Asset Price and Monetary Policy – the Effect of Expectation Formation∗

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Abstract

This paper studies the stabilization effect of monetary policy reacting to fluctuations in asset price, accounting for the expectation formation effect of policy regime shift, in a DSGE model calibrated to the U.S. economy. We find that, in contrast to the linear policy rule that generates negligible stabilization effect from responding to asset prices, the regime switching policy rule significantly shifts the “expected” inflation-output volatility frontier inwards, suggesting that expectation formation effect reinforces the stabilization effect from reacting to asset price. Moreover, the trade-off between the expected volatility of inflation and that of output, as demonstrated by the “Taylor curve,” substantially diminishes, implying that the taking into account of expectation formation effect expands the set of monetary policy choices available for monetary authority.

Key words: asset price, monetary policy, regime switching, DSGE

JEL classification: E3, E52, G1

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1 Introduction

Recent episodes concerning large fluctuations in asset prices have sparked debates among academics and policy makers regarding whether and how monetary policy should respond to fluctuations in asset prices. This paper studies the stabilization effect of monetary policy reacting to fluctuations in asset price in a dynamic, stochastic, general equilibrium (DSGE) model, accounting for the expectation formation effect, i.e., the impact arising from expectations of possible change in policy regime, à la Leeper and Zha (2003) and Davig and Leeper (2007). The model allows monetary policy reaction function to shift between “Hawkish” and “Dovish” regimes, according to the Markov process. We calibrate the model to the U.S. economy and investigate the significance of expectation formation effect on the volatilities of stock price, inflation, and output. We then study whether the policy rule that reacts to asset price and accounts for expectation formation effect, performs better than a conventional linear policy rule in stabilizing economic aggregates.

Existing works that incorporate monetary policy rule in a DSGE model tend to specify constant parameter values of the reaction function, implying that the central bank will maintain the same policy rule in the future. In contrast, this paper emphasizes the role of expectation formation effect in monetary policy rule to characterize the responses of monetary policy to inflation, output, and particularly asset price for the following reasons.

First, a large empirical literature has confirmed the existence of regime switching in monetary policy across central banks (Clarida et al. (2000), Sargent et al. (2006), Sims and Zha (2006)). Thus, assuming constant responsiveness of reaction function in monetary policy is inconsistent with the behavior of monetary authorities. Second, since empirical evidence strongly suggests that monetary policy switches regimes from time to time, rational agents will form expectations of future policy shifts given all available information. Thus, the assumption of a constant policy rule is also inconsistent with the notion of rational expectations. Third, as demonstrated in Leeper and Zha (2003) and Davig and Leeper (2007), expectations that future policy might switch to an alternative regime will affect equilibrium under the current regime. Thus, without taking into account of the expectation formation effect may give misleading assessment of monetary policy effects. Finally, since asset prices are sensitive to expectations of economic fundamentals
and policy changes in the future and expectation formation effect significantly affects the current equilibrium, reacting to asset prices may alter the strength of expectation formation effect in different regimes, and therefore change the effectiveness of monetary policy in stabilizing output gap and inflation.

The main findings of the paper are as follows. First, the effect of expectation formation can substantially influence the movement of asset price, suggesting that, aside from those factors studied in the literature, expectation formation effect also plays an important role in asset pricing. Second, under the linear policy rule, the model calibrated to the U.S. economy generates very limited stabilization effect in responding to asset prices in terms of output and inflation volatilities. The negligible gains from responding to asset prices are reminiscent of the findings of Bernanke and Gertler (1999, 2001) and Iacoviello (2005).

Third, under the regime switching policy rule, reacting to asset price significantly shifts the “expected” inflation-output volatility frontier, i.e., the “Taylor curve,” inwards, thereby lowering both the volatilities of inflation and output for all possible policy choices. Moreover, as discussed in Friedman (2006) and Taylor (2006), the Taylor curve represents an efficiency frontier showing the trade-off between inflation-output volatility for optimal monetary policy. We find that given a linear policy rule, the model exactly exhibits this trade-off. However, the trade-off between the expected volatility of inflation and that of output in the Taylor curve substantially diminishes under the regime switching policy rule. A certain parameter range even exists where the trade-off between the volatilities of output and inflation vanishes. This means that the policy rule that accounts for expectation formation effect and asset price movement expands the set of monetary policy choices available for a monetary authority, by allowing the expected volatility of inflation to remain stable for an extended range of parameter values while stabilizing the output gap.

Finally, given a measure of weighted loss function, we find that overall gains from reacting to asset price can be substantial when accounting for expectation formation effect, while reacting to asset price tends to raise weighted losses under the linear policy rule. This suggests that ignoring the effect of expectation formation may underestimate the stabilization effect of reacting to asset price.

Existing works are divided on the issue of whether monetary policy should systematically react to change in asset prices. Borio and Lowe (2002) observed that sustained
rapid credit growth combined with large increases in asset prices appears to increase the probability of financial instability episodes. Thus a monetary response to credit and asset markets may be appropriate to preserve both financial and monetary stability. Mishkin (2008) argued that it is more likely for financial regulators and central banks to identify a credit-fueled bubble in real time because “they might have information that lenders have weakened their underwriting standards and that credit extension is rising at abnormally high rates.” In a similar view, Cecchetti (1998) and Cecchetti et al. (2000) contended that asset price misalignments are no more difficult to identify than other components of the Taylor rule, such as output gaps, and thus monetary policy should react when asset prices become misaligned with fundamentals.

In contrast, Assenmacher-Wesche and Gerlach (2008) used data from 1986 to 2006 of 17 countries to study the responses of residential property and equity prices, inflation, and economic activity to monetary policy shocks. They found that using monetary policy to offset asset price movements to guard against financial instability is likely to induce pronounced macroeconomic fluctuations. Bernanke and Gertler (1999, 2001) expressed doubt that policymakers can judge reliably whether asset prices are driven by “irrational exuberance” or whether an asset price collapse is imminent. They argue that central banks should consider fluctuations in stock price only to the extent that they affect primary monetary policy goals of price stability and output growth. Greenspan (2002) expressed similar views that bubbles in asset prices are very difficult to identify as they build up and monetary policy cannot successfully deflate asset price bubbles without triggering a recession. Based on Bernanke et al. (1999) together with collateral constraints, Iacoviello (2005) found that the gains from reacting to asset price are negligible in terms of output and inflation stabilization. Similarly, Faia and Monacelli (2006) found that when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes.

All the above works have investigated this issue with linear monetary policy rules. Bordo and Jeanne (2002), on the other hand, concluded that optimal monetary policy is contingent on the economic conditions in a complex, non-linear way, and cannot be summarized in a simple rule-type policy. This paper responds to this view by investigating

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1Similarly, Gruen et al. (2005) show that the reaction function of a central banker sensitively depends
the effect of reacting to asset price in a non-linear monetary policy rule.

The rest of the paper is organized as follows. Section 2 describes the environment of the model and the behaviors of the agents. Section 3 solves the equilibrium and examines the effect of expectation formation on asset price movements. Section 4 evaluates whether reacting to asset price performs better in lowering volatilities of output gap and inflation in a regime-switching monetary rule. Section 5 concludes.

2 Model

We consider a model in which time is discrete and the economy is populated with households, final goods-producing firms, intermediate goods-producing firms, and a central bank. Each intermediate goods-producing firm produces distinct intermediate goods, indexed by \( i \in [0, 1] \).

2.1 Households and Final Goods Producers

At the beginning of period \( t \), a household has dividends \( D_t S_{t-1} \) from holding the shares of intermediate goods firms, gross returns (repayments) from lending (borrowing) \( R_t B_{t-1} \), and wage income \( W_t L_t \) from supplying labor to intermediate goods-producing firms, where \( D_t \) is dividend per share, \( B_t \) is one-period bonds, and \( W_t \) denotes the nominal wage rate. With these incomes, the household purchases composite final goods \( C_t \) given the general price level \( \Pi_t \), and decides holdings of equities \( S_t \) given the nominal equity price \( Q_t \) and lending or borrowing \( B_t \) at the gross interest rate \( R_t \).

The household’s problem is to maximum the expected lifetime utility function

\[
\max E_0 \sum \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\zeta}}{1+\zeta} \right),
\]

subject to the budget constraint (in real terms)

\[
C_t + q_t(S_t - S_{t-1}) + b_t = w_t L_t + d_t S_{t-1} + R_t b_{t-1}/\pi_t,
\]

on the detailed stochastic properties of the asset bubble. This result highlights the need for stringent informational requirements for a central bank to react to asset-price bubbles.
where $w_t \equiv W_t / P_t$ is the real wage rate, $q_t \equiv Q_t / P_t$ is the real equity price, $d_t \equiv D_t / P_t$ is the dividend in real terms, and $b_t \equiv B_t / P_t$ is the amount of one-period bonds in real terms. The parameter $\sigma$ is the inverse of elasticity of substitution, $\zeta$ is the inverse elasticity of labor supply, and $\chi$ is the relative weight of leisure.$^2$

The final consumption good is a Dixit-Stiglitz aggregation of differentiated intermediate goods, $C_t = \left[ \int_0^1 Y_{j,t}^{\theta-1} \, dj \right]^{\frac{1}{\theta-1}}$, where $Y_{j,t}$ denotes the type-$j$ intermediate good and $\theta > 1$ measures the elasticity of substitution between differentiated intermediate goods.

