Prediction of A Multivariate Long Memory Model Subject to Structural Breaks

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A. Motivation

(I). The literature on multivariate long memory models that also take structural breaks into account is rather limited.

(II). The latest one with endogenously determined break date was proposed by Caporale and Gil-Alana (2009) use a procedure which minimizes the residual sum of squares (RSS). However, their testing method is quite time consuming even for a bivariate system. Besides, it also suffer from possible large size distortions when the breaks occur at different locations of each component of the system [1].

(III). The estimated parameters for a multivariate system are many. In general, the more parameters needed to estimate, the more possible bias causing the larger forecast errors.

[1] We conduct some experiments in our simulation sections.

Table A. Estimations of Parameters for Each Regime

Regime	*BreakDate*	d
	DEM/USD	
full	12/2/1986 – 12/1/1996	0.382
0		0.344
1	1989/5/11	0.288
2	1991/3/18	0.348
3	1993/3/5	0.392
4	1995/8/24	0.417
5	1997/7/10	0.385
	USD/YEN	
full	12/1/1986 – 6/30/1999	0.410
0		0.397
1	1989/5/1	0.315
2	1991/5/16	0.388
3	1993/3/30	0.420
4	1995/6/15	0.410
5	1997/9/7	0.392

B. Contribution

Several advantages of using the VAR approximation approach are worth noting here.

(1). Although several estimation methods for the multivariate long memory models are proposed, those methods are unable to reduce the computational burden and avoid the inaccurate estimation of the differencing parameter completely. Our methodology can tackle the above two essential concerns when forecasting a multivariate long memory model and the exact order of the VARFIMA (p, d, q) process is *unknown*. The only procedure we should do is to use the VAR(k) model to approximate the VARFIMA(p, d, q) model subject to structural breaks.

(2). When the locations of breaks of each series in a multivariate system are different and close to the end of the sample, the conventional structural break tests for multivariate time series models could result in the misleading spurious breaks and the uncertainty in choosing the estimation sample for forecasting. Moreover, another issue we concern about here is whether or not dating common breaks in the multivariate system could utilize enough information for forecasts. Using our VAR(k) approximation framework which allows for different means, long memory parameters and the locations of breaks across each time series of the VARFIMA(p, d, q) processes, this method avoids detecting break dates and solves the potential presence of spurious breaks.

(3). Finally, VAR which was advanced by Sims (1980) in theoretical studies and empirical applications allows researchers to study the relationship among different time series. Our Monte Carlo simulations also confirm that both the multivariate least squares (LS) coefficient estimator for the "semiparametric" VAR(k) approximation of the VARFIMA with breaks and the residual covariance matrix estimator are consistent. Thus, this finding could be viewed as the basis for future investigation about the impulse responses and causality analysis when modelling data series as a multivariate long memory process with structural breaks.

(4). Our results clearly demonstrate that a VAR(k) approximation model for forecasting a VARFIMA(p, d, q) with structural breaks outperforms conventional methods, namely the two naive VARFIMA-based methods and the post-break model, even in cases where the structure of a VARFIMA model, including its parameters and lag orders, changes dramatically after breaks. Furthermore, for the special case in which the structural breaks take place immediately prior to the forecast period, our VAR-approximation also performs better.

(5). An empirical forecasting exercise of the multivariate realized covariance matrices shows that our VAR-approximation dominates the existing methods.

C. Model and Theoretical Results

Suppose T time series observations and let

$$D^1(L)(Y_t^{(1)} - \mu_1) = \varepsilon_t^{(1)}, \tag{1}$$

and

$$D^2(L)(Y_t^{(2)} - \mu_2) = \varepsilon_t^{(2)}, \tag{2}$$

be two VARFIMA(0,d,0) processes where (i) $Y_t^{(1)} = (y_{11,t}, y_{12,t}, \cdots, y_{1r,t})', t = 1, 2, \cdots, T_1$; $Y_t^{(2)} = (y_{21,t}, y_{22,t}, \cdots, y_{2r,t})', t = T_1+1, T_1+2, \cdots, T$. Thus, $Y_t^{(1)}$ and $Y_t^{(2)}$ are $r \times T_1$ and $r \times (T - T_1)$ matrices, respectively. (ii) $\mu_1 = (\mu_{11}, \mu_{12}, \cdots, \mu_{1r})'$ as $t = 1, 2, \cdots, T_1$; $\mu_2 = (\mu_{21}, \mu_{22}, \cdots, \mu_{2r})'$ as $T = T_1+1, T_1+2, \cdots, T$. Thus, μ_1 and μ_2 are $r \times T_1$ and $r \times (T - T_1)$ matrices, respectively. (iii) $D^1(L) = diag\{(1-L)^{d_{1,1}}, \cdots, (1-L)^{d_{1,r}}\}$ and $D^2(L) = diag\{(1-L)^{d_{2,1}}, \cdots, (1-L)^{d_{2,r}}\}$ where $d_{i,j}, i = 1, 2$ and $j = 1, 2, \cdots, r$ are the degrees of fractional integration of each of the elements of the vector Y_t and $d_{i,j} \in (-0.5, 0.5), d_{i,j} \neq 0$; here, we also allow the integration of each component $y_{ji,t}$ is different across $i = 1, 2, \cdots, r$ and $j = 1, 2$ and $t = 1, 2, \cdots, T$; (iv) $\varepsilon_t^{(1)} = \{e_{11,t}, \cdots, e_{1r,t}\}' \sim N(0, \Sigma_1)$, t=1, 2, $\cdots, T_1$;$\varepsilon_t^{(2)} = \{e_{21,t}, \cdots, e_{2r,t}\}' \sim N(0, \Sigma_2)$, t=$\text{T}_1 + 1, T_1 + 2, \cdots, T; where$e$_{ij,t}, i = 1, 2$ and $j = 1, 2, \cdots, r$ are independently and identically distributed processes. $\varepsilon_t^{(1)}$ and $\varepsilon_t^{(2)}$ are $r \times T_1$ and $r \times (T - T_1)$ matrices respectively.

C. Model and Theoretical Results

Y_t then take the form

$$Y_t = Y_t^{(1)} \quad \text{for} \quad t = 1, 2, \cdots, T_1,$$

and

$$Y_t = Y_t^{(2)} \quad \text{for} \quad t = T_1 + 1, \cdots, T,$$

where $T_1 = \kappa T$, $\kappa \in (0, 1)$. We consider two scenarios:

Case I: Changes in the differencing parameter only, i.e., $D^1(L) \neq D^2(L), \quad \mu_1 = \mu_2 = \mu.$

Case II: Changes in both differencing parameter and mean, i.e., $D^1(L) \neq D^2(L), \mu_1 \neq \mu_2.$

C. Model and Theoretical Results

Lemma 1. *When the DGP satisfies the basic model, with* $T_1 = \kappa T$, $\kappa \in (0,1)$, *the observed system can be represented by a VARFIMA model with long memory parameter* $D^*(L)$ *that is a linear combination of the pre-and post-break parameters* D^1 *and* D^2, *and with a mean* μ^* *that is a linear combination of the pre-and post-break means* μ_1 *and* μ_2, *that is*

$$D(L)^{d^*}(Y_t - \mu^*) = \varepsilon_t, \tag{3}$$

where $\mu^* = \kappa\mu_1 + (1-\kappa)\mu_2$, $D^{d^*} = \lambda D^{d_1} + (1-\lambda)D^{d_2}$, $0 \leq \lambda \leq 1$. *When* T *is finite,* $\kappa \to 0$, $\lambda \to 0$, *and* $\kappa \to 1$, $\lambda \to 1$. *When* $T \to \infty$, $D(L)^{d^*} = max(D(L)^{d_1}, D(L)^{d_2})$ *and* $\mu^* = \kappa\mu_1 + (1-\kappa)\mu_2$.

