Macroeconomic (In)stability of Interest Rate Rules in a Model with Banking System and Reserve Markets*

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Abstract

This paper develops a general equilibrium model with a banking system and a reserves market and shows that (i) the macroeconomic stabilizing properties of the nominal interest rate rules change quite substantially when we move from a model without a banking system to one with a banking system and a reserves market; (ii) the interplay between fiscal and monetary policies, in particular inflation-indexed versus non-indexed bonds, is crucial in determining the macroeconomic stabilizing properties of monetary rules; (iii) active rules and passive rules perform equally in regard to their macroeconomic stabilizing properties; (iv) continuous- and discrete-time specifications deliver the same/different (in)determinacy results for both the labor-only model and the endogenous-capital model under forward-looking/current-looking rules; (v) the inclusion of physical investment narrows the indeterminacy region under forward-looking rules; and (vi) current-looking rules make equilibrium determinacy impossible for both the labor-only economy and the endogenous-capital economy. Economic intuitions are provided.

Keywords: Nominal Interest Rate Rules, Indeterminacy, Open Market Operations, Credit Channel of Monetary Policy Transmission.

JEL Classification: E51, E58.

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1 Introduction

Since the seminal work of Taylor (1993), a vast literature has extensively analyzed the relative performance of active interest rate rules versus passive interest rate rules. As is well-known, many authors have suggested that to avoid real indeterminacy the central bank should adhere to active rules. Nevertheless, many others have still demonstrated that steering under active rules may introduce real indeterminacy in an otherwise determinate economy. In all situations, the existing literature seems to point to the conclusion that active rules and passive rules perform differently and that the relative desirability of active rules and passive rules crucially depends on the model features.[1] It then becomes a crucial issue for central bank to carefully select the monetary policy rule – the choice of an unsuitable interest rate feedback rule would cause the economy endogenous business fluctuations.

The main purpose of this paper is to re-examine the (in)equivalence of active rules and passive rules in regard to their macroeconomic stabilizing properties in a model with a banking system and a reserves market. The rationale for analyzing this kind of model is that, as is well-known, central bankers whose monetary policies are described as nominal interest rate rules conduct open market operations to adjust the supply of reserves in the reserves market, with a view to achieving their targets for the overnight loan rate (the federal funds rate in the United States). However, none of the existing studies incorporate the banking system *and* the reserves market in their theoretical models.[2] Since there is no federal funds rate in the model economy, as a conventional manipulation in the literature, the various authors generally use the nominal interest rate on government bonds as a proxy for the federal funds rate.[3] Such a manipulation seems plausible since the nominal interest rates on alternative financial instruments tend to move together over time. Nevertheless, this paper will show that once the federal funds market is incorporated, so that we no longer need to find any proxy for the federal funds rate, the macroeconomic stabilizing properties of active rules and passive rules turn out to be very different from what they would

[1] See Benhabib and Farmer (1999), McCallum (2003), and Woodford (2003) for a literature review.

[2] There are some tractable banking models in the interest rate rules literature (see, for example, Weder (2006) and Canzoneri et al. (2008)). However, none of the works establish the reserves market and hence are incapable of describing the central bank's open market operations.

[3] See, for example, Hirose (2013), Groshenny et al. (2013), and Anufriev et al. (2013).

be when we abstract from the model the banking system and the reserves market. In particular, in the model where there is no banking system and the reserves market, active rules and passive rules perform differently and hence it is crucial to choose a proper inflation coefficient in the nominal interest rate rules. Once the banking system and the reserves market are established, active rules and passive rules perform equally in regard to their macroeconomic stabilizing properties.

In addition to the aforementioned rationale, there is still another important reason, which has been approved in the literature, for the need to incorporate the banking system and the reserves market in the theoretical model. Through "the liquidity effect" of monetary policy, the central bank's open market operations affect the federal funds rate. Changes in the federal funds rate in turn influence the commercial banks' demand for excess reserves and hence the supply of loans to borrowers, which eventually affect aggregate demand in the economy. Such a channel of transmission for monetary policy, known as "the credit channel of monetary transmission," is absent in traditional models without the banking system and the reserves market. The literature on the credit channel of monetary policy transmission has demonstrated the important role of bank lending in explaining the length and the depth of business fluctuations [Bernanke 1983; Bernanke and Blinder 1988; Bernanke and Gertler 1995]. The development of this literature is in the light of the asymmetric treatment on bank assets and bank liabilities in traditional models. Specifically, as Bernanke and Blinder (1988, p.435) point out, "Money, the bank liability, is given a special role in the determination of aggregate demand. In contrast, bank loans are lumped together with other debt instruments in a "bond market," which is then conveniently suppressed by Walras' Law." In that paper, Bernanke and Blinder develop a variant of the IS/LM model which allows roles for both money and credit (bank loans). Both this paper and Bernanke and Gertler (1995) demonstrate the enhancement mechanism of the credit channel. Due to the existence of the credit channel, monetary policy still matters even in a liquidity trap. Bernanke and Blinder (1992), Kashyap et al. (1993), and Kashyap and Stein (1995) then provide empirical evidence that monetary transmission works through bank loans as well as bank deposits.[4]

[4] Other authors who analyze and support this credit view include Fuerst (1992), Li (2000), Einarsson and Marquis (2002), Li and Chang (2004), Gillman and Kejak (2004), Auray and Fève (2005), Chang et al. (2007), and Claus (2007), among others.

As the existing literature on the nominal interest rate rules lacks a model that incorporates the banking system and the reserves market, this paper attempts to develop a general equilibrium model of this kind, wherein the central bank sets the federal funds rate as a function of the inflation rate and affects the federal funds rate through open market operations. By means of the framework, this paper examines the local stability properties of the economy's steady state under active rules and passive rules, and compares the results with what we would obtain in a model without the banking system and the reserves market. Our key modeling strategy is based on some observations and facts. First, in view of the fact that commercial banks are the most important source of external funds for businesses, to simplify matters, we assume that firms have no access to the issuance of corporate bonds and/or equity. Therefore, bank loans are the only external funds that firms can acquire. Secondly, to characterize the central banker's open market operations, we assume that both the central bank and the commercial banks hold government bonds. The central bank adjusts the supply of reserves through open market operations. This together with commercial banks' demand for reserves (which equals the sum of required reserves plus excess reserves) determines the equilibrium federal funds rate.

Our numerical results show that regardless of the response of the federal funds rate to the inflation rate, there is always a unique steady state which exhibits saddle-path stability if inflation-indexed bonds are used for financing the government's deficits. In this case, worries about excessive volatilities under either active or passive regimes are unnecessary. However, with non-indexed bonds, both active rules and passive rules destabilizes the economy by giving rise to endogenous business fluctuations. The results are shown to be robust to changes in parameter values. We thus demonstrate (i) the important role of the interplay between fiscal and monetary policies, in particular inflation-indexed versus non-indexed bonds, in determining the macroeconomic stabilizing properties of monetary rules; and (ii) the equivalence of active rules and passive rules in regard to their macroeconomic stabilizing properties.

Our finding (i) is new in the literature. It is an interesting finding also because of the rising importance of inflation-indexed securities as an instrument of financing the governments' deficits in many countries including the U.S. and UK, among others [Campbell et al. 2009; Reschreiter 2010]. Regarding our finding (ii), it runs in sharp contrast to what we would obtain in a model without the banking system and the reserves market. The key reason is that in the present paper it is the federal funds rate that reacts to the inflation rate while in a model without the banking system and the reserves market the government bonds rate is supposed to react to inflation.

To understand the intuition, suppose that agents expect a higher inflation. In the model without the banking system, the monetary rule indicates that the nominal interest rate (of government bonds) increases. Under a passive policy, the real interest rate declines, which increases the shadow value of real financial wealth and hence the equilibrium marginal utility of consumption. The resulting higher consumption increases the inflation rate, thus validating the agents' initial inflationary expectations. By contrast, under an active policy, the real interest rate rises, thereby preventing the agents' inflationary expectations from become self-fulfilling.

In our model where the banking system and the reserves market are established, we note that the deposit rate is crucial in determining the actual inflation rate because it directly affects consumption demand. In addition, inflation indicates a loss of purchasing power of deposits; through the liquidity constraint, a decline in deposits caused by a higher expected inflation reduces consumption and hence the inflation rate.

There are two mechanisms by which a higher expected inflation affects the equilibrium deposit rate. First, upon the higher inflation expectations, the central bank will raise its target for the federal funds rate. To attain this higher federal funds rate target, the central bank will make an open market sale of government bonds which withdraws reserves from the banking system. This will reduce the supply of loans, which in turn increases the nominal loan rate and hence the nominal deposit rate. We demonstrate that regardless of the stance of monetary policy, in our model an open market sale by the central bank in response to a higher expected inflation reduces the real deposit rate.

Second, for non-indexed bonds, the principals and hence the nominal yields of government bonds are not indexed to inflation. Agents' inflation expectations therefore reduce the net (real) rate of return on government bonds. Through the arbitrage between government bonds and loans, the real loan rate declines as well. The resulting deterioration in commercial banks' real profits induces commercial banks to reduce the real deposit rate. It turns out that in the case of non-indexed bonds, the effect of a lower real deposit rate (which comes from both open market sales and non-indexed nominal yields of bonds) dominates that of a higher expected inflation. As a result, consumption increases, and agents' initial inflation expectations are validated in equilibrium. The whole process works for both active and passive rules. This makes the equivalence of active rules and passive rules in regard to their macroeconomic stabilizing properties.

If, by contrast, government bonds are inflation-indexed, their nominal yields increase with inflation, therefore the real yields of government bonds are unchanged. In this case, the effect of a lower real deposit rate comes solely from open market sales and hence is canceled by the effect of a higher expected inflation. As a consequence, agents' initial inflationary expectations will not be self-fulfilling and hence equilibrium indeterminacy and endogenous business fluctuations can never occur. Note again that the whole process works for both active and passive rules.

In view of the fact that the existing literature finds that due to other model features, the addition of endogenous physical capital may either narrow or enlarge the indeterminacy region, or switch the stabilizing properties of active and passive rules, we then include in the model the households' physical investment. By means of the model, our purpose is to find out how the inclusion of physical investment would affect the macroeconomic stabilizing properties of nominal interest rate rules in a banking model with the central bank's open market operations. Meanwhile, in response to the timing issue of monetary models with endogenous capital – for example, Carlstrom and Fuerst (2000*a*) versus Meng and Yip (2004), and Dupor (2001) versus Carlstrom and Fuerst (2005) and Li (2005) – we also carry out a robustness check for our result under a discrete-time setting.