Given the level of $C_t$, cost minimization by the households to achieve this level of composite consumption implies that the demand for intermediate good $j$ is

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta} C_t,$$

where $P_{j,t}$ is price of good $j$. Zero profit implies that these prices of differentiated goods are related to the aggregate price level $P_t$ as follows,

$$P_t = \left[ \int_0^1 P^{1-\theta}_{j,t} \, dj \right]^{\frac{1}{1-\theta}}.$$

The households then choose the level of composite consumption, asset demands, and labor supply, given goods prices, equity prices, and the wage rate. Solving the maximization problem yields a set of first order conditions. Together with the equity market clearing condition $S_t = 1$, for all $t$, we have

$$C_t^{-\sigma} = \beta R_t E_t \left( \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right),$$

$$C_t^{-\sigma} = \beta E_t \left( \frac{d_{t+1} + q_{t+1}}{q_t} C_{t+1}^{-\sigma} \right),$$

$$\frac{\chi L_t^\zeta}{C_t^{-\sigma}} = w_t,$$

$^2$Note that we abstract from the liquidity services of real money balances. This is done to simplify the exposition. This model can be viewed as a money-in-utility function model in which the household’s utility function includes a third term, $\theta M_t / P_t$, and we take $\theta$ to be arbitrarily small. A justification is the “cash-less” limit of a monetary economy, in which the cashless limit is a sufficiently good approximation (Woodford (1998)). For example, if financial innovation has proceeded enough to make cash balances of sufficiently small importance in carrying out transactions, then fluctuations in money demand have only negligible effects upon the equilibrium price level under a Wicksellian policy regime. Since we will be interested in formulating monetary policy by interest rate rules, explicit reference to the existence of money is not necessary.
2.2 Intermediate Goods Producers

The intermediate goods producers hire labor from households to produce intermediate goods using a linear technology as follows,

\[ Y_{i,t} = a_t L_{i,t}, \]

where \( a_t \) represents an aggregate productivity disturbance. These firms set prices for their differentiated products in a staggered fashion. Following Calvo (1983), we assume that in each period, a firm cannot adjust its price with a probability \( \tau \). By the law of large numbers, a fraction \( 1 - \tau \) of firms in a given period can re-optimize their pricing decisions.

The maximization problem of intermediate goods firm \( j \) is to choose \( P_{j,t} \) to maximize the expected discounted dividend flows

\[
E_t \sum_{i=0}^{\infty} \tau^i \Theta_{t+i} \left[ \left( \frac{P_{j,t}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{P_{j,t}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},
\]

subject to the demand schedule (1). The discount factor \( \Theta_{t+i} \) is given by \( \beta^i (C_{t+i}/C_t)^{-\sigma} \) and \( \varphi_t \) equals the firm’s real marginal cost. Since all firms that are able to adjust prices in period \( t \) face the same problem, these firms will set the same price. The optimal price setting \( P_t^* \) satisfies,

\[
\frac{P_t^*}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \tau^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} (P_{t+i}/P_t)^{\theta}}{E_t \sum_{i=0}^{\infty} \tau^i \beta^i C_{t+i}^{1-\sigma} (P_{t+i}/P_t)^{\theta-1}}.
\]  

The price level of economy at period \( t \) satisfies

\[
P_t^{1-\theta} = (1 - \tau)(P_t^*)^{1-\theta} + \tau P_{t-1}^{1-\theta}. \]

Clarida et al. (2001) suggests that one way to include a stochastic shock in the inflation adjustment equation is to add a stochastic wage markup \( u_t^w \), which represents deviations between the real wage and the marginal rate of substitution between leisure and consumption. Thus the labor supply condition (4) becomes

\[
\left( \frac{\chi L_t^\xi}{C_t^{1-\sigma}} \right) e^{u_t^w} = w_t,
\]

where \( u_t^w \) follows an i.i.d. normal process with mean zero and variance \( \sigma_{u_t}^2 \).
2.3 The Central Bank

The crucial feature of this model is the specification of central bank’s reaction function. To study the implications of regime shifts in monetary policy, we allow for the coefficients in the Taylor rule to vary with policy regime

\[
\left( \frac{R_t}{R} \right) = \left( \frac{\pi_t}{\pi^*} \right)^{\alpha_{\pi,s_t}} \left( \frac{x_t}{x} \right)^{\alpha_{x_s,t}} \left( \frac{q_t}{q} \right)^{\alpha_{q,s_t}} e^{u_t^M}, \tag{8}
\]

where \( \alpha_{\pi,s_t}, \alpha_{x_s,t} \) and \( \alpha_{q,s_t} \) are regime-dependent policy parameters that measure the aggressiveness of monetary policy against inflation \( \pi_t \), output gap \( x_t \), and the real equity price \( q_t \). The term \( \pi^* \) is the inflation target, \( R, x, \) and \( q \) are the steady states of \( R_t, x_t, \) and \( q_t \), respectively, and \( u_t^M \) is the shock to monetary policy which follows an i.i.d. normal process with mean zero and variance \( \sigma_M^2 \).

Monetary policy regime follows a Markov switching process between two states, Hawkish regime \( (s_t = 1) \) and Dovish regime \( (s_t = 2) \), that differ in the responsiveness to inflation, i.e., \( \alpha_{\pi,1} \geq \alpha_{\pi,2} \). The transition probabilities for the regime switching process are given by the following matrix

\[
P = \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix},
\]

where \( p_{ij} = \text{Prob}(s_{t+1} = i | s_t = j) \). Each column of \( P \) sums up to 1 so that \( p_{21} = 1 - p_{11} \) and \( p_{22} = 1 - p_{21} \). After identifying the two states, we compute the smoothed probability:

\[
P_{j,t} = \text{Pr}(s_t = j | \Gamma_T), \quad j = \{1, 2\},
\]

where \( \Gamma_T \) is all the information available in our sample periods. The idea is to compute the probability of a certain state of the economy from an ex-post point of view, thus utilizing the full set of information. If \( \text{Pr}(s_t = j | \Gamma_T) > 0.5 \), then we identify the economy most likely in state \( j, j = 1, 2 \).

2.4 Equilibrium

The labor market clear when the labor demand from intermediate goods producers equals the supply from households, \( L_t^D = L_t^S \), the equity market equilibrium requires \( S_t = 1 \), and
the one-period bond market clears when $b_t = 0$. Finally, the final goods market clears according to Walras Law.

The equilibrium consists of an allocation $\{Y_t, C_t, L_t, D_t, S_t, B_t\}$ and a sequence of prices $\{P_t, Q_t, W_t, R_t\}$ that satisfy the optimality conditions of households, intermediate goods producers, and final goods producers. Finally, all markets clear.

### 2.5 Linearization of the System

Let a variable with hat denote the percentage deviation for a variable $z_t$ around the steady state. Log-linearizing the optimal pricing rule (5), (6), and the labor supply (7), yields the expectations-augmented Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t^S, \quad (9)$$

where $\kappa \equiv (1-\tau)(1-\beta\tau)/\tau$, $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^f$ is the output gap, $\hat{Y}_t^f$ is the output when prices are fully flexible ($\tau = 0$), and $u_t^S = \kappa u_t^w$ represents the cost-push shock.

Log-linearizing the consumption Euler equation (2) leads to the forward-looking IS curve

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) + u_t^D, \quad (10)$$

where $1/\sigma$ is the elasticity of substitution and $u_t^D = E_t \hat{Y}_t^f - \hat{Y}_t^f$ represents the demand side shock.

Log-linearizing (2) and (3), together with (10), we obtain the equation for equity price determination,

$$\hat{q}_t = \beta E_t \hat{q}_{t+1} - \varpi_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \varpi_3 u_t^D - \varpi_4 u_t^S, \quad (11)$$

where $\varpi_2$, $\varpi_3$ and $\varpi_4$ are complex functions of model parameters, detailed in the appendix.

Finally, log-linearizing the interest rate rule (8), we have

$$\hat{R}_t = \alpha_{\pi,s_t} \hat{\pi}_t + \alpha_{x,s_t} \hat{x}_t + \alpha_{q,s_t} \hat{q}_t + u_t^M. \quad (12)$$

Recall that the central bank reacts more strongly to inflation in Hawkish regime ($s_t = 1$) than in Dovish regime ($s_t = 2$), i.e., $\alpha_{\pi,1} > \alpha_{\pi,2}$. 

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The disturbance terms $u_t^S$, $u_t^D$, and $u_t^M$, for (9), (10), and (12), respectively, are assumed to follow a first order autoregressive process

$$u_t^i = \rho_i u_{t-1}^i + \epsilon_t^i, i = S, D, M,$$

where $\epsilon_t^i$ is an i.i.d. normally distributed random variable with zero mean and unity variance.

In sum, the system of economy can be summarized by four equations, (9), (10), (11), and (12), that characterize the dynamics of inflation, output gap, equity price, and interest rate.