C. Model and Theoretical Results

Y_t can be rewrite as an infinite order VAR process,

$$Y_t = \sum_{j=1}^{\infty} A_j Y_{t-j} + \varepsilon_t,$$

where

$$A_j = \begin{bmatrix} \beta_{j,1} & 0 & 0 & \cdots \\ 0 & \beta_{j,2} & 0 & \cdots \\ 0 & 0 & \beta_{j,3} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \beta_{j,r} \end{bmatrix}_{r \times r} \tag{4}$$

and $\beta_{j,i} = O(j^{-d_i-1}), i = 1, 2, \cdots, r$ as $j \to \infty$. Denote the autocovariance function of Y_t by $\Omega(j) = \mathbf{E}(Y_t Y'_{t+j})$ for $j = 0, \pm 1, \pm 2, \ldots$.

C. Model and Theoretical Results

Lemma 2. *When the DGP satisfies the basic model, with* $T_1 = \kappa T$, $\kappa \in (0,1)$, *when* $d^* \in (-0.5, 0.5)$ *there exists an VAR(k) approximation of* Y_t, *as* $T \to \infty$, $k = O(T^r), 1 > r > 2d^*/(1+2d^*)$, *such that*

1. $\|\widehat{A}(k) - A(k)\| = O_p((klogT/T)^{0.5-d^*})$,

2. $\widehat{\Sigma}^2_{t,k} = \Sigma^2 + O_p(k^{-2d^*-1}T^{2d^*}) = \Sigma^2 + o_p(1)$, *where* $A(k) = (A_0, A_1, A_2, \cdots, A_k)$, $\widehat{A}(k)$ *is the OLS estimator of* $A(k)$ *and* $\widehat{e}_{t,k}$ *the OLS residual.*

D. Comparison of Forecasting Methods: Simulation Results

4.1 Simulation Design I: A Bivariate Model With One Break

we consider any two of the following three series

$$\text{DGP (a):} \quad (1-L)^d(y_t-\mu)=e_t,$$

$$\text{DGP (b):} \quad (1-0.7L)(1-L)^d(y_t-\mu)=(1+0.5L)e_t,$$

$$\text{DGP (c):} \quad (1-0.7L)(1-L)^d(y_t-\mu)=e_t,$$

to construct a bivariate model, i.e., $r=2$, subject to one break as follows.

$$Y_t^{(1)}=\begin{bmatrix} y_{1t}^1 \\ y_{2t}^1 \end{bmatrix} \quad \text{for} \quad t=1,2,\cdots,T_1,$$

and

$$Y_t^{(2)}=\begin{bmatrix} y_{1t}^2 \\ y_{2t}^2 \end{bmatrix} \quad \text{for} \quad t=T_1+1,2,\cdots,T.$$

To construct $T\times 2$ values of a stationary bivariate $I(d)$ process, we first generate $T\times 2$ independent values from the standard bivariate normal distribution, such that

$$\Sigma=\begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}. \tag{5}$$

D. Comparison of Forecasting Methods: Simulation Results

One of the important contributions of Lewis and Reinsel (1985) is that they derive the approximation of the mean square error of the $h \geq 1$ step ahead predictor $\widehat{Y}_{t,k}(h)$ with $\Sigma_k(h) \equiv (1 + kr/T)\Sigma(h)$, where $\Sigma(h)$ is the mean square error of the "optimal" predictor $Y_t^*(h)$ (Lewis and Reinsel, 1985, p.402). For purpose of comparison, we first calculate the $\Sigma_k(h) \equiv (1 + kr/T)\Sigma(h)$ for the experiments 1-2, which could be the benchmark of the comparison, since the DGPs of these 2 experiments follow the fractional white noise process in which the long memory parameter combinations, d_1^* and d_2^*, computed by the pre ($d_{1,1}$ and $d_{1,2}$) and post-break long memory parameters ($d_{2,1}$ and $d_{2,2}$) using the Lemma 1, of y_{1t} and y_{2t} are 0.35 and 0.25, respectively. We then report their corresponding elements of $(1 + kr/T)\Sigma(h)$ and $\Sigma(h)$ in these Table 3 [4].

[4] Even though here we cannot follow Lewis and Reinsel (1985) to derive such a great approximation factor, $(1 + kr/T)$, for our multivariate long memory model exactly at this moment, we show through Monte Carlo simulations, the results reported in our Table 3 are similar to those in Table 1 of Lewis and Reinsel (1985) where they consider a multivariate short memory system.

D. Comparison of Forecasting Methods: Simulation Results

The first one is to use the Mincer and Zarnowitz (1969) encompassing regression for the one-step-ahead forecasts of four forecast models with the rolling forecast window:

$$y_{i,t+1} = \alpha_0 + \alpha_1 y^{VAR}_{i,t+1|t} + \alpha_2 y^{AM}_{i,t+1|t} + \varepsilon_t,$$

where $i = 1, 2$ and $y^{VAR}_{i,t+1|t}$ denotes predictions by our benchmark VAR approximation model, and $y^{AM}_{i,t+1|t}$ denotes predictions from four alternative models.

D. Comparison of Forecasting Methods: Simulation Design II

To check the robustness of our methodology, we further design the following procedure. The bivariate model is

$$Y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} \tag{6}$$

where there is a break at $T_1^1 = \tau_1 T$ and $T_1^2 = \tau_2 T$ for $y_{1,t}$ and $y_{2,t}$, respectively, $\tau_1 \neq \tau_2, \tau_1, \tau_2 \in (0,1)$, i.e., the breaks occur at different locations for $y_{1,t}$ and $y_{2,t}$. In fact, the breaks usually occur at different time points for each time series in a multivariate system in reality. This could potentially lead to the issue of spurious break and inaccurate break-detection results.

4.3 Comparison When the Break Occurs at the End of the Sample

we allow the breaks occur at different locations for y_{1t} and y_{2t}. The DGP are generated as follows:

$$Y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} \tag{7}$$

where there exists a break at $T_1^{(1)} = T - 1$ and $T_1^{(2)} = T - 3$ for y_{1t} and y_{2t}, respectively. Table 8 presents the performance of the VAR-approximation (VAR) and VNVNO of observations $T_1 + h$ for $h = 1, 2, \ldots, 5$, assuming the breaks are unknown. The results clearly indicate that the VAR-approximation method produces the lowest RMSFEs.

E. Forecasting Multivariate Realized Volatility

(1.) Chiriac and Voev (2011) develop an approach for the dynamics of the whole realized covariance matrix to forecast realized volatility, by which the matrix constructed from the variance and correlation forecasts is guaranteed to be positive definite.

(2). The main procedure of their approach has three steps: (i) decomposing the series of covariance matrices into Cholesky factors; (ii) forecasting the Cholesky series with a suitable time series model and (iii) reconstructing the matrix forecast.

