Leveraged Firms, Patent Licensing, and Limited Liability

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Abstract

In a seminal paper on patent licensing, Kamien and Tauman (1986) show that fixed-fee licensing is always superior to royalty licensing for the outsider patentee. However, empirical studies demonstrate that royalty licensing is much more popular than fixed-fee licensing in practice. This paper attempts to reconcile with this controversy *within* Kamien and Tauman's (1986) theoretical framework by taking into account the financial structure. In particular, this paper focuses on an important feature of the financial structure of the firm in the modern corporation that has received no attention in the patent licensing literature. The main contribution of the paper is as follows. Provided that the leveraged firms produce a homogeneous product and engage in Cournot competition, this paper shows that the optimal licensing contract for the outsider patentee is royalty licensing when the mean-preserving variance of demand is large in the presence of debt financing, while it is non-exclusive fixed-fee licensing otherwise.

Leveraged Firms, Patent Licensing, and Limited Liability

1. Introduction

In a seminal paper on patent licensing, Kamien and Tauman (1986) show that fixed-fee licensing is always superior to royalty licensing for the outsider patentee owning a cost-reducing innovation, when firms produce a homogeneous good and engage in Cournot competition in the commodity market. Subsequently, Kamien *et al*. (1992) confirm the same result by using a generalized demand function. Nevertheless, the empirical literature, such as Rostoker (1984), shows that royalties alone account for 39%, fixed fees alone for 13%, and royalties plus fixed fees for 46% of licensing contracts. In addition, Macho-Stadler *et al*. (1996) and Jensen and Thursby (2001) also consistently point to the prevalence of royalty licensing. Thus, the findings of empirical studies demonstrate that royalty licensing is much more popular than fixed-fee licensing. This creates significant interest in explaining the rationale for choosing royalty licensing in licensing contracts.

The superiority of royalty over fixed-fee licensing has been justified in the literature by appealing to the following aspects. For the case where the patentee stands outside the market, Beggs (1992) emphasizes the role of asymmetric information; Muto (1993) points out the importance of product differentiation; Bousquet *et al*. (1998) highlight risk sharing; Macho-Stadler *et al*. (1996) and Jensen and Thursby

(2001) focus on moral hazard; Saracho (2002) shows that a sales delegation game is crucial; and Poddar and Sinha (2004) refer to the influence of spatial competition. For the case where the patentee is an insider competing with the licensees in the industry, Katz and Shapiro (1985) and Wang (1998) show that royalty licensing is definitely superior to fixed-fee licensing. Unfortunately, the above analyses completely ignore an important feature of the modern corporation that has received no attention in the treatment of the firm in the patent licensing literature, namely, the financial structure of the firm.

It is well recognized that firms usually issue debt to finance their production in a modern economy. In a pioneering paper, Brander and Lewis (1986) argue that the choice of financial structure can affect output markets through the limited liability effect of debt financing, in which shareholders will ignore reductions in returns in bankrupt states, since debt holders become the residual claimants.[1] The second channel for finances to affect output markets is the strategic bankruptcy effect, in which firms might make output market decisions that raise the chances of driving their rivals into insolvency. Brander and Lewis (1986) show that both the limited liability effect and the strategic bankruptcy effect may commit a leveraged firm to a more aggressive output stance.

[1] Other studies related to the issue that the choice of financial structure can affect output markets include: Brander and Lewis (1988), Showalter (1995), Hughes *et al.* (1998), and Damania (1999), etc.

As a matter of fact, the issue that whether or not royalty licensing is superior over fixed-fee licensing is very important in theory and in practice. This paper attempts to reconcile with these arguments *within* Kamien and Tauman's (1986) theoretical framework by taking into account the financial structure, when the leveraged firms produce a homogeneous good and engage in Cournot competition in the commodity market. In particular, we reason that the outsider patentee may decide to choose royalty licensing in order to earn a larger licensing profit caused by a more aggressive output strategy adopted by the leveraged firm, when the firm can issue debt to the public and financial institutions. To the best of our knowledge, a study on this issue has not yet been touched upon. This paper aims to fill this gap.

The main finding of this paper is as follows. By taking into account the debt financing of the leveraged firms, the optimal licensing contract for the outsider patentee is royalty licensing in the case of a homogeneous product and Cournot competition when the mean-preserving variance of demand is large, while it is non-exclusive fixed-fee licensing otherwise. Thus, this paper provides a new explanation to previous literature to justify the superiority of the royalty over the fixed-fee licensing contract.

The remainder of the paper is organized as follows. Section 2 sets up a basic model to analyze the case where patent licensing is absent. Section 3 and Section 4 examine the optimal number of licenses under fixed-fee licensing and the optimal royalty rate under royalty licensing, respectively. Section 5 explores the outsider patentee's optimal licensing contracts in terms of fixed-fee and royalty licensing. The final section concludes the paper.

2. The Basic Framework

Consider two leveraged firms denoted as 1 and 2 in an industry that sell a homogeneous product with a quadratic cost function $C_i = q_i^2/2,\ i = 1,2$ where q_i is firm i's output.[2] The firms can issue debt to the public and to financial institutions, and engage in Cournot competition in the commodity market. In addition, there is a patent holder standing outside the market and having a process innovation, which can reduce the production cost of the licensee by (εq_i) where ε denotes the innovation size. The output decisions are made before the realization of a random variable reflecting the variation in demand. Once profits are determined, the leveraged firms are obliged to repay the debt they have incurred by using the operating profit that they earn. When the operating profits are insufficient to meet the debt obligations, the firms go bankrupt and will then be taken over by the debt holders.

[2] In order to ensure an interior solution for the optimal debt level, the cost function has to be quadratic in output.

Assume that the firms face demand uncertainty, in which the inverse demand function takes the linear form as follows:

$$p = a - Q + z, \tag{1}$$

where a is a constant and $a > \varepsilon$; p denotes price; $Q = q_1 + q_2$ is market demand; and the random variable z reflects the demand uncertainty.