3 Solving the Equilibrium with a Regime-Switching Policy Rule

The system of the economy can be expressed as

$$\begin{align*}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t^S, \\
\hat{x}_t &= E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) + u_t^D, \\
\hat{R}_t &= \alpha_{x,s} \hat{\pi}_t + \alpha_{x,s} \hat{x}_t + \alpha_{q,s} \hat{q}_t + u_t^M, \\
\hat{q}_t &= \beta E_t \hat{q}_{t+1} - \omega_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \omega_3 u_t^D - \omega_4 u_t^S.
\end{align*}$$

To rewrite the model in a more compact form, we substitute (15) into (14) and re-arrange (13), (14) and (16) to characterize the economy by a three-variable system of inflation, output gap, and equity price, $(\hat{\pi}_t, \hat{x}_t, \hat{q}_t)$.

Define $\hat{\pi}_{i,t} \equiv \hat{\pi}_t(s_t = i)$, $\hat{x}_{i,t} \equiv \hat{x}_t(s_t = i)$, and $\hat{q}_{i,t} \equiv \hat{q}_t(s_t = i)$, $i = 1, 2$, which denote the state-dependent inflation, output gap and equity price. Let $\Gamma_t$ be the information set up to time $t$. The conditional expectation of $\hat{\pi}_{i,t+1}$ is $E_t [\hat{\pi}_{i,t+1}|\Gamma_t]$. Integrating over possible future regimes, we can write

$$E_t \hat{\pi}_{t+1} = E \left[ \hat{\pi}_{t+1}|s_t = i, \Gamma_t^{-s} \right] = p_{11} E \left[ \hat{\pi}_{1,t+1}|\Gamma_t^{-s} \right] + p_{12} E \left[ \hat{\pi}_{2,t+1}|\Gamma_t^{-s} \right],$$

where $i = 1, 2$, and $\Gamma_t^{-s}$ is the information set up to time $t$, excluding the current regime, i.e., $\Gamma_t = \Gamma_t^{-s} \cup \{s_t\}$. The conditional expectations of $\hat{x}_{t+1}$, and $\hat{q}_{t+1}$ can be calculated in the same way.
Denoting $E_t\hat{\pi}_{i,t+1} = E [\hat{\pi}_{i,t+1} | \Gamma_t^{-1}]$, $i = 1, 2$, and similarly for $E_t\hat{x}_{i,t+1}$ and $E_t\hat{q}_{i,t+1}$, define the forecasting errors:

$$
\eta^i_{i,t+1} \equiv \hat{\pi}_{i,t+1} - E_t\hat{\pi}_{i,t+1}, \\
\eta^q_{i,t+1} \equiv \hat{x}_{i,t+1} - E_t\hat{x}_{i,t+1}, \\
\eta^q_{i,t+1} \equiv \hat{q}_{i,t+1} - E_t\hat{q}_{i,t+1}, \quad i = 1, 2.
$$

We can then express the system in the following form,

$$
AY_{t+1} = BY_t + A\eta_{t+1} - C u_t, 
$$

where $Y_t = [\hat{\pi}_{1,t}, \hat{\pi}_{2,t}, \hat{x}_{1,t}, \hat{x}_{2,t}, \hat{q}_{1,t}, \hat{q}_{2,t}]$, $\eta_t = [\eta^i_{1,t}, \eta^q_{1,t}, \eta^i_{2,t}, \eta^q_{2,t}, \eta^q_{1,t}, \eta^q_{2,t}]$, and $u_t = [u^S_t, u^D_t, u^M_t]$, and the matrices, $A$, $B$ and $C$, are defined in the Appendix. A unique equilibrium requires six unstable roots to generate six linear restrictions that determine the regime-dependent forecasting errors for inflation, output gap and asset price. We compute the generalized eigenvalues for the system to check the existence and uniqueness of the model’s equilibrium following Sims (2002).

To solve the model, we employ the method of undetermined coefficients that delivers solutions as functions of the smallest set of state variables, $u^S_t$, $u^D_t$, $u^M_t$, and $s_t$ (more detail can be found in appendix). We posit the equilibrium solution with a regime-switching policy rule as follows:

$$
\hat{\pi}^A_t = a^S(s_t)u^S_t + a^D(s_t)u^D_t + a^M(s_t)u^M_t, \\
\hat{x}^A_t = b^S(s_t)u^S_t + b^D(s_t)u^D_t + b^M(s_t)u^M_t, \\
\hat{q}^A_t = c^S(s_t)u^S_t + c^D(s_t)u^D_t + c^M(s_t)u^M_t,
$$

which can be compactly expressed as,

$$
Z^A_t = G^A_{1,s_t}Z_{t-1} + G^A_{2,s_t}\epsilon_t, 
$$

where $Z^A_t = [\hat{\pi}^A_t, \hat{x}^A_t, \hat{q}^A_t, u^S_t, u^D_t, u^M_t]'$, $\epsilon_t = [\epsilon^S_t, \epsilon^D_t, \epsilon^M_t]'$, with the covariance matrix $\Omega^A_{s_t} = E [Z^A_t Z^A_t']$, and the superscript $A$ denotes the solution reacting to asset price.

Similarly, we consider the economy in which the central bank follows a regime-switching policy rule but without reacting to asset price. Replacing (15) by

$$
\hat{R}_t = \alpha_\pi, s_t\hat{\pi}_t + \alpha_x, s_t\hat{x}_t + u^M_t,
$$

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and solve for the equilibrium solution, which is expressed as
\[
Z_t = G_{1,s_t} Z_{t-1} + G_{2,s_t} \epsilon_t, \quad (19)
\]
where
\[
Z_t = [\widehat{\pi}_t, \widehat{x}_t, \widehat{q}_t, u_t^S, u_t^D, u_t^M]' , \quad \epsilon_t = [\epsilon_t^S, \epsilon_t^D, \epsilon_t^M]',
\]
with the covariance matrix \( \Omega_{s_t} = E[Z_t Z_t'] \).

### 3.1 The Equilibrium with a Fixed-Regime Policy Rule

We also solve the counterpart of (18) under which the central bank follows a specific regime \( j \) indefinitely. When the current regime of monetary policy is expected to stick to a specific regime, which is equivalent to imposing \( p_{jj} = 1 \) for \( j \in \{1, 2\} \), \( \alpha_{\pi,j} \), \( \alpha_{x,j} \), and \( \alpha_{q,j} \) \( (j = 1, 2) \) in the reaction function are time-invariant.

In this case, we can solve for the model analytically. The expressions of the solution are stated in the appendix. Here we rewrite the solution, currently given at regime \( j \), as the same form in (18):
\[
Z_{t,j}^A = G_{1,j}^A Z_{t-1,j} + G_{2,j}^A \epsilon_t, \quad j \in \{1, 2\}, \quad (20)
\]
where
\[
Z_{t,j}^A = [\widehat{\pi}_{t,j}^A, \widehat{x}_{t,j}^A, \widehat{q}_{t,j}^A, u_t^S, u_t^D, u_t^M]', \quad \epsilon_t = [\epsilon_t^S, \epsilon_t^D, \epsilon_t^M]',
\]
with the covariance matrix given the policy regime \( j \), \( \Omega_j^A = E[Z_{t,j}^A Z_{t,j}^A]' \).

Similarly, we consider the economy in which the central bank follows a fixed-regime policy rule but without reacting to asset price, i.e., \( \alpha_{q,j} = 0 \) for \( j = 1, 2 \). The appendix states the analytical solution of the system. Here we denote the system, currently given at regime \( j \), as follows:
\[
Z_{t,j} = G_{1,j} Z_{t-1,j} + G_{2,j} \epsilon_t, \quad j \in \{1, 2\}, \quad (21)
\]
where
\[
Z_{t,j} = [\widehat{\pi}_{t,j}, \widehat{x}_{t,j}, \widehat{q}_{t,j}, u_t^S, u_t^D, u_t^M]', \quad \epsilon_t = [\epsilon_t^S, \epsilon_t^D, \epsilon_t^M]',
\]
and the covariance matrix denoted as \( \Omega_j = E[Z_{t,j} Z_{t,j}] \).

### 3.2 Parameters Values

This section empirically estimates the structural parameter values in the reaction function (15), and then choose the remaining parameter values from the literature.
Before estimating the reaction function, note that there is a feedback effect in the system (13)-(16), so that \( \hat{\pi}_t, \hat{x}_t, \) or \( \hat{q}_t \) may also react to changes in the interest rate. To check for this potential endogeneity problem, we conduct the Hausman (1978) test to see whether any of these three variables is correlated with the error term. Let \( X_t \in [\hat{\pi}_t, \hat{x}_t, \hat{q}_t] \) be the variable that may correlate with the error term. Given an instrumental vector \( V_t \) that contains the lags of \( X_t \), we define \( \hat{X}_t = P_z X_t \), where \( P_z = V_t(V_t'V_t)^{-1}V_t' \), and estimate the following model

\[
\hat{R}_t = \delta_0 X_t + \delta_1 \hat{X}_t + \varepsilon_t.
\]

We test the null hypothesis that the coefficient \( \delta_1 \) is zero, i.e., an omitted variable test for the variables \( \hat{X}_t \). We estimate this equation using quarterly U.S. data from 1975Q1 to 2008Q4. For the explanatory variable, the inflation rates measured by changes in the GDP deflator, output gap is measured by detrended real GDP, \( \hat{q}_t \) is represented by the de-trended stock price index of S&P500, and the interest rate is represented by the federal funds rate.

