.

	Size of crisis period (T_y)					
Tests	500	400	300	200	100	50
Single-channel tests of contagion						
FR	0.054	0.054	0.049	0.052	0.051	0.057
CS_{12}	0.049	0.050	0.052	0.052	0.048	0.043
CK_{13}	0.049	0.048	0.048	0.048	0.043	0.035
CV_{22}	0.049	0.045	0.046	0.045	0.040	0.027
Multiple-channel tests of contagion						
$JLAD_1$	0.050	0.051	0.049	0.053	0.053	0.053
$JLAD_3$	0.051	0.050	0.047	0.056	0.051	0.053
$JLAED_1$	0.054	0.051	0.051	0.053	0.055	0.051

Table 4: The results of linear and asymmetric dependence joint tests of contagion during the subprime mortgage crisis of 2007-08, the great recession of 2008-09, and the European debt crisis of 2010-12.

Tests	AT	BG	DM	FR	GE	GR	IT	IR	NL	NW	PG	SP	SW	SZ	UK
	Subprime mortgage crisis of 2007 - 2008 (i = US)														
$JLAD_1$	3.95	0.89	1.05	0.85	3.18	1.42	1.14	9.37	2.15	1.98	3.30	3.33	0.95	0.64	0.60
pv	0.14	0.64	0.59	0.65	0.20	0.49	0.57	0.01*	0.34	0.37	0.19	0.19	0.62	0.73	0.74
$JLAD_2$	1.56	1.10	0.72	0.84	3.69	0.93	1.13	6.83	1.55	2.76	4.21	2.72	0.58	0.67	0.83
pv	0.46	0.58	0.70	0.66	0.16	0.63	0.57	0.03*	0.46	0.25	0.12	0.26	0.75	0.72	0.66
$JLAD_3$	5.49	1.98	1.13	0.88	4.25	1.44	1.74	15.00	2.28	3.06	5.79	4.67	0.98	1.24	1.33
pv	0.14	0.58	0.77	0.83	0.24	0.70	0.63	0.00*	0.52	0.38	0.12	0.20	0.81	0.74	0.72
	Great recession crisis of 2008 - 2009 (i = US)														
$JLAD_1$	24.20	29.24	47.49	27.00	22.73	15.31	13.49	22.61	75.40	80.50	18.39	13.05	28.33	30.89	52.29
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
$JLAD_2$	36.69	31.76	54.71	34.16	27.64	16.28	24.48	33.50	60.15	76.73	24.31	23.52	33.81	37.08	46.00
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
$JLAD_3$	46.40	54.34	90.59	56.02	44.49	22.69	30.58	47.15	128.84	128.90	27.66	31.24	56.60	57.42	88.53
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
	AT	BG	DM	FR	GE	IT	IR	NL	NW	PG	SP	SW	SZ	UK	US
	European debt crisis of 2010 - 2012 (i = Greece)														
$JLAD_1$	10.33	2.09	3.04	1.14	10.24	1.12	1.51	1.72	5.07	1.26	6.48	3.09	6.02	5.62	0.95
pv	0.01*	0.35	0.22	0.56	0.01*	0.57	0.47	0.42	0.08	0.53	0.04*	0.21	0.05*	0.06	0.62
$JLAD_2$	24.80	8.64	15.61	5.20	19.45	9.25	6.63	4.66	10.81	4.10	11.72	6.35	10.09	7.37	5.06
pv	0.00*	0.01*	0.00*	0.07	0.00*	0.01*	0.04*	0.10	0.00*	0.13	0.00*	0.04*	0.01*	0.03*	0.08
$JLAD_3$	32.25	8.67	17.87	5.29	24.14	9.25	7.20	5.83	14.27	5.35	16.48	7.45	10.40	8.38	5.07
pv	0.00*	0.03*	0.00*	0.15	0.00*	0.03*	0.07	0.12	0.00*	0.15	0.00*	0.06	0.02*	0.04*	0.17

Note: The reported values of the test statistics ($JLAD_1$, $JLAD_2$ and $JLAD_3$) are based on (12), (13), and (14). P-values are based on the chi-square distribution with two degrees of freedom for the $JLAD_1$and $JLAD_2$ tests and three degrees of freedom for the $JLAD_3$ test. An * denotes significance at the 5% level. The source crisis country is selected as the US during the subprime mortgage crisis and great recession, and as Greece during the European debt crisis.

Inspection of Table 4, which presents the joint tests of correlation and co-skewness, shows that the great recession crisis is the worst among the three crises since all transmission channels of contagion are operating. The key channels detected by the $JLAD_1$, $JLAD_2$ and $JLAD_3$ tests are from the mean returns in the US equity market to both mean returns and return volatility of the European equity markets ($r^1_{US} \rightarrow (r^1_{EU}, r^2_{EU})$), from both mean returns and return volatility in the US equity market to the returns of the European equity markets ($(r^1_{US}, r^2_{US}) \rightarrow r^1_{EU}$), as well as from the mean returns in the US equity market to both the mean returns and return volatility of the European equity markets and from the return volatility in the US equity market to the level returns of the European equity market ($r^1_{US} \rightarrow r^1_{EU}$, $r^1_{US} \rightarrow r^2_{EU}$ and $r^2_{US} \rightarrow r^1_{EU}$) during the great recession. The subprime mortgage crisis, however, did not result in significant evidence of contagion between the US equity market and the European equity markets except for Ireland at the 5% significant level of contagion detected by the p-values in terms of three types of linear and asymmetric dependence joint tests ($JLAD_1$, $JLAD_2$ and $JLAD_3$).