It turns out that in our banking model with the central bank's open market operations, continuous- and discrete-time specifications deliver the same (in)determinacy results under forward-looking rules. This is because in this case the continuous-time modelling and the discrete-time modelling share the same no-arbitrage conditions. The discrete-time model with a forward-looking rule does not have a zero eigenvalue and hence the dimension of indeterminacy is not increased. Our result thus tends to support Meng and Yip's (2004) viewpoint that the role of endogenous physical capital is that it adds into the model an additional initial condition. Nevertheless, while in Meng and Yip (2004) the type of government bonds that are used for financing budget deficits does not play a role in determining the macroeconomic stabilizing properties of monetary rules, in our endogenous-capital model we still reach the conclusion that (i) the interplay between fiscal and monetary policies is crucial for the macroeconomic stabilizing properties of monetary rules; and (ii) active rules and passive rules perform equally in regard to their macroeconomic stabilizing properties.

Finally, we show that the current-looking rule exerts two effects on the model's equilibrium conditions. First, it introduces a zero eigenvalue to the dynamical system through the no-arbitrage condition between physical capital and government bonds. Second, through the equation which connects the rates of deposits and loans, the required reserves ratio, and the excess reserves ratio, it introduces an additional difference equation of the inflation rate (which is a jump variable) and an eigenvalue which lies inside the unit circle. The second effect exists in the labor-only economy, while in the endogenous-capital economy both effects present. As a consequence, in both economies all the determinate equilibria are eliminated by current-looking rules. Such a viewpoint against the current-looking rule is also made by Carlstrom and Fuerst (2000*a*).

The remainder of this paper is organized as follows. Section 2 develops a general equilibrium model with the banking system and the reserves market and analyzes the existence and number of the economy's steady states, together with the local stability properties under the nominal interest rate rules. Section 3 adds physical capital investment into the model in Section 2. Section 4 concludes.

2 The Model

The model builds on a simplified version of Meng (2002) in that we assume inelastic labor supply and a log utility.[5] The reason why we choose this as the starting point is that, as we will show later in this section, the model is simple and gives a standard result in the literature: equilibrium uniqueness is ensured only under active rules. We then borrow quite substantially from Agénor (1997) for the description of the

[5] As Meng (2002) demonstrates, endogenous labor supply along with the CRRA utility function complicate the macroeconomic stabilizing property of the interest rate rules.

borrowing-lending activities between firms and commercial banks, and develop in our own way the reserves market and the central bank's open market operations. We assume that there are no fundamental uncertainties present in the economy.

2.1 Producers

There is a continuum of identical competitive firms in the economy, with the total number normalized to one. The specification of the representative firm's production technology follows Benhabib et al. (2001): $y_t = h_t^\alpha$, $0 < \alpha < 1$, where y_t is output and h_t is labor hours. Following Agénor (1997), we assume for simplicity that firms have no access to capital markets. Since they cannot raise external funds by issuing corporate bonds and/or equity, the only way they can finance their working capital is by borrowing from commercial banks. Working capital needs consist solely of labor costs and must be financed prior to the sale of output. Total production costs faced by firms equal the wage bill plus the interest payments made on bank loans.

Given the production technology, the representative firm's objective is to choose a sequence $\{h_t, l_t^d\}_{t=0}^{\infty}$ so as to maximize its real (net) profits

$$\Pi_{ft} = y_t - w_t h_t - r_{lt} l_t^d, \tag{1}$$

subject to the financial constraint:

$$w_t h_t \leq l_t^d, \tag{2}$$

where w_t is the real wage rate, l_t^d is the real amount of loans obtained from commercial banks, and r_{lt} is the real loan rate charged by commercial banks.

Assume that the firm's financial constraint (2) is continuously binding since, given that borrowing is costly, there is no reason for the firm to borrow excess funds from commercial banks. As a result, we can re-write the representative firm's profit function as follows:

$$\Pi_{ft} = h_t^\alpha - (1 + r_{lt}) w_t h_t. \tag{3}$$

Under the assumption that the labor market is perfectly competitive, the representative firm's profit maximization leads to

$$(1 + r_{lt})w_t = \frac{\alpha y_t}{h_t}, \tag{4}$$

which shows that labor demand is inversely related to the effective cost of labor, $(1 + r_{lt})w_t$.

By combining (2) and (4), we derive the firm's demand for credit as

$$l_t^d = \frac{\alpha y_t}{1 + r_{lt}}, \tag{5}$$

which is increasing in output and is decreasing in the loan rate.

Let $m_{ft}(= l_t^d)$ denote cash held outside the banking system by firms on which the inflation tax is levied at the rate π_t. Firms transfer their net income, q_{ft}, to their owners, households:

$$q_{ft} = \Pi_{ft} - l_t^d - \pi_t m_{ft}. \tag{6}$$

2.2 Households

The economy is also populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time and supplies its time inelastically to the production of output. The representative household maximizes a stream of discounted utilities over sequences of consumption:

$$U = \int_0^\infty \ln c_t e^{-\rho t} dt, \tag{7}$$

where c_t is consumption, and $\rho \in (0, 1)$ is the rate of time preference.

The liquidity constraint faced by the representative household is given by:[6]

$$c_t \leq m_{ht} + \theta d_t, \tag{8}$$

which states that all cash holdings m_{ht} and a fraction $\theta \in [0, 1)$ of deposits d_t are used for financing the household's consumption purchases. The assumption that all cash holdings but only a fraction θ of deposits are used for financing consumption purchases is to capture the fact that, due to the transactions costs of withdrawing

[6] See Goodfriend and McCallum (2007) for a similar formulation.

deposits (for example, looking for an ATM or going to bank counters), deposits provide fewer liquidity services than do cash balances.

The representative household also faces the following flow budget constraint:

$$\dot{m}_{ht} + \dot{d}_t = w_t - c_t + r_{dt}d_t - \pi_t m_{ht} + q_{ft} + q_{bt}, \tag{9}$$

where r_{dt} is the real deposit rate, and q_{ft} and q_{bt} respectively represent real income received from firms and commercial banks.

The representative household's objective is to choose a sequence $\{c_t, d_t, m_{ht}\}_{t=0}^{\infty}$ so as to maximize its life-time utility (7), subject to the liquidity constraint (8) and the budget constraint (9), taking as given M_{h0}, D_0, and the time paths of w_t, r_{dt}, π_t, q_{ft} and q_{bt}. Let λ_t denote the shadow value of real financial wealth and η_t the Lagrange multiplier for the CIA constraint (8). As is common in the literature, we assume that the CIA constraint (8) is strictly binding in equilibrium, thus $\eta_t > 0$ for all t. The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t \quad : \quad c_t^{-1} = \lambda_t + \eta_t, \tag{10}$$

$$d_t \quad : \quad \frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_{dt} - \theta\frac{\eta_t}{\lambda_t}, \tag{11}$$

$$m_{ht} \quad : \quad \frac{\dot{\lambda}_t}{\lambda_t} = \rho + \pi_t - \frac{\eta_t}{\lambda_t}, \tag{12}$$

$$\mathrm{TVC}_1 \quad : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{mt}m_{ht} = 0, \tag{13}$$

$$\mathrm{TVC}_2 \quad : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{dt}d_t = 0. \tag{14}$$

Equation (10) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional real dollar. From (10)-(12) we obtain the following relationship:

$$R_{dt} = (1-\theta)\left(\frac{1}{c_t\lambda_t} - 1\right), \tag{15}$$

where $R_{dt} = r_{dt} + \pi_t$ denotes the nominal deposit rate. To understand (15), let us first notice that we assume for simplicity that households have no access to investing in government bonds. This assumption helps simplify our mathematical derivation and is not harmful at all since the assumption that both commercial banks and the central bank hold government bonds is enough to characterize the central bank's open market operations in a regime of nominal interest rate rules, which is the central focus of this paper. In addition, if we allow households' holdings of government bonds, the household's first-order conditions will not change except that $\left(\frac{1}{c_t \lambda_t} - 1\right)$ in (15) will be pinned down as the nominal interest rate on government bonds. Therefore, (15) actually describes a linkage between the nominal deposit rate and the nominal government bond rate in the society. It is obvious that the transactions cost of withdrawing deposits (which is captured by θ) drives a wedge between the two rates.

2.3 Commercial banks

Assets of commercial banks consist of required reserves, RR_t, excess reserves, ER_t, credit extended to firms, l_t^s, and the real stock of government bonds, b_{pt}. Assume that commercial banks have no access to money and capital markets. Thus, bank liabilities consist solely of deposits held by households, d_t. We further follow Agénor (1997) in assuming for simplicity that banks have no net worth. Commercial banks' balance sheets can then be expressed as

$$RR_t + ER_t + l_t^s + b_{pt} = d_t. \tag{16}$$

Interest is not paid on required reserves held at the central bank which are determined by

$$RR_t = vd_t, \tag{17}$$

where $v \in (0, 1)$ is the reserve requirement ratio. Neither is interest paid on excess reserves. Commercial banks hold excess reserves to insure against deposit outflows. The opportunity cost of holding excess reserves is the interest rate that could have been earned on lending these reserves out in the reserves market, which is the federal funds rate, R_{fft}. Thus, an increase in the federal funds rate will induce a reduction in commercial banks' demand for excess reserves.[7] Following Taylor (2001), Carpenter and Demiralp (2006) and Mishkin (2007), we capture this behavior of commercial

[7] Although we assume that there are no fundamental uncertainties present in the economy, there are sunspot shocks which may alter the agents' consumption and investment decisions (Chin et al. 2012; Harrison and Weder, 2013). Thus, even though the opportunity cost of holding reserves is positive, commercial banks have incentives to hold excessive reserves.

banks by specifying that the excess reserve ratio, $e_t \equiv \frac{ER_t}{d_t}$, is a decreasing function of the federal funds rate, R_{fft}:

$$e_t = e(R_{fft}), \;\; e' < 0. \tag{18}$$

From (16)-(18), we obtain the supply of credit as

$$l_t^s = (1 - v - e_t)d_t - b_{pt}. \tag{19}$$

Assume that banks have no operating costs. The net profits of the representative bank are

$$\Pi_{bt} = r_{lt}l_t^s + r_t b_{pt} - r_{dt}d_t, \tag{20}$$

where r_t is the real interest rate on government bonds. Given the no-arbitrage condition between holding government bonds and making loans, $r_t = r_{lt}$, it then follows from the zero-net-profit condition of the representative bank that

$$r_{dt}d_t = r_{lt}(l_t^s + b_{pt}). \tag{21}$$

By using (19) to substitute out $(l_t^s + b_{pt})$ in (21), we then obtain the following relationship between the real rates of lending and deposits:

$$r_{lt} = \frac{r_{dt}}{1 - v - e_t}. \tag{22}$$

The above equation clearly shows that the real lending rate (r_{lt}) is *ceteris paribus* positively related to the real deposit rate (r_{dt}), the reserve requirement ratio (v), and the excess reserve ratio (e_t). In particular, a higher real deposit rate represents higher interest costs of commercial banks. Therefore, commercial banks will respond by raising the real lending rate. On the other hand, a higher reserve requirement ratio or a higher excess reserve ratio denotes a reduction in the supply of loans. This will result in a higher loan rate.