[Table 1]

From Table 1, inflation rate is the only variable that is endogenous to the federal funds rate. To account for the endogeneity problem, we follow Kim (2009) to estimate \( \hat{\pi}_t = z_t' \delta_x + u_t \) by OLS, where the vector \( z_t \) includes the instrumental variables, selected to be the lags of inflation rate, and \( \delta_x \) is the vector of parameters for these lagged inflation rate. According to the AIC criterion, we select four instruments with five to eight lag periods. After estimating this equation, we incorporate the residual term \( \hat{u}_t \) into (15), and then estimate the following modified policy rule:

\[
\hat{R}_t = \alpha_{x,1} \hat{\pi}_t + \alpha_{x,2} \hat{x}_t + \alpha_{q,1} \hat{q}_t + \psi_{\pi,q} \hat{u}_t + u_t^M.
\]

Table 2 report our estimation results using U.S. quarterly data from 1975Q1 to 2008Q4. The estimated transition probabilities are \( p_{11} = 0.97 \) and \( p_{22} = 0.96 \). The parameter values of the policy rule are \( \alpha_{x,1} = 2.6, \alpha_{x,2} = 0.07, \) and \( \alpha_{q,1} = 0.02 \) in Hawkish regime, and \( \alpha_{x,2} = 1.5, \alpha_{x,2} = 0.01, \) and \( \alpha_{q,2} = 0.03 \) in Dovish regime. Note that the reactions to inflation under both regimes are significant at the one percent level, and the reaction
under Hawkish regime ($\alpha_{x,1} = 2.6$) is clearly more active than that under Dovish regime ($\alpha_{x,2} = 1.5$). The reactions to output gap and stock price are significant under Hawkish regime, while insignificant under Dovish regime, suggesting that the Federal Reserve has been actively reacting to output gap and asset price in Hawkish regime, while remaining passive in Dovish regime.

We also estimate a linear model for comparison, which yields $\alpha_{x} = 1.9$, $\alpha_{x} = 0.07$, and $\alpha_{q} = 0.03$.\(^3\) where the reactions to inflation and output gap are both significant, while the reaction to asset price is not. The reaction to asset price appears insignificant under linear specification, implying that the linear policy rule fails to detect the Federal Reserve’s active response to asset price in Hawkish regime, albeit small in magnitude ($\alpha_{q,1} = 0.02$).\(^4\)

As listed in Table 2, the model allowing for regime switching attains a higher log-likelihood value ($-543.52$) than the linear model ($-578.90$). The $LR$ statistic, proposed by Garcia (1988), is given by $LR = 2(LU - LR) = 70$, which is significant at the 1% level (with a critical value 14.2 ), suggesting that the Markov Switching model performs better than the linear model.

Other structural parameters in this model are standard in the literature. We select their values from previous works. Table 3 reports these parameter values.

---

\(^3\)The estimate of $\alpha_{q}$ by Rigobon and Sack (2003) in a linear model is round 0.02. Our estimation result for the linear model is close to Rigobon and Sack’s (2003) findings.

\(^4\)The result here is relevant for the debate as to whether central banks have been reacting to asset prices. Empirical evidence has shown mixed results. For example, Bernanke and Gertler (1999) found that the Fed has focused its attention on expected inflation and the output gap and has not actively attempted to stabilize stock prices during the Volcker and Greenspan eras ($1979M10 - 1997M12$). However, Rigobon and Sack (2003) found that the Fed has systematically responded significantly to stock price movements. For the cross-country evidence, see, for example, Bohl et al. (2007) and Furlanetto (2008).

\(^5\)We also measure the probability that a regime occurs by computing the filtered probability:

$$P_{j,t} = \Pr(s_t = j|\Psi_t), \ j = \{1, 2\},$$
the monetary policy switched to Hawkish regime after the early 1980s recession, and then shifted back to Dovish regime in response to the aftermath of the savings and loans crisis in the early 1990s. This is in line with literature findings on the monetary policy environment in the pre-Volcker and Volcker-Greenspan eras (Clarida et al. (2000) and Lubik and Schorfheide (2004)). In around 2002, the monetary policy shifted once more to Dovish regime, ending Hawkish regime since 1994. As Taylor (2007) and others argued, the lax monetary environment was one of the major causes leading to the run-up in house price in the early 2000s before it collapsed in late 2006.

[Figure 1]

3.3 Effect of Expectation Formation on the Asset Price

This section investigates how the effect of expectation formation influences the movement of the asset price. Recall that \( \hat{q}_{t,s}^A \) denotes the equilibrium asset price when the policy rule is regime-switching and reacts to asset price, and \( \hat{q}_{t,j}^A \) denotes the equilibrium asset price when the policy rule maintains at a fixed regime and reacts to asset price, from systems (18) and (20), respectively. The simulation of the asset prices consists of the following three steps. First, given the benchmark values of structural parameters with the central bank reacting to asset price, we randomly draw from the three disturbance terms for each system, (18) and (20), respectively, and obtain a pair of asset prices \( (\hat{q}_{t,s}, \hat{q}_{t,j}) \). Second, we draw 1000 times to generate two series of asset prices. Given a regime \( j \) and \( N = 1000 \), we compute the following,

\[
\frac{1}{N} \sum_{i=1}^{N} (\hat{q}_{t,s}^A - \hat{q}_{t,j}^A)^2,
\]

which measures the averaged quadratic difference between the asset price generated under the model allowing for regime shift and that under the model given a fixed regime \( j \), i.e., the difference in these two asset prices caused by the effect of expectation formation. Finally, we repeat step 1 and 2 for 3000 times to obtain 3000 observations for each regime \( j \), and then plot their distributions in Figure 2.

where only the information up to time \( t (\Psi_t) \) is used. We find that the regimes identified by both methods are almost identical.
If expectation formation effect is substantial, then the difference between these two distributions should be sizeable. On the other hand, if expectation formation effect is getting negligible, these two distributions will move closer to each other and eventually collapse into a single point at the mean value.\(^6\)

The solid line in Figure 2 represents the distribution of (22) under Dovish regime and the dashed line represents that under Hawkish regime. The distribution under Dovish regime has a higher mean than that under Hawkish regime, suggesting that the expectation formation has a larger effect on stock price in Dovish regime than in Hawkish regime.\(^7\) Furthermore, these two distributions are completely separated, implying that expectation formation effect substantially influences asset price dynamics.

The results suggest that, aside from those factors studied in the literature, expectation formation effect also plays an important role in asset pricing.

[Figure 2]

4 Can Responding to Asset Price Lower Volatilities of Inflation and Output?

To evaluate the stabilization effect of the monetary policy reacting to asset price and accounting for expectation formation effect, we compute the following measure:

\[
\frac{\Omega_{ii,s_t}^A - \Omega_{ii,j}^A}{\Omega_{ii,j}} 
\]

where \(\Omega_{ii,s_t}^A\) is the diagonal element of the matrix \(\Omega_{s_t}^A\) and \(\Omega_{ii,j}^A\) is the diagonal element of the matrix \(\Omega_j\). Recall that \(\Omega_{s_t}^A\) is the covariance matrix of each element in \(Z_t^A\) when the regime-switching policy rule reacts to asset price, and \(\Omega_j\) is the corresponding covariance.

\(^6\)When \(N\) is large enough, the mean value is given by

\[
E\left(\bar{q}_{st}^A - \bar{q}_{ij}^A\right)^2 = E(\bar{q}_{st}^A)^2 + E(\bar{q}_{ij}^A)^2 - 2\text{cov}(\bar{q}_{st}^A, \bar{q}_{ij}^A).
\]

\(^7\)Note that the dispersion of the distribution will surely increase with its mean because of the quadratic metric in (22).
matrix of each element in $Z_{t,j}$ under the fixed regime $j$ when the policy rule does not react to asset price. The first three elements we are interested are inflation, output gap, and equity price.

For comparison, we also compute the relative volatilities under the linear monetary policy rule. Let $\Omega^A$ be the covariance matrix of the equilibrium solution under the linear model when the central bank reacts to asset price, while $\Omega$ corresponds to that when the central bank does not react to asset price. Thus, the effect of reacting to asset price under the linear policy rule is

$$\frac{\Omega^A_{ii} - \Omega_{ii}}{\Omega_{ii}}, i = 1, 2, 3,$$  \hspace{1cm} (24)

where $\Omega^A_{ii}$ and $\Omega_{ii}$ are the corresponding diagonal element of the matrix $\Omega^A$ and $\Omega$, respectively.

Table 4 and 5 display the results. A positive (negative) entry means that reacting to asset price combined with the expectation formation effect raises (lowers) the volatility of a variable proportionally relative to the fixed regime equilibrium, in which each variable is measured in terms of percentage deviation from the steady state.

[Table 4, 5]

Examining the overall effects of reacting to asset under these two different policy rules in Table 4 and 5 yields the following notable findings. Table 4 shows that reacting to asset price under the linear monetary rule reduces all the volatilities of inflation, output gap, and asset price. However, gains from responding to asset prices are small in terms of output and inflation stabilization under the linear monetary rule. The negligible gains from responding to asset prices are reminiscent of the findings of Bernanke and Gertler (1999, 2001) and Iacoviello (2005).