(3). The VAR-RV-Break (VRB) model is a break-adjusted VAR, denoted as

$$\Phi(L)(Y^* - \mu_{)} = \varepsilon_t,$$

where Y^* is the $(r \times 1)$vector of logarithmic realized exchange rate volatilities after mean break adjustments, μ is the unconditional mean, ε_t is a vector white noise process, r is the dimension of Y^* and the lag order is equal to five following Choi et al. (2010). The detailed procedure of the mean break adjustments is explained in Choi et al. (2010).

F. Concluding Remarks

(1). Our contribution of this paper is to develop a forecasting approach for the multivariate long memory VARFIMA(p, d, q) model subject to structural breaks. Using an easy-to-implement VAR(k)-approximating model, we avoid the inaccurate estimation of the long memory parameter inherent in the existing methods, and the problem of potential spurious breaks.

(2). Our simulation results and theoretical insights illustrated that (i) a VARFIMA(p, d, q) model subject to structural breaks can be well approximated by a VAR(k) model when k is chosen appropriately, and (ii) our VAR-based forecasting method provides a better out-of-sample forecasting performance than conventional methods for forecasting multivariate long memory processes under structural breaks.

(3). Our investigations can serve as a platform for future studies about the impulse responses and causality analysis concerning the multivariate long memory process subject to structural breaks. Furthermore, we show this method is very useful for forecasting the multivariate realized volatilities of stocks.

Table 1.1 Breakpoint Specifications by Experiment (EX) (One Break)

EX	y_{1t}^1	y_{1t}^2	τ	$d_{1,1}$	$d_{1,2}$	$\mu_{1,1}$	$\mu_{1,2}$
1	(a)	(a)	0.5	0.25	0.45	0.0	0.0
2	(a)	(a)	0.5	0.25	0.45	0.3	0.9
3	(a)	(a)	0.4	0.25	$0,45$	0.0	0.0
4	(a)	(a)	0.4	0.25	0.45	0.3	0.9
5	(a)	(b)	0.5	0.25	0.45	0.0	0.0
6	(a)	(b)	0.5	0.25	0.45	0.3	0.9
7	(a)	(b)	0.4	0.25	0.45	0.0	0.0
8	(a)	(b)	0.4	0.25	0.45	0.3	0.9
9	(a)	(c)	0.5	0.25	0.45	0.0	0.0
10	(a)	(c)	0.5	0.25	0.45	0.3	0.9
11	(a)	(c)	0.4	0.25	0.45	0.0	0.0
12	(a)	(c)	0.4	0.25	0.45	0.3	0.9
13	(b)	(b)	0.5	0.25	0.45	0.0	0.0
14	(b)	(b)	0.5	0.25	0.45	0.3	0.9
15	(b)	(b)	0.4	0.25	0.45	0.0	0.0
16	(b)	(b)	0.4	0.25	0.45	0.3	0.9
17	(c)	(c)	0.5	0.25	0.45	0.0	0.0
18	(c)	(c)	0.5	0.25	0.45	0.3	0.9
19	(c)	(c)	0.4	0.25	0.45	0.0	0.0
20	(c)	(c)	0.4	0.25	0.45	0.3	0.9
21	(a)	(c)	0.5	0.25	0.45	0.0	0.0
22	(a)	(c)	0.5	0.25	0.45	0.3	0.9
23	(a)	(c)	0.4	0.25	0.45	0.0	0.0
24	(a)	(c)	0.4	0.25	0.45	0.3	0.9
25	(a)	(c)	0.5	0.25	0.45	0.0	0.0
26	(a)	(c)	0.5	0.25	0.45	0.3	0.9
27	(a)	(c)	0.4	0.25	0.45	0.0	0.0
28	(a)	(c)	0.4	0.25	0.45	0.3	0.9

Table 1.2

EX	y_{2t}^1	y_{2t}^2	τ	$d_{2,1}$	$d_{2,2}$	$\mu_{2,1}$	$\mu_{2,2}$
1	(a)	(a)	0.5	0.15	0.35	0.0	0.0
2	(a)	(a)	0.5	0.15	0.35	0.1	0.6
3	(a)	(a)	0.4	0.15	0.35	0.0	0.0
4	(a)	(a)	0.4	0.15	0.35	0.1	0.6
5	(a)	(b)	0.5	0.15	0.35	0.0	0.0
6	(a)	(b)	0.5	0.15	0.35	0.1	0.6
7	(a)	(b)	0.4	0.15	0.35	0.0	0.0
8	(a)	(b)	0.4	0.15	0.35	0.1	0.6
9	(a)	(c)	0.5	0.15	0.35	0.0	0.0
10	(a)	(c)	0.5	0.15	0.35	0.1	0.6
11	(a)	(c)	0.4	0.15	0.35	0.0	0.0
12	(a)	(c)	0.4	0.15	0.35	0.1	0.6
13	(b)	(b)	0.5	0.15	0.35	0.0	0.0
14	(b)	(b)	0.5	0.15	0.35	0.0	0.0
15	(b)	(b)	0.4	0.15	0.35	0.0	0.0
16	(b)	(b)	0.4	0.15	0.35	0.1	0.6
17	(c)	(c)	0.5	0.15	0.35	0.0	0.0
18	(c)	(c)	0.5	0.15	0.35	0.1	0.6
19	(c)	(c)	0.4	0.15	0.35	0.1	0.6
20	(c)	(c)	0.4	0.15	0.35	0.1	0.6
21	(a)	(b)	0.5	0.15	0.35	0.0	0.0
22	(a)	(b)	0.5	0.15	0.35	0.1	0.6
23	(a)	(b)	0.5	0.15	0.35	0.1	0.6
24	(a)	(b)	0.5	0.15	0.35	0.1	0.6
25	(c)	(b)	0.5	0.15	0.35	0.0	0.0
26	(c)	(b)	0.5	0.15	0.35	0.1	0.6
27	(c)	(b)	0.5	0.15	0.35	0.1	0.6
28	(c)	(b)	0.5	0.15	0.35	0.1	0.6

Table 2.1 Breakpoint Specifications by Experiment (EX)
(Two Breaks of y_{1t})

EX	y_{1t}^1	y_{1t}^2	y_{1t}^3	τ_1	τ_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$\mu_{1,1}$	$\mu_{1,2}$	$\mu_{1,3}$
29	(a)	(a)	(a)	0.25	0.75	0.15	0.45	0.25	0.3	0.9	0.5
30	(b)	(b)	(b)	0.25	0.75	0.15	0.45	0.25	0.3	0.9	0.5
31	(a)	(c)	(b)	0.25	0.75	0.15	0.45	0.25	0.3	0.9	0.5
32	(a)	(a)	(a)	0.25	0.70	0.15	0.45	0.25	0.3	0.9	0.5
33	(b)	(b)	(b)	0.25	0.70	0.15	0.45	0.25	0.3	0.9	0.5
34	(a)	(c)	(b)	0.25	0.70	0.15	0.45	0.25	0.3	0.9	0.5

Table 2.2 Breakpoint Specifications by Experiment (EX)
(Two Breaks of y_{2t})

