A Simple Analytic Approximation Approach for Estimating the True Random Effects and True Fixed Effects Stochastic Frontier Models

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1. Introduction

Introduction

- We consider the estimation problems of the true random effects stochastic frontier analysis (TRESFA) and the closed related true fixed effects SFA (TFESFA) models proposed by Greene (2005).

- Both TRESFA model and TFESFA model are useful workhorses for efficiency estimation provided that the associated estimation is not costly.

- However, the full likelihood function of the TRESFA model does not admit a close-form presentation, and the maximum likelihood estimation (MLE) of the TFESFA model might be subject to the presence of incidental parameters when the number of cross units, *N*, are huge.

2. A True Random Effects Stochastic Frontier Model

A True Random Effects Stochastic Frontier Model

- Consider the following true random effects stochastic frontier model:

$$y_{it}=\alpha+x_{it}^{\mathrm{T}}\beta+\omega_i+v_{it}-Su_{it},\ i=1,\ 2,\ ...,N \text{ and } t=1,\ ...,\ T, \quad (1)$$

where y_{it} is the performance of firm i in period t, x_{it} is the vector of inputs or input prices, w_i is the random firm specific effect, and S=1 or -1, depending on the context under investigation.

- v_{it} and u_{it} are the symmetric and one sided components to the stochastic frontier model proposed by ALS (1977):

$$v_{it}\sim N[0,\sigma_v^2],\ u_{it}=|U_{it}| \text{ where } U_{it}\sim N[0,\ \sigma_u^2]\perp v_{it}. \quad (2)$$

- Define $\varepsilon_{it}=v_{it}-Su_{it}$ and rewrite the model in (1) as:

$$y_{it}=\alpha+x_{it}^{\mathrm{T}}\beta+\omega_i+\varepsilon_{it}, \quad (3)$$

A True Random Effects Stochastic Frontier Model

- We first note that

$$f(y_{it}|w_i) = \frac{2}{\sigma}\phi(\frac{\varepsilon_{it}}{\sigma})\Phi(\frac{-S\lambda\varepsilon_{it}}{\sigma}),\ \varepsilon_{it} = y_{it} - \alpha - x_{it}^{T}\beta - \omega_i = Z_{it} - \omega_i, \qquad (4)$$

where $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\varphi(.)$ and $\Phi(.)$ are the probability density function (pdf) and the cumulative distribution function (cdf) of a standard normal distribution, respectively, and $\lambda = \sigma_u / \sigma_v$.

- Conditional on w_i, the T observations for firm i are independent with each other, thus, the joint density of the T observations is:

$$f(y_{i1}, \ldots, y_{iT}|w_i) = \prod_{t=1}^{T} \frac{2}{\sigma}\phi(\frac{\varepsilon_{it}}{\sigma})\Phi(\frac{-S\lambda\varepsilon_{it}}{\sigma}). \qquad (5)$$

A True Random Effects Stochastic Frontier Model

- It follows that the unconditional joint density is:

$$L^i = \int_{\omega_i} \prod_{t=1}^{T} \frac{2}{\sigma} \phi\left(\frac{\varepsilon_{it}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right) g(\omega_i) d\omega_i, \quad (6)$$

where $g(.)$ denotes some probability density function.

- Assume that w_i is generated as a normal distribution with a variance σ_w^2, we derive from (6) that

$$L^i = \int_{-\infty}^{\infty} \prod_{t=1}^{T} \frac{2}{\sigma} \phi\left(\frac{\varepsilon_{it}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right) \frac{1}{\sigma_\omega} \phi\left(\frac{\omega_i}{\sigma_\omega}\right) d\omega_i. \quad (7)$$

- The likelihood function (7) can be evaluated with the Gaussian quadrature procedure suggested by Butler and Moffitt (1982).

2.1 An Analytic Formula for TRESFA with $T = 2$

An Analytic Formula for TRESFA with *T* = 2

- Consider the TRESFA model with T=2, we see from (7) that

$$L^i_{s,t} = \frac{4}{\sigma^2}\int_{-\infty}^{\infty}\Phi(\frac{-S\lambda\varepsilon_{is}}{\sigma})\Phi(\frac{-S\lambda\varepsilon_{it}}{\sigma})\phi(\frac{\varepsilon_{is}}{\sigma})\phi(\frac{\varepsilon_{it}}{\sigma})\frac{1}{\sigma_\omega}\phi(\frac{\omega_i}{\sigma_\omega})d\omega_i. \quad (8)$$

- Define $A^* = \frac{4}{\sqrt{8\pi^3}\sigma^2\sigma_w}$,

(9)

then we rewrite $L^i_{s,t}$ in (8) as

$$L^i_{s,t} = A^*\int_{-\infty}^{\infty} e^{-\nu^2/2\sigma_w^2} e^{(-Z_{is}^2+2Z_{is}\nu-\nu^2)/2\sigma^2} e^{(-Z_{it}^2+2Z_{it}\nu-\nu^2)/2\sigma^2}\Phi(P_1\nu+Q_1)\Phi(P_2\nu+Q_2)d\nu, \quad (10)$$

where

$$P_1 = \frac{S\lambda}{\sigma}, Q_1 = \frac{-S\lambda Z_{is}}{\sigma}, P_2 = \frac{S\lambda}{\sigma}, Q_2 = \frac{-S\lambda Z_{it}}{\sigma}, \quad (11)$$

and Z_{it} is observable and is defined in (4).