In contrast, Table 5 shows that introducing the possibility of regime shift can amplify the gains from reacting to asset price by significantly lowering the volatilities of output gap in both regimes and generating asymmetric responses for the volatilities of inflation and asset price, with the stabilizing effect larger than the destabilizing effect. Take inflation as an example, the total effect of reacting to asset price significantly lowers inflation volatility in Dovish regime by 37.05%, while it only moderately raises inflation volatility in Hawkish
regime by 16.25%.

To understand the intuition behind the result, we decompose the total effect of reacting to asset price under regime switching into two components:

\[
\begin{align*}
\Omega_{i,s}^A - \Omega_{i,j}^A &= (\Omega_{i,i,j}^A - \Omega_{i,i,j}^A) + (\Omega_{i,i,s}^A - \Omega_{i,i,j}^A),
\end{align*}
\]

where the former is the direct effect of reacting to asset price for a given fixed regime, and the latter is the effect of expectation formation. Note that the direct effect of reacting to asset price is calculated for a given fixed regime, and is different from the effect in a linear policy rule.

As shown in Table 5, the direct effect is stabilizing for all variables under both regimes when the regime is expected to prevail indefinitely. However, expectation formation effect introduces a volatility trade-off between the two regimes. For inflation and asset price, the anticipation of regime shift raises the volatility in Hawkish regime and lowers it in Dovish regime; and the effect is opposite for output gap. For example, inflation volatility is increased by 29.77% in Hawkish regime and decreased by 27.37% in Dovish regime; and output gap volatility is decreased by 31.44% in Hawkish regime and increased by 3.88% in Dovish regime. The results are intuitive: anticipating a regime shift from Hawkish to Dovish raises inflation volatility while it helps stabilize output gap volatility; and vice versa for anticipating a regime shift from Dovish to Hawkish.\(^8\)

\(^8\)We experiment with different combinations of parameter values to see how the effect of expectation formation changes. We find that when \(p_{22} = 0\) (i.e., when the policy regime falls into Dovish, it is believed that the regime will shift back to Hawkish for sure), while everything else remains the same, the expectation formation effect attains its maximum. The expectation formation effect appears larger for inflation and stock price, working more in the direction of reducing volatilities when the current regime is Dovish. This means that if a central bank has a reputation of maintaining low inflation and strictly abides by the Hawkish regime, i.e., \(p_{11}\) is close to unity and \(p_{22}\) is close to zero, the expectation formation effect will work most effectively when the current regime is Dovish, because the current regime is only transitory.
4.1 Taylor Curve and the Trade-off in “Expected” Volatilities of Output Gap and Inflation

The trade-off for stabilizing volatilities involves the probability that a regime occurs at a certain point in time, therefore the stabilization effects have to be weighted over the probability that a particular state will occur at a certain period of time. We consider the following measures of “expected” volatilities of output gap and inflation under the regime-switching policy rule,

\[
\Sigma^\text{RS}_x \equiv \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{2} \Pr(s_t = j|\Gamma_T) \left(\frac{(\bar{\sigma}^A_{x,s_t})^2 - \sigma^2_{x,j}}{\sigma^2_{x,j}}\right),
\]

\[
\Sigma^\text{RS}_\pi \equiv \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{2} \Pr(s_t = j|\Gamma_T) \left(\frac{(\bar{\sigma}^A_{\pi,s_t})^2 - \sigma^2_{\pi,j}}{\sigma^2_{\pi,j}}\right),
\]

where the proportional deviation from the variance under a fixed regime \( j \) without reacting to asset price is weighted by the smoothed probabilities over the two regimes and then averaged over the whole sample period (\( T = 127 \)).

The corresponding measures for the linear policy rule are given by

\[
\Sigma_x \equiv \frac{[ (\sigma^A_x)^2 - \sigma^2_x ]}{\sigma^2_x},
\]

\[
\Sigma_\pi \equiv \frac{[ (\sigma^A_\pi)^2 - \sigma^2_\pi ]}{\sigma^2_\pi}.
\]

We allow the asset price coefficient in (15) for Hawkish regime, \( \alpha_{q,1} \), to vary within the range [0.02, 0.12], and \( \alpha_{q,2} \), for Dovish regime, to vary within [0.03, 0.12]. The corresponding coefficient for the linear policy rule, \( \alpha_q \), varies within the range [0.03, 0.12]. Note that the benchmark estimated values of parameters are \( \alpha_{q,1} = 0.02 \), \( \alpha_{q,2} = 0.03 \), and \( \alpha_q = 0.03 \), as listed in Table 2.

Figure 3 shows the inflation-output volatility frontiers – the “Taylor Curve” – using (25)-(28). The horizontal axis is the volatility of output gap under the linear rule \( \Sigma_x \) and the expected volatility under the regime switching policy rule \( \Sigma^\text{RS}_x \), relative to the respective monetary rule without reacting to asset price; and the vertical axis is the volatility of inflation under the linear rule \( \Sigma_\pi \) and the expected volatility under the regime...
switching policy rule $\Sigma_{\pi}^{RS}$, relative to the respective monetary rule without reacting to asset price.

Consider first the linear policy rule, which is represented by the dashed line. As $\alpha_\pi$ rises from 0.03 to 0.12, the inflation-output volatility frontier stretches from the southeast end (at which $\alpha_\pi = 0.03$), toward the northwest end, which means that when the Federal Reserve reacts to asset price more actively, it stabilizes the output gap at the expense of price stabilization. The trade-off between volatilities of inflation and output is a standard feature of the Taylor curve, representing an efficiency frontier of fluctuations in output and inflation for policy choice (Friedman (2006) and Taylor (2006)).

Under the regime-switching policy rule, we allow either $\alpha_{\pi,1}$ or $\alpha_{\pi,2}$ to vary, starting with their benchmark values, 0.02 and 0.03, at which the solid line (Hawkish regime, varying $\alpha_{\pi,1}$) and the dotted line (Dovish regime, varying $\alpha_{\pi,2}$) meets. Clearly the Taylor curve for the regime switching policy rule, either for Hawkish or Dovish regime, lies far below the Taylor curve for the linear policy rule, i.e., accounting for expectation formation generates lower expected inflation volatilities both for output gap and inflation. This suggests that ignoring the effect of expectation formation may underestimate the stabilization effect of reacting to asset price.

Interestingly, for certain ranges of parameter values, i.e., $\alpha_{\pi,1} \in [0.02, 0.06]$ and $\alpha_{\pi,2} \in [0.03, 0.06]$, the trade-off between the expected volatilities of output gap and inflation vanishes. Within these ranges, reacting to asset price strictly lowers both the expected volatilities of output gap and inflation, and thus the expected inflation-output volatility frontiers become positively sloped.

As the intensiveness of reacting to asset price is getting larger, i.e., $\alpha_{\pi,1} \geq 0.06$ and $\alpha_{\pi,2} \geq 0.06$, output stabilization begins to take its toll on inflation volatility, thereby restoring the trade-off between the expected volatilities of output gap and inflation. This means that accounting for expectation formation effect can substantially diminish the trade-off implied by the Taylor curve and enlarge the set of monetary policy choices available for the monetary authority.

[Figure 3]
4.2 The Loss Function

Suppose the monetary authority is concerned with stabilizing the expected volatilities of inflation and output gap. Define the loss function $L^{RS}$ to be the weighted average in the expected volatilities of output gap and inflation relative to the volatilities under a fixed regime $j$:

$$L^{RS} = \omega \Sigma_x^{RS} + \Sigma_i^{RS},$$

where $\omega$ is the relative weight on the output gap stabilization. We also compute the loss function under the linear policy rule, which measures the weighted average in volatilities of inflation and output gap relative to the monetary rule without reacting to asset price:

$$L = \omega \Sigma_x + \Sigma_i.$$

The calibrated value of the relative weight on the output gap stabilization $\omega$ in the literature varies from 0.05 to one-third (Jensen (2002)). We follow Jensen (2002) and McCallum and Nelson (2004b) by setting $\omega = 0.2$. Given our benchmark parameter values listed in Table 2 and 3, we plot the loss functions of $L^{RS}$ and $L$, varying the reaction coefficient to asset price under the linear policy rule ($\alpha_q$) and that under the regime switching policy rule ($\alpha_{q,1}$ or $\alpha_{q,2}$) in Figure 4.

For the linear policy rule, the value of loss function $L$ is negative only for a very limited range of $\alpha_q$ and becomes positive as the policy responds more actively to asset price, implying that strongly reacting to asset price raises the value of loss function. On the other hand, reacting to asset price generates negative values of $L^{RS}$ throughout, no matter whether varying $\alpha_{q,1}$ or $\alpha_{q,2}$, which says that when taking into account of expectation formation effect reacting to asset price can effectively lowers the value of loss function. Corresponding to Figure 3, responding to asset price in Hawkish regime, represented by the solid line in Figure 4, generates the lowest schedule of the loss function.

Moreover, we find that the minimum value of the loss function $L^{RS}$ exists under the regime switching policy rule when $\alpha_{q,1} = 0.07$ (and all other parameter values remain the same in the benchmark case). In sum, accounting for expectation formation effect greatly strengthens the gains from reacting to asset price. Among those concerns reviewed in the Introduction, this result provides an alternative rationale for asset price stabilization.
5 Concluding Remarks

This paper sheds light on recent debates regarding whether the central bank should react to fluctuations in asset prices. As discussed in Leeper and Zha (2003) and Davig and Leeper (2007), expectations that possible regime shifts in monetary policy may occur in the future will affect equilibrium under the current regime. We analyze the effect of expectation formation on movements of asset prices, inflation, and output gap in a non-linear monetary policy rule to evaluate the gains or losses in reacting to asset prices.