Finally, since banks do not accumulate assets, net income transferred to households is

$$q_{bt} = \Pi_{bt} + l_t^s - \pi_t(RR_t + ER_t), \tag{23}$$

where the term $\pi_t(RR_t + ER_t)$ measures the inflation tax paid on reserves.

2.4 The government and the central bank

The specification of the central bank's interest rate feedback rule is very standard in the literature:

$$R_{fft} = \psi(\pi_t), \tag{24}$$

where the function $\psi(\cdot)$ is positive, increasing, and differentiable. Let π^* denote the steady-state inflation rate. In line with Leeper (1991), Meng (2002), and Benhabib et al. (2001), we refer to monetary policy as passive at π^* if $\psi'(\pi^*) < 1$ and as active at π^* if $\psi'(\pi^*) > 1$. Notice that since our model incorporates the reserves market, we no longer need to use the government bonds rate as the proxy for the federal funds rate; we let the federal funds rate respond to the inflation rate.

Assume that the central bank lends only to the government. The central bank's balance sheet is thus given by

$$b_{gt} = m_t + RR_t + ER_t, \tag{25}$$

where b_{gt} is the real stock of government bonds held by the central bank, $m_t = m_{ft} + m_{ht}$ is currency in circulation.

The government's expenditure consists of interest payments to its bond holders: commercial banks and the central bank. As a convention, the central bank transfers its revenue, which consists only of interest receipts from the government, to the government. The government's deficits are then financed by the issuance of government bonds. The flow budget constraint of the government is thus given by[8]

$$\dot{b}_t = r_t b_{pt} - \pi_t b_{gt} = r_t b_t - (r_t + \pi_t) b_{gt}, \tag{26}$$

[8] Since (26) is not very standard, we would like to set aside some space to prove it. The government budget constraint in nominal terms is given by:

$$\dot{B}_t = R_{bt} B_{pt}.$$

Given the definitions $b_t \equiv B_t/P_t$, $b_{gt} \equiv B_{gt}/P_t$, and $b_{pt} \equiv B_{pt}/P_t$, where B_t, B_{gt}, and B_{pt} respectively denote the corresponding nominal stocks and P_t is the price level, it is straightforward to demonstrate that the budget constraint in real terms is expressed as (26).

where $b_t = b_{pt} + b_{gt}$ is the aggregate stock of real government bonds. Notice from the central bank's balance sheet (25) that b_{gt} equals the sum of currency in circulation (m_t) and reserves ($RR_t + ER_t$) which equals high-powered money. Therefore, $\pi_t b_{gt}$ in the first equality of (26) measures the inflation taxes on high-powered money.

2.5 Open market operations and reserve market equilibrium

An open market purchase of government bonds from commercial banks increases the stock of government bonds held by the central bank and decreases the stock of government bonds held by commercial banks by the same amount. Therefore, it does not affect the outstanding stock of government bonds. However, an open market purchase injects reserves into the banking system, and thereby increases the available funds that can be used for making new loans. This is the so-called lending channel of monetary policy transmission.

Since our focus is on how the central bank implements the interest rate rules through open market operations, rather than on how the extension of credit by the central bank to the government causes inflation, we assume for simplicity and without loss of generality that for every issuance of bonds the government allocates a fixed proportion $\phi \in (0,1)$ of the government bonds to the central bank and the remaining proportion $1-\phi \in (0,1)$ to commercial banks.[9] The realized stock of government bonds held by the central bank, b_{gt}, is thus ϕb_t plus the amount due to open market purchases. This indicates that the amount of open market purchases and hence the amount of reserves injected into the reserves market is $b_{gt} - \phi b_t$.

The quantity of reserves demanded equals the sum of required reserves and excess reserves. We assume that the federal funds rate is below the discount rate, which is true at almost every date for every country. Thus, commercial banks will not borrow from the discount window and hence the supply of reserves will equal the nonborrowed reserves, that is the amount of reserves supplied by the central bank through open market operations: $b_{gt} - \phi b_t$. The reserves market equilibrium requires that the quantity of reserves demanded equals the quantity of reserves supplied, which is written as

[9] This can be interpreted as an extremely passive fiscal policy rule.

$$RR_t + ER_t = b_{gt} - \phi b_t. \tag{27}$$

According to (27), the demand for reserves is a downward-sloping curve in a diagram with the federal funds rate on the vertical axis. On the other hand, the supply of reserves is a vertical line in the diagram. Such a viewpoint is also made by Taylor (2001) and Mishkin (2007).

The central bank's balance sheet (25) and the reserves market equilibrium condition (27) together imply that

$$m_t = \phi b_t, \tag{28}$$

which states that, at each instant in time, currency in circulation, m_t, equals the real credit allocated by the central bank to the government, ϕb_t.

2.6 Credit market equilibrium and the resource constraint

Credit market equilibrium requires that the firms' demand for credit equals the credit extended to firms by commercial banks. By equating (5) with (19), we obtain this condition as follows:

$$\underbrace{\frac{\alpha y_t}{1 + r_{lt}}}_{l_t^d} = \underbrace{(1 - v - e_t)d_t - b_{pt}}_{l_t^s} = l_t\,, \tag{29}$$

The above credit market equilibrium condition determines the equilibrium lending rate.

By combining the representative commercial bank's balance sheet (16) with the central bank's balance sheet (25) and given that $l_t^d = l_t^s = l_t$ holds when the credit market is in equilibrium, we obtain

$$m_{ht} + d_t = b_t, \text{ or } m_t + d_t = l_t + b_t, \tag{30}$$

which indicates that the money supply, $m_t + d_t$, equals the quantity of loans plus the stock of government bonds. Notice that the quantity of loans is the credit extended to firms by commercial banks, and the stock of government bonds equals the credit extended to the government by commercial banks and the central bank.

By taking the time derivation on both sides of the first equation in (30), we obtain

$$\dot{m}_{ht} + \dot{d}_t = \dot{b}_t. \tag{31}$$

By substituting q_{ft} in (6) and q_{bt} in (23) into the household's budget (9), and given that $l_t^d = l_t^s = l_t$ holds when the credit market is in equilibrium, with (25) and (26), we have

$$\dot{m}_{ht} + \dot{d}_t = y_t - c_t + \dot{b}_t. \tag{32}$$

Equations (31) and (32) together give the economy's resource constraint as follows:

$$y_t = c_t. \tag{33}$$

2.7 Analysis of local dynamics

This subsection analyzes the existence and uniqueness of the model's steady state, together with the associated local dynamics. We start with defining the equilibrium:

Definition 1. *A monetary equilibrium is a set of paths* $\{c_t, m_{ht}, d_t, \pi_t, \lambda_t, r_{dt}, r_{lt}, R_{fft}, b_{gt}, b_t, l_t, y_t\}_{t=0}^{\infty}$ *that satisfies:*

(i) the firm's production function $y_t = h_t^{\alpha}$*, financial constraint (2), optimization (4), and net income transfers to the household (6);*

(ii) the household's liquidity constraint (8), budget constraint (9), optimization (10)-(12), and transversality conditions (13) and (14);

(iii) the commercial bank's balance sheet (19), optimization (21), and net income transfers to the household (23);

(iv) the central bank's monetary policy rule (24) and balance sheet (25), and the government's budget constraint (26);

(v) the market clearing conditions (27), (29), and $h_t = 1$.

Under Definition 1, Appendix 5.1 provides the detailed derivation of the dynamical system that governs the dynamics of the model which is presented in the following proposition:

Proposition 1. *The dynamics of the economy are fully characterized by the following differential equations:*

$$\dot{b}_t = r_{lt} b_t - (r_{lt} + \pi_t) b_{gt}, \tag{34}$$

$$\dot{\lambda}_t = (\rho + \pi_t + 1)\lambda_t - 1, \tag{35}$$

where $r_{lt} = \frac{(1-\theta)\alpha}{[\phi+\theta(1-\phi)]b_t} - 1$; $\pi_t = \psi^{-1}(R_{fft})$, *with* $\psi^{-1\prime} = \frac{1}{\psi'} > 0$; $R_{fft} = R_{ff}(\underset{+}{b_t}, \underset{-}{\lambda_t})$ *and* $b_{gt} = b_g(\underset{\pm}{b_t}, \underset{+}{\lambda_t})$, *with the partial derivatives given in the Appendix* 5.1*.*

Given the above dynamical system (34) and (35), the steady state is characterized by a pair of positive real numbers (b^*, λ^*) that satisfy $\dot{b}_t = \dot{\lambda}_t = 0$. It is straightforward to obtain

$$b^* = \frac{(1-\theta)\alpha}{[\phi + \theta(1-\phi)]\left[1 + \frac{(1-\theta)\rho - \theta\psi^{-1}(R^*_{ff})}{1-v-e(R^*_{ff})}\right]}, \tag{36}$$

$$\lambda^* = \frac{1}{\rho + \psi^{-1}(R^*_{ff}) + 1}, \tag{37}$$

where the steady-state federal funds rate R^*_{ff} is the solution to the following equation:

$$\underbrace{r^*_l}_{LHS} = \underbrace{\frac{[r^*_l + \psi^{-1}(R^*_{ff})][(1-\theta)\phi + v + e(R^*_{ff})]}{1-\theta}}_{RHS}, \tag{38}$$

where $r^*_l = \frac{(1-\theta)\rho - \theta\psi^{-1}(R^*_{ff})}{1-v-e(R^*_{ff})}$.[10] The remaining endogenous variables at the economy's steady state can then be derived accordingly.

To examine the existence and number of the economy's steady state in a transparent manner, we let $f(R^*_{ff}) = LHS - RHS$ from (38). Therefore, the equilibrium federal funds rate R^*_{ff} will be located at the intersection of $f(R^*_{ff})$ and the horizontal axis. Due to the complicated functional form of $f(R^*_{ff})$, we need to resort to a numerical method to plot $f(R^*_{ff})$. We will carry out this task later on.

