Exit in Vertical Relationships Under Uncertainty

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Motivation

There exist various non-integrated vertical relationships in a supply chain.

- Honda and its upstream firm in the Thailand flood during year 2011.
- The spillover effects during the financial crisis in 2008.
  E.g. The rise in popularity of securitized products $\rightarrow$ cheap credit and fall of lending stardards$\rightarrow$ the subprime mortgage delinquencies and foreclosures.
- Retailers pass the pain of price cut up along the supply chain.
  E.g. Frozen pizza and Walmart.
  Or the rise of cost in ingredients: the pain passed down the supply chain.

Question

When one firm is considering exit, how will its upstream or downstream counterpart react to this exit?

Could the firm strategically delay the exit of its counterparty and thus prevent its own exit?

This paper studies the impact of exit spillover between vertically related firms in the supply chain. In a model with

- demand uncertainty and
- dynamic game of infinite horizon,

we want to analyse

- the optimal timing of exit of both upstream and downstream firms;
- the strategic play of the upstream firm(price of the intermediate good);
- spillover effects of downstream to the upstream firm and vice versa.

Related literature

- Vertical integration and control
- Static entry/exit model:
  Ghemawat and Nalebuff(1985);
  Fudenberg and Tirole(1985).
- Real options:
  Boyarchenko and Levendorski(2007);
  Dixit and Pindyck(1994)
- Game-theoretic Real options:
  Murto(2004); Weeds(2002); Lambrecht(2001).

Results and Contributions

- Spillover Effect of Exit in a Vertical Relationship: the exit of one party will lead to the exit of the other party.
- Strategic delay is possible but depends on the role of the firm.
- The strategies of firms are not only timing decisions, but also include quantity and pricing decisions.
- A unique equilibrium is obtained.
- The expected delay in exit timing is derived.

Why Real Options?

Three types of entry decision rules:

1. Deterministic Envrionment: enter when profit $\pi(x) > 0$.
2. NPV, Net present value (Myopic agent):
   $\mathbb{E}^x[\int e^{-\gamma t}\pi(x_t)dt] > 0$.
3. Real options method: waiting has value.

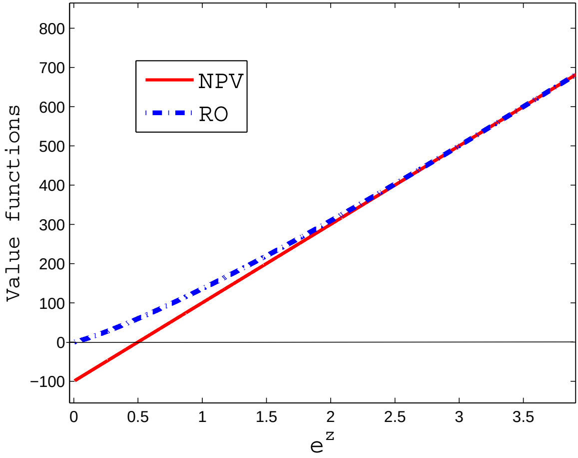

Figure: NPV v.s. Real Options

Real Options Approach

- The real options method is used to study the players' timing decision under demand uncertainty.
- Non-myopic players: considers the opportunity cost of immediate action.

The Environment I

- Two independent and vertically related firms:
  - a downstream monopolist D, and
  - an upstream monopolist firm U.
- Infinite and continuous time;
- common discount rate $\gamma > 0$.
- Firm U produces the intermediate good with a constant marginal cost: mc.
- Firm D buys the intermediate good from firm U.
- Firm D sells the final good to the consumers.

The Environment II

- There exist demand uncertainty of the final good and the inverse demand function of the final good:

$$D(p_t; X_t) = e^{X_t} p_t^{\delta},$$

where

  - $\delta < -1$: the constant demand elasticity;
  - X_t: the demand uncertainty that follows a Brownian motion with drift μ and variance σ^2. The dynamic of $\{X_t\}_{t \geq 0}$ is expressed by

$$dX_t = \mu dt + \sigma dW_t,$$

  and W_t is the standard Wiener process.