We find that anticipation of possible change in policy regime has a potentially large effect on asset price movements, implying the importance of considering the effect of expectation formation in pricing assets. In contrast to the linear policy rule that generates negligible stabilization effect from responding to asset prices, the regime switching policy rule significantly shifts the “expected” inflation-output volatility frontier (i.e., the Taylor curve) inwards, suggesting that expectation formation effect reinforces the gains from reacting to asset price. Furthermore, the trade-off between expected volatility of inflation and that of output, as implied by the Taylor curve, substantially diminishes under the regime switching policy rule. This implies that the policy rule accounting for expectation formation effect and reacting to asset price expands the set of policy choices available for the monetary authority.

Given a measure of weighted loss function, we find that overall gains from reacting to asset price can be substantial when accounting for expectation formation effect, while reacting to asset price raises weighted losses under the linear policy rule. Finally, given our calibrated economy, an interior optimal monetary policy in reacting to asset price exists.

Our results, however, requires cautious interpretation in terms of policy implications. This paper intentionally maintains the model as parsimonious as possible, by abstracting from several features concerning monetary policy and asset price. For example, we do not consider the existence of asset price “bubbles.” The model also abstracts from financial
market frictions, and thus the transmission mechanism of asset prices in affecting economic aggregates is primitive. A more thorough assessment of the problem calls for a more elaborate model. Incorporating any of these features, in our view, can only provide a stronger rationale for reacting to asset price and therefore strengthen our results.

Nevertheless, our model serves as a preliminary step to evaluate whether monetary policy should respond to asset prices when the expectation formation effect is important. The bottom line is that the significance of expectation formation effect can strengthen net gains for reacting to asset price, and thus provide an alternative perspective for policymakers in how to respond to fluctuations in asset prices.
Appendix

A.1. Derivation of (10) and (11)

First, we assume that intermediate goods price setting is flexible and the superscript $f$ denotes a variable with flexible prices. Linearizing the first order condition of labor supply $\frac{\chi(L_t^f)^\zeta}{(C_t^f)^\sigma} = a_t$ and the intermediate goods production function $Y_t^f = a_tL_t^f$, we have $\zeta L_t^f + \sigma \hat{C}_t^f = a_t$ and $\hat{Y}_t^f = \hat{L}_t^f + a_t$. Using the goods market clearing condition $\hat{Y}_t^f = \hat{C}_t^f$, combining with the above two equations, we have the flexible of price equilibrium output $\hat{Y}_t^f = \frac{1+\zeta}{\sigma+\zeta}a_t$. Define $u_t^D$ as

$$u_t^D = E_t\hat{Y}_{t+1}^f - \hat{Y}_t^f = \frac{1+\zeta}{\sigma+\zeta}(\rho_a - 1)a_t.$$

The relationship between the aggregate productivity disturbance $a_t$ and $u_t^D$ can be rewritten as

$$a_t = \frac{u_t^D}{(1+\zeta)(\rho_a - 1)} \left( \frac{\sigma + \zeta}{(1+\zeta)(\rho_a - 1)} \right).$$

Turning to the staggered price model, linearizing (2) and using the goods market clearing condition $\hat{Y}_t = \hat{C}_t$, we have

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - E_t\hat{p}_{t+1}).$$

Expressing this in terms of the output gap $\tilde{x}_t \equiv \hat{Y}_t - \hat{Y}_t^f$, we have

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - E_t\hat{p}_{t+1}) + u_t^D,$$

which is (10) in the main text.

The real dividends from intermediate good firm are

$$d_t \equiv \frac{D_t}{P_t} = Y_t - w_tL_t = \left[ 1 - \frac{\chi Y_t^{\zeta+\sigma}}{a_t^{\zeta+1}} \frac{u_t^S}{\chi} \right] Y_t.$$

Obviously, the dividends are negatively associated with cost-push shocks. Log-linearizing the above equation and normalizing $Y = 1$, we obtain

$$\tilde{d}_t = \frac{(1-\chi) - (\zeta + \sigma)\chi}{(1-\chi)} \tilde{Y}_t + \frac{(1+\zeta)\chi a_t - \chi u_t^S}{k}.$$
Let \( \varepsilon = \frac{x}{\Gamma x} \) and use \( \hat{Y}_t = \hat{x}_t + \hat{Y}_t \), then \( \hat{d}_t \) can be written as:

\[
\hat{d}_t = (1 - (\zeta + \sigma)\varepsilon)\hat{x}_t + \left[(1 + \zeta)\varepsilon + (1 - (\zeta + \sigma)\varepsilon)\frac{1 + \zeta}{\sigma + \zeta}\right] a_t - \frac{x}{k} u_t^S
\]

where \( \omega_0 \equiv (1 - (\zeta + \sigma)\varepsilon) > 0, \omega_1 \equiv (1 + \zeta)\varepsilon + \omega_0 \left(\frac{1 + \zeta}{\sigma + \zeta}\right) > 0 \), which guarantees that dividends are positively correlated with output gap and the technology shock. Linearizing (2) and (3) and using (10), we obtain

\[
\hat{q}_t = \beta E_t \hat{q}_{t+1} - [\sigma - (1 - \beta)\omega_0] E_t \hat{x}_{t+1} + \sigma \hat{x}_t - \frac{x}{k} (1 - \beta) u_t^S
\]

\[
+ \left[\sigma + (1 - \beta)\omega_1 \rho_a \frac{\sigma + \zeta}{(1 + \zeta)(\rho_a - 1)}\right] u_t^D,
\]

\[
= \beta E_t \hat{q}_{t+1} - \omega_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \omega_3 u_t^D - \omega_4 u_t^S.
\]

where \( \omega_2 \equiv \sigma - (1 - \beta)\omega_0, \omega_3 \equiv \sigma + (1 - \beta)\omega_1 \rho_a \frac{\sigma + \zeta}{(1 + \zeta)(\rho_a - 1)} \) and \( \omega_4 \equiv \frac{x}{k} (1 - \beta) \). This is the equation (11) in the main text.

**A.2. The System of the Model**

Given the system (13)-(16), we rewrite the model in a more compact form by substituting (15) into (14) and re-arrange (13), (14) and (16) to characterize the economy by a three-variable system of inflation, output gap, and equity price, \((\hat{\pi}_t, \hat{x}_t, \hat{q}_t)\).

Define \( \hat{\pi}_{i,t} \equiv \hat{\pi}_t(s_t = i), \hat{x}_{i,t} \equiv \hat{x}_t(s_t = i), \) and \( \hat{q}_{i,t} \equiv \hat{q}_t(s_t = i), i = 1, 2, \) which denote the state-dependent inflation, output gap, and equity price. Let \( \Gamma_t \) be the information set up to time \( t \). The conditional expectation of \( \hat{\pi}_{t+1} \) is \( E_t [\hat{\pi}_{t+1} | \Gamma_t] \). Integrating over possible future regimes, we can write

\[
E_t \hat{\pi}_{t+1} = E \left[ \hat{\pi}_{t+1} | s_t = i, \Gamma_{t}^{-s} \right] = p_{i1} E \left[ \hat{\pi}_{1,t+1} | \Gamma_{t}^{-s} \right] + p_{i2} E \left[ \hat{\pi}_{2,t+1} | \Gamma_{t}^{-s} \right],
\]

where \( i = 1, 2, \) and \( \Gamma_{t}^{-s} \) is the information set up to time \( t \), excluding the current regime, i.e., \( \Gamma_t = \Gamma_{t}^{-s} \cup \{s_t\} \). The conditional expectations of \( \hat{x}_{t+1} \), and \( \hat{q}_{t+1} \) can be calculated in the same way.

Denoting \( E_t \hat{\pi}_{i,t+1} = E \left[ \hat{\pi}_{i,t+1} | \Gamma_{t}^{-s} \right], i = 1, 2, \) and similarly for \( E_t \hat{x}_{i,t+1} \) and \( E_t \hat{q}_{i,t+1}, \)
define the forecasting errors:
\[ \eta_{t,t+1}^\pi \equiv \hat{\pi}_{t,t+1} - E_t \hat{\pi}_{t,t+1}, \]
\[ \eta_{t,t+1}^\tau \equiv \hat{\tau}_{t,t+1} - E_t \hat{\tau}_{t,t+1}, \]
\[ \eta_{t,t+1}^q \equiv \hat{q}_{t,t+1} - E_t \hat{q}_{t,t+1}, \]
\[ i = 1, 2. \]