[10] Equation (38) is obtained from (34) with $\dot{b}_t = 0$ and (80). From (35) with $\dot{\lambda}_t = 0$, we obtain $\lambda^* = \frac{1}{\rho+\psi^{-1}(R^*_{ff})+1}$. This, together with (79), lead to $r^*_l = \frac{(1-\theta)\rho-\theta\psi^{-1}(R^*_{ff})}{1-v-e(R^*_{ff})}$.

In terms of the steady-state's local stability properties, we linearize the dynamical system (34) and (35) around the steady state to obtain the following linear system:

$$\begin{bmatrix} \dot{b}_t \\ \dot{\lambda}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} b_t - b^* \\ \lambda_t - \lambda^* \end{bmatrix}, \tag{39}$$

where $\mathbf{J}_{11} = -1 - [r_l^* + \psi^{-1}(R_{ff}^*)]b_{g,b} + \frac{(1-\theta)\alpha b_g^*}{[\phi+\theta(1-\phi)](b^*)^2} - \frac{b_g^* R_{ff,b}}{\psi'}$, $\mathbf{J}_{12} = -[r_l^* + \psi^{-1}(R_{ff}^*)]b_{g,\lambda} - \frac{b_g^* R_{ff,\lambda}}{\psi'}$, $\mathbf{J}_{21} = \frac{\lambda^* R_{ff,b}}{\psi'}$, and $\mathbf{J}_{22} = \frac{\lambda^* R_{ff,\lambda}}{\psi'} + \frac{1}{\lambda^*}$.

The stability of a steady state is determined by comparing the eigenvalues of $\mathbf{J}$ that have negative real parts to the number of initial conditions in the dynamical system (34) and (35). λ_t is a jump variable. However, whether b_t is pre-determined or not depends on whether government bonds are inflation-indexed or not. For inflation-indexed bonds, since the principal of bonds is indexed to inflation, b_t is pre-determined. In this case, the steady state displays saddlepath stability and equilibrium uniqueness when the two eigenvalues of $\mathbf{J}$ have opposite signs. If more than one eigenvalue has a negative real part, then the steady state is locally indeterminate (a sink) and can be exploited to generate endogenous business fluctuations driven by agents' self-fulfilling expectations or sunspots. If both eigenvalues have positive real parts, then the steady state is a source. If government bonds are not inflation-indexed, b_t is a jump variable. In this case, the steady state displays equilibrium uniqueness if and only if both eigenvalues of $\mathbf{J}$ have positive real parts; otherwise, the steady state will exhibit local indeterminacy. Still, we need to resort to a numerical method to calculate the eigenvalues of $\mathbf{J}$.

To carry out the quantitative analysis, we first need to specify explicit functional forms for the excess reserve ratio function $e(\cdot)$ and the interest rate feedback function $\psi(\cdot)$. Following Taylor (2001), we specify a linear excess reserve ratio function as follows:

$$e_t = e_0 - e_1 R_{fft}, \tag{40}$$

where e_0 is a constant intercept term and $e_1 > 0$ measures the slope of the excess reserve ratio function.

We then follow McCallum and Nelson (1999) and Kurozumi (2006), among others, in specifying the following interest rate feedback function:

$$\psi(\pi_t) = \psi_0 + \psi_1 \pi_t, \tag{41}$$

where ψ_0 is a constant intercept term, and $\psi_1 > 1$ and $0 < \psi_1 < 1$ respectively represent the cases of active rules and passive rules.

Our benchmark parameterization is as follows. The time unit is assumed to be a quarter. The labor share α and the rate of time preference ρ are set at standard values used in the literature: $\alpha = 0.66$ and $\rho = 0.0045$, where the latter is chosen to imply an annual 1.8% discount rate [see, for example, Benhabib et al. (2001), Dupor (2001)].

In order to obtain values for the intercept and the slope of the excess reserve ratio function, i.e., e_0 and e_1 in (40), we estimate (40) using monthly data for the effective federal funds rate and aggregate reserves and deposits of depository institutions provided by the Board of Governors of the Federal Reserve System, over the period 1980:1-2009:12. The estimation result is presented as follows (standard errors in parentheses):

$$\begin{aligned} e_t &= \underset{(0.002)}{0.015482} - \underset{(0.00028)}{0.00178} R_{fft} \qquad \sigma_e = 0.02 \qquad R^2 = 0.099 \end{aligned} \tag{42}$$

It is clear that both the intercept and the coefficient of the federal funds rate have very low estimated standard errors and are very significantly different from zero. The estimation result is consistent with Taylor's (2001, p. 23) view that "...transactions costs and high penalties for overnight overdrafts suggest that α (which is e_1 in (40)) should be less than infinity and possibly quite small." In addition, the fact that the coefficient on the federal funds rate is significantly different from zero supports Taylor's (2001, p. 23) view that the coefficient is greater than zero.

The reserve requirement ratio is set at its average value in the same sample period: $v = 0.01482$. According to (28), in our model ϕ is pinned down as the currency to government debt ratio: $\phi = m_t/b_t$. The average value of ϕ in the sample period 1980:1-2009:12 is 0.084.[11] To obtain θ, we notice from (15) that θ determines the

[11] While data of currency is obtained from the Data Download Program of the Board of Governors of

wedge between the rates of deposit and government bonds: $\theta = 1 - R_{dt}/R_t$. Since we consider deposits as interest-bearing, providing liquidity services, and being subjected to reserve requirement, interest checking accounts are suitable choices. Due to the availability of data, we take the average value of θ over the period 2000:12-2009:12, which is 0.958.[12]

The response of the federal funds rate to the inflation rate, ψ_1, is set at 1.5 for active rules, so that at the steady state the interest rate rule has the slope suggested by Taylor (1993) [Benhabib et al. (2001) and Dupor (2001), among others]. For passive rules, we follow Dupor (2001) in adopting a value $\psi_1 = 0.99$. Finally, we set the intercept of the interest rate feedback function as $\psi_0 = 0.015$. The benchmark parameterization implies that the federal funds rate is 6% per year, which equals the average effective federal funds rate in the period 1980:1-2009:12.

We are now in a position to analyze the existence and uniqueness of the model's steady state and the associated local dynamics. While plotting $f(R^*_{ff})$, we allow the federal funds rate to vary between 0% and 25%. Note that this range covers all the possibilities of the federal funds rate since the time unit is assumed to be a quarter.

Figure 1(a) illustrates the the results of the benchmark case: $\alpha = 0.66$, $\rho = 0.0045$, $e_0 = 0.015482$, $e_1 = 0.00178$, $v = 0.01482$, $\phi = 0.084$, $\theta = 0.958$, and $\psi_0 = 0.015$. In addition to $\psi_1 = 1.5$ for active rules and $\psi_1 = 0.99$ for passive rules, Figure 1(a) also plots $f(R^*_{ff})$ for other values of ψ_1 to see how the result changes to ψ_1. Under each parameterization, we calculate the eigenvalues of the Jacobian matrix J in (39). The result is presented in Table 1(a). As Figure 1(a) clearly depicts, in the benchmark case, $f(R^*_{ff})$ always intersects the horizontal axis once and there is therefore a unique steady state. We then see from Table 1(a) that the steady state is always characterized by one positive root and one negative root. This indicates that regardless of the response of the federal funds rate to the inflation rate, ψ_1, the steady

---

the Federal Reserve System, data of government debt is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis.

[12] Data of the treasury yields is obtained from the Board of Governors of the Federal Reserve System. Data of the rate on interest checking accounts is obtained from FRED of the Federal Reserve Bank of St. Loui. Since data of the rate on interest checking accounts is not available until 2000:12, we use the average value of θ over the period 2000:12-2009:12. Our benchmark value $\theta = 0.958$ is obtained by using the market yield on U.S. Treasury securities at 20-year constant maturity. If the yield on inflation-indexed securities is used, then $\theta = 0.93$. If the Treasury long-term (over 10 years) average rate is used, then $\theta = 0.93$ as well. No matter which bonds rate is used, θ is very stable over the sample period, with the lowest value at around 0.88.

state always exhibits saddlepath stability if government bonds are inflation-indexed. If, by contrast, government bonds are not inflation-indexed, the steady state always exhibits equilibrium indeterminacy. This result holds for cases where ψ_1 is bigger than 1.5.

The above result demonstrates (i) the important role of the interplay between fiscal and monetary policies, in particular inflation-indexed versus non-indexed bonds, in determining the macroeconomic stabilizing properties of monetary rules; and (ii) the equivalence of active rules and passive rules in regard to their macroeconomic stabilizing properties. To assess the robustness of the benchmark parameterization result, in what follows we consider variations in some parameter values. In Figure 1(b) and Table 1(b), we first consider a reduction in θ from 0.9 to 0.5, which represents a higher transactions cost of financing consumption purchases using deposits. It turns out that the equilibrium federal funds rate increases, but the steady-state's stability properties remain unchanged. Figure 1(c) and Table 1(c) illustrate the extreme case where only cash is used for financing consumption purchases. While the steady-state's stability properties are not changed, the equilibrium federal funds rate for the cases where $\psi_1 = 1.5$ and 0.99 are too high to be empirically plausible since the monthly data for the annual effective federal funds rate never exceed 20% during the period 1980:1-2009:12 (actually since when the data become available at the Board of Governors of the Federal Reserve System).[13] We then consider in Figure 1(d) and Table 1(d) an increase in ϕ from 0.5 to 0.9, and in Figure 1(e) and Table 1(e) a reduction in ϕ from 0.5 to 0.01. It is obvious that everything that we obtained under the benchmark parameterization, including the equilibrium federal funds rate and the steady-state's stability properties, remains unchanged.

2.8 Mechanism of equilibrium (in)determinacy

The result obtained in the previous subsection runs in sharp contrast to what we would obtain in a model without the banking system and the reserves market. To understand why, in what follows we first present the model without the banking system. We then, by comparing the models with and without the banking system, explain the mechanism of why different (in)determinacy results are obtained.

[13] The highest annual federal funds rate appears in 1980:01 which is 19.08%.

As a conventional manipulation in the literature, in the absence of the banking system and the reserves market, we need households to invest in government bonds so that the model has a nominal interest rate that can serve as a proxy for the federal funds rate. In this case, the representative household's objective is to choose a sequence $\{c_t, m_t, b_t\}_{t=0}^{\infty}$ so as to maximize its life-time utility (7), subject to the liquidity and the budget constraints:

$$
\begin{aligned}
c_t &\leq m_t, && (43) \\
\dot{m}_t + \dot{b}_t &= w_t - c_t + (R_t - \pi_t)b_t - \pi_t m_t + \Pi_{ft}, && (44)
\end{aligned}
$$

where R_t is the nominal interest rate on government bonds, and $\Pi_{ft} = (1 - \alpha)y_t$ denotes the receipt of dividends from firms. Since there is no banking system, the representative firm's problem is standard.