As for the European debt crisis, three types of linear and asymmetric dependence joint tests ($JLAD_1$, $JLAD_2$ and $JLAD_3$) all indicate significant evidence of contagion between the Greek equity markets and each of the European equity markets including Austria, Germany, Spain and Switzerland through the three combinations of multiple channels of correlation and co-skewness. Moreover, Belgium, Denmark, Italy, Norway, the UK are also affected by the European debt crisis with source market that specifies to be Greek equity markets based on both $JLAD_2$ and $JLAD_3$ tests, and Ireland and Sweden are affected by this crisis only based on the $JLAD_2$ test. Surprisingly, France, Netherlands, Portugal, and the US did not show evidence of contagion from the Greek equity markets based on three types of the linear and asymmetric dependence joint tests ($JLAD_1$, $JLAD_2$ and $JLAD_3$) during the financial crisis of 2010-12.

5.2.2 Linear, Asymmetric and Extremal Dependence Joint Tests of Contagion

Table 5 provides the results of two joint tests based on changes in linear, asymmetric and extremal dependence channels, where the $JLAED_1$ test detects contagion from the mean returns to the mean returns, return volatility and return skewness, and the $JLAED_2$ test detects the spillover of financial shocks from the mean returns, return volatility, and return skewness to the mean returns during the three financial crises of 2007-12. Given the Monte Carlo experiments earlier, the p-values reported in parentheses are based on the asymptotic distribution.

Table 5: The results of linear, asymmetric and extremal dependence joint tests of contagion during the subprime mortgage crisis of 2007-08, the great recession of 2008-09, and the European debt crisis of 2010-12.

	Country														
Tests	AT	BG	DM	FR	GE	GR	IT	IR	NL	NW	PG	SP	SW	SZ	UK
Subprime mortgage crisis of 2007 - 2008 (i = US)															
$JLAED_1$	8.70	1.80	12.93	1.14	28.81	24.26	2.80	15.78	10.68	8.88	3.62	7.78	12.27	5.90	9.04
pv	0.03*	0.62	0.00*	0.77	0.00*	0.00*	0.42	0.00*	0.01*	0.03*	0.31	0.05*	0.01*	0.12	0.03*
$JLAED_2$	21.78	29.47	38.33	27.07	60.13	42.14	22.95	33.44	25.90	17.84	15.10	28.41	29.63	29.97	27.43
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
Great recession crisis of 2008 - 2009 (i = US)															
$JLAED_1$	87.10	118.58	102.67	91.82	30.75	20.40	65.89	69.26	263.01	204.19	93.00	60.13	50.34	113.61	207.34
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
$JLAED_2$	135.69	144.72	140.84	124.35	69.43	52.86	74.85	66.63	217.30	227.03	128.34	78.74	76.29	158.72	225.67
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
	AT	BG	DM	FR	GE	IT	IR	NL	NW	PG	SP	SW	SZ	UK	US
European debt crisis of 2010 - 2012 (i = Greece)															
$JLAED_1$	60.35	18.06	48.35	5.63	127.56	5.72	54.87	8.87	46.45	2.86	13.96	44.21	13.97	10.27	10.52
pv	0.00*	0.00*	0.00*	0.13	0.00*	0.13	0.00*	0.03*	0.00*	0.41	0.00*	0.00*	0.00*	0.02*	0.01*
$JLAED_2$	149.30	105.40	103.69	91.18	190.41	84.31	129.50	81.85	60.54	20.58	74.44	98.14	131.89	113.01	19.84
pv	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*

Note: The reported values of the test statistics ($JLAED_1$ and $JLAED_2$) are based on (18) and (19). P-values are based on the chi-square distribution with three degrees of freedom. An * denotes significance at the 5% level. The source crisis country is selected as the US during the subprime mortgage crisis and great recession, and as Greece during the European debt crisis.

The results of the joint tests of contagion in Table 5, show that both the $JLAED_1$ and $JLAED_2$ tests provide significant evidence of contagion from the US equity market to the European equity markets during the subprime mortgage crisis and great recession, and from the Greek equity market to the European equity markets during the European debt crisis. In particular, the joint channels of correlation, co-skewness$_{21}$ and co-kurtosis$_{31}$ detected by the $JLAED_2$ test are operating with the transmission of contagion from the mean returns, return volatility and return skewness in the US equity market to the mean returns of all European equity markets during the two crises of 2007-09, as well as from the returns, return volatility and return skewness in the Greek equity market to the returns of all European equity markets during the European debt crisis. Compared with the results of the $JLAED_2$ test, the $JLAED_1$ test also indicates evidence of contagion at the 5% significant level during the three crises, but the multiple transmission channels of correlation, co-skewness$_{12}$ and co-kurtosis$_{13}$ are not operating for all cases. For example, there is no evidence of contagion detected by the $JLAED_1$ test between the US equity market and the equity markets in Belgium, France, Italy, Poland, and Switzerland during the subprime mortgage crisis and between the Greek equity market and the equity markets in France, Italy, and Poland during the European debt crisis.

6 Conclusions

This paper introduced a new class of multiple-channel tests of contagion based on changes in both linear and non-linear co-moments of the distribution of returns during financial crises. The proposed approach enables the measurement of five different combinations of channels through the correlation, co-skewness and co-kurtosis of financial market contagion. To develop five multiple-channel tests of contagion, the test statistics of joint co-moments need to be derived by using the Lagrange multiplier test. Following the work of Fry et al. (2010), the bivariate generalized normal distribution with the higher order co-moments is used to derive three forms of test statistics. Then, the joint tests of contagion are proposed by comparing the test statistic for joint co-moments during the non-crisis period with that during the crisis period. Notably, the test statistic of joint co-moments during the crisis period needs to be adjusted by using the adjusted correlation coefficient in order to adjust for the bias due to the heteroskedasticity of asset returns (Forbes and Rigobon, 2002). The new approach captures a range of possible transmission channels of contagion such as cross-market mean returns, cross-market mean returns and return volatility, and cross-market mean returns and return skewness.