The system of the model can be expressed by:
\[ BY_t = AE_t Y_{t+1} + C u_t \]
\[ = AY_{t+1} - A \eta_{t+1} + C u_t, \]
where \( \eta_{t+1} = Y_{t+1} - E_t Y_{t+1} \). Rearranging the equation, we have
\[ AY_{t+1} = BY_t + A \eta_{t+1} - C u_t, \]
\[ u_t = \rho u_{t-1} + \varepsilon_t \]
where
\[ Y_t = \begin{bmatrix} \hat{\pi}_{1,t} \\ \hat{\pi}_{2,t} \\ \hat{x}_{1,t} \\ \hat{x}_{2,t} \\ \hat{q}_{1,t} \\ \hat{q}_{2,t} \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \eta_{1,t}^\pi \\ \eta_{2,t}^\pi \\ \eta_{1,t}^\tau \\ \eta_{2,t}^\tau \\ \eta_{1,t}^q \\ \eta_{2,t}^q \end{bmatrix}, \quad u_t = \begin{bmatrix} u_t^S \\ u_t^D \\ u_t^M \end{bmatrix}, \]
\[ A = \begin{bmatrix} \beta \otimes \mathbf{P} & \mathbf{0}_{2	imes 2} & \mathbf{0}_{2	imes 2} \\ \sigma^{-1} \otimes \mathbf{P} & \mathbf{P} & \mathbf{0}_{2	imes 2} \\ \mathbf{0}_{2	imes 2} & -\varpi_2 \otimes \mathbf{P} & \beta \otimes \mathbf{P} \end{bmatrix}, \quad \rho = \begin{bmatrix} \rho_S & 0 & 0 \\ 0 & \rho_D & 0 \\ 0 & 0 & \rho_M \end{bmatrix}, \]
\[ B = \begin{bmatrix} \mathbf{I}_{2	imes 2} & -\kappa \mathbf{I}_{2	imes 2} & \mathbf{0}_{2	imes 2} \\ \sigma^{-1} \alpha_{\pi,1} & 0 & 1 + \sigma^{-1} \alpha_{x,1} \\ 0 & \sigma^{-1} \alpha_{\pi,2} & 0 - \sigma^{-1} \alpha_{x,2} \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -\sigma^{-1} \\ 0 & 1 & -\sigma^{-1} \\ -\varpi_4 & \varpi_3 & 0 \\ -\varpi_4 & \varpi_3 & 0 \end{bmatrix}. \]
A.3. Undetermined Coefficient Method

Following McCallum (2004a), the solution of the Undetermined Coefficient method in our context is given by

\[ b_{\tau} = \alpha \sigma_{\tau} + \beta \Delta_{\tau} + \gamma \Xi_{\tau}, \]

where \( b_{\tau}, b_{\tau}^D, b_{\tau}^M \) are functions of the smallest set of state variables. Rewriting the system as the following form:

\[ \mathbf{Y}_t = \text{Sol}^{un} u_t, \text{where Sol}^{un} = \begin{bmatrix} a^1 & a^D & a^M \\ b^1 & b^D & b^M \\ c^1 & c^D & c^M \end{bmatrix}. \]

Plugging \( \text{Sol}^{un} u_t \) into \( \mathbf{Y}_t \) in (A1), we have

\[ B \text{Sol}^{un} u_t = A \text{Sol}^{un} \mathbf{p} u_t + C u_t. \]

Rewriting the equation as \( \text{Sol}^{un} = B^{-1} A \text{Sol}^{un} \mathbf{p} + B^{-1} C \) we thus have

\[ \text{vec} (\text{Sol}^{un}) = (\mathbf{I} - \mathbf{p} \otimes B^{-1} A)^{-1} \text{vec} (B^{-1} C). \]

Redefining \( Z^A_t = [\hat{\pi}_t, \hat{x}_t, \hat{\eta}_t, u_t^S, u_t^D, u_t^M]^T, s_t = 1, 2, \) the solution of the system can be expressed by:

\[ Z^A_t = G_{1,s_t} Z^A_{t-1} + G_{2,s_t} \epsilon_t, \]

where \( \epsilon_t = [\epsilon^S_t, \epsilon^D_t, \epsilon^M_t]^T. \) The solution implies that the covariance matrix of \( Z^A_t \), measured by \( \Omega^A_{s_t} = E [Z^A_t Z^A_t^T] \), which is a 6 \times 6 matrix, can be expressed as the stacked column vector given as follows,

\[ \text{vec}[\Omega_{s_t}] = (\mathbf{I} - G_{1,s_t} \otimes G_{1,s_t})^{-1} \text{vec} [G_{2,s_t} G_{2,s_t}^T]. \]
Similarly, the covariance matrix of $Z_t^A$ at fixed regime $j$, given by $\Omega_j$, can be expressed as

$$vec [\Omega_j] = (I - G_{1,j} \otimes G_{1,j})^{-1} vec(G_{2,j}G_{2,j}^\prime).$$

### A.4. Fixed Regime Equilibrium

**Case 1: Without Responding to Asset Price**

Let $p_{jj} = 1, j \in \{1, 2\}$, we obtain the fixed regime solution using the undetermined coefficient method

$$\hat{\pi}_{j,t} = \frac{\alpha_{x,j} + \sigma(1 - \rho_S)}{\Delta_{j,S}} u^S_t + \frac{\kappa \sigma}{\Delta_{j,D}} u^D_t + \frac{-\kappa}{\Delta_{j,M}} u^M_t,$$

$$\hat{x}_{j,t} = \frac{(\rho_S - \alpha_{x,j})}{\Delta_{j,S}} u^S_t + \frac{(1 - \beta \rho_D) \sigma}{\Delta_{j,D}} u^D_t + \frac{(\beta \rho_M - 1)}{\Delta_{j,M}} u^M_t,$$

$$\hat{q}_{j,t} = \frac{\psi_j}{\Delta_{j,S}} u^S_t + \frac{\phi_j}{\Delta_{j,D}} u^D_t + \frac{(\omega_2 \rho_M - \sigma)}{\Delta_{j,M}} u^M_t,$$

where $\Delta_{j,c} \equiv \kappa(\alpha_{x,j} - \rho_c) + (\alpha_{x,j} + \sigma(1 - \rho_c)) (1 - \beta \rho_c) > 0$, $\psi_j \equiv [\rho_S \omega_2 - \sigma - \omega_4 \kappa (\alpha_{x,j} - \rho_S)] - \omega_4 [\alpha_{x,j} + \sigma(1 - \rho_S)] < 0$ and $\phi_j \equiv \omega_3 \kappa (1 - \beta \rho_D) + (1 - \beta \rho_D) \sigma + \alpha_{x,j} + (\sigma - \beta \rho_D \omega_2) \sigma > 0$, $(\omega_2 \rho_M - \sigma) < 0$, $c = S, D, M$ and $j = 1, 2$. To understand the solution, we first consider the responses of inflation, output gap, and asset price when the central bank reacts more rigorously to inflation (a higher $\alpha_{x,j}$) or output gap (a higher $\alpha_{x,j}$). We have checked that $\rho_S - \alpha_{x,j} < 0$ (if $\alpha_{x,j} > 1$), $1 - \beta \rho_D > 0$, $\beta \rho_M - 1 < 0$, and $\partial \Delta_{j,k} / \partial \alpha_{x,j} > 0$, for all $k$.

To examine the signs of those coefficients in the system, consider $z_t = A(\gamma)u_t$, where $A$ is a function of structural parameters $\gamma$ and $u_t$ following an AR(1) process. Since $\sigma_z^2 = \frac{A^2}{1 - \rho_u^2} \sigma_u^2$, $\partial A^2 / \partial \gamma > 0$ implies that the volatility of $z_t$ resulting from $u_t$ will rise. Thus, we have

$$\frac{\partial (\psi_j / \Delta_{j,S})^2}{\partial \alpha_{x,j}} = \frac{2(\sigma - \omega_2 \rho_S) [\alpha_{x,j} + (1 - \rho_S) \sigma]}{\Delta_{j,S}^3 (1 - \rho_S \beta)} > 0,$$

$$\frac{\partial (\phi_j / \Delta_{j,D})^2}{\partial \alpha_{x,j}} = \frac{2 \kappa \sigma (\omega_2 \rho_D - \sigma) \Xi_2}{\Delta_{j,D}^3 (1 - \rho_D \beta)} < 0,$$

$$\frac{\partial (\psi_j / \Delta_{j,S})^2}{\partial \alpha_{x,j}} = \frac{2 (\omega_2 \rho_S - \sigma) (\alpha_{x,j} - \rho_S) \Xi_1}{\Delta_{j,S}^3 (1 - \rho_S \beta)} < 0,$$

$$\frac{\partial (\phi_j / \Delta_{j,D})^2}{\partial \alpha_{x,j}} = \frac{2 \omega_2 \rho_D - \sigma) \Xi_2}{\Delta_{j,D}^3 (1 - \rho_D \beta)} < 0.$$
where \( \Xi_1 = (\alpha_{x,j} - \rho_S)(\sigma - \omega_2 \rho_S + \omega_4 \kappa) + \omega_4 (1 - \rho_S) \beta (\alpha_{x,j} + \sigma (1 - \rho_S)) > 0 \) and \( \Xi_2 = (\alpha_{x,j} - \rho_D) \kappa \omega_3 + (1 - \rho_D) \beta [\omega_3 (\alpha_{x,j} + (1 - \rho_D) \sigma) + \sigma (\sigma - \omega_2 \rho_D)] > 0 \). Next, we have

\[
\frac{\partial ((\alpha_{x,j} + \sigma (1 - \rho_S))/\Delta_{j,S})^2}{\partial \alpha_{x,j}} = \frac{2 \kappa (\alpha_{x,j} - \rho_S) (\alpha_{x,j} + (1 - \rho_S) \sigma)}{\Delta_{j,S}^3} > 0
\]

\[
\frac{\partial ((\rho_S - \alpha_{x,j})/\Delta_{j,S})^2}{\partial \alpha_{x,j}} = \frac{2 (\alpha_{x,j} - \rho_S) (\alpha_{x,j} + (1 - \rho_S) \sigma) (1 - \rho_S \beta)}{\Delta_{j,S}^3} > 0.
\]

finally, it is straightforward to check that \( \partial [(\omega_2 \rho_M - \sigma)/\Delta_{j,M}]^2 / \partial \alpha_{x,j} < 0 \) and \( \partial [(\omega_2 \rho_M - \sigma)/\Delta_{j,M}]^2 / \partial \alpha_{x,j} < 0 \). We summarize the results in Table A1.