An Analytic Formula for TRESFA with *T* = 2

- The likelihood function in (10) belongs to a subcase of the following more general integral function:

$$L_{s,t}^{i} = A^{*} I_{i,s,t}, \qquad (12)$$

where

$$I_{i,s,t} = \int_{-\infty}^{\infty} e^{-q_1 \nu^2 + q_2 \nu + q_3} \Phi(P_1 \nu + Q_1) \Phi(P_2 \nu + Q_2) d\nu, \; q_1 > 0, \qquad (13)$$

and

$$q_1 = \frac{1}{\sigma^2} + \frac{1}{2\sigma_w^2}, \; q_2 = \frac{(Z_{is} + Z_{it})}{\sigma^2}, \; q_3 = \frac{-(Z_{is}^2 + Z_{it}^2)}{2\sigma^2}. \qquad (14)$$

An Analytic Formula for TRESFA with *T* = 2

- By (7.1.1) of Abramowitz and Stegun (1970), we define the error function as:

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt = 2\int_0^{\sqrt{2}z} \phi(t)dt, \quad (15)$$

where $$erf(-z) = -erf(z). \quad (16)$$

- By (13), we observe that:

$$I_{i,s,t} = \int_{-\infty}^{\infty} e^{-q_1\nu^2 + q_2\nu + q_3} [\int_{-\infty}^{P_1\nu+Q_1} \phi(\zeta)d\zeta][\int_{-\infty}^{P_2\nu+Q_2} \phi(\zeta)d\zeta]\phi(\nu)d\nu$$

$$= \int_{-\infty}^{\infty} e^{-q_1\nu^2 + q_2\nu + q_3} [\frac{1}{2} + \frac{1}{2} erf(\frac{P_1\nu + Q_1}{\sqrt{2}})][\frac{1}{2} + \frac{1}{2} erf(\frac{P_2\nu + Q_2}{\sqrt{2}})]\phi(\nu)d\nu.$$

An Analytic Formula for TRESFA with $T = 2$

- If the condition $-Q_1/P_1 < -Q_2/P_2$ holds, then v in (13) is located in three mutually exclusive segments:

$$v \in (-\infty, -\frac{Q_1}{P_1}), \text{or } v \in [-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2}), \text{or } v \in [-\frac{Q_2}{P_2}, \infty), \quad (17)$$

such that the value of $I_{i,s,t}$ can be evaluated as:

$$\begin{aligned} I_{i,s,t} = & \int_{-\infty}^{-Q_1/P_1} e^{-q_1 v^2 + q_2 v + q_3} [\frac{1}{2} + \frac{1}{2} erf(\frac{P_1 v + Q_1}{\sqrt{2}})][\frac{1}{2} + \frac{1}{2} erf(\frac{P_2 v + Q_2}{\sqrt{2}})]\phi(v) dv \\ & + \int_{-Q_1/P_1}^{-Q_2/P_2} e^{-q_1 v^2 + q_2 v + q_3} [\frac{1}{2} + \frac{1}{2} erf(\frac{P_1 v + Q_1}{\sqrt{2}})][\frac{1}{2} + \frac{1}{2} erf(\frac{P_2 v + Q_2}{\sqrt{2}})]\phi(v) dv \\ & + \int_{-Q_2/P_2}^{\infty} e^{-q_1 v^2 + q_2 v + q_3} [\frac{1}{2} + \frac{1}{2} erf(\frac{P_1 v + Q_1}{\sqrt{2}})][\frac{1}{2} + \frac{1}{2} erf(\frac{P_2 v + Q_2}{\sqrt{2}})]\phi(v) dv \\ & = A_1 + B_1 + C_1. \end{aligned} \quad (18)$$

An Analytic Formula for TRESFA with *T* = 2

- Tsay *et al.* (2012) show that $erf(x)$ can be well approximated with a function, $g(x) = 1 - e^{c_1 x + c_2 x^2}$ if $x \geq 0$:

$$erf\,(x \geq 0) \sim 1 - e^{c_1 x + c_2 x^2}, \quad c_1 = \text{-}1.0950081470333 \text{ and } c_2 = \text{-}0.75651138383854 \quad (19)$$

- Technically, after substituting the approximating function in (19) for the error functions in (18), we find the integrand in each of A_1, B_1, and C_1 of (18) is a form of $e^{-(av^2+2bv+c)}$. Using (7.4.32) of Abramowitz and Stegun (1970):

$$\int e^{-(kx^2+2mx+\text{n})}dx = \frac{1}{2}\sqrt{\frac{\pi}{k}}e^{\frac{m^2\text{-}kn}{k}}erf\,(\sqrt{k}x + \frac{m}{\sqrt{k}}) + C, k \neq 0,$$

where C denotes some finite constant, we obtain an analytic formula for $I_{i,s,t}$.

- **Theorem 1**. *Given that the condition $-Q_1/P_1 < -Q_2/P_2$ holds, and defining $a_1 = P_1$, $b_1 = Q_1$, $a_2 = P_2$, and $b_2 = Q_2$, $I_{i,s,t}$ in (18) can be approximated by $I^{app}_{i,s,t}(a_1, b_1; a_2, b_2)$ as such:*