The proposed multiple-channel tests of contagion had two advantages over the existing single-channel tests of contagion in the literature (the co-skewness tests of contagion, the co-kurtosis tests of contagion, and the co-volatility test of contagion). First, the proposed tests show better size properties compared to the nominal size of 5% with small sample sizes during the crisis period often observed during financial crises. The joint tests size was better than these for the single-channel tests. The existing single-channel tests of contagion based on changes in either co-skewness (Fry et al., 2010) or co-kurtosis (Fry-McKibbin and Hsiao, 2014) produced empirical sizes far smaller than the nominal size given the crisis sample period being relatively short. Second, the proposed multiple-channel tests of contagion display the second highest power following the single-channel tests investigated if the DGP contains the transmission channel of contagion consistent with the single-channel tests.

Applying the proposed five multiple-channel tests of contagion to three financial crises of 2007-12, showed that the great recession crisis is the worst crisis among the three crises as the results of five joint tests show significant evidence of contagion between the US equity market and the European equity markets. The subprime mortgage crisis, however, did not reveal evidence of contagion transmitting from the US equity market to the European equity markets except for to the Irish equity market based on the joint tests of linear and asymmetric dependence. The European debt crisis also provided evidence of contagion through multiple channels, about half of European equity markets being affected based on the joint tests of correlation and co-skewness and nearly all European equity markets being affected based on the joint tests of correlation, co-skewness and co-kurtosis. Overall, the joint tests of correlation, co-skewness and co-kurtosis provided more evidence of contagion than the joint tests of correlation and co-skewness among three financial crisis of 2007-12, indicating the importance of changes in multiple non-linear cross-market dependence channels during financial crises.

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A Information Matrix Derivations

The following results are used to derive the information matrix of test statistics for joint co-moments below. Consider the following bivariate normal distribution with higher order co-moments, written as

$$\begin{aligned} h &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2+\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\ &+\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2+\theta_5\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^1 \\ &+\theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3+\theta_7\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^3\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^1 \\ &+\theta_8\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2. \end{aligned} \tag{25}$$

We take the expectations of the first and second derivatives of the distribution in (25) under the null hypothesis of bivariate normality, then the elements of the information matrix at observation t are

$$\begin{aligned} I_{1,1,t} &= E\left[\left(\frac{\partial h}{\partial\mu_1}\right)^2\right]-E\left[\frac{\partial h}{\partial\mu_1}\right]E\left[\frac{\partial h}{\partial\mu_1}\right] \\ &= \left(\frac{1}{\sigma_1^2}\right)\left(\frac{1}{1-\rho^2}\right), \end{aligned}$$

$$\begin{aligned} I_{1,2,t} &= E\left[\frac{\partial h}{\partial\mu_1}\frac{\partial h}{\partial\mu_2}\right]-E\left[\frac{\partial h}{\partial\mu_1}\right]E\left[\frac{\partial h}{\partial\mu_2}\right] \\ &= \left(-\frac{\rho}{\sigma_1\sigma_2}\right)\left(\frac{1}{1-\rho^2}\right), \end{aligned}$$

$$\begin{aligned} I_{1,3,t} &= E\left[\frac{\partial h}{\partial\mu_1}\frac{\partial h}{\partial\sigma_1^2}\right]-E\left[\frac{\partial h}{\partial\mu_1}\right]E\left[\frac{\partial h}{\partial\sigma_1^2}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned} I_{1,4,t} &= E\left[\left(\frac{\partial h}{\partial\mu_1}\right)\left(\frac{\partial h}{\partial\sigma_2^2}\right)\right]-E\left[\frac{\partial h}{\partial\mu_1}\right]E\left[\frac{\partial h}{\partial\sigma_2^2}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned}
I_{1,5,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \rho}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
I_{1,6,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \theta_4}\right] \\
&= \frac{1}{\sigma_1},
\end{aligned}$$

$$\begin{aligned}
I_{1,7,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= \frac{2\rho}{\sigma_1},
\end{aligned}$$

$$\begin{aligned}
I_{1,8,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
I_{1,9,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
I_{1,10,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
I_{2,2,t} &= E\left[\left(\frac{\partial h}{\partial \mu_2}\right)^2\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \mu_2}\right] \\
&= \left(\frac{1}{\sigma_2^2}\right)\left(\frac{1}{1-\rho^2}\right),
\end{aligned}$$

$$\begin{aligned}
I_{2,3,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \sigma_1^2}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \sigma_1^2}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
I_{2,4,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \sigma_2^2}\right] \\
&= 0,
\end{aligned}$$

$$\begin{aligned} I_{2,5,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \rho}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned} I_{2,6,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \theta_4}\right] \\ &= \frac{2\rho}{\sigma_2} \end{aligned}$$

$$\begin{aligned} I_{2,7,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \theta_5}\right] \\ &= \frac{1}{\sigma_2} \end{aligned}$$

$$\begin{aligned} I_{2,8,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \theta_6}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned} I_{2,9,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned} I_{2,10,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\ &= 0, \end{aligned}$$