The government's budget constraint and the central bank's interest rate feedback rule are also standard:

$$
\begin{aligned}
\dot{m}_t + \dot{b}_t &= (R_t - \pi_t)b_t - \pi_t m_t, && (45) \\
R_t &= \varphi(\pi_t), && (46)
\end{aligned}
$$

where we refer to monetary policy as passive at π^* if $\varphi'(\pi^*) < 1$ and as active at π^* if $\varphi'(\pi^*) > 1$.

The model's equilibrium conditions can be reduced to a single differential equation:

$$
\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \underbrace{\left(\frac{1}{\lambda_t} - 1\right)}_{R_t} + \pi_t, \tag{47}
$$

where $\pi_t = \psi^{-1}(R_t)$, with $\psi^{-1\prime} = \frac{1}{\psi'} > 0$, is obtained from (46). It is straightforward to demonstrate that the eigenvalue of the above differential equation is $\frac{\psi'-1}{\psi'\lambda^*}$, which is positive (negative) if $\psi' > (<)0$. Since λ_t is a jump variable, the model generates a standard result that active rules maintain saddle-path stability while passive rules must give rise to local indeterminacy. It is noticeable that whether government bonds are inflation-indexed or not does not matter for the (in)determinacy result.

The counterpart of (47) for the model with the banking system is (35), which can be re-written as

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \left(\frac{1}{\lambda_t} - 1\right) + \pi_t, \tag{48}$$

where $\pi_t = \psi^{-1}(R_{fft})$, with $\psi^{-1\prime} = \frac{1}{\psi'} > 0$. As we have mentioned in section 2.2, in the model with the banking system, if we allow households' holdings of government bonds, $\left(\frac{1}{\lambda_t} - 1\right)$ in (48) will be pinned down as the nominal interest rate on government bonds. Obviously, (47) and (48) look exactly the same except that in the model without the banking system the government bonds rate R_t reacts to the inflation rate, therefore $\pi_t = \psi^{-1}(R_t)$; whereas, in the model with the banking system, it is the federal funds rate R_{fft} that reacts to the inflation rate, therefore $\pi_t = \psi^{-1}(R_{fft})$.

Suppose that agents expect a higher inflation. In the model without the banking system, the monetary rule (46) indicates that the nominal interest rate R_t will be increased. Under a passive policy, the real interest rate $R_t - \pi_t$ declines as a result, which causes the shadow value of real financial wealth λ_t and hence the equilibrium marginal utility of consumption to rise. This leads to an increase in agents' consumption and hence an increase in the inflation rate, thus validating the agents' initial inflationary expectations. By contrast, under an active policy, the real interest rate rises, therefore preventing the agents' inflationary expectations from becoming self-fulfilling.

To understand the intuition for equilibrium (in)determinacy in the model with the banking system, note that the deposit rate is crucial in determining the actual inflation rate because it directly affects consumption demand. In particular, (11), which is equivalent to (48), can be re-written as

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \frac{r_{dt} + \theta\pi_t}{1-\theta}. \tag{49}$$

The above equation indicates that: (i) a higher real deposit rate will decrease the shadow value of real financial wealth, which will in turn decrease the equilibrium marginal utility of consumption (equation (10)), thereby causing a lower desired consumption and hence a lower inflation; (ii) inflation indicates a loss of purchasing power of deposits; this causes the households to reduce their deposits and subsequently, through the liquidity constraint (8), consumption and hence the inflation rate decrease.

Let us start the economy from its steady state, and consider a slight deviation caused by a higher expected inflation by agents. There are two mechanisms by which the higher expected inflation affects the equilibrium deposit rate. First, upon the belief, the central bank will raise its target for the federal funds rate by $\psi' \Delta\pi$. To attain this higher federal funds rate target, the central bank will make an open market sale of government bonds which withdraws reserves from the banking system. This will reduce the supply of loans, which in turn increases the nominal loan rate. Given that in equilibrium the nominal loan rate equals the nominal government bonds rate, from (37) we obtain that in the neighborhood of the steady state a one percentage point increase in the funds rate causes the nominal loan rate to increase by $\frac{1}{\psi'}$ percentage points. Therefore, the open market sale by the central bank causes the nominal loan rate to increase by $\Delta\pi$ percentage points. Equation (15) indicates that the nominal deposit rate will then increase by $(1-\theta)\,\Delta\pi$ percentage points. Eventually, the inflation adjusted real deposit rate decreases by $\theta\Delta\pi$ percentage points.

Second, for non-indexed bonds, the principals and hence the nominal yields of government bonds are not indexed to inflation. Agents' inflation expectations therefore reduce the net (real) rate of return on non-indexed bonds. Through the arbitrage between government bonds and loans, the real loan rate declines as well. The resulting deterioration in commercial banks' real profits induces commercial banks to reduce the real deposit rate. Combining with the effect of open market sales, we tell that in the case of non-indexed bonds, the real deposit rate decreases by more than $\theta\Delta\pi$ percentage points. By referring to (49), the effect on the shadow value of real financial wealth of a lower real deposit rate dominates that of a higher expected inflation. As a result, consumption increases, and agents' initial inflation expectations are validated in equilibrium. Note that the whole process works for both active and passive rules.

If government bonds are inflation-indexed, their nominal yields increase with inflation, therefore the real yields of government bonds are unchanged. Since the mechanism described in the previous paragraph by which the government bonds rate affects the loan rate, the deposit rate, consumption, and hence the inflation rate is not at work in this case, the effect on the shadow value of real financial wealth of a lower real deposit rate (caused solely by open market sales) cancels that of a higher expected inflation. Therefore, agents' initial inflationary expectations will not be self-fulfilling and hence equilibrium indeterminacy and endogenous business fluctuations can never occur. Note again that the whole process works for both active and passive rules.

3 Model with Investment

In the literature, many efforts have been devoted to developing the implications of interest rate feedback rules on aggregate stability in circumstances with capital accumulation [See, for example, Carlstrom and Fuerst (2000a, b), Dupor (2001, 2002), Lubik (2003), Meng and Yip (2004), Carlstrom and Fuerst (2005), Li (2005), Huang and Meng (2007), Huang et al. (2009), and Gliksberg (2009)]. These contributions together point to the conclusion that due to other model features, the addition of endogenous physical capital may either narrow or enlarge the indeterminacy region, or switch the stabilizing properties of active and passive rules.[14] It is thus of our interest to explore how the inclusion of physical investment would affect the macroeconomic stabilizing properties of nominal interest rate rules in a banking model with the central bank's open market operations.

For this purpose, we incorporate physical investment into the model we constructed in the previous section. To make the least modification to the model, we assume that households invest in new capital, and rent the capital stock in a competitive market to firms for production purposes. Firms borrow from commercial banks to finance their working capital, which consists of labor wage and capital rental costs. Other model features remain exactly the same as we described in the previous section. In what follows we only illustrate the related equations that need modification.

First, the representative firm's production technology takes the Cobb-Douglas form: $y_t = h_t^{\alpha} k_t^{\beta}$, where k_t is the capital stock, and $0 < \alpha < 1$ and $0 < \beta < 1$

[14] These model features include the adjustment costs of investment, flexible versus sticky prices, the monopolistic distortions, productive externalities, the cost share of capital, the steady-state inflation, the labor supply elasticity, non-separable leisure, productive money, liquidity-constrained investment purchases, the cash-in-advance timing versus the cash-when-I'm-done timing, and whether the policy is forward-looking, current-looking, or backward-looking.

represent the labor and capital shares of national income, respectively. Given this production technology, the representative firm's objective is to choose a sequence $\left\{h_t, k_t, l_t^d\right\}_{t=0}^{\infty}$ so as to maximize its real (net) profits

$$\Pi_{ft} = y_t - w_t h_t - r_{kt} k_t - r_{lt} l_t^d, \tag{50}$$

subject to the financial constraint:

$$w_t h_t + r_{kt} k_t \leq l_t^d, \tag{51}$$

where r_{kt} is the rental rate of capital.

As in the previous section, profit maximization of the firm leads to the following equations which state that factor demands are inversely related to the effective cost of production factors:

$$(1 + r_{lt}) w_t = \frac{\alpha y_t}{h_t} \quad \text{and} \quad (1 + r_{lt}) r_{kt} = \frac{\beta y_t}{k_t}. \tag{52}$$

By combining (51) and (52), we obtain the firm's demand for credit as $l_t^d = \frac{(\alpha+\beta) y_t}{1+r_{lt}}$. Therefore, the credit market equilibrium condition is given by:

$$\underbrace{\frac{(\alpha + \beta) y_t}{1 + r_{lt}}}_{l_t^d} = \underbrace{(1 - v - e_t) d_t - b_{pt}}_{l_t^s} = l_t\,. \tag{53}$$

The related modification to the representative household's problem involves the budget constraint and the addition of the law of motion of physical capital:

$$\dot{m}_{ht} + \dot{d}_t = w_t + r_{kt} k_t - c_t - i_t + r_{dt} d_t - \pi_t m_{ht} + q_{ft} + q_{bt}, \tag{54}$$

$$\dot{k}_t = i_t - \delta k_t, \;\; k_0 \; > 0 \;\text{ given}, \tag{55}$$

where i_t is investment, and $\delta \in [0, 1]$ is the capital depreciation rate. This completes the description of the modification of the model.

From the representative household's first-order conditions we obtain the following no-arbitrage conditions between physical capital, deposits, and government bonds:

$$r_{kt} - \delta = r_{dt} + \theta\left(\frac{1}{c_t\lambda_t} - 1\right) = \left(\frac{1}{c_t\lambda_t} - 1\right) - \pi_t. \tag{56}$$

Recall from the previous section that $\left(\frac{1}{c_t\lambda_t} - 1\right)$ in (56) represents the nominal interest rate of government bonds.

In addition, the economy's resource constraint can be obtained as follows:

$$y_t = c_t + i_t. \tag{57}$$

As in the previous section, we first define the equilibrium:

Definition 2. *A monetary equilibrium in the model with endogenous capital is a set of paths* $\{c_t, m_{ht}, d_t, \pi_t, \lambda_t, r_{dt}, r_{lt}, R_{fft}, b_{gt}, b_t, l_t, y_t, k_t, i_t\}$ *that satisfies:*

(i) the firm's production function, financial constraint, optimization, and net income transfers to the household;

(ii) the household's liquidity constraint, budget constraint, the law of motion of physical capital, optimization, and transversality conditions;

(iii) the commercial bank's balance sheet, optimization and net income transfers to the household;

(iv) the central bank's monetary policy rule and balance sheet, and the government's budget constraint;

(v) the market clearing conditions for the reserves market, the credit market, and the labor market.

