Money, Barter, and Hyperinflation

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Outline

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Motivation
Economist vs. Anthropologist

- On explaining the transition from using money to bartering, economists commonly concentrate on the large money supply and its extreme consequence of losing purchasing power of money, i.e., hyperinflation.

- For example, various models developed by Engineer and Bernhardt (1991), Ritter (1995), and Banerjee and Maskin (1996) have an implication that the economy with large money supply or high inflation rate could drive out the monetary equilibrium.

- However, Cellarius (2000), an anthropologist, finds that the barter activities actually take place before and after the period of hyperinflation in Bulgaria in the 1990s.
Evidence from Russia

- Similarly, in an empirical study of barter and inflation in Russia during 1994 and 2000, Kim and Pirttilä (2004) find that no positive relation exists between changes in inflation and bartering transactions.

- The relation is even strongly negative in Russia from 1995 to 1997. During that period, the inflation rate was declining to zero but the bartering transactions were rapidly increasing and approached half of total industrial transactions.

- Therefore, if hyperinflation is indeed a crucial factor in the transition from using money to bartering, then other complementary explanations are needed to identify such contrary evidences.
We have an unifiable explanation

- In this paper, we apply a search model to describe the transition from using money to bartering.

- In particular, we assume that individuals' utility are derived from consuming two different goods and introduce a parameter called the degree of consumption interdependence.

- With different degrees of consumption interdependence, we show that an economy with higher consumption interdependence is always in a barter equilibrium during hyperinflation.

- However, the model contains a possibility that people may switch from using money to bartering even though the inflation rate is low.
The Model
Agents with consumption interdependence

- There is a continuum of agents living eternally with a unit mass. A proportion $\mu$ of agents are type 1 and others are type 2.
- Time is discrete. In each period, for $i = 1, 2$, a type $i$ agent produces one unit of good $x_i$ with zero cost.
- The utility function of type $i$ agent is:

$$u_i = u\left(x_i^\theta \cdot x_j^{1-\theta}\right), \quad (1)$$

where $i, j \in \{1, 2\}$, $i \neq j$, and $0 < \theta < 1$. We further assume that $u''(\cdot) < 0 < u'(\cdot)$ and $u(0) = 0$.
- Agents with a higher $(1 - \theta)$ would have a greater marginal utility of consuming others’ goods. In this sense, we call $(1 - \theta)$ the degree of consumption interdependence.
The bargaining game

- Agents randomly meet and trade bilaterally.
- Without money, they barter with each other by the alternating-offers bargaining process.
- Once the agreement is not reached immediately, the object is decaying with a losing rate of $\delta$ at the end of each proposition.
- We call $\delta$ the decaying rate and it satisfies $0 < \delta < 1$.
- Following Rubinstein (1982), the bargaining game has a unique subgame perfect Nash equilibrium in which the agreement would be immediately reached.
- In this equilibrium, the share for seller is $1/(1 + \delta)$ and the remainder for buyer is $\delta/(1 + \delta)$. 
Fiat money

- We introduce the government into the model. The function of the government is to issue fiat money uniformly to a fraction $M \in (0,1)$ of all agents.

- A money holder only has one unit of money when she or he obtains it. Money is intrinsically valueless but can buy goods if sellers accept it.

- Money holders can also produce their specialized goods and their final consumption will contain one unit of their production in the monetary exchanges.

- Once money is used in a transaction, the money holder will become a producer without money and the counterparty will become a money holder in the next period.
In the steady state

- Let $q > 0$ be the purchasing power of money, i.e., the inverse of goods’ price, and $\Pi$ be the common belief of the probability that other agents would accept money.

- In the steady state, we have Bellman equations as follows:

$$\begin{align*}
    rV_1 &= (1 - \mu)(1 - M)u\left(\delta^{1-\theta}(1 + \delta)^{-1}\right) + (1 - \mu)M \max_{\pi_1} \{V_{1M} - V_1, 0\}, \\
    rV_{1M} &= (1 - \mu)(1 - M)\Pi\left(u\left(q^{1-\theta}\right) + V_1 - V_{1M}\right), \\
    rV_2 &= \mu(1 - M)u\left(\delta^{1-\theta}(1 + \delta)^{-1}\right) + \mu M \max_{\pi_2} \{V_{2M} - V_2, 0\}, \\
    rV_{2M} &= \mu(1 - M)\Pi\left(u\left(q^{1-\theta}\right) + V_2 - V_{2M}\right),
\end{align*}$$

where $r \in (0,1)$ is the discount factor and $\pi_i$ are agents’ strategies for whether to accept money or not.
The monetary equilibrium

- For a type 1 agent, she or he would always accept money if $V_{1M} > V_1$, and not accept it if $V_{1M} < V_1$.

- Similarly, for type 2 agents, they would also use such decision rule depending on $V_{2M}$ and $V_2$.