$$\begin{aligned} I_{3,3,t} &= E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)^2\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right]E\left[\frac{\partial h}{\partial \sigma_1^2}\right] \\ &= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2+2}{4\sigma_1^4}\right), \end{aligned}$$

$$\begin{aligned} I_{3,4,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right]E\left[\frac{\partial h}{\partial \sigma_2^2}\right] \\ &= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2}{4\sigma_1^2\sigma_2^2}\right), \end{aligned}$$

$$
\begin{aligned}
I_{3,5,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \rho}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho}{2\sigma_1^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
I_{3,6,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_4}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{3,7,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{3,8,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= \frac{3\rho}{2\sigma_1^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{3,9,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= \frac{9\rho}{2\sigma_1^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{3,10,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= \frac{1+2\rho^2}{\sigma_1^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{4,4,t} &= E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)^2\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \sigma_1^2}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{2-\rho^2}{4\sigma_2^4}\right),
\end{aligned}
$$

$$
\begin{aligned}
I_{4,5,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \sigma_2^2}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2}{2\sigma_2^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
I_{4,6,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right]E\left[\frac{\partial h}{\partial \theta_4}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{4,7,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right]E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{4,8,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right]E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= \frac{9\rho}{2\sigma_2^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{4,9,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= \frac{3\rho}{2\sigma_2^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{4,10,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= \frac{2\rho^2+1}{\sigma_2^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{5,5,t} &= E\left[\left(\frac{\partial h}{\partial \rho}\right)^2\right] - E\left[\frac{\partial h}{\partial \rho}\right]E\left[\frac{\partial h}{\partial \rho}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{1+\rho^2}{1-\rho^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
I_{5,6,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \rho}\right]E\left[\frac{\partial h}{\partial \theta_4}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{5,7,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \rho}\right]E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{5,8,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 3,
\end{aligned}
$$

$$
\begin{aligned}
I_{5,9,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 3,
\end{aligned}
$$

$$
\begin{aligned}
I_{5,10,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 4\rho,
\end{aligned}
$$

$$
\begin{aligned}
I_{6,6,t} &= E\left[\frac{\partial h}{\partial \theta_4}\frac{\partial h}{\partial \theta_4}\right] - E\left[\frac{\partial h}{\partial \theta_4}\right] E\left[\frac{\partial h}{\partial \theta_4}\right] \\
&= 3 + 12\rho^2,
\end{aligned}
$$

$$
\begin{aligned}
I_{6,7,t} &= E\left[\frac{\partial h}{\partial \theta_4}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \theta_4}\right] E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= 6\rho^3 + 9\rho,
\end{aligned}
$$

$$
\begin{aligned}
I_{6,8,t} &= E\left[\frac{\partial h}{\partial \theta_4}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \theta_4}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{6,9,t} &= E\left[\frac{\partial h}{\partial \theta_4}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \theta_4}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{6,10,t} &= E\left[\frac{\partial h}{\partial \theta_4}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \theta_4}\right] E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{7,7,t} &= E\left[\frac{\partial h}{\partial \theta_5}\frac{\partial h}{\partial \theta_5}\right] - E\left[\frac{\partial h}{\partial \theta_5}\right] E\left[\frac{\partial h}{\partial \theta_5}\right] \\
&= 3 + 12\rho^2,
\end{aligned}
$$

$$
\begin{aligned}
I_{7,8,t} &= E\left[\frac{\partial h}{\partial \theta_5}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \theta_5}\right]E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{7,9,t} &= E\left[\frac{\partial h}{\partial \theta_5}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \theta_5}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{7,10,t} &= E\left[\frac{\partial h}{\partial \theta_5}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \theta_5}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{8,8,t} &= E\left[\frac{\partial h}{\partial \theta_6}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \theta_6}\right]E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 15 + 81\rho^2,
\end{aligned}
$$

$$
\begin{aligned}
I_{8,9,t} &= E\left[\frac{\partial h}{\partial \theta_6}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \theta_6}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 24\rho^4 + 63\rho^2 + 9,
\end{aligned}
$$

$$
\begin{aligned}
I_{8,10,t} &= E\left[\frac{\partial h}{\partial \theta_6}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \theta_6}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 54\rho^3 + 42\rho,
\end{aligned}
$$

$$
\begin{aligned}
I_{9,9,t} &= E\left[\frac{\partial h}{\partial \theta_7}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \theta_7}\right]E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 15 + 81\rho^2,
\end{aligned}
$$

$$
\begin{aligned}
I_{9,10,t} &= E\left[\frac{\partial h}{\partial \theta_7}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \theta_7}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 53\rho^3 + 42\rho,
\end{aligned}
$$

$$
\begin{aligned}
I_{10,10,t} &= E\left[\frac{\partial h}{\partial \theta_8}\frac{\partial h}{\partial \theta_8}\right] - E\left[\frac{\partial h}{\partial \theta_8}\right]E\left[\frac{\partial h}{\partial \theta_8}\right] \\
&= 8 + 68\rho^2 + 20\rho^4.
\end{aligned}
$$

B Derivation of Test Statistics for Joint Co-moments

B.1 Correlation and Co-skewness$_{12}$ Joint Test

Consider the following bivariate normal distribution with first form of co-skewness given as

$$\begin{aligned} f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - \eta_1\right], \end{aligned} \tag{26}$$

where

$$\begin{aligned} \eta_1 &= \ln \iint \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2\right] dr_1 dr_2 \\ &= \ln \iint \exp\left[h_1\right] dr_1 dr_2, \end{aligned} \tag{27}$$

and

$$\begin{aligned} h_1 &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\ &\quad +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2. \end{aligned} \tag{28}$$

