The Seller's Listing Strategy in Online Auctions: A Simple Theory

[VERY PRELIMINARY]

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Abstract

This paper proposes a simple model to characterize the sellers' optimal listing strategy as a function of their rates of time-impatience. Specifically, it is shown that the fixed-price listing, the pure auction, and the buy-it-now auction are each a solution of the seller's single optimization problem under different values of rate of time-impatience. We also show that the optimal posted price in the fixed-price listing is greater than the optimal reserve price for the buy-it-now auction, which in turn is greater than that of the pure auction.

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1 Introduction

The rise of the online auctions firms such as eBay or Yahoo! has provided potential sellers with a vast arrays of opportunities to sell commodities which otherwise cannot find any outlay of being disposed of.[1] More important for the researchers who are interested in transaction or matching mechanism is the fact that the online auction sites offer not only the opportunity to sell a great variety of commodities, but also different formats to sell them.

Although the Revenue Equivalence Theorem has identified a very general condition under which all auction formats yield exactly the same revenue to the seller, in reality the sellers have many other considerations than revenue itself. They include transaction risk, affiliation between the bidders' valuation of the item, budget constraint, or bidder symmetry.[2] In online auctions the seller's consideration are even more varied and diversified. For example, the duration of a listing might run from 1 to 7 days, and the seller or buyer who have time preference might prefer the transaction to complete earlier than its duration. There are three major listing formats in the online auctions. The first is the usual ascending price auction, either with or without the reserve price.[3] Since most auction sites offer the choice of a proxy bid, this is essentially a second-price auction. In this paper we will call it the "pure auction." The second format is the fixed-price listing: An item is listed with a fixed posted price, and the bidders can only buy the item with the posted price. The third format, which is a specific online auction innovation, is the buy-it-now (BIN) auction. On top of the pure auction, the seller sets a price, called the buy-it-now price, by which the bidder can win the item immediately by agreeing to pay that price. In the eBay auction (which is the main concern in our paper), the BIN option is temporary: If any bidder

[1] See, for example, Anderson (2006).

[2] See Krishna (2012), Chapter 4, for details.

[3] In the online auction, there are two types of reserve price. The publicly observed reserve price is the starting price. The first bidder for the item must bid a price at least as high as the starting price. Another types of reserve price, the secret reserve price, is a price set by the seller which cannot be publicly observed. A bidder who bids below the secret reserve price will only be informed as such, without knowing the value of the minimum eligible bid.

places a bid before any other bidder buys out the item with BIN, then the BIN option disappears, and the auction become a pure auction.[4]

All the three formats of listing mentioned above are prevalent in online auctions. For example, Hacker and Sickles (2010) show that pure auction, BIN auction and fixed-price listing all occupy a substantial portion in their eBay auction data. An obvious question to ask therefore: What determines a seller's choice of one format over the others? There is surprisingly few research that addresses this question. On the theoretical side, Wang (1993) compares price postings and auctions, and under the assumptions that buyers arrive randomly and that their valuations are independent, he shows that auction is more likely to be adopted, relative to fixed-price posting, if the buyer's marginal-revenue curves become steeper. Our paper is closest to Wang et al. (2008). They assume that bidders have different costs in participating a regular auction and a BIN auction. Depending on the relative cost, the bidders decide which type of auction to participate in. This in turn determines the threshold value of the bidder's valuation above which he prefers exercising BIN to bidding. This also characterizes two out-off values, one high and one low. If the BIN price is set to be higher than the high out-off, no bidder wants to exercise BIN, and the BIN auction essentially becomes a pure auction. If the BIN price is set lower than the low out-off, then all bidders prefer exercise BIN than bidding, and the BIN auction is essentially a fixed-price posting. Only if the BIN price is set between the two out-off values can BIN to effective. Since the number of bidders, the costs in exercising BIN and participating in bidding are all exogenous, it is difficult to completely characterize the optimal listing strategy of the seller.

On the empirical side, Hammond (2010) compares the pure auctions and the fixed-price listings of CDs, and shows that the sellers who have larger size and less heterogeneous inventory are more likely to adopt the posted price format than the pure auction. Moreover, pure auctions result

[4] There is also another format, called the "Best Offer", in which the bidders and seller can bargain for the price in a fixed protocol designed by the auction platforms. But the categories of commodities which can use this format are rare.

in higher sale rate but, given sale, lower transaction price. Chen et al. (2012) also use data of auctions of iPods in eBay to show that the seller's characteristics have significant influence on their adoption of auction format. In particular, among the three types of format discussed in this paper, the sellers who are more experienced are more likely to adopt fixed-price format, while the sellers who have more listings of identical items are more likely to adopt the BIN format. In terms of auction result, pure and BIN auction have greatest sale rate, while the fixed-price format has the greatest transaction price, given sale. In a word, the choice between the three formats reflect the seller's preference between transaction price and sale rate, given risks consideration. Einav et al. (2012) show that the proportion of pure auction listing in eBay has substantial reduced in the past ten years or so. Moreover, that of the fixed-price listings has greatly increased. They also show that this is mainly caused by the shift of the buyers preference toward posted-price format. Bauner (2011) use auction data of baseball tickets in eBay to show that, in addition to opportunity, buyers heterogeneity is also an important factor in explaining the simultaneous existence of auction and fixed-price sales.