EX	y^1_{2t}	y^2_{2t}	y^3_{2t}	τ_1	τ_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$\mu_{1,1}$	$\mu_{1,2}$	$\mu_{1,3}$
29	(a)	(a)	(a)	0.25	0.75	0.45	0.15	0.25	0.6	0.2	0.8
30	(b)	(b)	(b)	0.25	0.75	0.45	0.15	0.25	0.6	0.2	0.8
31	(c)	(a)	(b)	0.25	0.75	0.45	0.15	0.25	0.6	0.2	0.8
32	(a)	(a)	(a)	0.40	0.80	0.45	0.15	0.25	0.6	0.2	0.8
33	(b)	(b)	(b)	0.40	0.80	0.45	0.15	0.25	0.6	0.2	0.8
34	(c)	(a)	(b)	0.40	0.80	0.45	0.15	0.25	0.6	0.2	0.8

Table 3. Diagonal Elements of $\Sigma_k(h)$ and $(1+2k/T)\Sigma(h)$
of Corresponding d^*

d^*	*Lead*	T	*VAR order*	Theoretical $(1+2k/T)\Sigma(h)$		Theoretical $\Sigma(h)$	
				y_{1t}	y_{2t}	y_{1t}	y_{2t}
0.35	$h=1$	100	$k=3$	1.060	1.060	1.000	1.000
		200	$k=5$	1.050	1.050	1.000	1.000
	$h=2$	100	$k=3$	1.190	1.190	1.123	1.123
		200	$k=5$	1.179	1.179	1.123	1.123
	$h=3$	100	$k=3$	1.249	1.249	1.178	1.178
		200	$k=5$	1.237	1.237	1.178	1.178
	$h=4$	100	$k=3$	1.285	1.285	1.213	1.213
		200	$k=5$	1.273	1.273	1.213	1.213
	$h=5$	100	$k=3$	1.311	1.311	1.237	1.237
		200	$k=5$	1.298	1.298	1.237	1.237
0.15	$h=1$	100	$k=3$	1.060	1.060	1.000	1.000
		200	$k=5$	1.050	1.050	1.000	1.000
	$h=2$	100	$k=3$	1.084	1.084	1.023	1.023
		200	$k=5$	1.074	1.074	1.023	1.023
	$h=3$	100	$k=3$	1.092	1.092	1.030	1.030
		200	$k=5$	1.081	1.081	1.030	1.030
	$h=4$	100	$k=3$	1.096	1.096	1.034	1.034
		200	$k=5$	1.085	1.085	1.034	1.034
	$h=5$	100	$k=3$	1.098	1.098	1.036	1.036
		200	$k=5$	1.088	1.088	1.036	1.036

Table 4.1. Mean Square Error (Diagonal Elements of $\Sigma_k(h)$) of predicting $y_{1,t+h}$

T	EX	1	2	3	4	5	6	7	8	9	10
100	$h=1$	1.18	1.17	1.18	1.18	1.20	1.21	1.20	1.19	1.28	1.31
	$h=2$	1.31	1.33	1.34	1.34	1.16	1.19	1.18	1.18	1.20	1.22
	$h=3$	1.54	1.56	1.56	1.55	1.44	1.47	1.47	1.45	1.58	1.62
	$h=4$	1.59	1.61	1.61	1.62	1.41	1.45	1.43	1.43	1.44	1.50
	$h=5$	1.71	1.73	1.73	1.73	1.56	1.60	1.58	1.58	1.65	1.71
200	$h=1$	1.11	1.13	1.10	1.10	1.11	1.12	1.11	1.11	1.18	1.22
	$h=2$	1.31	1.35	1.30	1.33	1.15	1.17	1.14	1.17	1.17	1.18
	$h=3$	1.46	1, 48	1.45	1.48	1.32	1.35	1.31	1.34	1.42	1.44
	$h=4$	1.65	1.68	1.64	1.67	1.46	1.49	1.45	1.48	1.52	1.53
	$h=5$	1.63	1.65	1.62	1.66	1.46	1.50	1.45	1.49	1.59	1.62

T	EX	11	12	13	14	15	16	17	18	19	20
100	$h=1$	1.26	1.20	1.26	1.21	1.19	1.19	1.20	1.20	1.19	1.20
	$h=2$	1.20	1.19	1.19	1.18	1.18	1.18	1.17	1.17	1.18	1.18
	$h=3$	1.59	1.54	1.58	1.53	1.45	1.44	1.53	1.53	1.53	1.54
	$h=4$	1.47	1.45	1.47	1.44	1.42	1.42	1.49	1.41	1.42	1.45
	$h=5$	1.68	1.65	1.67	1.64	1.58	1.58	1.50	1.64	1.64	1.65
200	$h=1$	1.17	1.19	1.11	1.13	1.11	1.11	1.12	1.13	1.12	1.12
	$h=2$	1.14	1.17	1.14	1.16	1.13	1.16	1.13	1.15	1.12	1.15
	$h=3$	1.39	1.42	1.32	1.35	1.31	1.34	1.37	1.39	1.36	1.38
	$h=4$	1.48	1.52	1.46	1.48	1.44	1.48	1.47	1.48	1.47	1.50
	$h=5$	1.51	1.60	1.45	1.50	1.45	1.49	1.51	1.53	1.50	1.56

T	EX	21	22	23	24	25	26	27	28	29	30
100	$h=1$	1.28	1.20	1.25	1.20	1.28	1.31	1.25	1.31	1.13	1.14
	$h=2$	1.19	1.15	1.18	1.17	1.18	1.20	1.17	1.20	1.22	1.12
	$h=3$	1.57	1.39	1.57	1.53	1.56	1.61	1.57	1.61	1.29	1.25
	$h=4$	1.43	1.48	1.46	1.44	1.44	1.49	1.46	1.48	1.33	1.27
	$h=5$	1.64	1.52	1.66	1.64	1.65	1.71	1.67	1.71	1.30	1.26
200	$h=1$	1.20	1.22	1.17	1.11	1.19	1.22	1.16	1.18	1.18	1.19
	$h=2$	1.15	1.18	1.13	1.17	1.14	1.17	1.12	1.16	1.16	1.06
	$h=3$	1.39	1.43	1.38	1.34	1.39	1.42	1.38	1.40	1.29	1.26
	$h=4$	1.49	1.52	1.47	1.48	1.48	1.52	1.47	1.50	1.19	1.12
	$h=5$	1.52	1.61	1.51	1.49	1.52	1.60	1.51	1.58	1.25	1.23

T	EX	31	32	33	34
100	$h=1$	1.16	1.13	1.14	1.15
	$h=2$	1.11	1.22	1.12	1.10
	$h=3$	1.24	1.29	1.25	1.23
	$h=4$	1.26	1.32	1.26	1.24
	$h=5$	1.25	1.31	1.27	1.25
200	$h=1$	1.25	1.15	1.17	1.20
	$h=2$	1.16	1.26	1.14	1.16
	$h=3$	1.33	1.33	1.30	1.32
	$h=4$	1.33	1.38	1.32	1.34
	$h=5$	1.34	1.36	1.32	1.34

Table 4.2. Mean Square Errors (Diagonal Elements of $\Sigma_k(h)$) of Predicting $y_{2,t+h}$