- Applying the notion of Nash equilibrium, the monetary equilibrium exists if and only if

$$\Pi > \frac{u(\delta^{1-\theta}(1 + \delta)^{-1})}{u(q^{1-\theta})}.$$  \hspace{1cm} (6)

- Inequality (6) says that if agents’ belief $\Pi$ is large enough, the monetary equilibrium would be sustained.
Proof for the equilibrium condition (→)

For a type $i$ agent, she or he would always accept money if $V_{iM} > V_i$, and not accept it if $V_{iM} < V_i$. In the special case of $V_{iM} = V_i$, the agent would use any of $\pi_i \in [0,1]$.

Therefore, if $V_{iM} > V_i$ for $i = 1,2$, then we would have the monetary equilibrium that all agents accept money for sure.

We first show that this monetary equilibrium implies (6).

Suppose that $V_{iM} > V_i$ for $i = 1,2$.

For simplicity, we define $\delta^{1-\theta}(1+\delta)^{-1} \equiv a$ and $q^{1-\theta} \equiv b$. 
Subtracting (2) from (3) yields

\[ r(V_{1M} - V_1) = (1 - \mu)(1 - M)[\Pi u(b) - u(a)] \\
- [(1 - \mu)(1 - M)\Pi + (1 - \mu)M](V_{1M} - V_1), \]

which implies

\[ [r + (1 - \mu)(1 - M)\Pi + (1 - \mu)M](V_{1M} - V_1) = (1 - \mu)(1 - M)[\Pi u(b) - u(a)]. \]

Since \( r, \mu, M \in (0,1), \) \((V_{1M} - V_1) > 0,\) and \( \Pi \in [0,1],\) we must have \( \Pi u(b) - u(a) > 0,\) which implies

\[ \Pi > \frac{u(\delta^{1-\theta}(1 + \delta)^{-1})}{u(q^{1-\theta})}. \]

Similarly, by subtracting (4) from (5), we also obtain monetary equilibrium \( \rightarrow (6) \) in the case of \( i = 2. \)
Proof for the equilibrium condition (←)

We now show that (6) implies the monetary equilibrium, i.e.,

\[ V_{iM} > V_i \text{ for } i = 1, 2. \]

Suppose this is not true, and (6) implies \( V_{iM} \leq V_i \) for \( i = 1, 2 \).

Subtracting (2) form (3) yields

\[
\begin{align*}
\rho (V_{1M} - V_1) &= (1 - \mu)(1 - M)[\Pi u(b) - u(a)] - [(1 - \mu)(1 - M)\Pi](V_{1M} - V_1),
\end{align*}
\]

which implies

\[
[\rho + (1 - \mu)(1 - M)\Pi](V_{1M} - V_1) = (1 - \mu)(1 - M)[\Pi u(b) - u(a)].
\]

Since \( \rho, \mu, M \in (0, 1), (V_{1M} - V_1) \leq 0 \), and \( \Pi \in [0, 1] \), it follows that \( \Pi u(b) - u(a) \leq 0 \), which contradicts (6).
Similarly, subtracting (4) from (5) also yields a contradictory result of \( \Pi u(b) - u(a) \leq 0 \) for the case of \( i = 2 \).

Therefore, (6) must imply \( V_{iM} > V_i \) for \( i = 1,2 \), i.e., the monetary equilibrium \( \leftarrow (6) \).

Hence, we conclude that the monetary equilibrium exists if and only if the common belief \( \Pi \) satisfying (6).
Rational expectation hypothesis

- By the rational expectation hypothesis, agents’ strategies must be consistent with the common belief in the monetary equilibrium. That is, $\pi_i = \Pi = 1$ for all $i$.

- However, if $u(\delta^{1-\theta}(1+\delta)^{-1}) / u(q^{1-\theta}) \geq 1$, inequality (6) would not be satisfied.

- This imposes crucial constraint for the existence of monetary equilibrium that

\[ u(q^{1-\theta}) > u(\delta^{1-\theta}(1+\delta)^{-1}). \]  

(7)

- If constraint (7) is not satisfied, inequality (6) would not be valid even though $\pi_i = \Pi = 1$, and then the economy must be in a pure barter equilibrium.
Discussion
The threshold value for monetary equilibria

- According to equation (1), we interpret the value of \( (1 - \theta) \) as the degree of consumption interdependence.
- Since \( u''(\cdot) < 0 < u'(\cdot) \), constraint (7) could be reduced to
  \[
  q > \delta (1 + \delta)^{-1/(1-\theta)}.
  \] (8)
- Let the right hand side of (8) be the threshold value of the existence of monetary equilibria, denoted by \( q^* \). That is, \( q^*(\theta) = \delta (1 + \delta)^{-1/(1-\theta)} \).
- In general, economies with any \( (1 - \theta) \) could always have monetary equilibria if the purchasing power of money \( q \) is greater than \( q^*(\theta) \), i.e., \( q > q^*(\theta) \). We illustrate \( q^*(\theta) \) in the following figure.
Barter and monetary equilibria

Examples with $\delta = 0.2$

Examples with $\delta = 0.4$

Examples with $\delta = 0.6$

Examples with $\delta = 0.8$
Implications of the result

- An economy with an extremely low degree of consumption interdependence is always in a monetary equilibrium, because people only need few others’ goods and they always accept money in order to trade some basic goods for living.