$$
\begin{aligned}
I^{app}_{i,s,t}(a_1,b_1;a_2,b_2) &= \frac{1}{8}\left[1-sign(a_1)\right]\left[1-sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\,e^{\frac{q_2^2/4+q_1q_3}{q_1}} \\
&\times \left[erf\left(\sqrt{q_1}\frac{-b_1}{a_1}-\frac{q_2}{2\sqrt{q_1}}\right)+1\right] \\
&+ \frac{1}{8}sign(a_1)\left[1-sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\,e^{\frac{\eta_3^2-\eta_1\eta_5}{\eta_1}} \\
&\times \left[erf\left(\sqrt{\eta_1}\frac{-b_1}{a_1}+\frac{\eta_3}{\sqrt{\eta_1}}\right)+1\right] \\
&+ \frac{1}{8}\left[1-sign(a_1)\right]sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\,e^{\frac{\eta_4^2-\eta_2\eta_6}{\eta_2}} \\
&\times \left[erf\left(\sqrt{\eta_2}\frac{-b_1}{a_1}+\frac{\eta_4}{\sqrt{\eta_2}}\right)+1\right] \\
&+ \frac{1}{8}sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_3+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_5+\eta_6}{\eta_1+\eta_2-q_1}} \\
&\times \left[erf\left(\sqrt{\eta_1+\eta_2-q_1}\frac{-b_1}{a_1}+\frac{\eta_3+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right)+1\right] \\
&+ \frac{1}{8}\left[1+sign(a_1)\right]\left[1-sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\,e^{\frac{q_2^2/4+q_1q_3}{q_1}} \\
&\times \left[erf\left(\sqrt{q_1}\frac{-b_2}{a_2}-\frac{q_2}{2\sqrt{q_1}}\right)-erf\left(\sqrt{q_1}\frac{-b_1}{a_1}-\frac{q_2}{2\sqrt{q_1}}\right)\right] \\
&+ \frac{1}{8}\left[1+sign(a_1)\right]sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\,e^{\frac{\eta_4^2-\eta_2\eta_6}{\eta_2}} \\
&\times \left[erf\left(\sqrt{\eta_2}\frac{-b_2}{a_2}+\frac{\eta_4}{\sqrt{\eta_2}}\right)-erf\left(\sqrt{\eta_2}\frac{-b_1}{a_1}+\frac{\eta_4}{\sqrt{\eta_2}}\right)\right] \\
&- \frac{1}{8}sign(a_1)\left[1-sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\,e^{\frac{\eta_7^2-\eta_1\eta_8}{\eta_1}} \\
&\times \left[erf\left(\sqrt{\eta_1}\frac{-b_2}{a_2}+\frac{\eta_7}{\sqrt{\eta_1}}\right)-erf\left(\sqrt{\eta_1}\frac{-b_1}{a_1}+\frac{\eta_7}{\sqrt{\eta_1}}\right)\right] \\
&- \frac{1}{8}sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_7+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_6+q_3)}{\eta_1+\eta_2-q_1}} \\
&\times erf\left(\sqrt{\eta_1+\eta_2-q_1}\frac{-b_2}{a_2}+\frac{\eta_7+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right) \\
&+ \frac{1}{8}sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_7+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_6+q_3)}{\eta_1+\eta_2-q_1}} \\
&\times erf\left(\sqrt{\eta_1+\eta_2-q_1}\frac{-b_1}{a_1}+\frac{\eta_7+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right) \\
&+ \frac{1}{8}\left[1+sign(a_1)\right]\left[1+sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\,e^{\frac{q_2^2/4+q_1q_3}{q_1}} \\
&\times \left[1-erf\left(\sqrt{q_1}\frac{-b_2}{a_2}-\frac{q_2}{2\sqrt{q_1}}\right)\right] \\
&- \frac{1}{8}\left[1+sign(a_1)\right]sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\,e^{\frac{\eta_9^2-\eta_2\eta_{10}}{\eta_2}} \\
&\times \left[1-erf\left(\sqrt{\eta_2}\frac{-b_2}{a_2}+\frac{\eta_9}{\sqrt{\eta_2}}\right)\right] \\
&- \frac{1}{8}sign(a_1)\left[1+sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\,e^{\frac{\eta_7^2-\eta_1\eta_8}{\eta_1}} \\
&\times \left[1-erf\left(\sqrt{\eta_1}\frac{-b_2}{a_2}+\frac{\eta_7}{\sqrt{\eta_1}}\right)\right] \\
&+ \frac{1}{8}sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_7+\eta_9+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_{10}+q_3)}{\eta_1+\eta_2-q_1}} \\
&\times \left[1-erf\left(\sqrt{\eta_1+\eta_2-q_1}\frac{-b_2}{a_2}+\frac{\eta_7+\eta_9+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\eta_1 &= \frac{2q_1-c_2a_1^2}{2}; \\
\eta_2 &= \frac{2q_1-c_2a_2^2}{2}; \\
\eta_3 &= \frac{-\sqrt{2}c_2a_1b_1+c_1sign(a_1)a_1-\sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_4 &= \frac{-\sqrt{2}c_2a_2b_2+c_1sign(a_2)a_2-\sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_5 &= \frac{-\sqrt{2}c_2b_1^2+2c_1sign(a_1)b_1-2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_6 &= \frac{-\sqrt{2}c_2b_2^2+2c_1sign(a_2)b_2-2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_7 &= \frac{-\sqrt{2}c_2a_1b_1-c_1sign(a_1)a_1-\sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_8 &= \frac{-\sqrt{2}c_2b_1^2-2c_1sign(a_1)b_1-2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_9 &= \frac{-\sqrt{2}c_2a_2b_2-c_1sign(a_2)a_2-\sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_{10} &= \frac{-\sqrt{2}c_2b_2^2-2c_1sign(a_2)b_2-2\sqrt{2}q_3}{2\sqrt{2}},
\end{aligned}
$$

An Analytic Formula for TRESFA with $T = 2$

- **Theorem 2**. *Given that the condition* $-Q_1/P_1 \geq -Q_2/P_2$ *holds, then* $I_{i,s,t}$ *in (18) can be approximated by* $I_{i,s,t}{}^{app}(a_1, b_1; a_2, b_2)$ *in Theorem 1 with* $a_1 = P_2$, $b_1 = Q_2$, $a_2 = P_1$, *and* $b_2 = Q_1$.
- With Theorems 1 and 2, the approximate joint likelihood function for firm *i* at periods *s* and *t* is computed as:

$$\widetilde{L^{i}_{s,t}} = A*I\{\frac{-Q_1}{P_1} < \frac{-Q_2}{P_2}\} I^{app}_{i,s,t}(a_1 = P_1,\ b_1 = Q_1;\ a_2 = P_2,\ b_2 = Q_2)$$
$$+A*I\{\frac{-Q_1}{P_1} \geq \frac{-Q_2}{P_2}\} I^{app}_{i,s,t}(a_1 = P_2,\ b_1 = Q_2;\ a_2 = P_1,\ b_2 = Q_1), \qquad (20)$$

where $I\{.\}$ is the indicator function taking the value one if the statement in the bracket is true and zero otherwise.