T	EX	1	2	3	4	5	6	7	8	9	10
100	$h = 1$	1.08	1.11	1.08	1.10	1.10	1.11	1.10	1.12	1.18	1.22
	$h = 2$	1.26	1.29	1.30	1.32	1.16	1.20	1.20	1.22	1.29	1.33
	$h = 3$	1.29	1.33	1.35	1.36	1.22	1.27	1.27	1.29	1.46	1.55
	$h = 4$	1.32	1.36	1.38	1.40	1.24	1.31	1.31	1.33	1.54	1.62
	$h = 5$	1.33	1.35	1.42	1.45	1.24	1.34	1.33	1.36	1.65	1.69
200	$h = 1$	1.14	1.16	1.14	1.16	1.14	1.16	1.14	1.17	1.25	1.26
	$h = 2$	1.20	1.22	1.20	1.25	1.08	1.12	1.08	1.13	1.19	1.22
	$h = 3$	1.26	1.27	1.26	1.29	1.18	1.21	1.18	1.22	1.40	1.45
	$h = 4$	1.32	1.35	1.31	1.36	1.19	1.24	1.20	1.24	1.37	1.44
	$h = 5$	1.47	1.51	1.44	1.48	1.35	1.39	1.35	1.40	1.60	1.68

T	EX	11	12	13	14	15	16	17	18	19	20
100	$h = 1$	1.18	1.16	1.10	1.12	1.10	1.12	1.10	1.12	1.13	1.16
	$h = 2$	1.30	1.25	1.14	1.17	1.18	1.20	1.22	1.25	1.25	1.25
	$h = 3$	1.54	1.56	1.22	1.26	1.27	1.29	1.45	1.47	1.52	1.56
	$h = 4$	1.61	1.62	1.24	1.27	1.31	1.33	1.55	1.58	1.60	1.62
	$h = 5$	1.68	1.71	1.23	1.25	1.33	1.36	1.57	1.59	1.67	1.71
200	$h = 1$	1.25	1.31	1.13	1.15	1.14	1.16	1.13	1.16	1.14	1.17
	$h = 2$	1.20	1.24	1.07	1.10	1.07	1.12	1.15	1.19	1.15	1.20
	$h = 3$	1.41	1.47	1.18	1.21	1.18	1.21	1.39	1.42	1.39	1.42
	$h = 4$	1.39	1.46	1.19	1.20	1.19	1.24	1.38	1.44	1.38	1.45
	$h = 5$	1.61	1.69	1.35	1.38	1.35	1.40	1.62	1.66	1.62	1.68

T	EX	21	22	23	24	25	26	27	28	29	30
100	$h = 1$	1.11	1.12	1.11	1.29	1.23	1.24	1.20	1.24	1.16	1.18
	$h = 2$	1.16	1.20	1.19	1.17	1.13	1.17	1.16	1.17	1.24	1.14
	$h = 3$	1.22	1.27	1.26	1.34	1.26	1.31	1.29	1.31	1.34	1.30
	$h = 4$	1.26	1.31	1.30	1.32	1.26	1.31	1.31	1.31	1.38	1.33
	$h = 5$	1.25	1.34	1.32	1.37	1.27	1.36	1.33	1.36	1.37	1.32
200	$h = 1$	1.13	1.15	1.14	1.17	1.24	1.25	1.30	1.30	1.20	1.22
	$h = 2$	1.07	1.11	1.08	1.13	1.08	1.12	1.10	1.13	1.20	1.10
	$h = 3$	1.18	1.20	1.18	1.22	1.20	1.20	1.21	1.23	1.29	1.27
	$h = 4$	1.19	1.23	1.20	1.24	1.20	1.23	1.20	1.24	1.20	1.20
	$h = 5$	1.35	1.39	1.35	1.40	1.37	1.40	1.38	1.42	1.26	1.32

T	EX	31	32	33	34
100	$h = 1$	1.25	1.16	1.17	1.21
	$h = 2$	1.16	1.27	1.15	1.17
	$h = 3$	1.33	1.33	1.30	1.33
	$h = 4$	1.34	1.39	1.33	1.34
	$h = 5$	1.35	1.37	1.33	1.34
200	$h = 1$	1.27	1.22	1.21	1.24
	$h = 2$	1.12	1.11	1.21	1.13
	$h = 3$	1.28	1.28	1.30	1.29
	$h = 4$	1.20	1.20	1.27	1.20
	$h = 5$	1.31	1.31	1.34	1.31

Table 5.1. Relative Root Mean Squared Forecast Error of y_{1t}

EX	*h*	*PBK*	*VNVK*	*PBUK*	*VNVUK*	*VNVNO*	*RWOK*	*RWOUK*
1	1	1.010	1.099	1.077	1.176	1.051	0.997	1.034
	3	1.007	1.119	1.086	1.189	1.072	1.000	1.046
2	1	1.037	1.147	1.116	1.212	1.083	1.002	1.076
	3	1.056	1.174	1.142	1.246	1.078	1.004	1.092
5	1	1.049	1.164	1.127	1.211	1.190	1.003	1.100
	3	1.055	1.182	1.128	1.232	1.192	1.007	1.119
6	1	1.127	1.208	1.182	1.301	1.272	1.011	1.146
	3	1.129	1.221	1.177	1.318	1.279	1.014	1.153
9	1	1.063	1.153	1.132	1.188	1.187	1.002	1.102
	3	1.080	1.182	1.149	1.180	1.190	1.009	1.118
10	1	1.132	1.189	1.166	1.321	1.292	1.010	1.142
	3	1.139	1.192	1.159	1.338	1.299	1.016	1.150
13	1	1.020	1.069	1.117	1.219	1.114	1.001	1.103
	3	1.013	1.076	1.100	1.227	1.121	1.002	1.119
14	1	1.152	1.123	1.212	1.343	1.231	1.011	1.152
	3	1.182	1.134	1.200	1.363	1.222	1.014	1.157
21	1	1.099	1.204	1.137	1.362	1.326	1.008	1.129
	3	1.104	1.223	1.152	1.396	1.355	1.012	1.136
22	1	1.162	1.289	1.148	1.466	1.407	1.010	1.133
	3	1.188	1.301	1.169	1.487	1.416	1.013	1.142
23	1	1.101	1.198	1.145	1.358	1.321	1.007	1.122
	3	1.098	1.214	1.162	1.391	1.351	1.010	1.131
24	1	1.154	1.276	1.158	1.427	1.417	1.018	1.129
	3	1.175	1.298	1.177	1.472	1.426	1.020	1.136
25	1	1.109	1.226	1.142	1.382	1.366	1.003	1.118
	3	1.118	1.239	1.158	1.402	1.382	1.007	1.127
26	1	1.178	1.302	1.156	1.477	1.404	1.014	1.129
	3	1.188	1.316	1.171	1.489	1.419	1.016	1.141
27	1	1.107	1.201	1.153	1.360	1.329	1.004	1.116
	3	1.101	1.216	1.164	1.391	1.363	1.005	1.130
28	1	1.165	1.288	1.162	1.451	1.421	1.021	1.135
	3	1.181	1.301	1.179	1.469	1.432	1.026	1.142
29	1	1.139	1.367	1.328	2.026	1.099	1.007	1.183
	3	1.158	1.396	1.354	1.987	1.122	1.012	1.194
30	1	1.168	1.400	1.439	2.225	1.129	1.016	1.199
	3	1.192	1.426	1.420	2.256	1.147	1.022	1.214
31	1	1.231	1.656	1.449	2.818	2.037	1.019	1.244
	3	1.288	1.688	1.427	2.865	2.089	1.027	1.252
32	1	1.132	1.374	1.342	2.033	1.111	1.008	1.166
	3	1.147	1.389	1.366	1.998	1.138	1.016	1.182
33	1	1.186	1.419	1.437	2.312	1.127	1.017	1.201
	3	1.222	1.438	1.419	2.294	1.141	1.020	1.218
34	1	1.296	1.757	1.532	2.924	2.074	1.020	1.257
	3	1.363	1.579	1.546	2.916	2.126	1.034	1.269

Notes : PBK and PBUK represent the post-break method with the known and unknown break respectively. VNVK and VNVUK represent the Naive VARFIMA-based method with the known and unknown break respectively. VNOVO represents the ARFIMA(d^*)-based method. ROWK and ROWUK represent the robust optimal weight averaging method with the known and unknown break, respectively.