It is assumed that the random variable z follows a uniform distribution over the interval $[\underline{z}, \bar{z}]$, where $\underline{z} = -\bar{z}$ and $\bar{z}$ denotes the upper bound of the demand uncertainty, whose density function is $f(z) = 1/(2\bar{z})$, for which the mean is $E(z) = 0$ and the variance is $Var(z) = \bar{z}^2/3$. It follows that a rise in the upper bound of the random variable $\bar{z}$ can be denoted as a mean-preserving spread that leaves the mean fixed but increases the variance of demand.

The game in question consists of three stages. In the first stage, the outsider patentee chooses the optimal licensing contract in terms of fixed-fee and royalty licensing to maximize its profit. Then, the firms decide whether or not to accept the license. By following Brander and Lewis (1986), the owners choose optimal debt levels to maximize the total value of the firm in the second stage. Finally, the leveraged firms determine their outputs to maximize the equity value, engaging in Cournot competition in the commodity market in the third stage. The game can be solved by backward induction, beginning with the final stage.

In what follows, we first analyze the equilibria for the case where patent licensing is absent, and then the equilibria for the case where the innovation is licensed in terms of fixed-fee and royalty licensing, respectively. Note that the game in the former case will be reduced to a two-stage game, in which the owners choose optimal debt levels in stage 1, and then the managers determine outputs in stage 2.

In stage 2, as argued by Brander and Lewis (1986), given the debt levels (D_1, D_2), the managers are assumed to choose output levels with the objective of maximizing the equity value of the firm to the shareholders. This is what an owner-manager would choose to do, and is certainly what wealth-maximizing shareholders would want the managers to do. Assume that the shareholders of the firm are risk neutral with respect to the firm's returns and therefore have their interests served by the maximization of the equity value. The value to the shareholders is referred to as the equity value and is represented by V_i as:

$$\begin{aligned} V_{Ni} &= \int_{\hat{z}_i}^{\bar{z}} (\Pi_i - D_i) f(z) dz \\ &= \int_{\hat{z}_i}^{\bar{z}} \left[(a - q_i - q_j + z) q_i - C_i - D_i \right] \frac{1}{2\bar{z}} dz, \, i = 1,2, \, i \neq j, \end{aligned} \tag{2}$$

where the subscript "*N*" denotes variables associated with the case where patent licensing is absent; D_i is firm *i*'s debt level; Π_i is firm *i*'s operating profit; and $\hat{z}_i$ represents the break-even state of the demand uncertainty where firm *i* can just meet its debt obligations with nothing left over.

Eq. (2) represents the expected profits net of debt obligations in good states ($z \geq \hat{z}_i$). In bad states ($z < \hat{z}_i$), the firm earns zero profit as all of its earnings are paid to debt holders.

Define the break-even state of the demand uncertainty, $\hat{z}_i$, in which the leveraged firm's net profit equals nil as follows:

$$\Pi_i(\hat{z}_i) - D_i = (a - q_i - q_j + \hat{z}_i)q_i - C_i - D_i = 0, i = 1,2, i \neq j. \tag{3}$$

By differentiating (2) with respect to the output q_i, respectively, and then letting them equal zero, we can solve for the equity value maximizing outputs as follows:

$$q_{Ni} = \frac{1}{16}(4a + 2\bar{z} + 3\hat{z}_i - \hat{z}_j), i = 1,2, i \neq j. \tag{4}$$

Substituting (4) into (3), we can rewrite (3) as follows:

$$\frac{1}{512}(12a - 10\bar{z} + 25\hat{z}_i - 3\hat{z}_j)(4a + 2\bar{z} + 3\hat{z}_i - \hat{z}_j) = D_i,\ i = 1, 2,\ i \neq j. \tag{3.1}$$

Eqs. (4) and (3.1) show that, in stage 2, given the financial composition (D_1, D_2) determined in stage 1, a rise in firm *i*'s debt level increases (decreases) firm *i*'s (*j*'s) output by increasing (decreasing) $\hat{z}_i(\hat{z}_j)$. The intuition is the same as that derived in Brander and Lewis (1986) as follows. As debt rises, low marginal value (bad) states become irrelevant, for in those states the firm is turned over to the debt holders, and the equity holders receive zero in any case. Since the firm restricts its attention to higher marginal profit (good) states, it adopts a more aggressive output strategy. Thus, we have Lemma 1 as follows:

Lemma 1. *Given the financial composition* (D_1, D_2) *determined in stage* 1, *a rise in firm* i's *debt level increases* (*decreases*) *firm* i's (j's) *output*.

Next, firm i's debt value W_i can be expressed as:[3]

$$\begin{aligned} W_{Ni} &= \int_{\hat{z}_i}^{\bar{z}} D_i f(z)dz + \int_{z_i'}^{\hat{z}_i} \Pi_i f(z)dz \\ &= \int_{\hat{z}_i}^{\bar{z}} D_i f(z)dz + \int_{z_i'}^{\hat{z}_i} [(a - q_i - q_j + z)q_i - C_i] f(z)dz, \; i = 1,2, \; i \neq j, \end{aligned} \tag{5}$$

where z_i' denotes firm i's zero operating profit state of the demand uncertainty.

Following Brander and Lewis (1986), the operating profit earned by the leveraged firms is sufficient to reimburse the debt in good states denoted by the first term on the right-hand side of (5), while it is smaller than the debt obligation in that the whole of the operating profit is given to the debt holders in bad states as represented by the second term.[4]

The zero-operating profit state z_i' can be derived as follows:

$$\Pi_i(z_i') = \left(a - q_i - q_j + z_i' - \frac{1}{2} q_i \right) q_i = 0, \; i = 1,2, \; i \neq j. \tag{6}$$

Substituting (4) into (6), we can solve for z_i' as follows:

[3] When the demand situations are in bad states where $z \in [\underline{z}, z']$, the operating profit is less than zero so that there is nothing left for the debt holders.