An Analytic Formula for TRESFA with *T* = 2

- **Theorem 3**. *When T=2, The log-likelihood function for the true random effects stochastic frontier model in (1) for firm i at time s and t is approximated as:*

$$\ln L_{s,t}^{app} = \sum_{i=1}^{N} \ln \widetilde{L_{s,t}^{i}},$$

where $\widetilde{L_{s,t}^{i}}$ *is defined in (20).*

- It is clear that Gaussian quadrature method or simulation-based procedure are not needed when computing $\ln L^{app}{}_{s,t}$, because the error function *erf* (.) can be directly calculated with a standard statistic package.

2.2 Dealing with TRESFA with $T > 2$

Dealing with TRESFA with $T > 2$

- The analytic procedure to evaluate the log-likelihood function of the model in (1) is not feasible when T is large. We can easily circumvent this difficulty by using the pairwise likelihood principle which has been used for the pseudolikelihood methods of Besag (1975) under spatial data.

- We pool all pairwise log-likelihood functions under various combinations of time s and time t for each unit i as:

$$\ln L \sim \sum_{i=1}^{N} \sum_{t \neq s} \ln \widetilde{L_{s,t}^{i}} = \sum_{t \neq s} \ln L_{s,t}^{app} \quad (21)$$

Clearly, the cost of conducting PLE is limited, because the computational cost of is mild.

Dealing with TRESFA with $T > 2$

- Once all the parameters of the TRESFA model are in hand, we observe from (1) that

$$\varepsilon_{\mathrm{i,t}} = w_i + v_{i,t} - Su_{i,t}. \quad (22)$$

- At time t, we note that

$$\varepsilon_{i,t} = e_{i,t} - Su_{i,t}, \quad e_{i,t} \sim N\left[0, \sigma_{\omega}^2 + \sigma_{v}^2\right], \quad u_{i,t} \sim \left|N\left[0, \sigma_{u}^2\right]\right|, \quad (23)$$

  so we can use the method of Jondrow et al. (1982) to extract the inefficiency part of the composite error as $E(u_{i,t} \mid \varepsilon_{i,t})$ for firm i at time t. To rank the efficiency level across firms, we might use the average of $E(u_{i,t} \mid \varepsilon_{i,t})$ across t.

3. A True Fixed Effects Stochastic Frontier Model

A True Fixed Effects Stochastic Frontier Model

- Consider the estimation of the true fixed effects SFA models of the form in (3):

$$y_{it}=\alpha+x_{it}^{\mathrm{T}}\beta+\omega_i+\varepsilon_{it}=\alpha_i+x_{it}^{\mathrm{T}}\beta+\varepsilon_{it} \quad (24)$$

- To extract more information from the TFESFA data, we suggest using a pairwise differencing operator as follows:

$$y_{i,s}\text{-}y_{i,t}=(x_{i,s}-x_{i,t})^{\mathrm{T}}\beta+(\varepsilon_{i,s}-\varepsilon_{i,t}). \;\; s\neq t. \quad (25)$$

- The coefficient β and the parameters characterizing the composite error $\varepsilon_{i,t}$ can be estimated with likelihood-based estimator if the pdf of $\varepsilon_{i,s}-\varepsilon_{i,t}$ is known. Since $\varepsilon_{i,t}$ and $\varepsilon_{i,s}$ are mutually independent, the joint density function of them is the product of their individual densities:

$$f(\varepsilon_{i,s},\varepsilon_{i,t})=\frac{4}{\sigma^2}\phi(\frac{\varepsilon_{i,s}}{\sigma})\Phi(\frac{-S\lambda\varepsilon_{i,s}}{\sigma})\phi(\frac{\varepsilon_{i,t}}{\sigma})\Phi(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}). \quad (26)$$

A True Fixed Effects Stochastic Frontier Model

- Using (26) and making the transformation

$$h^i_{s,t} = \varepsilon_{i,s} - \varepsilon_{i,t} \qquad (27)$$

- The joint density function of $\varepsilon_{i,t}$ and $h^i_{s,t}$ is

$$f(\varepsilon_{i,t}, h^i_{s,t}) = \frac{4}{\sigma^2}\phi\left(\frac{h^i_{s,t}+\varepsilon_{i,t}}{\sigma}\right)\Phi\left(\frac{-S\lambda(h^i_{s,t}+\varepsilon_{i,t})}{\sigma}\right)\phi\left(\frac{\varepsilon_{i,t}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}\right). \qquad (28)$$

- Thus, the marginal density function of $h^i_{s,t}$ is computed by

$$f(h^i_{s,t}) = \frac{4}{\sigma^2}\int_{-\infty}^{\infty}\phi\left(\frac{h^i_{s,t}+\varepsilon_{i,t}}{\sigma}\right)\Phi\left(\frac{-S\lambda(h^i_{s,t}+\varepsilon_{i,t})}{\sigma}\right)\phi\left(\frac{\varepsilon_{i,t}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}\right) d\varepsilon_{i,t}. \qquad (29)$$

A True Fixed Effects Stochastic Frontier Model

- Without loss of clarity, we can write above marginal density function as:

$$f(h)=\frac{4}{\sigma^2}\int_{-\infty}^{\infty}\phi\left(\frac{h+\varepsilon}{\sigma}\right)\Phi\left(\frac{-S\lambda(h+\varepsilon)}{\sigma}\right)\phi\left(\frac{\varepsilon}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon}{\sigma}\right)d\varepsilon. \quad (30)$$