Table 5.2. Relative Root Mean Squared Forecast Error of y_{2t}

EX	h	PBK	$VNVK$	$PBUK$	$VNVUK$	$VNVNO$	$ROWK$	$ROWUK$
1	1	1.008	1.119	1.101	1.176	1.044	0.997	1.021
	3	1.001	1.121	1.090	1.189	1.039	1.000	1.029
2	1	1.048	1.140	1.122	1.212	1.115	1.005	1.089
	3	1.064	1.166	1.138	1.220	1.089	1.009	1.092
5	1	1.033	1.124	1.111	1.231	1.214	1.003	1.079
	3	1.057	1.112	1.132	1.229	1.222	1.014	1.103
6	1	1.107	1.248	1.202	1.322	1.292	1.009	1.124
	3	1.120	1.251	1.197	1.308	1.303	1.016	1.137
9	1	1.044	1.143	1.098	1.204	1.180	1.002	1.059
	3	1.067	1.162	1.120	1.233	1.201	1.009	1.065
10	1	1.149	1.199	1.189	1.342	1.345	1.007	1.112
	3	1.157	1.190	1.171	1.356	1.329	1.014	1.129
13	1	1.018	1.059	1.100	1.179	1.094	1.001	1.056
	3	1.023	1.066	1.109	1.200	1.110	1.003	1.081
14	1	1.147	1.134	1.231	1.331	1.190	1.010	1.160
	3	1.172	1.146	1.219	1.352	1.193	1.018	1.178
21	1	1.121	1.266	1.180	1.380	1.300	1.008	1.131
	3	1.107	1.252	1.172	1.399	1.321	1.013	1.140
22	1	1.150	1.302	1.172	1.433	1.377	1.011	1.142
	3	1.162	1.332	1.181	1.467	1.392	1.020	1.157
23	1	1.119	1.208	1.156	1.366	1.344	1.007	1.123
	3	1.100	1.227	1.166	1.379	1.359	1.011	1.139
24	1	1.161	1.296	1.162	1.431	1.399	1.006	1.120
	3	1.182	1.302	1.183	1.462	1.416	1.010	1.131
25	1	1.117	1.255	1.172	1.377	1.327	1.011	1.172
	3	1.100	1.247	1.184	1.410	1.339	1.016	1.181
26	1	1.166	1.316	1.187	1.455	1.389	1.016	1.201
	3	1.194	1.327	1.221	1.481	1.406	1.022	1.212
27	1	1.105	1.217	1.176	1.416	1.312	1.009	1.159
	3	1.118	1.232	1.192	1.439	1.333	1.013	1.172
28	1	1.172	1.272	1.170	1.401	1.410	1.015	1.164
	3	1.189	1.296	1.188	1.411	1.418	1.024	1.171
29	1	1.121	1.344	1.318	2.000	1.079	1.008	1.141
	3	1.158	1.380	1.334	1.972	1.118	1.014	1.158
30	1	1.168	1.400	1.439	2.225	1.129	1.019	1.170
	3	1.192	1.426	1.420	2.256	1.147	1.018	1.189
31	1	1.267	1.689	1.473	2.678	2.172	1.015	1.196
	3	1.289	1.647	1.459	2.725	2.202	1.027	1.203
32	1	1.129	1.355	1.347	2.011	1.119	1.010	1.154
	3	1.140	1.369	1.352	1.954	1.125	1.021	1.172
33	1	1.179	1.429	1.422	2.367	1.227	1.021	1.208
	3	1.207	1.435	1.426	2.350	1.242	1.027	1.223
34	1	1.316	1.747	1.586	2.801	2.236	1.028	1.271
	3	1.372	1.692	1.544	2.826	2.267	1.036	1.284