[4] The bankruptcy cost is assumed to be zero in the paper.

$$z_i' = -\frac{1}{32}\left(12a - 10\bar{z} - 7\hat{z}_i - 3\hat{z}_j\right), i = 1,2, i \neq j. \tag{7}$$

In stage 1, the owners determine the optimal debt levels to maximize the total value of the firm Y_i, which is the sum of the equity value V_i and the debt value W_i. Thus, the total value of firm i can be derived from (2) and (5) as follows:[5]

$$Y_{Ni} = V_{Ni} + W_{Ni} = \int_{z_i'}^{\bar{z}} \Pi_i f(z)dz = \int_{z_i'}^{\bar{z}} [(a - q_i - q_j + z)q_i - C_i]\left(\frac{1}{2\bar{z}}\right)dz, i = 1,2, i \neq j. \tag{8}$$

As in Hughes *et al.* (1998), we observe that $\hat{z}_i$ is related to D_i through the break-even condition (3.1). For convenience, we solve for equilibrium debt levels by differentiating (8) with respect to $\hat{z}_i$ instead of D_i. Thus, by substituting (4) and (7) into (8), and then differentiating (8) with respect to $\hat{z}_i$ by setting them equal to zero, we can solve for the optimal $\hat{z}_i$ as follows:

$$\hat{z}_{Ni}^* = \frac{1}{29}\left(-10a + 19\bar{z}\right), i = 1,2, \tag{9.1}$$

By substituting (9.1) into (4), (3.1), (7), and (8), we can derive the equilibria under the no licensing case as follows:

$$q_{Ni}^* = \frac{6}{29}\left(a + \bar{z}\right), i = 1,2, \tag{9.2}$$

$$D_{Ni}^* = \frac{24}{(29)^2}\left(a + \bar{z}\right)^2, i = 1,2, \tag{9.3}$$

$$z_{Ni}'^* = -\frac{1}{29}\left(14a - 15\bar{z}\right), i = 1,2, \tag{9.4}$$

$$Y_{Ni}^* = \frac{294}{(29)^3\bar{z}}\left(a + \bar{z}\right)^3, i = 1,2, \tag{9.5}$$

It should be noted that (9.1)-(9.5) apply to the case where $z_{Ni}'^* > \underline{z} = -\bar{z}, i = 1,2.$

5 It is noteworthy that the total value of the firm equals the firm's expected operating profit.

Otherwise, the variable z' in (5) and (8) has to be replaced by $z'^{*}_{Ni} = \underline{z} = -\bar{z}, i = 1,2.$

Thus, we can derive the critical state of the upper bound of the demand uncertainty, $\bar{z}_1$, for which $z'^{*}_{Ni} = \underline{z} = -\bar{z}, i = 1,2,$ by using (9.4) as follows:

$$\bar{z} = \bar{z}_1 = 7a/22, \tag{9.6}$$

In order to ensure that the zero-operating profit state, z'_i, is greater than the lower bound of the demand uncertainty, we find from (9.6) that the upper bound of the demand uncertainty has to be greater than $\bar{z}_1$, i.e., $z'^{*}_{Ni} > \underline{z}, i = 1,2,$ when $\bar{z} > \bar{z}_1$.

Since a rise in the upper bound of the demand uncertainty can be denoted as a rise in the mean-preserving variance of demand, it follows from (9.3) that a rise in the mean-preserving variance of demand increases firm *i*'s optimal debt level. The intuition is as follows. A rise in the upper bound of the demand uncertainty enlarges the mean-preserving variance of demand and the range of the good states. Since the leveraged firm cares only about good states, it will increase the optimal debt level. Thus, we have:

Proposition 1. *A rise in the mean-preserving variance of demand increases the optimal debt levels of the two firms*.

3. Fixed-fee Licensing

Suppose that the outsider patentee licenses the patent to firm 1 only under exclusive fixed-fee licensing. It follows that the production costs of firm 1 and firm 2 change to $C_1 = q_1^2/2 - \varepsilon q_1$ and $C_2 = q_2^2/2$, respectively, when firm 1 accepts the licensing. Note that the game in question turns into a three-stage game, in which a licensing stage is added prior to the debt financing stage. This game can be solved by backward induction.

In stage 3, by substituting the production costs of the firms into the equity value, we can solve for the optimal outputs as follows:

$$q_{F1}^{O} = \frac{1}{16}\left(4a + 2\overline{z} + 6\varepsilon + 3\hat{z}_1 - \hat{z}_2\right), \tag{10.1}$$

$$q_{F2}^{O} = \frac{1}{16}\left(4a + 2\overline{z} - 2\varepsilon + 3\hat{z}_2 - \hat{z}_1\right), \tag{10.2}$$

where the superscript "*O*" denotes variables associated with the case where the patent is licensed exclusively; and the subscript "*F*" represents variables in the case of fixed-fee licensing.

In stage 2, by substituting these production costs of the firms into the total value of the firm, we can solve for the optimal $\hat{z}_i$ as follows:

$$\hat{z}_{F1}^{O*} = \frac{1}{493}\left(-170a - 230\varepsilon + 323\overline{z}\right), \tag{11.1}$$

$$\hat{z}_{F2}^{O*} = \frac{1}{493}\left(-170a + 60\varepsilon + 323\overline{z}\right). \tag{11.2}$$

Eq. (11) shows that a rise in the innovation size ε reduces $\hat{z}_{F1}^{O*}$ while increasing $\hat{z}_{F2}^{O*}$. This result occurs because the licensee becomes more efficient in production by accepting the licensing. Thus, the licensee can earn a higher profit, leading to a lower break-even state of the demand uncertainty. On the contrary, the unlicensed firm will incur a loss and this will result in a higher break-even state of the demand uncertainty.