- The integral function in (30) admits the following familiar form:

$$f(h)=\frac{2}{\pi\sigma^2}\int_{-\infty}^{\infty}e^{-v^2/\sigma^2-hv/\sigma^2-h^2/2\sigma^2}\Phi(P_1v+Q_1)\Phi(P_2v)dv, \quad (31)$$

where

$$P_1=\frac{-S\lambda}{\sigma},\ Q_1=\frac{-S\lambda h}{\sigma},\ P_2=\frac{-S\lambda}{\sigma}. \quad (32)$$

A True Fixed Effects Stochastic Frontier Model

- The function in (31) can be recast as:

$$f(h)=\frac{2}{\pi\sigma^2}\int_{-\infty}^{\infty} e^{-q_1v^2+q_2v+q_3}\Phi(P_1v+Q_1)\Phi(P_2v)dv, \quad q_1>0, \qquad (33)$$

and

$$q_1=\frac{1}{\sigma^2}, \quad q_2=\frac{-h}{\sigma^2}, \quad q_3=\frac{-h^2}{2\sigma^2}. \qquad (34)$$

- Pairwise differencing makes the direct estimation of the fixed effect, α_i, impossible. However, it can be estimated as:

$$\hat{\alpha}_{i,TFE}=\bar{y}_i-\bar{x}_i^{\mathrm{T}}\hat{\beta}_{TFE}+S\sqrt{\frac{2}{\pi}}\hat{\sigma}_u, \qquad (35)$$

Where $\bar{y}_i$ and $\bar{x}_i$ are within group sample means of the dependent variable and the stochastic regressors, respectively; $\hat{\beta}_{TFE}$ denotes the coefficient estimation from the first stage estimation based on our formula.

A True Fixed Effects Stochastic Frontier Model

- $\hat{\alpha}_{i,TFE}$ in (35) can be used to rank the relative efficiency level across firms via the Schmidt and Sickles' (1984) method, i.e., the firm specific inefficiency is measured as a deviation from the benchmark level:

$$\hat{u}_{i,TFE} = \max_i(\hat{\alpha}_{i,TFE}) - \hat{\alpha}_{i,TFE} \geq 0. \qquad (36)$$

- Our method is a two-stage approach as compared to the one-step stimulated MLE procedure of Greene (2005) who proposes using brute force computation to estimate all the fixed effects parameters along with the other ones.
- The estimate of β using the pairwise differencing method is not affected by the incidental parameters problems especially when N is huge.

4. Monte Carlo Experiment

Monte Carlo Experiment

- Following Olson *et al.* (1980, p. 76), we consider a set of experiments with two regressors model:

$$y_{it}^{l} = \alpha + \beta_1 x_{it}^{l} + \omega_i^{l} + v_{it}^{l} - u_{it}^{l}, \;\; u_{it} = \left|U_{it}\right| \tag{37}$$

  where x_{it}'s are generated from standard normal distribution, and l denotes the l-th replication of the data.

- All the programs are written in GAUSS. Two hundred additional values are generated in order to obtain random starting values. The optimization algorithm used to implement the MLE is the quasi-Newton algorithm of Broyden, Fletcher, Goldfarb, and Shanno (BFGS) contained in the GAUSS MAXLIK library. The maximum number of iterations for each replication is 100.

4.1 Gaussian Quadrature versus Analytic Approximation for TRESFA Model

Gaussian Quadrature versus Analytic Approximation for TRESFA Model

- We engage the Gaussian quadrature procedure as a benchmark for the proposed analytic approximation method. Moreover, when *T*=2, the full likelihood function can be evaluated with both models, so we can assess the accuracy of the analytic formula via the case *T*=2.

- The simulation results are contained in Table 1 and Table 2 where the true parameters are:

$$\xi = (\alpha, \beta_1, \sigma_u, \sigma_v, \sigma_w)^{\mathrm{T}}, \ \alpha = \beta_1 = 1, \ \sigma_w = 1, \qquad (38)$$

and

$$\{\sigma_u = 2, \sigma_v = 1\}, \ \text{or} \ \{\sigma_u = 0.5, \sigma_v = 0.25\}, \ \text{or} \ \{\sigma_u = 0.4, \sigma_v = 0.2\} \qquad (39)$$

Table 1. Maximum Likelihood Estimation for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	Bias (Analytic)					Bias (Quadrature)				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 2$ and $\sigma_v = 1$					
100	0.0994	-0.0046	0.1266	0.0131	0.0350	0.1010	-0.0045	0.1291	0.0100	0.0339
200	0.0419	0.0003	0.0487	0.0080	0.0149	0.0430	0.0002	0.0506	0.0056	0.0149
400	0.0108	-0.0006	0.0102	0.0081	0.0075	0.0122	-0.0006	0.0126	0.0055	0.0075
800	0.0006	-0.0006	-0.0003	0.0098	0.0038	0.0021	-0.0006	0.0022	0.0071	0.0038
					$\sigma_u = 0.5$ and $\sigma_v = 0.25$					
100	0.1075	-0.0026	0.1372	-0.0059	0.0086	0.0834	-0.0026	0.1094	0.0088	0.0061
200	0.0755	-0.0001	0.0928	0.0003	0.0043	0.0698	-0.0003	0.0847	0.0026	0.0030
400	0.0456	0.0000	0.0551	0.0012	0.0021	0.0492	0.0000	0.0580	-0.0048	0.0019
800	0.0224	-0.0003	0.0264	0.0019	0.0010	0.0251	-0.0004	0.0299	-0.0019	0.0010
					$\sigma_u = 0.4$ and $\sigma_v = 0.2$					
100	0.0932	-0.0012	0.1181	-0.0071	0.0088	0.0600	-0.0013	0.0804	0.0067	0.0050
200	0.0720	-0.0003	0.0896	-0.0033	0.0041	0.0676	-0.0005	0.0846	-0.0116	0.0026
400	0.0495	0.0000	0.0605	-0.0018	0.0023	0.0669	0.0000	0.0816	-0.0226	0.0020
800	0.0226	-0.0002	0.0274	0.0016	0.0012	0.0489	-0.0002	0.0609	-0.0230	0.0014

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (40).