As the table shows, when \( \alpha_{x,j} \) is higher, a trade-off exists in supply shocks: a more active policy in reacting to inflation reduces the elasticities of inflation, but it raises the responsiveness of output and stock price. An increase in \( \alpha_{x,j} \) reduces the responsiveness of all variables in response to both demand shock and monetary shock. A more active policy in reacting to output gap (a higher \( \alpha_{x,j} \)) reduces the responsiveness of output and stock price, but it raises the elasticities of inflation, which mirrors the effect of an increase in \( \alpha_{x,j} \). However, an increase in \( \alpha_{x,j} \) reduces the responsiveness of all variables in response to both demand shock and monetary shock.

Table A1 The Effects of \( \alpha_{x,j} \) and \( \alpha_{x,j} \) on Inflation, Output Gap, and Stock Price Given Various Sources of Disturbances

<table>
<thead>
<tr>
<th>Source of Disturbance</th>
<th>( \alpha_{x,j} )</th>
<th>( u_t^S ), ( u_t^D ), ( u_t^M )</th>
<th>Source of Disturbance</th>
<th>( \alpha_{x,j} )</th>
<th>( u_t^S ), ( u_t^D ), ( u_t^M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{j,t} )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \pi_{j,t} )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{q}_{j,t} )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Case 2: Responding to Asset Price

When the central bank responds to asset price, we can obtain the fixed regime solution
with \( p_{ij} = 1, j \in \{1, 2\} \):

\[
\hat{\pi}_{j,t}^A = \frac{\alpha_{x,j} + (1 - \rho_S)\sigma + \left(\frac{\omega_4 + \sigma - \rho_S}{\beta - \rho_S}\omega_2\right)\alpha_{q,j}}{\Delta_{2,S}} u_t^S + \frac{\kappa [\sigma - \left(\frac{\omega_3 \alpha_{q,j}}{\beta - \rho_D}\right)]}{\Delta_{2,D}} u_t^D + \frac{-\kappa}{\Delta_{2,M}} u_t^M,
\]

\[
\hat{x}_{j,t}^A = \frac{\omega_4 \alpha_{q,j} + \rho_S - \alpha_{x,j}}{\Delta_{2,S}} u_t^S + \frac{(1 - \beta \rho_M)\sigma - \omega_3 \alpha_{q,j}}{\Delta_{2,D}} u_t^D + \frac{(\beta \rho_M - 1)}{\Delta_{2,M}} u_t^M,
\]

\[
\hat{q}_{j,t}^A = \frac{\psi_j}{\Delta_{2,S}} u_t^S + \frac{\phi_j}{\Delta_{2,D}} u_t^D + \frac{(\omega_2 \rho_M - \sigma)}{\Delta_{2,M}} u_t^M,
\]

where \( \Delta_{2,c} = \kappa (\alpha_{x,j} - \rho_c) + (\alpha_{x,j} + \sigma (1 - \rho_c))(1 - \beta \rho_c) + \alpha_{q,j} (\sigma - \omega_2 \rho_c), c = S, D, M \). The responses of endogenous variables to various disturbances can be analyzed as in Case 1.

**References**


Table 1 Hausman Test for Endogeneity

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>$\delta_1$</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t$</td>
<td>Inflation Rate</td>
<td>1.7</td>
<td>8.7</td>
</tr>
<tr>
<td>$x_t$</td>
<td>Detrended GDP</td>
<td>0.006</td>
<td>0.05</td>
</tr>
<tr>
<td>$q_t$</td>
<td>Detrended stock price</td>
<td>0.05</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2 Estimated Parameter Values of Monetary Policy Rule
When the Fed React to Asset Price ($\alpha_q > 0$)

<table>
<thead>
<tr>
<th></th>
<th>Linear model</th>
<th>Markov Switching Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hawkish regime</td>
<td>Dovish regime</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>1.9***</td>
<td>2.6***</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.06</td>
</tr>
<tr>
<td>$\alpha_q$</td>
<td>0.07*</td>
<td>0.07**</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.0266</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\psi_x$</td>
<td>0.2</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma^M$</td>
<td>0.6</td>
<td>0.11***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.22</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.6</td>
<td>0.97***</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.02</td>
<td>0.96***</td>
</tr>
<tr>
<td>LogLik</td>
<td>-578.90</td>
<td>-543.52</td>
</tr>
</tbody>
</table>

Note: $2(L_u - L_c) = 70$ is larger than 99% critical value 14.2, *** is significant at 1%, ** is significant at 5% and * is significant at 10%.

Table 3 Model Parameter Values

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Other Preference Parameters</td>
<td>$\beta = 0.99$</td>
<td>$\sigma = 1$</td>
<td>$\eta = 0.1$</td>
<td>$\chi = 0.2$</td>
<td></td>
<td>$\tau = 2/3$</td>
<td></td>
<td>$\rho_S = 0.85$</td>
<td>$\sigma_S = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\rho_D = 0.85$</td>
<td>$\sigma_D = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\rho_M = 0.6$</td>
<td>$\sigma_M = 1$</td>
</tr>
</tbody>
</table>
Table 4 The Effect of Reacting to Stock Price Under Linear Model

<table>
<thead>
<tr>
<th>$\pi_t$</th>
<th>$x_t$</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0029</td>
<td>-0.0523</td>
<td>-0.0440</td>
</tr>
</tbody>
</table>

Note: The entries are given by the benchmark estimated parameter values, which are $\alpha_x = 0.07$, $\alpha_\pi = 1.9$ and $\alpha_q = 0.03$.

Table 5 The Effect of Reacting to Asset Price Taking Into Account of the Expectation Format Effect

<table>
<thead>
<tr>
<th>Regime</th>
<th>Hawkish</th>
<th>Dovish</th>
<th>Hawkish</th>
<th>Dovish</th>
<th>Hawkish</th>
<th>Dovish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Effect</td>
<td>0.1625</td>
<td>-0.3705</td>
<td>-0.5176</td>
<td>-0.4994</td>
<td>0.0123</td>
<td>-0.3067</td>
</tr>
<tr>
<td>Direct Effect</td>
<td>-0.1352</td>
<td>-0.0968</td>
<td>-0.2032</td>
<td>-0.5382</td>
<td>-0.0685</td>
<td>-0.1920</td>
</tr>
<tr>
<td>Expectation Format Effect</td>
<td>0.2977</td>
<td>-0.2737</td>
<td>-0.3144</td>
<td>0.0388</td>
<td>0.0808</td>
<td>-0.1147</td>
</tr>
</tbody>
</table>

Note: The entries are given by the estimated parameter values in Table 2, which are $p_{11} = 0.97$, $p_{22} = 0.96$, $\alpha_{\pi,1} = 2.6$, $\alpha_{x,1} = 0.07$, $\alpha_{q,1} = 0.02$, $\alpha_{x,2} = 1.5$, $\alpha_{x,2} = 0.01$ and $\alpha_{q,2} = 0.03$. It is measured by the difference between the two sets of solutions, one allowing for regime shift and the other under fixed regime, as the volatility of a variable deviating proportionally from a given regime $j$. A positive(negative) entry means that the expectation formation effect raises (lowers) the volatility of a variable, given a fixed policy regime.
Figure 1  The Federal Fund Rate and Smoothed Probability of Hawkish Regime

Note: The solid line represents the federal funds rate, the dashed line represents the smoothed probability of the Hawkish regime, and the shaded areas are periods of recession identified by NBER.

Figure 2  The Effect of Expectation Formation on Asset Price

Note: Each observation on a distribution comes from 1000 random drawings, measuring the average quadratic difference between the asset price generated under the model allowing for regime shift and that under the model given a fixed regime $j$, i.e., the difference in these two asset prices caused by the effect of expectation formation under each regime. We then repeat the procedure for 3000 times to obtain 3000 observations for each given regime $j$. 

Figure 3  The Taylor Curve:  the Frontier of Inflation-Output Volatility

Note: The horizontal (vertical) axis is the volatility of output gap (inflation) under the linear rule and the expected volatility of output gap (inflation) under the regime-switching policy rule, relative to the respective monetary rule without reacting to asset price.

Figure 4  The Loss Functions

Note: The horizontal axis is the coefficient of reacting to asset price under the linear policy rule ($\alpha_q$) and that under the regime-switching policy rule ($\alpha_{q,1}$ or $\alpha_{q,2}$). The vertical axis is the loss function under the linear policy rule ($L$) and that under the regime-switching policy rule ($L^{RS}$) varying $\alpha_{q,1}$ or $\alpha_{q,2}$, respectively. The relative weight on the output gap stabilization, $\omega$, is taken to be 0.2.