Next, by substituting (11) and the cost functions $C_1 = q_1^2/2 - \varepsilon q_1$ and $C_2 = q_2^2/2$ into (3), (7), (8), and (10), we can obtain the equilibria under exclusive fixed-fee licensing as follows:

$$D_{F1}^{O*} = \frac{24}{(493)^2}(17a + 23\varepsilon + 17\bar{z})^2, \tag{12.1}$$

$$D_{F2}^{O*} = \frac{24}{(493)^2}(17a - 6\varepsilon + 17\bar{z})^2, \tag{12.2}$$

$$q_{F1}^{O*} = \frac{6}{493}(17a + 23\varepsilon + 17\bar{z}), \tag{12.3}$$

$$q_{F2}^{O*} = \frac{6}{493}(17a - 6\varepsilon + 17\bar{z}), \tag{12.4}$$

$$Y_{F1}^{O*} = \frac{294}{(493)^3\bar{z}}(17a + 23\varepsilon + 17\bar{z})^3, \tag{12.5}$$

$$Y_{F2}^{O*} = \frac{294}{(493)^3\bar{z}}(17a - 6\varepsilon + 17\bar{z})^3, \tag{12.6}$$

$$z_{F1}'^{O*} = -\frac{1}{493}(238a + 322\varepsilon - 255\bar{z}), \tag{12.7}$$

$$z_{F2}'^{O*} = -\frac{1}{493}(238a - 84\varepsilon - 255\bar{z}). \tag{12.8}$$

By definition, the innovation is drastic if the licensee can drive the unlicensed firm out of the market and meanwhile charge the monopoly price under exclusive fixed-fee licensing. However, we find from (12.4) that firm 2's output is always greater than zero due to ($\varepsilon < a$). Thus, we can obtain the following lemma:

Lemma 2. *The innovation can never be a drastic innovation, regardless of the innovation size in the presence of debt financing.*

The result derived in Lemma 2 is sharply different from the existing literature, in which there always exists a sufficiently large innovation size making the innovation drastic. This result occurs because limited liability commits a leveraged firm to a more aggressive output stance, leading to the result that the output of the unlicensed firm is always greater than zero.

Likewise, (12.1)-(12.8) apply to the case where $z'^{O*}_{Fi} > \underline{z} = -\overline{z}, i = 1,2$. Otherwise, the variable z' in (5) and (8) has to be replaced by $z' = \underline{z} = -\overline{z}$. By (12.7) and (12.8), we define:

$$\overline{z} = \overline{z}_2 = 7a/22 + 161\varepsilon/374, \tag{12.9}$$

$$\overline{z} = \overline{z}_3 = 7a/22 - 21\varepsilon/187. \tag{12.10}$$

where $\overline{z}_2$ is defined as the critical state for which $z'^{O*}_{F1} = \underline{z}$, while $\overline{z}_3$ is the critical state for which $z'^{O*}_{F2} = \underline{z}$. Thus, $z'^{O*}_{F1} > \underline{z}$ if $\overline{z} > \overline{z}_2$, while $z'^{O*}_{F2} > \underline{z}$ if $\overline{z} > \overline{z}_3$.

We find from (12.1) and (12.2) that a rise in the innovation size ε increases the optimal debt level of the licensee, while it decreases that of the unlicensed firm. The intuition can be stated as follows. We have shown that a rise in the innovation size lowers the licensee's break-even state of the demand uncertainty, while it increases that of the unlicensed firm. Hence, the good states faced by the licensee become larger, while those of the unlicensed firm become smaller. Since the leveraged firms only care about the good states, it follows that the licensee will increase the optimal debt level, while the unlicensed firm will decrease the debt level. Next, a rise in the upper bound of the demand uncertainty, i.e., a rise in the mean-preserving variance of demand, increases both firm*s*' optimal debt levels. The intuition derived in the case of the absence of licensing carries over to this case. Thus, we can establish:

Proposition 2. *Provided that the outsider patentee chooses exclusive fixed-fee licensing, a rise in the innovation size will increase the debt level of the licensee, while reducing that of the unlicensed firm. Moreover, a rise in the mean-preserving variance of demand increases the optimal debt levels of the two firms.*

In stage 1, when the outsider patentee chooses exclusive fixed-fee licensing, the patentee's profit equals the maximal fixed-fee that the licensee would like to pay for accepting the license. Following the definition by Kamien and Tauman (1986), the outsider patentee's profit can be defined as the difference in the licensee's profit between accepting and not accepting the license, which is derivable by subtracting

(9.5) from (12.5) as:

$$\Omega_F^{O*} = F_1^{O*} = \frac{6762\varepsilon}{(493)^3 \bar{z}}\left[867(a+\bar{z})^2 + 1173\varepsilon(a+\bar{z}) + 529\varepsilon^2\right], \tag{13}$$

where $\Omega_F{}^{O*}$ and $F_1{}^{O*}$ denote the outsider patentee's profit and the licensee's fixed-fee under exclusive fixed-fee licensing, respectively.

We move on to the case where the outsider patentee licenses its patent non-exclusively under fixed-fee licensing. The production costs of both firms change to $C_i = q_i^2/2 - \varepsilon q_i, i = 1,2$ when both firms accept the licensing.

By the same procedures, we can derive the equilibria under non-exclusive fixed-fee licensing as follows:

$$\hat{z}_{Fi}^{T*} = \frac{1}{29}(-10a - 10\varepsilon + 19\bar{z}), i = 1,2, \tag{14.1}$$

$$D_{Fi}^{T*} = \frac{24}{(29)^2}(a + \varepsilon + \bar{z})^2, \ i = 1, 2, \tag{14.2}$$

$$q_{Fi}^{T*} = \frac{6}{29}(a + \varepsilon + \bar{z}), i = 1,2, \tag{14.3}$$

$$Y_{Fi}^{T*} = \frac{294}{(29)^3 \bar{z}}(a + \varepsilon + \bar{z})^3, i = 1,2, \tag{14.4}$$

$$z_{Fi}'^{T*} = -\frac{1}{29}(14a + 14\varepsilon - 15\bar{z}), i = 1,2, \tag{14.5}$$

$$\Omega_F^{T*} = \frac{13524\varepsilon}{(493)^3 \bar{z}}\left[867(a+\bar{z})^2 + 561\varepsilon(a+\bar{z}) + 223\varepsilon^2\right], \tag{14.6}$$

where the superscript "*T*" denotes variables associated with the case where the patent is licensed non-exclusively.