A test of correlation and co-skewness$_{12}$ is based on the null hypothesis

$$H_0 : \rho = \theta_4 = 0. \tag{29}$$

Under the null hypothesis of independence and bivariate normality, the maximum likelihood estimators of the unknown parameters are simply

$$\widehat{\mu}_i = \frac{1}{T}\sum_t r_{i,t}; \widehat{\sigma}_i^2 = \frac{1}{T}\sum_t \left(r_{i,t} - \widehat{\mu}_i\right)^2; \widehat{\rho} = \sum_t \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right).$$

Let the parameters of equation (26) be

$$\Theta_1 = \left\{\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho, \theta_4\right\}.$$

By taking the log function of equation (26), the log likelihood function at time t is given by

$$\begin{aligned}
\ln L_t(\Theta_1) &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2+\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\
&\quad +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-\eta_1 \\
&= h_1-\eta_1, \qquad (30)
\end{aligned}$$

where h_1 and η_1 are given by equations (28) and (27).

Asymptotic information matrix, derived by Fry et al. (2010), can be shown as

$$\begin{aligned}
I_t(\Theta_1) &= -E\left[\frac{\partial^2 \ln L_t(\Theta_1)}{\partial\Theta_1\partial\Theta_1'}\right] \qquad (31) \\
&= E\left[\frac{\partial h_1}{\partial\Theta_1}\frac{\partial h_1}{\partial\Theta_1'}\right]-E\left[\frac{\partial h_1}{\partial\Theta_1}\right]E\left[\frac{\partial h_1}{\partial\Theta_1'}\right].
\end{aligned}$$

Using equation (31) and the results from Appendix A, the information matrix under H_0 ($H_0:\rho=\theta_4=0$) is

$$\begin{aligned}
I(\Theta_1) &= \sum_{t=1}^{T} I_t(\Theta_1) \qquad (32) \\
&= T\times\left(E\left[\frac{\partial h_1}{\partial\Theta_1}\frac{\partial h_1}{\partial\Theta_1'}\right]_{\rho=\theta_4=0}-E\left[\frac{\partial h_1}{\partial\Theta_1}\right]_{\rho=\theta_4=0}E\left[\frac{\partial h_1}{\partial\Theta_1'}\right]_{\rho=\theta_4=0}\right) \\
&= T\times\begin{bmatrix}
I_{1,1,t} & I_{1,2,t} & I_{1,3,t} & I_{1,4,t} & I_{1,5,t} & I_{1,6,t} \\
I_{1,2,t} & I_{2,2,t} & I_{2,3,t} & I_{2,4,t} & I_{2,5,t} & I_{2,6,t} \\
I_{1,3,t} & I_{2,3,t} & I_{3,3,t} & I_{3,4,t} & I_{3,5,t} & I_{3,6,t} \\
I_{1,4,t} & I_{2,4,t} & I_{3,4,t} & I_{4,4,t} & I_{4,5,t} & I_{4,6,t} \\
I_{1,5,t} & I_{2,5,t} & I_{3,5,t} & I_{4,5,t} & I_{5,5,t} & I_{5,6,t} \\
I_{1,6,t} & I_{2,6,t} & I_{3,6,t} & I_{4,6,t} & I_{5,6,t} & I_{6,6,t}
\end{bmatrix},
\end{aligned}$$

where the element of $I_{i,j}$ in the information matrix at observation t is shown in Appendix A. Replacing the unknown population parameters by consistent estimators under the null hypothesis, the inverse asymptotic information matrix is

$$I^{-1}\left(\widehat{\Theta}_1\right)=\frac{1}{T}\begin{bmatrix}
\frac{1}{\widehat{\sigma}_1^2} & 0 & 0 & 0 & 0 & \frac{1}{\widehat{\sigma}_1} \\
0 & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2\widehat{\sigma}_1^4} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2\widehat{\sigma}_2^4} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\frac{1}{\widehat{\sigma}_1} & 0 & 0 & 0 & 0 & 3
\end{bmatrix}^{-1}. \qquad (33)$$

Evaluating the gradient for ρ and θ_4 under the null hypothesis gives

$$\begin{aligned} \frac{\partial \ln L}{\partial \rho}|_{\rho=\theta_4=0} &= \sum_{t=1}^{T}\left(\frac{\partial h_1}{\partial \rho}\right) - T\left(\frac{\partial \eta_1}{\partial \rho}\right) \\ &= \sum_{t=1}^{T}\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right), \end{aligned} \tag{34}$$

$$\begin{aligned} \frac{\partial \ln L}{\partial \theta_4}|_{\rho=\theta_4=0} &= \sum_{t=1}^{T}\left(\frac{\partial h_1}{\partial \theta_4}\right) - T\left(\frac{\partial \eta_1}{\partial \theta_4}\right) \\ &= \sum_{t=1}^{T}\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2. \end{aligned} \tag{35}$$

The score function under H_0 is given as

$$\begin{aligned} Q\left(\widehat{\Theta}_1\right) &= \frac{\partial \ln L}{\partial \Theta_1}|_{\rho=\theta_4=0} \\ &= \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & \sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right) & \sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 \end{array}\right]'. \end{aligned} \tag{36}$$

The Lagrange multiplier statistic is obtained by substituting equations (33) and (36) into

$$LM_1 = Q\left(\widehat{\Theta}_1\right)' I^{-1}\left(\widehat{\Theta}_1\right) Q\left(\widehat{\Theta}_1\right). \tag{37}$$

The pertinent statistic is

$$LM_1 = \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1}{T}}}\right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2}{T}}}\right)^2, \tag{38}$$

which is asymptotically distributed as χ^2_2 under the null hypothesis.