Table 2. Maximum Likelihood Estimation for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	RMSE (Analytic)					RMSE (Quadrature)				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 2$ and $\sigma_v = 1$					
100	0.4784	0.1272	0.5834	0.3016	0.1974	0.4804	0.1273	0.5865	0.2962	0.1926
200	0.3035	0.0902	0.3653	0.2117	0.1328	0.3033	0.0902	0.3650	0.2106	0.1328
400	0.1917	0.0622	0.2246	0.1483	0.0913	0.1920	0.0622	0.2250	0.1480	0.0913
800	0.1307	0.0440	0.1515	0.1038	0.0617	0.1309	0.0440	0.1519	0.1034	0.0617
					$\sigma_u = 0.5$ and $\sigma_v = 0.25$					
100	0.2488	0.0385	0.2857	0.1256	0.0759	0.2507	0.0402	0.2750	0.1240	0.0757
200	0.2041	0.0278	0.2402	0.1139	0.0541	0.2078	0.0288	0.2399	0.1066	0.0541
400	0.1558	0.0189	0.1841	0.0935	0.0393	0.1583	0.0192	0.1854	0.0820	0.0393
800	0.1096	0.0135	0.1297	0.0665	0.0262	0.1109	0.0135	0.1308	0.0621	0.0263
					$\sigma_u = 0.4$ and $\sigma_v = 0.2$					
100	0.2121	0.0311	0.2357	0.1026	0.0731	0.2109	0.0340	0.2153	0.0926	0.0734
200	0.1796	0.0224	0.2076	0.0942	0.0525	0.1826	0.0229	0.1987	0.0748	0.0526
400	0.1469	0.0153	0.1709	0.0805	0.0384	0.1551	0.0153	0.1753	0.0634	0.0383
800	0.1008	0.0109	0.1184	0.0591	0.0256	0.1158	0.0110	0.1322	0.0510	0.0257

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (40).

Gaussian Quadrature versus Analytic Approximation for TRESFA Model

- The typical numerical-integral procedure suggested by Butler and Moffitt (1982) for the random effects probit model becomes biased when the correlation coefficient within each unit (ρ) is relatively large. (Lee (2000))
- We investigate the performance of the quadrature method in Table 3 under the setup that ρ is relatively large, i.e.,

$$\{\sigma_u = 0.4, \sigma_v = 0.2\} \tag{40}$$

- The correlation between individuals within a group is $\rho \approx$ 0.9106 under the desgin in (40).

Gaussian Quadrature versus Analytic Approximation for TRESFA Model

- Before discussing the results in Table 3, we emphasize here that, when $T > 2$, the estimation based on quadrature method is still equivalent to the full MLE of the TRESFA models.
- In Table 3, the PLE is found to possess a well-defined asymptotic behavior, because the associated RMSE decreases with the increasing sample size T. On the contrary, we cannot observe a similar pattern from the quadrature method.
- This observation leads us to further check the performance of the quadrature procedure when T is relatively large.

Table 3. Maximum Likelihood Estimation for the True Random Effects SFA Model

$\alpha = \beta_1 = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.2$, $\sigma_w = 1$, and $N = 1000$

	Bias					RMSE				
$T = 3$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
Analytic										
	0.0066	0.0001	0.0122	-0.0017	-0.0007	0.0557	0.0071	0.0615	0.0347	0.0228
Quadrature										
	0.0175	0.0000	0.0234	-0.0172	-0.0016	0.0600	0.0073	0.0379	0.0241	0.0230
$T = 4$										
Analytic										
	0.0117	0.0001	0.0101	-0.0028	0.0010	0.0526	0.0056	0.0486	0.0313	0.0215
Quadrature										
	0.0224	0.0000	0.0169	-0.0181	-0.0021	0.0822	0.0056	0.0265	0.0216	0.0221

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (44).

Table 4. MLE for the True Random Effects SFA Model using Gaussian Quadrature $\alpha = \beta_1 = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.2$, $\sigma_w = 1$, and $N = 1000$

	Bias					RMSE				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$T = 10$	-0.0047	-0.0001	0.0071	-0.0266	-0.0331	0.2567	0.0033	0.0125	0.0272	0.0708
$T = 20$	-0.0065	-0.0001	0.0036	-0.0316	-0.0970	0.4619	0.0023	0.0077	0.0318	0.1678
$T = 30$	0.0201	0.0000	0.0017	-0.0331	-0.1245	0.5267	0.0020	0.0060	0.0333	0.2116

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (44).

4.2 PLE with Analytic Formula for TRESFA Models

PLE with Analytic Formula for TRESFA Models

- Table 5 considers the model with the following parameter values:

$$\xi = \left(\alpha, \beta_1, \sigma_u, \sigma_v, \sigma_w\right)^{\mathrm{T}}, \ \alpha = \beta_1 = 1, \ \sigma_u = \sigma_w = 1, \ \sigma_v = \{1/2, 1, 3/2\}. \quad (41)$$

- The results show that the analytic formula is computationally efficient in that it easily handles the simulations with a sample size of 800 and 1000 replications.
- Table 5 also reveals that the performance of the MLE improves with the value λ. This phenomenon is typically found in the stochastic frontier literature.