Similarly, (14.1)-(14.6) apply to the case where $z_{Fi}'^{T*} > \underline{z} = -\bar{z}, i = 1,2.$ By (14.5), we define:

$$\bar{z} = \bar{z}_4 = 7(a+\varepsilon)/22, \tag{14.7}$$

where $\bar{z}_4$ is defined as the critical state for which $z_{Fi}'^{T*} = \underline{z}, i = 1,2$. Thus, $z_{Fi}'^{T*} > \underline{z}$ if $\bar{z} > \bar{z}_4$.

Eq. (14.2) shows that both a rise in the innovation size and a rise in the mean-preserving variance of demand increase the optimal debt levels of the two firms. The same intuition as that derived in the case of exclusive fixed-fee licensing applies to this result. Thus, we have:

Proposition 3. *Provided that the outsider patentee chooses non-exclusive fixed-fee licensing, both a higher innovation size and a larger mean-preserving variance of demand increase the optimal debt levels of the two firms*.

We are now in a position to examine the optimal number of licenses under fixed-fee licensing. By subtracting (14.6) from (13), we obtain:

$$\Omega_F^{O*} - \Omega_F^{T*} = \frac{6762\varepsilon}{(493)^3 \bar{z}}\left[-867(a+\bar{z})^2 + 51\varepsilon(a+\bar{z}) + 83\varepsilon^2\right] > (<) 0,$$
$$\text{if } \bar{z} < (>) \bar{z}_5 = \frac{-17\left(-3+\sqrt{1005}\right)a + 166\varepsilon}{17\left(-3+\sqrt{1005}\right)}. \tag{15}$$

By taking into account the restriction imposed in the case of exclusive fixed-fee licensing, i.e., $\bar{z} > \bar{z}_2 = 7a/22 + 161\varepsilon/374$, we can figure out that the inequality $\bar{z} < \bar{z}_5 = \left[-17\left(-3+\sqrt{1005}\right)a + 166\varepsilon\right]\Big/\left[17\left(-3+\sqrt{1005}\right)\right]$ in (15) is invalid. Thus, the outsider patentee will license its innovation non-exclusively under fixed-fee licensing, when the mean-preserving variance of demand is relatively large, say, $\overline{z} > \overline{z}_5 = \left[-17\left(-3+\sqrt{1005}\right)a + 166\varepsilon\right] \big/ \left[17\left(-3+\sqrt{1005}\right)\right]$. Intuitively, when the innovation size relative to the mean-preserving variance of demand is large, the competition between firms can be mitigated dramatically if the patent is licensed exclusively. This will result in higher extra profit being earned by the licensee via accepting the licensing. Thus, the outsider patentee can capture a larger profit by licensing the patent exclusively rather than non-exclusively under fixed-fee licensing. On the contrary, it will license its innovation non-exclusively under fixed-fee licensing, when the mean-preserving variance of demand is relatively large.

Based on the above analysis, we can establish the following proposition:

Proposition 4. *The outsider patentee will license its innovation non-exclusively under fixed-fee licensing, when the mean-preserving variance of demand is relatively large, say,* $\overline{z} > \overline{z}_5 = \left[-17\left(-3+\sqrt{1005}\right)a + 166\varepsilon\right] \big/ \left[17\left(-3+\sqrt{1005}\right)\right]$.

4. Royalty Licensing

In this section, we explore the outsider patentee's optimal royalty rate under royalty licensing. The same result as that derived in Kamien and Tauman (1986) and Kamien *et al*. (1992), where the outsider patentee will definitely license the patent non-exclusively under royalty licensing, can be obtained in this paper.[6] Thus, in what follows we shall study the case of non-exclusive royalty licensing directly. Since both firms have the patent, their production cost functions can be expressed as $C_i = q_i^2/2 - \varepsilon q_i + rq_i, i = 1,2,$ where r is the royalty rate and $r \le \varepsilon$.

By taking into account the cost functions under royalty licensing, we can derive the equilibria as follows:

$$\hat{z}_{Ri}^T = \frac{1}{29}\left(-10a - 10\varepsilon + 10r + 19\bar{z}\right), i = 1,2, \tag{16.1}$$

$$D_{Ri}^T = \frac{24}{(29)^2}\left(a + \varepsilon - r + \bar{z}\right)^2, \ i = 1,2, \tag{16.2}$$

$$q_{Ri}^T = \frac{6}{29}\left(a + \varepsilon - r + \bar{z}\right), i = 1,2, \tag{16.3}$$

where the subscript "*R*" denotes variables associated with the case of royalty licensing.

In stage 1, the outsider patentee's profit-maximizing problem is to choose an optimal royalty rate to maximize its profit under the constraint $r \le \varepsilon$, which can be expressed as:

$$\begin{aligned} &\max_r \quad \Omega_R^T = r\left(q_{Ri}^T + q_{Rj}^T\right) \\ &s.t. \quad r \le \varepsilon. \end{aligned} \tag{17}$$

6 The solution procedures are available from the authors upon request.

Substituting (16.3) into (17), and then differentiating (17) with respect to *r*, we obtain:

$$\frac{\partial \Omega_R^T}{\partial r} = \frac{12}{29}\left(a + \varepsilon - 2r + \overline{z}\right), \tag{18}$$

Recall that $a > \varepsilon$, $\varepsilon \geq r$ and $\overline{z} > 0$. It follows that the sign of (18) is definitely positive. Thus, the outsider patentee's profit is monotonically increasing in the royalty rate, leading to the result that the optimal royalty rate equals the innovation size by appealing to the constraint in (17) as follows:

$$r^{T*} = \varepsilon. \tag{19}$$

By substituting (19) into (16.2), we find that the optimal debt levels of the firms remain unchanged under royalty licensing when the licensees accept the licensing. This result emerges because the production costs of the firms also remain unchanged. Next, the optimal debt levels of the firms are increasing in the mean-preserving variance of demand. The same intuition carries over to this case. Thus, we have the following proposition:

Proposition 5. *Provided that the outsider patentee chooses royalty licensing, the optimal debt levels of the firms remain unchanged, regardless of the innovation size, while they are increasing in the mean-preserving variance of demand.*

By substituting (19) into (17), we can figure out the outsider patentee's profit under royalty licensing as follows:

$$\Omega_R^{T*} = 12\varepsilon\left(a+\bar{z}\right)/29. \tag{20}$$

Note that the licensees' production costs by accepting royalty licensing are identical to those in the absence of licensing. It follows that the restriction, that (16.1)-(16.3) apply to the case where $z'^{T*}_{Ri} > \underline{z} = -\bar{z}, i = 1,2$, will be the same as (9.6).

5. The Optimal Licensing Contract

We are now in a position to explore the outsider patentee's optimal licensing contract in terms of fixed-fee and royalty licensing. First of all, we analyze the case where the zero operating profit state of the demand uncertainty is greater than the lower bound of the demand uncertainty in all licensing regimes, i.e., $z'^{j*}_{ki} > \underline{z}, i = 1,2, j = O,T, k = N,F,R.$

Recall that the outsider patentee will license the patent non-exclusively under fixed-fee licensing. By subtracting (14.6) from (20), we can obtain the difference in the outsider patentee's profit between royalty and non-exclusive fixed-fee licensing as:

$$\begin{aligned}\Omega_R^{T*} - \Omega_F^{T*} &= \frac{12}{29}\varepsilon\left(a+\bar{z}\right) - \frac{13524\varepsilon}{(493)^3\bar{z}}\left[867\left(a+\bar{z}\right)^2 + 561\varepsilon\left(a+\bar{z}\right) + 223\varepsilon^2\right] \\ &> (<) 0, \text{if } \bar{z} > (<) \bar{z}_6,\end{aligned} \tag{21}$$

where

$$\bar{z}_6 = \frac{-7535a}{21832} + \frac{37191\varepsilon}{371144} + \frac{\sqrt{59072816401a^2 + 18078470718\varepsilon a + 12356850625\varepsilon^2}}{371144}.$$

Eq. (21) shows that the difference in the outsider patentee's profit between royalty and non-exclusive fixed-fee licensing is positive when the mean-preserving variance of demand is large, say, $\bar{z} > \bar{z}_6$, while it is negative otherwise. However, by taking into account the restriction in exclusive fixed-fee licensing, i.e., $\bar{z} > \bar{z}_2 = 7a/22 + 161\varepsilon/374$, the inequality $\bar{z} < \bar{z}_6$ will be ruled out. Thus, the optimal licensing contract is royalty licensing, when the mean-preserving variance of demand is large, say, $\bar{z} > \bar{z}_6$.

The intuition behind the result can be stated as follows. When the mean-preserving variance of demand is large, the competition between firms in the commodity market is severe because the leveraged firms will commit to an aggressive strategy by increasing the debt levels. This situation emerges because a larger mean-preserving variance of demand increases the debt levels of the firms in all licensing regimes, and a higher debt level increases the output. Moreover, we have proved that the optimal debt levels remain unchanged regardless of the innovation size under royalty licensing so that the aggressive effect caused by the innovation size vanishes, while they are increasing with the innovation size under non-exclusive fixed-fee licensing. It follows that non-exclusive fixed-fee licensing will cause the competition to be more intense than under royalty licensing. As a result, in order to mitigate the competition between firms so that the outsider patentee can wrest a larger profit from the licensees, the outsider patentee will choose royalty licensing.

Next, as the above intuition carries over to other cases where at least one of the zero operating profit state of the demand uncertainty is no greater than the lower bound of the demand uncertainty, i.e., $\bar{z} < \bar{z}_2,$, the detailed derivations of the optimal licensing contracts in these cases are contained in Appendix A for saving on space. We illustrate the outsider patentee's optimal licensing contracts by using Figure 1 and Table 1. The optimal licensing contracts derived in areas I-V of Figure 1 can be summarized in Table 1. We can conclude from Table 1 that the optimal licensing is royalty licensing when the mean-preserving variance of demand is large, while it is non-exclusive fixed-fee licensing otherwise. Thus, we can establish:

(Insert Figure 1 here)

(Insert Table 1 here)

Proposition 6. *Suppose that the leveraged firms manufacture a homogeneous product and engage in Cournot competition. The optimal licensing contract for the outsider patentee in terms of fixed-fee and royalty licensing is royalty licensing when the mean-preserving variance of demand is large in the presence of debt financing,* *while it is non-exclusive fixed-fee licensing otherwise*.

The result derived in Proposition 6 is sharply different from that of Kamien and Tauman (1986) as well as Kamien *et al*. (1992), in which the optimal licensing contract of the outsider patentee is always non-exclusive fixed-fee licensing when firms produce homogeneous products and engage in Cournot competition. The difference emerges because the leveraged firms will become more aggressive in terms of their output strategy in the presence of debt financing so that the outsider patentee may choose royalty licensing to reduce the competition between firms.

6. Concluding Remarks

This paper has developed a licensing model to reconcile with the controversy of royalty licensing being superior over fixed-fee licensing in theory and in practice *within* Kamien and Tauman's (1986) theoretical framework by taking into account the financial structure of the firm. The focus of the paper is on the limited liability effect of debt financing and the strategic effect of the licensing decision on output strategies. Several striking results are derived as follows.