Timing and Strategy Space I

At $t = 0$,

- a contract of the intermediate good price is signed:$p_u(X_t)$.
- This contract price may only be changed once: the new price function: $\hat{p}_u(X_t)$.(Irreversible change)
- Either firm may propose the price change.

Timing and Strategy Space II

At time t, both firms observe the realization of the demand shock is $X_t = x$.

- Firm U charges firm D $p_u(x)$ per unit of the intermediate good.
- Firm D decides $q_t(p_u, x)$.
- Firm i receives its revenue π_i and pay its operational cost F_i.
- Firm i decides whether or not to exit its market. The exit threshold of firm i: $h_i \in \mathbb{R}$.
- Firm i decides whether or not to propose a price change. If firm i proposes a price change, it also decides the new price $\hat{p}_u$. The price change threshold: $h_s \in \mathbb{R}$.

where $i = D, U$.

Timing and Strategy Space III

- Given p_{ut} and the realization of the state variable $X_t = x$, firm D maximizes its profit at time t by solving the following problem:

$$\max_{q_t} \quad q_t(p - p_{ut}) - F_D, \tag{1}$$

$$\Rightarrow q_t = \left(\frac{\delta}{\delta + 1}\right)^{\delta} p_{ut}^{\delta} e^{x}, \tag{2}$$

where $p(q_t; x) = (q_t / e^x)^{1/\delta}$.

- Firm U takes (2) as given and maximizes its current profit:

$$\max_{p_{ut}} \quad (p_{ut} - mc) \cdot q_t - F_U. \tag{3}$$

Lemma 1

At time t, given the realization of the stochastic state variable, x, the firms' time t profits are are maximized by

$$\bar{p_{ut}} = \bar{u} \cdot mc \quad and \quad \bar{q}_t = \left(\frac{\delta}{\delta + 1} \right)^{\delta} (\bar{u} \cdot mc)^{\delta} e^{x}, \tag{4}$$

where $\bar{u} = \delta/(\delta+1) > 1$ *with* $\delta < -1$.

Assumptions:

1. Price functions take the form $p_u = p_u(u, X_t) = u \cdot mc$, where $u \in [1, \infty)$.
2. No bubble condition:

$$\gamma - \Psi(1) > 0, \tag{5}$$

where $\Psi(z) = \mu z + \frac{\sigma^2}{2} z^2$.

Payoff Streams and Firms' Maximization Problem I

With $p_u = u \cdot mc$ and $u \in [1, \infty)$, the firms will receive profits of the form

$$\pi_i(u, x) = W_i(u)e^x - F_i, \quad i = D, U. \tag{6}$$

Payoff Streams and Firms' Maximization Problem II

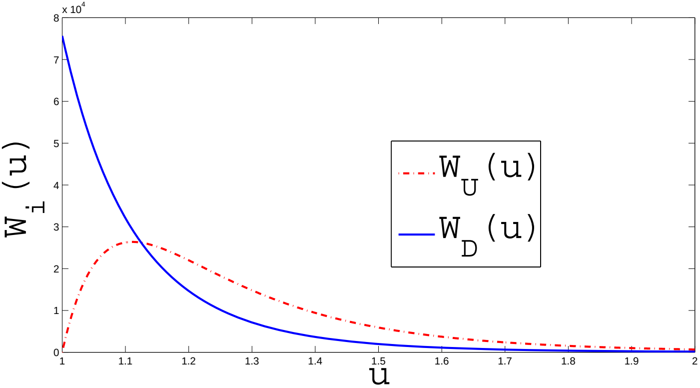

Figure: The shape of W_U and W_D

Payoff Streams and Firms' Maximization Problem III

Three possible cases:

1. both firms want to exit simultaneously;
2. firm D wants to exit before firm U, and
3. firm U wants to exit before firm D.