Table 5. PLE for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	Bias					RMSE				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 1$ and $\sigma_v = 1/2$					
100	0.1224	-0.0023	0.1546	0.0086	0.0133	0.3730	0.0720	0.4498	0.2154	0.0968
200	0.0707	-0.0003	0.0861	0.0068	0.0054	0.2766	0.0515	0.3339	0.1728	0.0680
400	0.0290	-0.0004	0.0335	0.0052	0.0026	0.1776	0.0350	0.2116	0.1225	0.0492
800	0.0138	-0.0007	0.0158	0.0003	0.0022	0.1101	0.0251	0.1292	0.0817	0.0343
					$\sigma_u = 1$ and $\sigma_v = 1$					
100	0.1235	-0.0049	0.1572	0.0525	0.0155	0.5405	0.1026	0.6625	0.2390	0.1359
200	0.1341	-0.0001	0.1658	0.0126	0.0089	0.4664	0.0715	0.5767	0.1663	0.0939
400	0.0979	-0.0011	0.1222	0.0049	0.0043	0.3930	0.0495	0.4846	0.1309	0.0668
800	0.0758	-0.0009	0.0920	0.0006	0.0021	0.3241	0.0357	0.3993	0.1060	0.0476
					$\sigma_u = 1$ and $\sigma_v = 3/2$					
100	0.0558	-0.0045	0.0746	0.0907	0.0319	0.6781	0.1352	0.8311	0.2698	0.2197
200	0.0961	0.0003	0.1180	0.0476	0.0162	0.5925	0.0940	0.7308	0.1979	0.1420
400	0.0986	-0.0006	0.1208	0.0279	0.0084	0.5296	0.0657	0.6545	0.1559	0.0988
800	0.1085	-0.0012	0.1327	0.0141	0.0032	0.4764	0.0462	0.5855	0.1264	0.0684

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38) and (45).

PLE with Analytic Formula for TRESFA Models

- We further investigate the performance of PLE by considering 2 additional choice of σ_w :

  $$\sigma_W = \{0.5, 1.3\},$$

  under 3 different value of T :

  $$T = \{2, 4, 8\}, \qquad (42)$$

  and

  $$\alpha = \beta_1 = 1, \ \sigma_u = 1, \ \sigma_v = 0.5, \qquad (43)$$

- The simulations are contained in Tables 6 and 7. As expected, the PLE works well for these TRESFA model. The changing patterns found in the table resembles closely to that found in Table 5.

Table 6. PLE for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_u = 1$, $\sigma_v = 1/2$, and $\sigma_w = 1/2$

	Bias					RMSE				
$T = 2$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$N = 50$	0.0976	-0.0056	0.1267	0.0174	0.0380	0.3494	0.0883	0.4232	0.2061	0.1518
$N = 100$	0.0562	-0.0023	0.0713	0.0069	0.0166	0.2527	0.0636	0.3085	0.1557	0.0991
$N = 150$	0.0275	-0.0011	0.0324	0.0073	0.0088	0.1773	0.0505	0.2141	0.1248	0.0774
$T = 4$										
$N = 50$	0.0468	0.0021	0.0564	0.0048	0.0209	0.2375	0.0598	0.2756	0.1396	0.0866
$N = 100$	0.0213	-0.0001	0.0263	0.0003	0.0110	0.1503	0.0417	0.1728	0.0966	0.0592
$N = 150$	0.0089	-0.0013	0.0117	0.0022	0.0087	0.1089	0.0340	0.1237	0.0754	0.0489
$T = 8$										
$N = 50$	0.0145	-0.0007	0.0161	0.0049	0.0169	0.1486	0.0429	0.1572	0.0892	0.0686
$N = 100$	0.0042	-0.0006	0.0038	0.0050	0.0101	0.1009	0.0306	0.1040	0.0616	0.0465
$N = 150$	0.0027	-0.0004	0.0012	0.0046	0.0063	0.0783	0.0248	0.0825	0.0497	0.0372

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (46), (47), and (48).

Table 7. PLE for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_u = 1$, $\sigma_v = 1/2$, and $\sigma_w = 1.3$

	Bias					RMSE				
$T=2$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$N=50$	0.2123	-0.0049	0.2789	0.0056	0.0321	0.5087	0.1019	0.5953	0.2672	0.1578
$N=100$	0.1564	-0.0026	0.1971	0.0058	0.0146	0.4224	0.0745	0.5030	0.2356	0.1110
$N=150$	0.1187	-0.0012	0.1455	0.0078	0.0081	0.3615	0.0592	0.4301	0.2156	0.0890
$T=4$										
$N=50$	0.1318	0.0029	0.1612	0.0028	0.0259	0.4108	0.0641	0.4509	0.2130	0.1450
$N=100$	0.0811	0.0001	0.1014	-0.0057	0.0143	0.3052	0.0443	0.3405	0.1607	0.1021
$N=150$	0.0451	-0.0009	0.0580	-0.0032	0.0116	0.2286	0.0364	0.2509	0.1272	0.0836
$T=8$										
$N=50$	0.0626	-0.0008	0.0743	0.0013	0.0268	0.3098	0.0453	0.3046	0.1448	0.1404
$N=100$	0.0296	-0.0007	0.0330	0.0015	0.0159	0.2045	0.0322	0.1970	0.1036	0.0958
$N=150$	0.0183	-0.0002	0.0177	0.0023	0.0103	0.1559	0.0259	0.1411	0.0834	0.0771

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (46), (47), and (48).

4.3 Analytic Formula for TFESFA Models

Analytic Formula for TFESFA Models

- This subsection investigates the finite sample performance of the PLE based on pairwise differencing and our analytic formula under the following TFESFA model:

$$\{\sigma_u = 1, \sigma_v = 0.5\}, \text{or } \{\sigma_u = 2, \sigma_v = 1\}, \ \sigma_w = 1.5, \ \alpha = \beta_1 = 1, \tag{44}$$

with 3 different value of T:

$$T = \{2, 4, 8\}, \tag{45}$$

- As discussed previously, pairwise differencing makes the estimation of time-invariant parameters infeasible. Nevertheless, provided that $x_{i,t}$ is time-varying and w_i is mean zero, we can estimate the global intercept as:

$$\hat{\alpha}_{TFE} = \overline{y} - \overline{x}^{\mathrm{T}} \hat{\beta}_{TFE} + \mathrm{S}\sqrt{\frac{2}{\pi}}\hat{\sigma}_u. \tag{46}$$

Analytic Formula for TFESFA Models

- The last item at the right side of (46) derives from the population mean of a half-normal distribution, and $\bar{y}$ and $\bar{x}$ denotes the sample average of the dependent variable and independent variables, respectively. The simulation findings are contained in Table 8.
- The major feature of the results in Table 8 is similar to what we observe for the TRESFA models. The analytic formula work very well for the TFESFA model in that the bias is small, and the RMSE of the PLE improves with the value of T across different model specifications as well.