- Given medium cases of \((1 - \theta)\), an economy with an extremely low value of \(q\) is always in a barter equilibrium. That is, people are better-off with direct bartering during hyperinflation.

- Finally, the model contains a possibility that people may switch from using money to bartering even though the purchasing power of money is unchanged.

- As a transitional economy, the consumption interdependence of Russia was rapidly increasing in the 1990s, so the bartering activities were more popular when the inflation rate is low.
Extension
**Bargaining power**

- We now assume agents could choose the decaying rate $\delta$ with the range of $\delta \in [\delta_L, \delta_H]$ in the production stage, where $0 < \delta_L < \delta_H < 1$.

- Following Rubinstein (1982), the game also has a unique subgame perfect Nash equilibrium in which the agreement is immediately reached.

- In the equilibrium, with a realization of $\delta$, the share of seller is $1/(1 + \delta)$ and the remainder for buyer is $\delta/(1 + \delta)$.

- Therefore, a lower value of $\delta$ represents a larger bargaining power for agents in selling their goods.
Corner solution

- Backward to the production stage, the decision problem for agents is that what \( \delta \) should be taken.

- Let \( F_i(\delta(j)) \), a cumulative distribution of type \( j \) agents’ decaying rates, be the belief of a type \( i \) agent, where \( i, j \in \{1, 2\} \) and \( i \neq j \).

- For a type \( i \) agent with a decaying rate of \( \delta(i) \), if she or he randomly meets a type \( j \) agent, the expected utility from a barter exchange is

\[
E_i[u(\delta(i))] = \int u\left( (1 + \delta(i))^{-\theta} \delta(j)^{1-\theta} (1 + \delta(j))^{\theta-1} \right) dF_i(\delta(j)).
\]
The first derivative of the expected utility is

$$\frac{dE_i[u(\delta(i))]}{d\delta(i)} = -\theta (1 + \delta(i))^{-(1+\theta)} \times$$

$$\int \delta(j)^{1-\theta} (1 + \delta(j))^{\theta-1} u'(1 + \delta(i))^{-\theta} \delta(j)^{1-\theta} (1 + \delta(j))^{\theta-1} dF_i(\delta(j)) ,$$

which is negative since $u'(\cdot) > 0$.

This shows that the optimal choice of $\delta(i)$ for the type $i$ agents is a corner solution, i.e., $\delta(i) = \delta_L$.

Hence, they would always produce goods with the lowest decaying rate $\delta_L$ and then obtain the largest bargaining power.

Symmetrically, for all type $j$ agents, they face the same decision problem and would also choose $\delta(j) = \delta_L$. 
Barter equilibrium

- Therefore, all agents produce goods with the lowest decaying rate and the expected utility in any exchange is reduced to $u(\delta_L^{1-\theta}(1 + \delta_L)^{-1})$ for sure.

- This barter equilibrium implies that people always barter too much to arrive at the social efficient allocations.

- In an extreme case, we have $\lim_{\delta_L \to 0} u(\delta_L^{1-\theta}(1 + \delta_L)^{-1}) = 0$ for any $(1 - \theta)$.

- Clearly, the welfare can be improved if all agents take $\delta_H$ simultaneously.

- However, no one has incentive to deviate such Nash equilibrium since they all play the dominant strategy of choosing $\delta_L$. 
Gains from using money

- Note that the monetary equilibrium constraint of 
  \[ q > \delta_L (1 + \delta_L)^{-1/(1-\theta)} \] could easily hold if \( \delta_L \to 0 \).

- As long as the monetary equilibrium is sustained, the 
purchasing power of money is strictly greater than that from 
barter exchanges.

- Since \( q \) is independent of \( \delta_L \), a seller has no bargaining power 
when the transaction is taken by money.

- As a result, the expected utility for money holders will be 
  \( u(q^{1-\theta}) \) for sure in a monetary exchange.

- Since \( u(q^{1-\theta}) > u(\delta_L^{1-\theta} (1 + \delta_L)^{-1}) \) and each agent has a 
positive probability to be a money holder, using money 
therefore would improve the social welfare.
Conclusion
In this paper, we apply a search model to describe the transition from using money to bartering.

The result shows that an economy with higher consumption interdependence is always in a barter equilibrium during hyperinflation.

In contrast, an economy with an extremely low degree of consumption interdependence is always in a monetary equilibrium.

Moreover, the model also contains a possibility that agents may switch from using money to bartering even though the inflation rate is low.

Introducing fiat money into the economy can improve the welfare from reducing sellers’ bargaining power.
Thank you.