Table 6.1. Forecast Evaluation of $y_{1,T+1}$

EX		α_0	α_1	α_2	R^2	$P(F)$	$P(GW)$
1	VAR	0.039(0.050)	0.961(0.109)		0.294		
	VAR+PBK	−0.029(0.112)	0.918(0.122)	0.189(0.126)	0.317	0.32	0.06
	VAR+PBUK	0.051(0.087)	0.907(0.127)	0.122(0.136)	0.307	0.36	0.05
	VAR+NVK	−0.022(0.069)	0.899(0.097)	0.160(0.114)	0.310	0.34	0.05
	VAR+NVUK	0.062(0.060)	0.901(0.116)	0.106(0.176)	0.292	0.39	0.04
	VAR+NVNO	−0.100(0.149)	0.876(0.136)	0.211(0.142)	0.323	0.28	0.06
	VAR+ROWK	−0.011(0.067)	0.702(0.203)	0.388(0.257)	0.378	0.19	0.20
	VAR+ROWUK	0.043(0.089)	0.889(0.114)	0.201(0.194)	0.311	0.33	0.06
6	VAR	0.100(0.177)	0.917(0.192)		0.279		
	VAR+PBK	−0.068(0.125)	0.886(0.144)	0.226(0.233)	0.297	0.44	0.03
	VAR+PBUK	−0.029(0.090)	0.870(0.119)	0.162(0.201)	0.281	0.47	0.02
	VAR+NVK	−0.021(0.121)	0.881(0.159)	0.140(0.182)	0.286	0.45	0.02
	VAR+NVUK	0.107(0.201)	0.866(0.124)	0.099(0.209)	0.282	0.50	0.02
	VAR+NVNO	−0.106(0.168)	0.854(0.187)	0.082(0.237)	0.270	0.66	0.00
	VAR+ROWK	0.033(0.123)	0.806(0.152)	0.275(0.194)	0.339	0.30	0.17
	VAR+ROWUK	0.079(0.109)	0.886(0.114)	0.261(0.192)	0.305	0.41	0.03
10	VAR	0.117(0.229)	0.928(0.213)		0.343		
	VAR+PBK	0.147(0.235)	0.849(0.229)	0.215(0.144)	0.366	0.33	0.05
	VAR+PBUK	−0.082(0.192)	0.849(0.248)	0.116(0.228)	0.352	0.44	0.03
	VAR+NVK	0.028(0.081)	0.850(0.172)	0.201(0.197)	0.360	0.35	0.03
	VAR+NVUK	0.080(0.144)	0.812(0.167)	0.129(0.241)	0.351	0.39	0.03
	VAR+NVNO	−0.122(0.066)	0.809(0.139)	0.101(0.233)	0.348	0.42	0.03
	VAR+ROWK	−0.105(0.101)	0.792(0.202)	0.288(0.119)	0.399	0.24	0.21
	VAR+ROWUK	−0.113(0.099)	0.899(0.114)	0.241(0.164)	0.373	0.31	0.07
21	VAR(k)	−0.067(0.097)	0.948(0.223)		0.379		
	VAR+PBK	0.109(0.145)	0.856(0.129)	0.229(0.207)	0.396	0.41	0.08
	VAR+PBUK	−0.093(0.101)	0.901(0.156)	0.150(0.165)	0.375	0.50	0.03
	VAR+NVK	−0.039(0.098)	0.863(0.179)	0.200(0.099)	0.371	0.46	0.03
	VAR+NVUK	−0.055(0.201)	0.819(0.237)	0.132(0.217)	0.368	0.53	0.01
	VAR+NVNO	0.090(0.089)	0.802(0.179)	0.108(0.211)	0.364	0.56	0.00
	VAR+ROWK	−0.022(0.055)	0.702(0.203)	0.363(0.257)	0.442	0.12	0.24
	VAR+ROWUK	−0.138(0.111)	0.873(0.114)	0.279(0.201)	0.402	0.39	0.15
29	VAR	0.074(0.096)	0.931(0.129)		0.328		
	VAR+PBK	−0.049(0.168)	0.900(0.185)	0.289(0.199)	0.362	0.32	0.07
	VAR+PBUK	0.172(0.139)	0.848(0.137)	0.198(0.173)	0.346	0.42	0.04
	VAR+NVK	−0.019(0.100)	0.852(0.226)	0.123(0.162)	0.332	0.45	0.04
	VAR+NVUK	0.201(0.262)	0.827(0.211)	0.090(0.189)	0.329	0.52	0.03
	VAR+NVNO	−0.219(0.277)	0.864(0.129)	0.242(0.279)	0.353	0.36	0.05
	VAR+ROWK	0.010(0.039)	0.713(0.203)	0.402(0.264)	0.427	0.21	0.19
	VAR+ROWUK	0.043(0.089)	0.886(0.114)	0.261(0.192)	0.313	0.33	0.07
34	VAR	0.094(0.195)	0.926(0.219)		0.244		
	VAR+PBK	−0.051(0.153)	0.876(0.287)	0.249(0.184)	0.266	0.50	0.04
	VAR+PBUK	0.071(0.141)	0.836(0.177)	0.156(0.241)	0.254	0.58	0.02
	VAR+NVK	−0.031(0.093)	0.847(0.185)	0.202(0.169)	0.260	0.53	0.02
	VAR+NVUK	0.111(0.100)	0.829(0.214)	0.100(0.233)	0.249	0.62	0.00
	VAR+NVNO	−0.169(0.230)	1.064(0.131)	0.062(0.279)	0.244	0.66	0.00
	VAR+ROWK	−0.034(0.081)	0.801(0.301)	0.257(0.162)	0.312	0.42	0.03
	VAR+ROWUK	−0.163(0.265)	0.889(0.266)	0.240(0.219)	0.261	0.54	0.04

Table 6.2. Forecast Evaluation of $y_{2,T+1}$

EX		α_0	α_1	α_2	R^2	$P(F)$	$P(GW)$
1	VAR	0.020(0.052)	0.977(0.209)		0.366		
	VAR+PBK	0.044(0.132)	0.842(0.192)	0.282(0.146)	0.388	0.22	0.06
	VAR+PBUK	−0.051(0.117)	0.857(0.149)	0.179(0.166)	0.370	0.29	0.04
	VAR+NVK	0.060(0.099)	0.876(0.212)	0.255(0.178)	0.375	0.24	0.04
	VAR+NVUK	−0.052(0.161)	0.867(0.156)	0.146(0.229)	0.374	0.32	0.04
	VAR+NVNO	−0.19(0.109)	0.868(0.123)	0.272(0.193)	0.380	0.24	0.05
	VAR+ROWK	0.056(0.075)	0.689(0.203)	0.429(0.270)	0.415	0.16	0.16
	VAR+ROWUK	−0.101(0.104)	0.860(0.187)	0.268(0.192)	0.384	0.21	0.08
6	VAR	−0.074(0.109)	0.901(0.095)		0.268		
	VAR+PBK	−0.068(0.125)	0.835(0.144)	0.244(0.293)	0.296	0.40	0.05
	VAR+PBUK	−0.129(0.085)	0.860(0.119)	0.164(0.245)	0.287	0.49	0.02
	VAR+NVK	−0.133(0.149)	0.873(0.192)	0.191(0.212)	0.290	0.44	0.05
	VAR+NVUK	0.111(0.211)	0.879(0.124)	0.069(0.233)	0.276	0.56	0.01
	VAR+NVNO	0.109(0.138)	0.867(0.187)	0.092(0.232)	0.280	0.53	0.01
	VAR+ROWK	0.033(0.123)	0.802(0.152)	0.275(0.194)	0.327	0.30	0.10
	VAR+ROWUK	0.079(0.109)	0.886(0.114)	0.261(0.192)	0.310	0.34	0.07
10	VAR	0.107(0.200)	0.932(0.253)		0.355		
	VAR+PBK	−0.147(0.235)	0.899(0.229)	0.231(0.144)	0.375	0.34	0.05
	VAR+PBUK	−0.199(0.298)	0.878(0.144)	0.142(0.109)	0.362	0.42	0.03
	VAR+NVK	−0.031(0.090)	0.818(0.222)	0.198(0.197)	0.368	0.36	0.05
	VAR+NVUK	0.080(0.144)	0.812(0.167)	0.100(0.241)	0.357	0.46	0.03
	VAR+NVNO	0.097(0.090)	0.829(0.149)	0.087(0.255)	0.355	0.48	0.02
	VAR+ROWK	−0.105(0.101)	0.793(0.202)	0.291(0.107)	0.399	0.25	0.16
	VAR+ROWUK	−0.113(0.099)	0.898(0.114)	0.242(0.164)	0.380	0.33	0.08
21	VAR(k)	0.116(0.199)	0.901(0.204)		0.319		
	VAR+PBK	0.042(0.125)	0.886(0.133)	0.173(0.221)	0.338	0.39	0.05
	VAR+PBUK	0.091(0.111)	0.851(0.166)	0.121(0.141)	0.317	0.45	0.04
	VAR+NVK	−0.032(0.090)	0.812(0.199)	0.187(0.104)	0.341	0.37	0.06
	VAR+NVUK	0.129(0.222)	0.822(0.246)	0.090(0.129)	0.321	0.51	0.04
	VAR+NVNO	0.111(0.099)	0.808(0.179)	0.110(0.181)	0.324	0.50	0.04
	VAR+ROWK	−0.022(0.055)	0.702(0.203)	0.367(0.254)	0.439	0.12	0.19
	VAR+ROWUK	−0.138(0.111)	0.873(0.114)	0.289(0.222)	0.408	0.28	0.09
29	VAR	0.084(0.126)	0.941(0.139)		0.321		
	VAR+PBK	0.049(0.168)	0.877(0.189)	0.261(0.182)	0.340	0.33	0.04
	VAR+PBUK	0.069(0.119)	0.838(0.130)	0.198(0.176)	0.330	0.40	0.03
	VAR+NVK	−0.021(0.109)	0.858(0.226)	0.239(0.169)	0.337	0.36	0.03
	VAR+NVUK	0.101(0.202)	0.827(0.129)	0.177(0.145)	0.320	0.46	0.01
	VAR+NVNO	−0.018(0.069)	0.813(0.109)	0.300(0.261)	0.350	0.33	0.06
	VAR+ROWK	0.010(0.039)	0.713(0.203)	0.402(0.264)	0.423	0.21	0.18
	VAR+ROWUK	−0.037(0.094)	0.824(0.171)	0.322(0.201)	0.366	0.30	0.09
34	VAR	0.114(0.165)	0.906(0.231)		0.244		
	VAR+PBK	−0.044(0.173)	0.836(0.292)	0.212(0.184)	0.260	0.52	0.05
	VAR+PBUK	−0.182(0.156)	0.810(0.227)	0.116(0.091)	0.250	0.60	0.03
	VAR+NVK	−0.044(0.076)	0.847(0.155)	0.188(0.169)	0.255	0.55	0.03
	VAR+NVUK	0.012(0.065)	0.877(0.151)	0.092(0.233)	0.246	0.64	0.01
	VAR+NVNO	0.066(0.059)	0.854(0.131)	0.052(0.279)	0.244	0.67	0.01
	VAR+ROWK	−0.087(0.111)	0.821(0.269)	0.279(0.189)	0.289	0.40	0.11
	VAR+ROWUK	0.101(0.125)	0.842(0.236)	0.221(0.178)	0.262	0.48	0.05