Under Definition 2, we obtain the dynamical system that governs the dynamics of the model which is presented in the following proposition:

Proposition 2. *The dynamics of the economy with endogenous capital are fully characterized by the following differential equations:*

$$\dot{b}_t = r_{lt}b_t - (r_{lt} + \pi_t)b_{gt}, \tag{58}$$

$$\dot{\lambda}_t = (\rho + \pi_t + 1)\lambda_t - \frac{1}{[\phi + \theta(1-\phi)]b_t - (1-\theta)l_t}, \tag{59}$$

$$\dot{k}_t = k_t^{\beta} - \underbrace{\{[\phi + \theta(1-\phi)]b_t - (1-\theta)l_t\}}_{c_t} - \delta k_t \tag{60}$$

where $r_{lt} = \frac{(\alpha+\beta)k_t^{\beta}}{l_t} - 1$*;* $\pi_t = \psi^{-1}(R_{fft})$*, with* $\psi^{-1\prime} = \frac{1}{\psi'} > 0$*;* $R_{fft} = R_{ff}(b_t, \lambda_t, k_t)$*,* $b_{gt} = b_g(b_t, \lambda_t, k_t)$*, and* $l_t = l(b_t, \lambda_t, k_t)$*.*

The way we analyze the existence and uniqueness of the model's steady state and the associated local dynamics is the same as what we did in the previous section. To parameterize the model, we set the capital share of national income β at 0.3 and the capital depreciation rate δ at 0.025, where the latter corresponds to a 10% annual rate. Other parameter values are the same as those we adopted in the previous section. We present the simulation results in Table 2 and Figure 2, where the equilibrium federal funds rate R_{ff}^* is located at the intersection of $g(R_{ff}^*)$ and the horizontal axis.[15] It is clear that the economy always has a unique steady state which is characterized by one negative root and two positive roots.[16] Since endogenous capital adds into the dynamical system (58)-(60) an additional initial condition, we obtain the results that there exists no equilibrium with inflation-indexed bonds and that with non-indexed

[15] The model's steady-state conditions are given by:

$$\begin{aligned} b^* &= \frac{(k^*)^{\beta} + (1-\theta)l^* - \delta k^*}{\phi + \theta(1-\phi)}, \\ \lambda^* &= \frac{1}{\{[\phi + \theta(1-\phi)]b^* - (1-\theta)l^*\}[\rho + \psi^{-1}(R_{ff}^*) + 1]}, \\ k^* &= \frac{\beta l^*}{(\delta+\rho)(\alpha+\beta)}, \end{aligned}$$

where $l^* = (\alpha+\beta)\left\{\left[\frac{(1-\theta)\rho - \theta\psi^{-1}(R_{ff}^*)}{1-v-e(R_{ff}^*)} + 1\right]\left(\frac{\delta+\rho}{\beta}\right)^{\beta}\right\}^{1/(\beta-1)}$, and the steady-state federal funds rate R_{ff}^* is the solution to the following equation:

$$\underbrace{[1 - v - e(R_{ff}^*)](1-\phi)r_l^* - \{[v + e(R_{ff}^*)](1-\phi) + \phi\}\psi^{-1}(R_{ff}^*)}_{LHS} = \underbrace{\frac{[r_l^* + \psi^{-1}(R_{ff}^*)][v + e(R_{ff}^*)]l^*}{b^*}}_{RHS},$$

where $r_l^* = \left(\frac{\beta}{\delta+\rho}\right)^{\beta}\left(\frac{\alpha+\beta}{l^*}\right)^{1-\beta} - 1$. We let $g(R_{ff}^*) = LHS - RHS$ from the above equation.

[16] For cases $2(a)$, $2(d)$, and $2(e)$, as we lower ψ_1 further, two steady states will emerge where the steady state associated with a lower equilibrium federal funds rate is characterized by one negative root and two positive roots, the steady state associated with a higher equilibrium federal funds rate is characterized by two negative roots and one positive root. However, the high-equilibrium federal funds rate steady state has an equilibrium federal fund rate that is too high to be empirically plausible. For example, when $\psi_1 = 0.25$, which represents a very passive rule, the high equilibrium federal fund rate equilibrium has a quarterly rate at 0.214 for cases $2(a)$, $2(d)$, and $2(e)$. Given the fact that the high-equilibrium federal funds rate steady state is not empirically plausible, we do not discuss the global indeterminacy issue.

bonds equilibrium uniqueness is ensured regardless of the stance of monetary policy. It is note worthy that although our result tends to support Meng and Yip's (2004) viewpoint that the role of endogenous physical capital is that it adds into the model an additional initial condition, in their paper whether it is indexed bonds or non-indexed bonds that are used for financing budget deficits does not matter for the macroeconomic stabilizing properties of monetary rules. In our endogenous-capital model, we still obtain the results that (i) the interplay between fiscal and monetary policies is crucial for the macroeconomic stabilizing properties of monetary rules; and (ii) active rules and passive rules perform equally in regard to their macroeconomic stabilizing properties.

Due to our flexible prices setting, the existing works that are most comparable with ours are Carlstrom and Fuerst (2000*a*) and Meng and Yip (2004). Carlstrom and Fuerst (2000*a*) prove in a cash-in-advance model that, with elastic labor supply and separable leisure, forward-looking interest rate rules ensure real determinacy if and only if monetary policy is passive; current-looking rules, on the other hand, make equilibrium determinacy impossible. Meng and Yip (2004) demonstrate in a money-in-the-utility function model that, with either inelastic labor supply or elastic labor supply and separable leisure, equilibrium uniqueness is ensured regardless of the stance of monetary policy.

To clarify the reason why Carlstrom and Fuerst (2000*a*) and Meng and Yip (2004) reach diverse conclusions, we notice that the key difference between their theoretical frameworks is that Carlstrom and Fuerst (2000*a*) adopt a discrete-time modelling while Meng and Yip (2004) adopt a continuous-time modelling. In the literature, there was also a dialogue between Dupor (2001) and Carlstrom and Fuerst (2005) regarding the (in)determinacy issue of nominal interest rate rules in continuous- versus discrete-time models with endogenous capital and sticky prices. Under Dupor's (2001) continuous-time modelling, only passive policies can ensure determinacy. By contrast, under Carlstrom and Fuerst's (2005) discrete-time modelling, passive policies lead to indeterminacy if the nominal interest rate reacts to current inflation; under forward-looking rules, all the determinate equilibria are eliminated. The key point is that a continuous time model cannot differentiate current and future rate of returns. This results in different no-arbitrage conditions and different (in)determinacy results in Dupor (2001) and Carlstrom and Fuerst (2005). Li (2005) provides a rigorous demonstration of why incorporating capital accumulation in a continuous-time model with the nominal interest rate feedback rules may dramatically change the (in)determinacy region(s); a necessary and sufficient condition under which continuous time limit can be a correct approximation of the behavior of the discrete time model is provided.

The above timing issue of monetary models points to the need of a robustness check for this paper's result within a discrete-time setting. Appendix 5.2 illustrates the modification of our endogenous-capital model for it to become a discrete-time one, along with the model's steady-state conditions and dynamical system for both the forward-looking rule and the current-looking rule. In what follows, we first assume that the discrete-time model is given by a forward-looking rule. We then comment on the current-looking rule.

As it turns out, under the same set of parameter values, our result survives the robustness check as long as parameters are within their empirically plausible values.[17] To understand why, let us look at the no-arbitrage conditions for our discrete-time model:

$$r_{kt+1} + 1 - \delta = \frac{1 + R_{dt+1}/(1-\theta)}{p_t} = \frac{1}{p_t c_{t+1}\lambda_{t+1}}. \tag{61}$$

where $\frac{1}{c_{t+1}\lambda_{t+1}}$ represents the gross nominal interest rate of government bonds; $p_t (\equiv P_{t+1}/P_t)$ denotes the expected gross inflation rate which, under forward-looking rules, equals $\psi^{-1}(R_{fft}) + 1$.

Comparison of (56) with (61) reveals that, for households who plan over their whole life horizon, $t = 0, ..., \infty$, the continuous- and the discrete-time modeling suggest the same set of no-arbitrage conditions. The discrete-time version of our model with a forward-looking rule does not have a zero eigenvalue. Therefore, the dimension of indeterminacy is not increased and continuous- and discrete-time specifications

[17] In particular, we first notice that $\phi = m_t/b_t$ has a value between 0.0658 and 0.1214 and $\theta = 1 - R_{dt}/R_t$ lies between 0.88 and 0.988 in the sample period. Our simulation result shows that the discrete-time model has a different (in)determinacy result only in three situations: (i) when $\theta \leq 0.55$, the Jacobian matrix $\hat{\mathrm{J}}$ in (90) has three roots lying inside the unit circle; (ii) when $\phi \leq 0.02$, $\hat{\mathrm{J}}$ has two roots lying inside the unit circle; and (iii) when $\phi \geq 0.9$, $\hat{\mathrm{J}}$ has three roots lying inside the unit circle. Note that all these cases are not empirically plausible.

deliver the same (in)determinacy results.[18]

Note also that although $\frac{1}{c_{t+1}\lambda_{t+1}}$ in (61) represents the government bonds rate, it does not respond to inflation. Therefore, the inflation coefficient in the nominal interest rate rules does not have a decisive effect on the (in)determinacy result. To see what determine the (in)determinacy result, we note first from (52) that in our model the equilibrium capital rental rate is affected by both the marginal product of capital and the loan rate. Second, credit market equilibrium gives the equilibrium loan rate as $r_{lt} = \frac{(\alpha+\beta)y_t}{(1-v-e_t)d_t-b_t+b_{gt}} - 1$. Finally, profit maximization of commercial banks leads to the relationship between the rates of deposits and loans, the required reserves ratio, and the excess reserves ratio (with the federal funds rate inside the excess reserve ratio function): $r_{dt} = r_{lt}(1 - v - e_t)$. Therefore, what enter the consumption Euler equation and determine the (in)determinacy result include the details in the financial system, including the banking system (the credit market), the reserves market (the central bank's open market operations), and the bonds market.

The fact that in our model it is the federal funds rate, rather than the government bonds rate, that reacts to the inflation rate, leads to different no-arbitrage conditions than the existing literature. For example, in Carlstrom and Fuerst (2005) and Huang and Meng (2007), the no-arbitrage condition is:

$$r_{kt+1} + 1 - \delta = \frac{1 + R_t}{p_t}, \tag{62}$$

where the nominal rate of government bonds R_t reacts to future (current) inflation p_t (p_{t-1}) if the monetary rule is a forward-looking (current-looking) one. The continuous-time counterpart of (62) is

$$r_{kt} - \delta = R_t - \pi_t, \tag{63}$$

which is (8) in Dupor (2001), where $\pi_t = p_t - 1$.