Payoff Streams and Firms' Maximization Problem IV

Given $p_u = u \cdot mc$, let firm i want to exit before firm j. Firm j's maximization problem:

$$V_j(u,x) = \max_{\tau_s, \hat{\tau}_j, \hat{u}} \mathbb{E}^x \left[\int_0^{\tau_s} e^{-\gamma t} \pi_j(u,x) dt \right] + \mathbb{E}^x \left[\int_{\tau_s}^{\hat{\tau}_j} e^{-\gamma t} \pi_j(\hat{u},x) dt \right] \quad (7)$$
$$\text{s.t.} \quad \tau_s \leq \tau_i \leq \hat{\tau}_j \leq \hat{\tau}_i,$$

where

- $\hat{u}$: changing $p_u = u \cdot mc$ to $\hat{p_u} = \hat{u} \cdot mc$;
- h_s: the threshold of when to change the price;
- $\tau_s = inf\{t > 0 | X_t \leq h_s\}$;
- $\hat{h}_i$: firm i's new exit thresholds under the new price function;
- $\hat{\tau}_i = inf\{t > 0 | X_t \leq \hat{h}_i\}$.

Payoff Streams and Firms' Maximization Problem V

Firm i solves the following problem:

$$V_i(u,x) = \max_{\tau_i,\hat{\tau}_i} \mathbb{E}^x \left[\int_0^{\tau_s \wedge \tau_i} e^{-\gamma t} \pi_i(u,x) dt \right] + 1_{\{\tau_s \geq \tau_i\}} \mathbb{E}^x \left[\int_{\tau_s}^{\hat{\tau}_i} e^{-\gamma t} \pi_i(\hat{u},x) dt \right]. \quad (8)$$

Optimal Exit Timing without the Price Change Option I

Firm i finds its optimal exit timing by solving the following system of equations, $i = D, U$:

$$\left(\gamma - \mu \frac{\partial}{\partial x} - \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}\right) V_i(u,x) = W_i(u)e^x - F_i \qquad \text{if } x > h_i$$

$$V_i(u,x) = 0 \qquad \text{if } x \leq h_i$$

The above second order differential equation is a standard real-options problem. The closed form solution to h_i is

$$e^{h_i} = \frac{F_i}{\kappa^+(1)W_i(u)}, \tag{9}$$

where $\kappa^+(1) = \beta^+/(\beta^+ - 1)$ and $\beta^\pm$ are the roots of $\gamma - \Psi(1) = 0$.

Optimal Exit Timing without the Price Change Option II

Theorem 2

Given that $p_{ut} = u \cdot mc, \forall t > 0$,

1. *both firms would exit simultaneously* ($h_D = h_U$) *iff* $\frac{F_D}{F_U} = \Delta \cdot \left(\frac{u}{u-1}\right)$;
2. *firm D would want to exit before firm U* ($h_D > h_U$) *iff* $\frac{F_D}{F_U} > \Delta \cdot \left(\frac{u}{u-1}\right)$, *and*
3. *firm U would want to exit before firm D* ($h_D < h_U$) *iff* $\frac{F_D}{F_U} < \Delta \cdot \left(\frac{u}{u-1}\right)$,

where $\Delta = -1/(\delta + 1)$.

Optimal Exit Timing without the Price Change Option III

Theorem 3

If the price signed in the contract is such that $\frac{F_D}{F_U} = \Delta \cdot \left(\frac{u}{u-1}\right)$, *both firms exit simultaneously at* $h^* = h_U = h_D$ *and the option to change the intermediate good price function is not exercised.*

Theorem 4

$\forall u \in [1, \infty)$, $\bar{u} = \underset{u}{\operatorname{argmin}} \quad h_U(u)$.

Theorem 5

$h_D(u)$ *is strictly decreasing in* u.

The Optimal Switching Time I

- Two asymmetric firms not only compete in how to share the pie of the final good market, but also rely on the existence of each other.
- When one party exits, its counterparty must exit as well.

WOLG, let firm i be the first firm which reaches its exit threshold under the price function specified in the contract, and firm j be the firm which wants to stay in the market longer, *i.e.* $h_i \geq h_j$.