Table 7. The Size Performance of Sup-Wald Test

Experiments	23	24	27	28	34
Size					
T=100	0.21	0.33	0.34	0.44	0.66
T=200	0.27	0.35	0.42	0.49	0.69

Table 8. Root Mean Squared Forecast Error Value of η_{T_1+1}

	Experiments	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
y_{1t}	VAR					
	23	1.18	1.16	1.39	1.49	1.55
	24	1.16	1.19	1.36	1.53	1.57
	27	1.17	1.14	1.38	1.50	1.53
	28	1.20	1.19	1.40	1.52	1.60
	$VNVNO$					
	23	1.65	1.72	1.86	1.91	2.01
	24	1.85	1.83	1.98	2.09	2.19
	27	1.71	1.79	1.89	1.97	2.11
	28	1.91	1.93	2.01	2.11	2.20
y_{2t}	VAR					
	23	1.18	1.11	1.20	1.21	1.38
	24	1.19	1.18	1.25	1.29	1.41
	27	1.29	1.13	1.22	1.20	1.40
	28	1.31	1.19	1.26	1.25	1.43
	$VNVNO$					
	23	1.58	1.54	1.61	1.66	1.83
	24	1.69	1.68	1.70	1.83	1.91
	27	1.72	1.61	1.69	1.72	1.90
	28	1.88	1.54	1.76	1.81	1.99

Table 9. Out-of-Sample Forecast Evaluation (Cholesky)

EX		α_0	α_1	α_2	R^2	$P(F)$	$P(GW)$
AXP	VAR	0.067(0.110)	0.917(0.205)		0.281		
	VAR+PB	0.090(0.146)	0.887(0.201)	0.253(0.199)	0.295	0.48	0.04
	VAR+VNV	0.102(0.121)	0.869(0.193)	0.201(0.178)	0.289	0.52	0.03
	VAR+VNVNO	−0.118(0.089)	0.892(0.129)	0.161(0.121)	0.284	0.62	0.03
	VAR+VRB	0.046(0.110)	0.802(0.157)	0.289(0.202)	0.306	0.39	0.10
	VAR+ROW	−0.033(0.095)	0.778(0.163)	0.373(0.234)	0.367	0.27	0.20
C	VAR	0.014(0.109)	0.893(0.213)		0.235		
	VAR+PB	−0.071(0.131)	0.862(0.185)	0.277(0.211)	0.241	0.62	0.02
	VAR+VNV	0.093(0.114)	0.842(0.202)	0.299(0.201)	0.246	0.57	0.03
	VAR+VNVNO	0.100(0.144)	0.878(0.211)	0.188(0.167)	0.237	0.68	0.02
	VAR+ VRB	−0.095(0.105)	0.820(0.174)	0.324(0.260)	0.270	0.42	0.08
	VAR+ROW	0.033(0.123)	0.802(0.152)	0.275(0.194)	0.329	0.30	0.03
GE	VAR	0.017(0.192)	0.908(0.236)		0.322		
	VAR+PB	−0.047(0.158)	0.866(0.219)	0.284(0.193)	0.337	0.39	0.04
	VAR+VNV	−0.131(0.199)	0.828(0.224)	0.216(0.167)	0.329	0.44	0.05
	VAR+VNVNO	−0.072(0.100)	0.878(0.158)	0.127(0.149)	0.324	0.52	0.03
	VAR+VRB	0.087(0.119)	0.792(0.199)	0.367(0.268)	0.362	0.33	0.18
	VAR+ROW	−0.113(0.099)	0.899(0.114)	0.241(0.164)	0.377	0.30	0.26
HD	VAR(k)	−0.025(0.075)	0.941(0.132)		0.412		
	VAR+PB	−0.049(0.119)	0.873(0.167)	0.263(0.182)	0.419	0.38	0.05
	VAR+VNV	0.062(0.093)	0.842(0.239)	0.285(0.191)	0.423	0.34	0.04
	VAR+VNVNO	−0.092(0.111)	0.821(0.251)	0.296(0.200)	0.428	0.30	0.04
	VAR+VRB	0.049(0.121)	0.806(0.157)	0.394(0.268)	0.441	0.21	0.17
	VAR+ROW	−0.138(0.111)	0.873(0.114)	0.279(0.222)	0.402	0.39	0.24
IBM	VAR	0.060(0.106)	0.955(0.146)		0.447		
	VAR+PB	0.050(0.152)	0.916(0.167)	0.191(0.172)	0.455	0.46	0.06
	VAR+VNV	−0.029(0.068)	0.918(0.226)	0.129(0.131)	0.448	0.53	0.04
	VAR+VNVNO	0.021(0.077)	0.900(0.144)	0.156(0.210)	0.451	0.48	0.04
	VAR+VRB	−0.017(0.056)	0.869(0.170)	0.284(0.235)	0.467	0.29	0.19
	VAR+ROW	0.043(0.089)	0.886(0.114)	0.261(0.192)	0.313	0.33	0.30
JPM	VAR	0.054(0.151)	0.901(0.217)		0.250		
	VAR+PB	0.101(0.161)	0.872(0.233)	0.171(0.201)	0.256	0.58	0.04
	VAR+VNV	−0.077(0.089)	0.857(0.183)	0.147(0.169)	0.254	0.62	0.02
	VAR+VNVNO	0.066(0.059)	0.854(0.131)	0.101(0.132)	0.251	0.66	0.02
	VAR+VRB	0.064(0.092)	0.832(0.182)	0.229(0.199)	0.268	0.37	0.09
	VAR+ROW	−0.050(0.101)	0.889(0.266)	0.240(0.219)	0.267	0.50	0.15