B.2 Correlation, Co-skewness$_{12}$ and Co-skewness$_{21}$ Joint Test

Consider the following bivariate normal distribution with two forms of co-skewness, given as

$$\begin{aligned} f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\left. +\theta_4\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_5\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^1 - \eta_2\right], \end{aligned} \tag{39}$$

where

$$\eta_2 = \ln \iint \exp \left[-\frac{1}{2} \left(\frac{1}{1-\rho^2} \right) \left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^2 - 2\rho \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right) \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right) \right) \right.$$
$$\left. +\theta_4 \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^2 + \theta_5 \left(\frac{r_{1,t}-\mu_1}{\sigma_1} \right)^2 \left(\frac{r_{2,t}-\mu_2}{\sigma_2} \right)^1 \right] dr_1 dr_2 \quad (40)$$

A test of correlation, co-skewness$_{12}$ and co-skewness$_{21}$ is based on the null hypothesis

$$H_0 : \rho = \theta_4 = \theta_5 = 0. \quad (41)$$

Using equation (31) and the results from Appendix A, the inverse asymptotic information matrix under H_0 is

$$I^{-1}\left(\widehat{\Theta}_2\right) = \frac{1}{T} \begin{bmatrix} \frac{1}{\widehat{\sigma}_1^2} & 0 & 0 & 0 & 0 & \frac{1}{\widehat{\sigma}_1} & 0 \\ 0 & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 & \frac{1}{\widehat{\sigma}_2} \\ 0 & 0 & \frac{1}{2\widehat{\sigma}_1^4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\widehat{\sigma}_2^4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{\widehat{\sigma}_1} & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & \frac{1}{\widehat{\sigma}_2} & 0 & 0 & 0 & 0 & 3 \end{bmatrix}^{-1}. \quad (42)$$

where $\Theta_2 = \{\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho, \theta_4, \theta_5\}$. The score function under H_0 is given as

$$Q\left(\widehat{\Theta}_2\right) = \frac{\partial \ln L}{\partial \Theta_2}|_{\rho=\theta_i=0}, \forall i = 4, 5 \quad (43)$$
$$= \left[\begin{matrix} 0 & 0 & 0 & 0 & \sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1} \right) \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2} \right) \end{matrix} , \right.$$
$$\left. \sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1} \right)^1 \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2} \right)^2 \quad \sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1} \right)^2 \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2} \right)^1 \right]'.$$

The Lagrange multiplier statistic is obtained by substituting equations (42) and (43) into

$$LM_2 = Q\left(\widehat{\Theta}_2\right)' I^{-1}\left(\widehat{\Theta}_2\right) Q\left(\widehat{\Theta}_2\right). \quad (44)$$

The pertinent statistic is

$$LM_2 = \left(\frac{\frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right) \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1}{T}}} \right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1 \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2}{T}}} \right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2 \left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^1}{\sqrt{\frac{2}{T}}} \right)^2, \quad (45)$$

which is asymptotically distributed as χ_3^2 under the null hypothesis.

B.3 Correlation, Co-skewness$_{12}$ and Co-kurtosis$_{13}$ Joint Test

Consider the following bivariate normal distribution with the first form of co-skewness and co-kurtosis given as

$$f(r_{1,t}, r_{2,t}) = \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right) \left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right) \right) + \theta_4 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_6 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 - \eta_3 \right], \quad (46)$$

where

$$\eta_3 = \ln \iint \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right) \left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right) \right) + \theta_4 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 + \theta_6 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 \right] dr_1 dr_2, \quad (47)$$

A test of correlation, co-skewness$_{12}$ and co-kurtosis$_{13}$ is based on the null hypothesis

$$H_0 : \rho = \theta_4 = \theta_6 = 0, \quad (48)$$

in equation (46).

Using the results from Appendix A, the inverse asymptotic information matrix under H_0

is

$$I^{-1}\left(\widehat{\Theta}_3\right)=\frac{1}{T}\left[\begin{array}{ccccccc} \frac{1}{\widehat{\sigma}_1^2} & 0 & 0 & 0 & 0 & \frac{1}{\widehat{\sigma}_1} & 0 \\ 0 & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2\widehat{\sigma}_1^4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\widehat{\sigma}_2^4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 3 \\ \frac{1}{\widehat{\sigma}_1} & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 15 \end{array}\right]^{-1}. \tag{49}$$

where $\Theta_3=\{\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho,\theta_4,\theta_6\}$. The score function under H_0 is given as

$$\begin{aligned} Q\left(\widehat{\Theta}_3\right) &= \frac{\partial \ln L}{\partial \Theta_3}|_{\rho=\theta_4=\theta_6=0} \qquad (50)\\ &= \Bigg[\ 0 \quad 0 \quad 0 \quad 0 \quad \sum_{t=1}^{T}\left(\tfrac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\tfrac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right) \\ &\qquad \sum_{t=1}^{T}\left(\tfrac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\tfrac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 \quad \sum_{t=1}^{T}\left(\tfrac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\tfrac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3\ \Bigg]'. \end{aligned}$$

Substituting equations (49) and (50) into the Lagrange multiplier statistic in equation (37) gives

$$\begin{aligned} LM_3 &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{2}{5T}}}\right)^2+\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2}{T}}}\right)^2 \\ &+\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3}{\sqrt{\frac{6}{T}}}\right)^2 \\ &-\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1}{T}}}\right)\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3}{\sqrt{\frac{1}{T}}}\right), \end{aligned} \tag{51}$$

which is asymptotically distributed as χ_3^2 under the null hypothesis.