- Firm j wants $\hat{h}_i \leq \hat{h}_j$.

The Optimal Switching Time II

- By changing the price to delay the exit of firm i, firm j is sacrificing its own value to subsidize the other firm so that it could be in the market longer.
  $\Rightarrow$ Given that the price signed in the contract is optimal, it is never optimal to change the price too early as firm j is better off by delaying the change until h_i.
- However, if firm j waits after $x < h_i$ to change the price function, firm i would have already exit at $x = h_i$ (exit is irreversible) and firm j will have to exit at the same time.
- In order to prevent firm i's exit, firm j maximizes (7) by $h_s = h_i$.

The Optimal New Price I

Firm j solves the following problem:

$$\hat{V}_j(\hat{u},x) = \max_{\hat{u},\hat{\tau}_j} \quad \mathbb{E}^x\left[\int_{\tau_s}^{\hat{\tau}_j} e^{-\gamma t}\pi_j(\hat{u},x)dt\right] \quad \text{s.t.} \quad \hat{h}_j \geq \hat{h}_i. \tag{10}$$

$$\hat{h_U} \geq \hat{h_D} \quad \text{iff} \quad W_D(\hat{u}) \geq \frac{F_D}{F_U} W_U(\hat{u}). \tag{11}$$

Lemma 6

$W_D(\hat{u}) = \frac{F_D}{F_U} W_U(\hat{u})$ has an unique solution $\hat{u}^*$, $\forall \frac{F_D}{F_U} > 0$.

Lemma 7

$\hat{u}^* < \bar{u}$ if $\frac{F_D}{F_U} > \bar{u}$ and $\hat{u}^* \geq \bar{u}$ if $\frac{F_D}{F_U} \leq \bar{u}$.

Conclusion

Theorem 8

Assume that the initial state is sufficiently high. When the downstream firm initially wishes to exit prior to the upstream, the unique equilibrium is that both firms receive the maximized profit until h_s and then firm U will propose to lower the intermediate price so that it is optimal for both firms exit simultaneously at h^*.

$$u^* = \begin{cases} \bar{u} & \text{if} \quad x > h_s, \\ \hat{u}^* & \text{if} \quad x \le h_s \end{cases}, \quad h_s = ln\left(\frac{F_D}{\kappa^+(1)W_D(\bar{u})}\right),$$

$$\text{and} \quad \hat{h_U} = \hat{h_D} = h^*,$$

where $\bar{u} > \hat{u}^*$ *and* $h^* = \frac{F_U}{\kappa^+(1)W_U(\hat{u})}$.

Theorem 9

At the date firm U change the intermediate good price to $\hat{p}_u$, it knows the expected delay in exit conditioned on the state level of that date is

$$\mathbb{E}_{\mathbb{P}}[\tau^*_{h_s}] = \frac{1}{\mu}(h^* - h_s) = \frac{1}{\mu} ln \left[\frac{F_U W_D(\bar{u})}{F_D W_U(\hat{u})} \right].$$

Theorem 10

When the upstream firm initially wishes to exit prior to the downstream, there is nothing the downstream firm can do to delay the exit of the upstream firm. Therefore the unique equilibrium is that both firms exit simultaneously at the upstream firms exit threshold.

$$u^* = \bar{u} \quad \forall x, \quad and \quad h^* = ln\left(\frac{F_U}{\kappa^+(1)W_U(\bar{u})}\right).$$

Contribution

- Extend the real-options literature by studying strategic interactions associated with abandonment options in vertical relationships.
- I enrich the firms strategy space by allowing the firms to decide on price and quantities to maximize its life-time expected present value.
- Study the interdependence of firms in vertical relationships during hard times whereas most of the literature concentrate on vertical integration and controls.

For both the game-theoretic real-options literature and vertical relation literature, by introducing uncertainty to the supply chain, this model brings a new view to the interdependence relationship in supply chains.

Thank you!