It is therefore obvious from (62) and (63) that in models where the government bonds rate serves as the proxy for the federal funds rate, the inflation rate enters

[18] Although this section only carries out the robustness check for our endogenous-capital model, our labor-only model also survives the robustness check. In addition, similar to the endogenous-capital model, the labor-only discrete-time model has a different (in)determinacy result only when $\theta \leq 0.1$. In this empirically implausible case, the eigenvalue of the difference equation of b_t lies outside the unit circle. The detailed proof is available upon request.

the no-arbitrage condition twice. This makes the inflation coefficient in the nominal interest rate rules to have a decisive effect on the (in)determinacy result. In addition, inspection of (62) and (63) reveals that continuous- and discrete-time models have different no-arbitrage conditions. As demonstrated by Carlstrom and Fuerst (2005) and Huang and Meng (2007), under forward-looking rules the no-arbitrage condition introduces a zero eigenvalue and therefore the indeterminacy region is enlarged. This is the reason why under Dupor's (2001) continuous-time modeling, passive policy can ensure equilibrium determinacy; in Carlstrom and Fuerst's (2005) discrete-time model, under forward-looking rules there is indeterminacy for essentially all values of the inflation coefficient in the interest rate rules. Similarly, under Meng's (2004) continuous-time modeling, determinacy is ensured regardless of the monetary policy stance; under Carlstrom and Fuerst's (2000*a*) discrete-time modeling, equilibrium indeterminacy can occur.

We then move to the discussion of the current-looking rule in the discrete-time version of our model. The current-looking rule exerts two effects on the model's equilibrium conditions. First, the expected gross inflation rate p_t in the no-arbitrage condition (61) equals $\psi^{-1}(R_{fft+1}) + 1$. This introduces a zero eigenvalue to the dynamical system (91).[19] Second, through the equation which connects the rates of deposits and loans, the required reserves ratio, and the excess reserves ratio, $r_{dt} = r_{lt}(1-v-e_t)$, the current-looking rule introduces an additional difference equation of the jump variable p_t to the dynamical system (91) and an eigenvalue which lies inside the unit circle, as long as empirically plausible parameter values are considered. As a consequence, all the determinate equilibria are eliminated by current-looking rules.[20] This viewpoint against the current-looking rule is also made by Carlstrom and Fuerst (2000*a*). Note that the federal funds rate in the policy rule (24) can respond to current inflation or future inflation. Therefore, the continuous-time limit of the discrete-time model is the same for current- or forward-looking policies.

[19] Mathematical proof is available upon request.

[20] The second effect also exists in the labor-only economy. Thus, the indeterminacy result of the current-looking rule also holds in the labor-only economy.

4 Conclusion

This paper is the first attempt in the literature to formally characterize the banking system *and* the reserves market in a general equilibrium monetary model. Within this framework, we are able to describe how the central banker's open market operations in the reserves market affect the federal funds rate when it adopts a regime of nominal interest rate rules. Our banking model is simple/basic in that we do not allow firms or commercial banks to issue debt or equity instruments to raise external funds; neither do we consider New Keynesian features of imperfect markets and nominal rigidities. We focus on traditional banking services and indirect finance through financial intermediaries. The reason why we make these assumptions is that commercial banks are the most important source of external funds for businesses in most countries. Furthermore, the central bank conducts open market operations mainly with commercial banks. How much of the reserves injected by the central bank into the banking system through open market operations will be released to firms and/or consumers depends on the willingness to borrow and lend between firms/consumers and commercial banks. By virtue of the model's simplified feature, we can easily understand the interplays between the central bank's open market operations, the overnight interbank market, and the extension of credit to private borrowers. The model can be extended to one that allows firms or commercial banks to obtain external funds by issuing debt or equity instruments. The theory of "the financial accelerator" can also be embedded. We plan to pursue these research projects in the near future.

5 Appendix

5.1 Derivation of the dynamical system of the labor-only economy

Under Definition 1, the macroeconomic equilibrium conditions are:

$$c_t = m_{ht} + \theta d_t, \tag{64}$$

$$\dot{\lambda}_t = (\rho + \pi_t + 1)\lambda_t - \frac{1}{c_t}, \tag{65}$$

$$
\begin{aligned}
r_{dt} &= (1-\theta)\left(\frac{1}{c_t\lambda_t}-1\right)-\pi_t, & (66)\\
r_{dt} &= r_{lt}[1-v-e(R_{fft})], & (67)\\
R_{fft} &= \psi(\pi_t), & (68)\\
l_t+m_{ht} &= \phi b_t, & (69)\\
\dot{b}_t &= r_{lt}b_t-(r_{lt}+\pi_t)b_{gt}, & (70)\\
[v+e(R_{fft})]d_t &= b_{gt}-\phi b_t, & (71)\\
\frac{\alpha}{1+r_{lt}} &= (1-v-e_t)d_t-b_t+b_{gt}=l_t, & (72)\\
m_{ht}+d_t &= b_t, & (73)\\
c_t &= y_t=1. & (74)
\end{aligned}
$$

It is obvious that we have the above 11 equations which contain 11 unknowns, namely, c_t, m_{ht}, d_t, λ_t, π_t, r_{dt}, r_{lt}, R_{fft}, b_{gt}, b_t, and l_t. We will have a two-dimensional dynamical system (b_t, λ_t) constituted by (65) and (70). Other endogenous variables have to be expressed as functions of (b_t, λ_t).

First, from (69), (73), (74), and (64) we derive l_t as a function of b_t:

$$l_t=\frac{[\phi+\theta(1-\phi)]b_t}{1-\theta}. \tag{75}$$

We then obtain from (73), with the help of (69) and (75), d_t as a function of b_t:

$$d_t=\frac{b_t}{1-\theta}. \tag{76}$$

From (68), we obtain $\pi_t=\psi^{-1}(R_{fft})$, $\psi^{-1\prime}=\frac{1}{\psi'}>0$. By using this function and (74) to substitute out π_t and c_t in (66), we have

$$r_{dt}=(1-\theta)\left(\frac{1}{\lambda_t}-1\right)-\psi^{-1}(R_{fft}). \tag{77}$$

From (72) we obtain the loan rate as

$$r_{lt}=\frac{\alpha}{l_t}-1, \tag{78}$$

where l_t is given in (75). Equations (67), (77), and (78) lead to

$$(1-\theta)\left[\frac{1}{\lambda_t}-1\right]-\psi^{-1}(R_{fft})=\left[\frac{(1-\theta)\alpha}{[\phi+\theta(1-\phi)]b_t}-1\right][1-v-e(R_{fft})]. \tag{79}$$

From the above equation we solve $R_{fft}=R_{ff}(b_t,\lambda_t)$, where the partial derivatives are $R_{ff,b}\equiv\frac{\partial R_{ff}}{\partial b_t}=\frac{(1-\theta)\alpha[1-v-e(R_{fft})]}{(1/\psi'-r_{lt}e')[\phi+\theta(1-\phi)]b_t^2}>0$ and $R_{ff,\lambda}\equiv\frac{\partial R_{ff}}{\partial\lambda_t}=\frac{1-\theta}{(r_{lt}e'-1/\psi')\lambda_t^2}<0.$

By using (76) to substitute out d_t in (71), we obtain

$$b_{gt}=\frac{[(1-\theta)\phi+v+e(R_{fft})]b_t}{1-\theta}. \tag{80}$$

Therefore, we obtain $b_{gt}=b_b(b_t,\lambda_t)$, where the partial derivatives are $b_{g,b}\equiv\frac{\partial b_g}{\partial b_t}=\frac{(1-\theta)\phi+v+e(R_{fft})+e'b_tR_{ff,b}}{1-\theta}\gtrless 0$ and $b_{g,\lambda}\equiv\frac{\partial b_g}{\partial\lambda_t}=\frac{e'b_tR_{ff,\lambda}}{1-\theta}>0$.

This completes the derivation.

5.2 The discrete-time model with endogenous capital

The equations that need modification include the representative household's budget constraint, the law of motion of physical capital, the monetary policy rule, and the government budget constraint:

$$\begin{aligned}
\frac{M_{ht+1}+D_{t+1}}{P_t} &= \frac{M_{ht}+(1+R_{dt})D_t}{P_t}+w_t+r_{kt}k_t-c_t-i_t+q_{ft}+q_{bt}, && (81)\\
k_{t+1} &= i_t+(1-\delta)k_t,\ \ k_0\ >0 \text{ given}, && (82)\\
R_{fft} &= \psi(\pi_{t-j}), && (83)\\
B_{t+1}-B_t &= R_{bt}B_{pt}, && (84)
\end{aligned}$$

where $j=0$ is forward-looking rule, and $j=1$ is current-looking rule.

It is straightforward to demonstrate that forward- and current-looking rules lead to the same steady-state conditions:

$$\begin{aligned}
l^* &= (\alpha+\beta)\left\{\left[\frac{[(1-\theta)(1+\rho)-1][\psi^{-1}(R^*_{ff})+1]+\theta}{1-v-e(R^*_{ff})}+1\right]\left(\frac{\delta+\rho}{\beta}\right)^{\beta}\right\}^{1/(\beta-1)} && (85)\\
b^* &= \frac{(k^*)^\beta+(1-\theta)l^*-\delta k^*}{\phi+\theta(1-\phi)}, && (86)\\
\lambda^* &= \frac{1}{\{[\phi+\theta(1-\phi)]b^*-(1-\theta)l^*\}[\psi^{-1}(R^*_{ff})+1](1+\rho)}, && (87)
\end{aligned}$$

$$k^* = \frac{\beta l^*}{(\delta+\rho)(\alpha+\beta)}, \tag{88}$$

where steady-state federal funds rate R^*_{ff} is the solution to the following equation:

$$r^*_l b^* = [r^*_l + \psi^{-1}(R^*_{ff})]\{[v + e(R^*_{ff})][l^* + (1-\phi)b^*] + \phi b^*\}, \tag{89}$$

where $r^*_l = \left(\frac{\beta}{\delta+\rho}\right)^{\beta} \left(\frac{\alpha+\beta}{l^*}\right)^{1-\beta} - 1$.