B.4 Co-skewness$_{12}$ and Co-skewness$_{21}$ Joint Test

A joint test of co-skewness$_{12}$ and co-skewness$_{21}$ is given by the restrictions

$$H_0:\theta_4=\theta_5=0. \tag{52}$$

in equation (39). Using the results from Appendix A, the inverse asymptotic information matrix is

$$I^{-1}\left(\widehat{\Theta}_4\right)=\frac{\left(1-\widehat{\rho}^2\right)}{T}\begin{bmatrix} \frac{1}{\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & 0 & 0 & 0 & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_1} & \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_1} \\ \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_2} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_2} \\ 0 & 0 & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_1^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_2^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{1+\widehat{\rho}^2}{1-\widehat{\rho}^2} & 0 & 0 \\ \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_1} & \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_2} & 0 & 0 & 0 & \begin{matrix}\left(3+12\widehat{\rho}^2\right)\times\\ \left(1-\widehat{\rho}^2\right)\end{matrix} & \begin{matrix}\left(6\widehat{\rho}^3+9\widehat{\rho}\right)\times\\ \left(1-\widehat{\rho}^2\right)\end{matrix} \\ \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_1} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_2} & 0 & 0 & 0 & \begin{matrix}\left(6\widehat{\rho}^3+9\widehat{\rho}\right)\times\\ \left(1-\widehat{\rho}^2\right)\end{matrix} & \begin{matrix}\left(3+12\widehat{\rho}^2\right)\times\\ \left(1-\widehat{\rho}^2\right)\end{matrix} \end{bmatrix}^{-1}. \tag{53}$$

where $\Theta_4=\{\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho,\theta_4,\theta_5\}$. The score function under H_0 is given as

$$\begin{aligned} Q\left(\widehat{\Theta}_4\right) &= \frac{\partial\ln L}{\partial\Theta_4}|_{\theta_i=0},\forall i=1,2 \qquad (54)\\ &= \Big[\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ , \\ &\quad \sum_{t=1}^{T}\left(\tfrac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\tfrac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 \ \ \sum_{t=1}^{T}\left(\tfrac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\tfrac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^1\ \Big]'. \end{aligned}$$

The Lagrange multiplier statistic is obtained by substituting equations (53) and (54) into

$$LM_4=Q\left(\widehat{\Theta}_4\right)'I^{-1}\left(\widehat{\Theta}_4\right)Q\left(\widehat{\Theta}_4\right), \tag{55}$$

The pertinent statistic is

$$\begin{aligned} LM_4 &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{2-2\widehat{\rho}^6}{\left(2\widehat{\rho}^2+1\right)T}}}\right)^2+\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{2-2\widehat{\rho}^6}{\left(2\widehat{\rho}^2+1\right)T}}}\right)^2 \qquad (56)\\ &+\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2}{\sqrt{\frac{1-\widehat{\rho}^6}{\left(\widehat{\rho}^3+2\widehat{\rho}\right)T}}}\right)\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)}{\sqrt{\frac{1-\widehat{\rho}^6}{\left(\widehat{\rho}^3+2\widehat{\rho}\right)T}}}\right), \end{aligned}$$

which is asymptotically distributed as χ_2^2 under the null hypothesis.

C Review of Contagion Tests

C.1 Forbes and Rigobon Adjusted Correlation Test

Forbes and Rigobon (2002) identify contagion as an increase in the correlation of returns between non-crisis and crisis periods having adjusted for market fundamentals and any increases in volatility of the source country. The test statistic is

$$FR\left(i \to j; r_i^1, r_j^1\right) = \left(\frac{\widehat{v}_{y|x_i} - \widehat{\rho}_x}{\sqrt{\frac{1}{T_y} + \frac{1}{T_x}}}\right)^2, \tag{57}$$

where $\widehat{v}_{y|x_i}$ is the adjusted correlation coefficient given by

$$\widehat{v}_{y|x_i} = \frac{\widehat{\rho}_y}{\sqrt{1 + \left(\frac{s_{y,i}^2 - s_{x,i}^2}{s_{x,i}^2}\right)\left(1 - \widehat{\rho}_y^2\right)}}, \tag{58}$$

C.2 Co-skewness Contagion Tests

Co-skewness contagion tests are developed by Fry et al. (2010) and contain two forms. The first version of the co-skewness contagion test statistic is denoted CS_{12}

$$CS_{12}\left(i \to j; r_i^1, r_j^2\right) = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^2\right) - \widehat{\psi}_x\left(r_i^1, r_j^2\right)}{\sqrt{\dfrac{4\widehat{\nu}_{y|x_i}^2 + 2}{T_y} + \dfrac{4\widehat{\rho}_x^2 + 2}{T_x}}}\right)^2, \tag{59}$$

where $\widehat{\psi}_y(r_i^m, r_j^n)$ and $\widehat{\psi}_x(r_i^m, r_j^n)$ are the estimated co-skewness ($m = 1, n = 2$) coefficient during the crisis and non-crisis period in equations (15) and (16) and $\widehat{\nu}_{y|x_i}$ is the adjusted crisis correlation in equation (58). The CS_{12} is to test for the transmission of contagion from the mean returns ($m = 1$) of asset i to the return volatility ($n = 2$) of asset j through changes in the non-crisis co-skewness coefficient $\left(\widehat{\psi}_x\right)$ compared to the crisis period co-skewness coefficient $\left(\widehat{\psi}_y\right)$.