First of all, provided that the leveraged firms produce a homogeneous product and engage in Cournot competition, this paper shows that the optimal licensing contract for the outsider patentee in terms of fixed-fee and royalty licensing is royalty licensing when the mean-preserving variance of demand is large in the presence of debt financing, while it is non-exclusive fixed-fee licensing otherwise. This paper thus provides a new explanation to justify the superiority of royalty licensing over the fixed-fee licensing contract. Secondly, the larger mean-preserving variance of demand always results in larger debt levels, regardless of the licensing means. Thirdly, an increase in the innovation size increases the debt level of the licensee under fixed-fee licensing, while it lowers that of the unlicensed firm. However, the debt levels remain unchanged under royalty licensing. Fourthly, the outsider patentee will license its innovation non-exclusively under fixed-fee licensing, when the mean-preserving variance of demand is relatively large. Lastly, the innovation can never be drastic, regardless of the innovation size in the presence of debt financing.

Appendix A. The Optimal Licensing Contracts in Areas of Figure 1

In Figure 1, area I denotes the situation where the zero operating profit state is larger than the lower bound of the demand uncertainty in all licensing regimes, i.e., $z_{ki}'^{j*} > \underline{z}, i = 1,2, j = O,T, k = N,F,R,$ which is restricted by $\overline{z} > \overline{z}_2$. The optimal licensing contract for this area is royalty licensing, which is determined by (21). Area II is for $z_{F1}'^{O*} < \underline{z},$ which is restricted by $\overline{z}_4 < \overline{z} < \overline{z}_2$. Area III is for $z_{F1}'^{O*} < \underline{z},$ and $z_{Fi}'^{T*} < \underline{z},$ and is restricted by $\overline{z}_1 < \overline{z} < \overline{z}_4$. Area IV is for $z_{F1}'^{O*} < \underline{z},\ z_{Fi}'^{T*} < \underline{z},$ and $z_{Ni}'^{*} < \underline{z},$ and is restricted by $\overline{z}_3 < \overline{z} < \overline{z}_1$, and area V is for $z_{F1}'^{O*} < \underline{z}, z_{Fi}'^{T*} < \underline{z}, z_{F2}'^{O*} < \underline{z}, z_{Ni}'^{*} < \underline{z},$ and $\hat{z}_{Fi}^{T*} < \overline{z},$ and is restricted by $\overline{z}_7 < \overline{z} < \overline{z}_3$. The critical value of the state $\overline{z}_7$ will be defined later in this section. The remaining area is for $\hat{z}_{Fi}^{T*} > \overline{z},$ and is restricted by $0 < \overline{z} < \overline{z}_7$, where the firms can not survive in this situation, because they incur a loss in every state of the demand uncertainty. In what follows, we examine the optimal licensing contract in areas II-V, respectively.

For area II, since $z_{F1}'^{O*} < \underline{z}$ and the zero operating profit state can not be smaller than $\underline{z},$ the endogenous variable $z_{F1}'^{O*}$ in (12.7) is thus replaced by $z_{F1}'^{O*} = \underline{z}$. By substituting $z_{F1}'^{O*} = \underline{z}$ into the model and then redoing the calculation, we can obtain the difference in the outsider patentee's profit between royalty and non-exclusive fixed-fee licensing as follows:

$$\Omega_R^{T*} - \Omega_F^{T*} = \frac{12}{29}\varepsilon(a+\bar{z}) - \frac{294}{\bar{z}}\left[\frac{2}{24389}(a+\varepsilon+\bar{z})^3 - \frac{1}{332(83)^2}(5a-3\varepsilon+8\bar{z})^3\right]. \quad (A.1)$$

We find from (A.1) that this profit difference is positive (negative) if $\bar{z} > (<) \bar{z}_8$.[7] Thus, the optimal licensing contract for area II is royalty licensing if the mean-preserving variance of demand is large, say, $\bar{z} > \bar{z}_8$, while it changes to non-exclusive fixed-fee licensing otherwise. The intuition is the same as that in (21). When the mean-preserving variance of demand is large, and the competition between firms in the commodity market is severe, the outsider patentee will choose royalty licensing to mitigate the competition. On the contrary, when the mean-preserving variance of demand is small, the competition between firms is not so intense, with the result that the outsider patentee will select non-exclusive fixed-fee licensing.

For area III, since $z_{F1}'^{O*} < \underline{z}$, and $z_{Fi}'^{T*} < \underline{z}$, the endogenous variable $z_{F1}'^{O*}$ in (12.7) and $z_{Fi}'^{T*}$ in (14.5) are replaced by $z_{F1}'^{O*} = z_{Fi}'^{T*} = \underline{z}$. By substituting $z_{F1}'^{O*} = z_{Fi}'^{T*} = \underline{z}$ into the model and then redoing the calculation, we can obtain:

$$\Omega_R^{T*} - \Omega_F^{T*} = \frac{12}{29}\varepsilon(a+\bar{z}) - \frac{21}{2}\left[\frac{2(a+\varepsilon)^2}{121} - \frac{7}{(83)^3\bar{z}}(5a-3\varepsilon+8\bar{z})^3\right]. \quad (A.2)$$

Eq. (A.2) shows that this profit difference is positive (negative) if $\bar{z} > (<) \bar{z}_9$.[8]

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[7] The exposition of $\bar{z}_8$ is too tedious. We do not present it in the paper, but it is available from the authors upon request.

[8] The exposition of $\bar{z}_9$ is too tedious. We do not present it in the paper, but it is available from the authors upon request.

Thus, the optimal licensing contract for area III is royalty licensing if the mean-preserving variance of demand is large, say, $\bar{z} > \bar{z}_9$, while it changes to non-exclusive fixed-fee licensing otherwise.