Table 8. PLE for the True Fixed Effects SFA Model $\alpha = \beta_1 = 1$, $\sigma_w = 1.5$, and $N = 1000$

	Bias				RMSE			
	α	β_1	σ_u	σ_v	α	β_1	σ_u	σ_v
				$\sigma_u = 1$ and $\sigma_v = 0.5$				
$T = 2$	0.1445	-0.0003	0.1760	-0.0225	0.3447	0.0235	0.4250	0.1860
$T = 4$	0.0434	0.0003	0.0517	-0.0138	0.1613	0.0135	0.1996	0.0928
$T = 8$	0.0271	0.0008	0.0334	-0.0144	0.1015	0.0093	0.1140	0.0585
				$\sigma_u = 2$ and $\sigma_v = 1$				
$T = 2$	0.2688	0.0003	0.3325	-0.0426	0.6567	0.0469	0.8182	0.3670
$T = 4$	0.0847	0.0005	0.1029	-0.0275	0.3169	0.0269	0.3973	0.1855
$T = 8$	0.0533	0.0016	0.0669	-0.0288	0.1861	0.0186	0.2282	0.1169

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (49), and (50). The estimate for α is based on the formula in (51).

5. Empirical Application to WHO Health Attainment

Empirical Application to WHO Health Attainment

- We apply the TRESFA model to the data used in Evans *et al.* (2000 a,b) (ETML) and Greene (2003) about the World Health Organization's (WHO) panel data on national health care systems.
- As clearly pointed out in ETML (2000 a,b) and Greene (2003), the rankings were produced using a form of the fixed effects, stochastic frontier methodology proposed by Schmidt and Sickles (1984).
- Greene (2003) emphasizes that one criticism of the fixed effects methodology used for the WHO 2000 report is that the model fails to distinguish between cross country heterogeneity unrelated to inefficiency and the inefficiency itself.

Empirical Application to WHO Health Attainment

- Two measures of health care attainment were analyzed for the WHO panel data, disability adjusted life expectancy (DALE) and a composite measure of health care delivery (COMP).
- It is shown in Gravelle *et al*. (GJJS) (2002 a,b) that 51 out of 191 countries are observed for only one year (1997). The results from the method of Schmidt and Sickles (1984) are based on 140 countries only.
- Thus, the findings of this section are generated from 700 observations spanning 1993-1997 of these 140 countries, because the method of Schmidt and Sickles (1984) is used as the benchmark for comparison.

Empirical Application to WHO Health Attainment

- The production function considered in Schmidt and Sickles (1984) is denoted

$$y_{it} = \alpha + x_{it}^{\mathrm{T}}\beta + v_{it} - u_{it}, \; i = 1, 2, \ldots\ldots, N, \; t = 1, \ldots\ldots, T, \qquad (47)$$

- i.e., σ_w is assumed to be zero, and the level of inefficiency is time-invariant. The preceding model is rewritten as:

$$y_{it} = (\alpha - u_i) + x_{it}^{\mathrm{T}}\beta + v_{it} = \alpha_i + x_{it}^{\mathrm{T}}\beta + v_{it}. \qquad (48)$$

- Schmidt and Sickles (1984) suggest using within estimator for the model in (53), and the country specific inefficiency is measured as a deviation from the benchmark level:

$$\widehat{u}_i = \max_i(\widehat{\alpha}_i) - \widehat{\alpha}_i \geq 0. \qquad (49)$$

Table 9. Descriptive Statistics for Variables, 1997 Observations

	Mean	Std. Dev.
Variable		
Dale (Disability adjusted life expectancy)	56.83	12.29
HEXP (Health expenditures per capita in 1997 PPP$)	445.37	616.36
EDUC (Average year of schooling)	6.00	2.62
TROPICS (Dummy variable for tropical location)	0.508	0.501
DGPC (Per capita GDP in 1997 PPP$)	6609.4	7614.8

Notes: Data are taken from Greene (2003).

Table 10. Health Care Outcome Analysis

	Schmidt-Sickles' (1984) Method	TRESFA
Variable		
HEXP	0.1342 (3.433)	0.1177 (1.331)
EDUC	2.2111 (4.897)	4.1358(7.128)
EDUC^2	-0.0344 (0.859)	-0.1801(3.238)
DGPC	-	0.4708 (3.308)
TROPICS	-	-3.7600 (2.602)
Constant	-	37.8239 (2.221)
σ_u	-	0.0009 (0.000)
σ_v	-	0.4795 (24.164)
σ_w	-	6.6862 (16.282)

Notes: Data are defined in Table 10. DALE is the dependent variable. HEXP is divieded by 100, and GDPC is divided by 1000 before estimation. The number in parathesis denotes the absolute value of t ratio statistic. The t ratio of the TRESFA model is computed from the covariance matrix estimators outlined in (5) and (6) of Kuk and Nott (2000).

6. Conclusions

Conclusions

- We consider the estimation issues of the true random effects and true fixed effects SFA models of Greene (2005).
- We first evaluate the effectiveness of Gaussian quadrature via Monte Carlo experiments.
- The simulations reveal that the performance of the quadrature procedure is not reliable when T is relatively large and the within group correlation is strong as found in the panel probit literature.
- The proposed analytic approximation method can easily deal with the models of Greene (2005).
- Furthermore, the analytic strategy can be easily carried out with standard statistics packages, and its implementation for the likelihood estimation is found to be stable for the experiments conducted in this paper.