The second version of the co-skewness contagion test statistic is denoted CS_{21}

$$CS_{21}\left(i \to j; r_i^2, r_j^1\right) = \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^1\right) - \widehat{\psi}_x\left(r_i^2, r_j^1\right)}{\sqrt{\dfrac{4\widehat{\nu}_{y|x_i}^2 + 2}{T_y} + \dfrac{4\widehat{\rho}_x^2 + 2}{T_x}}}\right)^2, \tag{60}$$

where $\widehat{\psi}_y(r_i^m, r_j^n)$ and $\widehat{\psi}_x(r_i^m, r_j^n)$ are the estimated co-skewness ($m = 2, n = 1$) coefficient during the crisis and non-crisis period in equations (15) and (16). The $CS_{21}\left(i \rightarrow j; r_i^2, r_j^1\right)$ tests for contagion through new spillovers from the return volatility ($m = 2$) of the source market i to the mean returns ($n = 1$) of a recipient country j.

C.3 Co-kurtosis Contagion Tests

The co-kurtosis contagion tests are developed by Hsiao (2012) and have two forms. The first version of the co-kurtosis contagion test statistic is denoted CK_{13}

$$CK_{13}\left(i \rightarrow j; r_i^1, r_j^3\right) = \left(\frac{\widehat{\xi}_y\left(r_i^1, r_j^3\right) - \widehat{\xi}_x\left(r_i^1, r_j^3\right)}{\sqrt{\frac{18\widehat{v}_{y|x_i}^2 + 6}{T_y} + \frac{18\widehat{\rho}_x^2 + 6}{T_x}}} \right)^2, \tag{61}$$

where

$$\widehat{\xi}_y\left(r_i^m, r_j^n\right) = \frac{1}{T_y}\sum_{t=1}^{T_y}\left(\frac{y_{i,t} - \widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}}\right)^m \left(\frac{y_{j,t} - \widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}}\right)^n - 3\widehat{v}_{y|x_i}, \tag{62}$$

$$\widehat{\xi}_x\left(r_i^m, r_j^n\right) = \frac{1}{T_x}\sum_{t=1}^{T_x}\left(\frac{x_{i,t} - \widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}}\right)^m \left(\frac{x_{j,t} - \widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}}\right)^n - 3\widehat{\rho}_x. \tag{63}$$

The CK_{13} is to test for the transmission of contagion from the mean returns ($m = 1$) of the source market i to the return skewness ($n = 3$) of recipient market j through changes in the non-crisis co-kurtosis coefficient $\left(\widehat{\psi}_x\right)$ compared to the crisis period co-kurtosis coefficient $\left(\widehat{\psi}_y\right)$.

The second version of the co-kurtosis contagion test statistic is denoted CK_{31}

$$CK_{31}\left(i \rightarrow j; r_i^3, r_j^1\right) = \left(\frac{\widehat{\xi}_y\left(r_i^3, r_j^1\right) - \widehat{\xi}_x\left(r_i^3, r_j^1\right)}{\sqrt{\frac{18\widehat{v}_{y|x_i}^2 + 6}{T_y} + \frac{18\widehat{\rho}_x^2 + 6}{T_x}}} \right)^2. \tag{64}$$

where $\widehat{\psi}_y(r_i^m, r_j^n)$ and $\widehat{\psi}_x(r_i^m, r_j^n)$ are the estimated co-kurtosis ($m = 3, n = 1$) coefficient during the crisis and non-crisis period in equations (62) and (63). The $CK_{31}\left(i \rightarrow j; r_i^2, r_j^1\right)$ tests for contagion through new spillovers from the return skewness ($m = 3$) of the source market i to the mean returns ($n = 1$) of the recipient country j.

C.4 Co-volatility Contagion Test

The co-volatility contagion test is developed by Fry-McKibbin and Hsiao (2014) and is denoted CV_{22}

$$CV_{22}(i \to j; r_i^2, r_j^2) = \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^2\right) - \widehat{\psi}_x\left(r_i^2, r_j^2\right)}{\sqrt{\frac{4\widehat{v}_{y|x_i}^4 + 16\widehat{v}_{y|x_i}^2 + 4}{T_y} + \frac{4\widehat{\rho}_x^4 + 16\widehat{\rho}_x^2 + 4}{T_x}}} \right)^2, \tag{65}$$

where

$$\begin{aligned}
\widehat{\psi}_y(r_i^2, r_j^2) &= \frac{1}{T_y}\sum_{t=1}^{T_y}\left(\frac{y_{i,t} - \widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}}\right)^2 \left(\frac{y_{j,t} - \widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}}\right)^2 - \left(1 + 2\widehat{v}_{y|x_i}^2\right) \\
\widehat{\psi}_x(r_i^2, r_j^2) &= \frac{1}{T_x}\sum_{t=1}^{T_x}\left(\frac{x_{i,t} - \widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}}\right)^2 \left(\frac{x_{j,t} - \widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}}\right)^2 - \left(1 + 2\widehat{\rho}_x^2\right).
\end{aligned}$$

The $CV_{22}\left(i \to j; r_i^2, r_j^2\right)$ tests for contagion through new spillovers from the volatility $(m = 2)$ of the returns of the crisis market i to the volatility $(n = 2)$ of the returns of the recipient country j. Under the null hypothesis of no contagion, the test statistics FR, CS_1, CS_2, CK_1, CK_2 and CV_{22} are asymptotically distributed as χ_1^2.