For area IV, since $z_{F1}^{\prime O*} < \underline{z}$, $z_{Fi}^{\prime T*} < \bar{z}$, and $z_{Ni}^{\prime *} < \underline{z}$, the endogenous variables $z_{F1}^{\prime O*}$ in (12.7), $z_{Fi}^{\prime T*}$ in (14.5), and $z_{Ni}^{\prime *}$ in (9.4) are replaced by $z_{F1}^{\prime O*} = z_{Fi}^{\prime T*} = z_{Ni}^{\prime *} = \underline{z}$. By substituting $z_{F1}^{\prime O*} = z_{Fi}^{\prime T*} = z_{Ni}^{\prime *} = \underline{z}$ into the model and then redoing the calculation, we can obtain:

$$\Omega_R^{T*} - \Omega_F^{T*} = \frac{6}{11}\varepsilon a - \frac{21}{2}\left[\frac{2(a+\varepsilon)^2}{121} - \frac{7}{(83)^3 \bar{z}}(5a - 3\varepsilon + 8\bar{z})^3\right]. \tag{A.3}$$

Similarly, (A.3) shows us that this profit difference is positive (negative) if $\bar{z} > (<) \bar{z}_{10}$.[9] However, the inequality $\bar{z} > \bar{z}_{10}$ is ruled out by the restriction $z_{Ni}^{\prime *} < \underline{z}$, i.e., $\bar{z} < 7a/22$. Thus, the optimal licensing contract for area IV is non-exclusive fixed-fee licensing.

For area V, since $z_{F1}^{\prime O*} < \underline{z}$, $z_{Fi}^{\prime T*} < \underline{z}$, $z_{Ni}^{\prime *} < \underline{z}$, and $z_{F2}^{\prime O*} < \underline{z}$, the endogenous variables $z_{F1}^{\prime O*}$ in (12.7), $z_{Fi}^{\prime T*}$ in (14.5), $z_{Ni}^{\prime *}$ in (9.4), and $z_{F2}^{\prime O*}$ in (12.8) are replaced by $z_{F1}^{\prime O*} = z_{Fi}^{\prime T*} = z_{Ni}^{\prime *} = z_{F2}^{\prime O*} = \underline{z}$. By substituting $z_{F1}^{\prime O*} = z_{Fi}^{\prime T*} = z_{Ni}^{\prime *} = z_{F2}^{\prime O*} = \underline{z}$ into the model and then redoing the calculation, we find that the restriction $\hat{z}_i^{T*}\ (i = 1, 2) < \bar{z}$ holds if $\bar{z} > \bar{z}_7 = (a + \varepsilon)/11$. Moreover, we derive:

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[9] The exposition of $\bar{z}_{10}$ is too tedious. We do not present it in the paper, but it is available from the authors upon request.

$$\Omega_R^{T*} - \Omega_F^{T*} = -\frac{30a\varepsilon + 336\varepsilon^2}{3025} < 0. \tag{A.4}$$

Eq. (A.4) shows that the optimal licensing contract for area V is non-exclusive fixed-fee licensing.

It should be noted that the critical value of the state $\bar{z}_7 > \bar{z}_5$. By referring to this relationship, we find from (15) and Figure 1 that the outsider patentee will license its innovation non-exclusively under fixed-fee licensing in areas I-V. Thus, it is not a problem for us to ignore the outsider patentee’s profit in the case of exclusive fixed-fee licensing in areas I-V while comparing the difference in profit between royalty and non-exclusive fixed-fee licensing.

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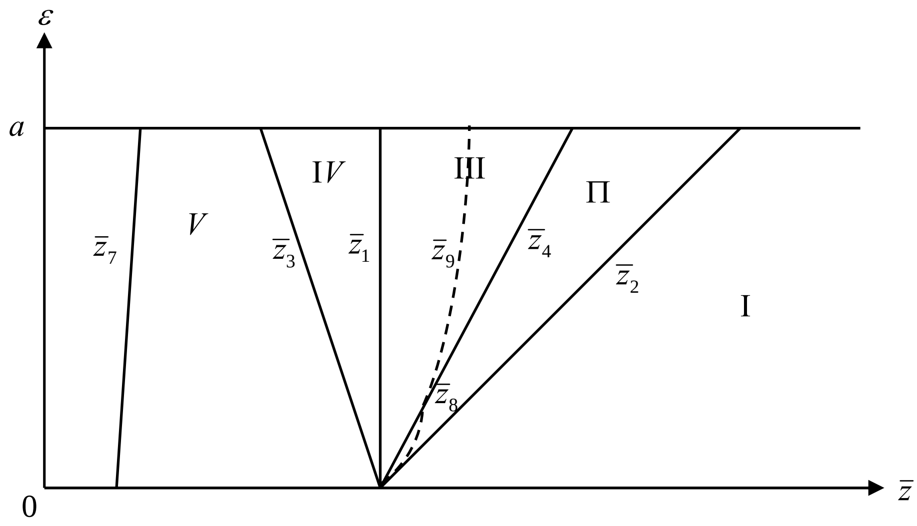

Figure 1. The Optimal Licensing Contracts

Table 1. Summary of Optimal Licensing Contracts in Areas I-V of Figure 1

Area	Restriction on z'_i	Restriction on $\bar{z}$	The optimal licensing contract
I	none	$\bar{z} > \bar{z}_2$	Royalty
II	$z'^{O*}_{F1} = \underline{z}$	$\bar{z}_4 < \bar{z} < \bar{z}_2$	Royalty if $\bar{z}_8 < \bar{z} < \bar{z}_2$ Non-exclusive fixed-fee if $\bar{z}_4 < \bar{z} < \bar{z}_8$
III	$z'^{O*}_{F1} = z'^{T*}_{Fi} = \underline{z}$	$\bar{z}_1 < \bar{z} < \bar{z}_4$	Royalty if $\bar{z}_9 < \bar{z} < \bar{z}_4$ Non-exclusive fixed-fee if $\bar{z}_1 < \bar{z} < \bar{z}_9$
IV	$z'^{O*}_{F1} = z'^{T*}_{Fi} = z'^{*}_{Ni} = \underline{z}$	$\bar{z}_3 < \bar{z} < \bar{z}_1$	Non-exclusive fixed-fee
V	$z'^{O*}_{F1} = z'^{T*}_{Fi} = z'^{*}_{Ni} = z'^{O*}_{F2} = \underline{z}$	$\bar{z}_7 < \bar{z} < \bar{z}_3$	Non-exclusive fixed-fee