By taking log-linear approximations to the equilibrium conditions in the neighborhood of the steady state, we obtain the following dynamical system for forward-looking rules:

$$z_{t+1} = \hat{\mathrm{J}} z_t, \tag{90}$$

where z_t denotes the vector $[\hat{l}_t, \hat{b}_t, \hat{\lambda}_t, \hat{k}_t]'$, hat variables denote percentage deviations from their steady-state values, and $\hat{\mathrm{J}}$ is the 4×4 Jacobian matrix. For current-looking rules, the dynamical system is:

$$x_{t+1} = \tilde{\mathrm{J}} x_t, \tag{91}$$

where x_t denotes the vector $[\hat{l}_t, \hat{b}_t, \hat{\lambda}_t, \hat{k}_t, \hat{p}_t]'$, and $\tilde{\mathrm{J}}$ is the 5×5 Jacobian matrix. In the case of indexed bonds, the model exhibits saddle path stability when two eigenvalues of the Jacobian matrix lies inside and the others outside the unit circle since both b_t and k_t are pre-determined. When more than two eigenvalues are inside the unit circle, the steady state will exhibit equilibrium indeterminacy. When more than two eigenvalues are outside the unit circle, there will exist no rational expectations equilibrium. By contrast, in the case of non-indexed bonds, the model exhibits saddle path stability when one eigenvalue of the Jacobian matrix lies inside and the others outside the unit circle since only k_t is pre-determined. When more than one eigenvalues are inside the unit circle, there will be equilibrium indeterminacy. When all eigenvalues are outside the unit circle, the steady state becomes a totally unstable source.

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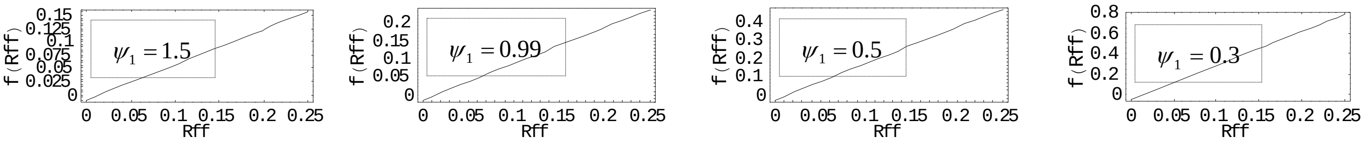
Table 1: Model without Investment

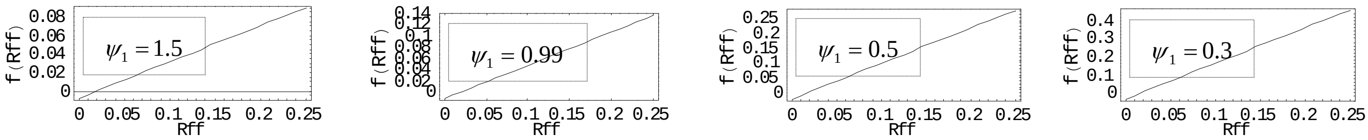
1(a): Benchmark case: $\phi = 0.084$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$
1(b): Reducing θ: $\phi = 0.084$, $\theta = 0.5$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.02 $(-+)$	0.018 $(-+)$	0.017 $(-+)$	0.016 $(-+)$
1(c): Only cash is used for purchasing: $\phi = 0.084$, $\theta = 0$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.07 $(-+)$	0.051 $(-+)$	0.033 $(-+)$	0.026 $(-+)$
1(d): Increasing ϕ: $\phi = 0.9$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$
1 (e): Reducing ϕ: $\phi = 0.01$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$	0.015 $(-+)$

Table 2: Model with Investment

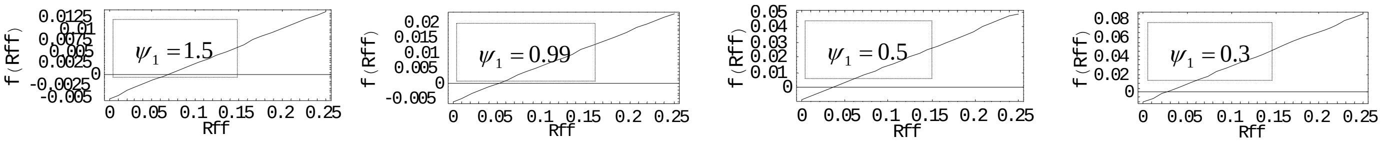
2(a): Benchmark case: $\phi = 0.084$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$
2(b): Reducing θ: $\phi = 0.084$, $\theta = 0.5$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.02 $(-++)$	0.018 $(-++)$	0.017 $(-++)$	0.016 $(-++)$
2(c): Only cash is used for purchasing: $\phi = 0.084$, $\theta = 0$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.07 $(-++)$	0.051 $(-++)$	0.033 $(-++)$	0.026 $(-++)$
2(d): Increasing ϕ: $\phi = 0.9$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$
2(e): Reducing ϕ: $\phi = 0.01$, $\theta = 0.958$				
ψ_1	1.5	0.99	0.5	0.3
R_{ff}^* (roots)	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$	0.015 $(-++)$



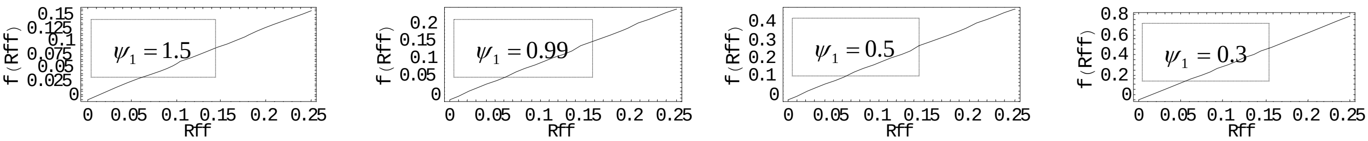
1(a): benchmark case: $\phi = 0.084$, $\theta = 0.958$



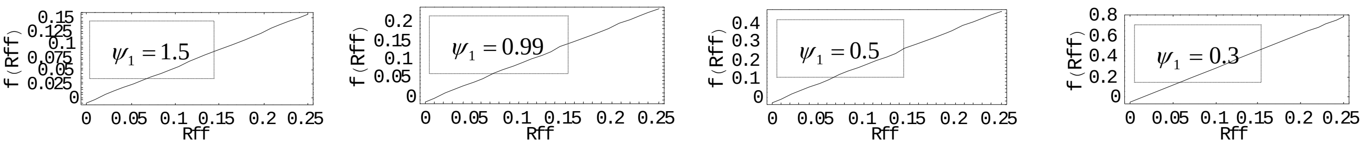
1(b): reducing θ *:* $\phi = 0.084$, $\theta = 0.5$



1(c): only cash is used for purchasing: $\phi = 0.084$, $\theta = 0$

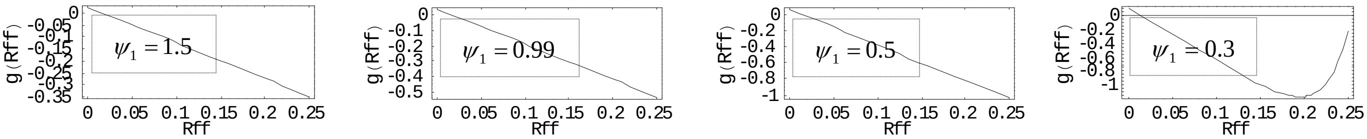


1(d): increasing ϕ *:* $\phi = 0.9$, $\theta = 0.958$

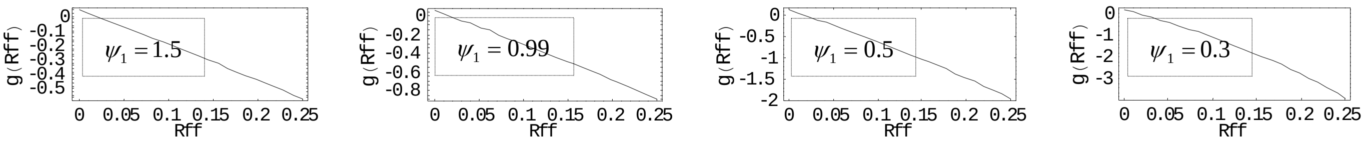


1(e): reducing ϕ *:* $\phi = 0.01$, $\theta = 0.958$

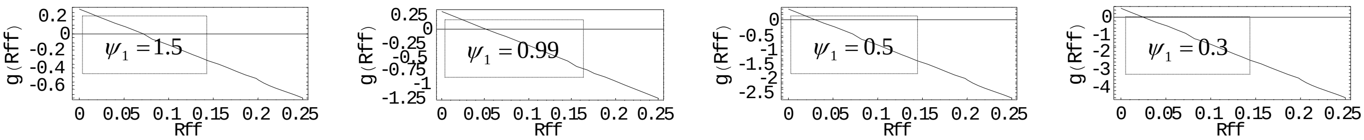
Figure 1: Existence of the Steady State of the Model without Investment



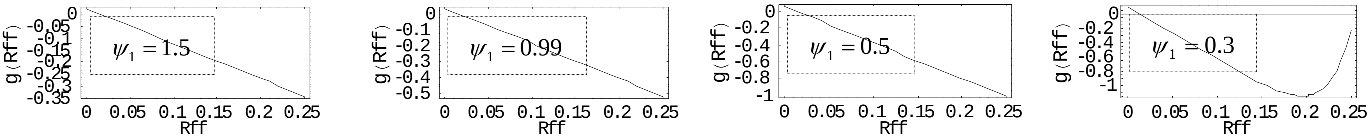
2(a): benchmark case: $\phi = 0.084$, $\theta = 0.958$



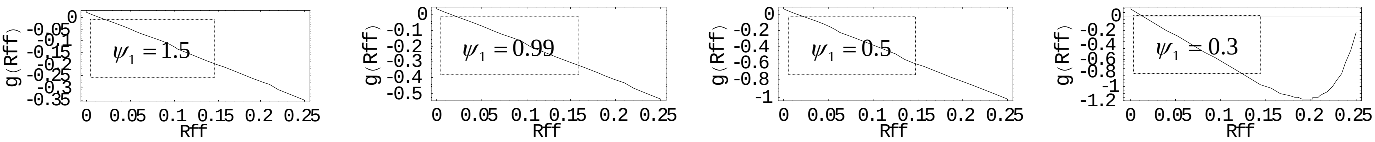
2(b): reducing θ *:* $\phi = 0.084$, $\theta = 0.5$



2(c): only cash is used for purchasing: $\phi = 0.084$, $\theta = 0$



2(d): increasing ϕ *:* $\phi = 0.9$, $\theta = 0.958$



2(e): reducing ϕ *:* $\phi = 0.01$, $\theta = 0.958$

Figure 2: Existence of the Steady State of the Model with Investment