In this paper, we argue that both posted price are the pure auction are a special case of the BIN auction. In a BIN auction, the seller has two choice variables, one is the BIN price, and the other the starting price.[5] Since the BIN price is the upper-bound for possible transaction prices and the reserve price the lower bound, a fixed-price format is simply an auction in which the seller sets the BIN price equal to its reserve price. Moreover, the seller can set a BIN price so high that even the bidder with the highest possible valuation will not exercise BIN. In that case the auction is equivalent to one in which no BIN is posted. In other words, it is essentially a pure auction.

The theoretical literature has identified time-preference as one of the reasons for the sellers to

[5] In the eBay auctions, the seller can also set the secret reserve price, under which the bidder only knows whether his bid surpasses the reserve price, but do not know its value during the auction. However, secret reserve price is rarely used by sellers, and literature also shows that it is actually inferior to the open reserve price. (See, e.g., Lucking-Reiley et al., 2007) In this paper, we only consider auctions with publicly observed reserve prices.

adopt BIN (Mathews, 2004). Under this theory, either the buyer or the seller is time-impatient and, other things being equal, prefers to obtain (or sell) the commodity sooner than later. In that case, BIN offers a chance for the transaction to complete earlier than the duration of the auction.[6] Using this model, this paper endogenizes the seller's optimal listing strategy in terms of their rate of time-impatience. Specifically, the three listing formats are each a solution (under certain values of the seller's rate of time-impatience) of the same optimization problem for the seller. The fixed-price listing is the solution in which the starting price is equal to the BIN price; and two pure auction is the solution in which the BIN price is so high that no bidder exercise it. Similar to Mathews (2006), when there is no time-impatience, BIN serves no function for the seller, and pure auction is the optimal listing format. Moreover, there exists a threshold value of time-impatience so that when the seller is more (less) time-impatient than this threshold, he will adopt fixed-price listing (BIN listing). We therefore completely characterize the sellers' listing strategy as a function of degrees of their time-impatience.

Besides characterizing the seller's optimal listing strategy, we also derive the optimal reserve price (the posted price in the case of fixed-price listing) for each format, and show that the optimal posted price is greater than the optimal reserve price for the BIN auction, which in turn is greater than that for the pure auction. This result offers a clear empirical implication of our model: Since the starting price is equivalent to the open reserve price, if we collect data of auctions of identical items, then after controlling for seller's heterogeneity, the fixed-price listing will have the greatest posted price, followed by the starting price of the BIN auctions, with the pure auction having the lowest starting price.[7]

[6] Another strand of literature rationalizes BIN by showing that it offers a chance to reduce price risk for either the buyer or the seller. See Budish and Takeyama (2001), Mathews and Katzman (2006), Hidvegi et al. (2006), Reynolds and Wooders (2009), and Chen et al. (forthcoming).

[7] Using data from eBay's auction of iPods, Chen et al. (2012) confirm this prediction.

2 Theoretical Model

An item is auctioned among n bidders. The valuation of the item to each bidder i, v_i, is private information, and is independently drawn from $[0, \bar{v}]$ according to density function $f(\cdot)$, with $F(\cdot)$ its distribution function.[8] We make the usual assumption of increasing hazard rate for $f(\cdot)$, that $f(v)/(1 - F(v))$ is strictly increasing in v. The seller has two decision variables. One is the buy-it-now price B, the other is the publicly observed reserve price r, in the form of the starting price. As a rule, the value of r must not be greater than B, i.e., it must be that $r \leq B$.

We are concerned with the eBay-type temporary BIN, so that BIN will disappear whenever a bidder places a bid.[9] We therefore adopt a two-stage model first proposed by Mathews (2004) for the eBay auctions. In the first stage, every bidder can decide whether to purchase the item with the BIN price. If at least one bidder is willing to buy out, the item is sold with price B. In case more than one bidder is willing to do so, they win with equal probability. If no bidder is willing to buy out the item, then BIN disappears and the auction enters into the second stage, in which the bidder compete via a second-price auction with reserve price r. There is a discount between the two stages, with discount rate δ $(0 < \delta \leq 1)$.

The optimal symmetric bidding strategy has been derived by Mathews (2004), and for the reader's convenience we summarized it in the follows.

Proposition 1. *Given B and r, there exists a threshold value of valuation, $\tilde{v} \in [0, \bar{v}]$, so that, for any bidder i, if $v_i \geq \tilde{v}$, it is optimal to buy out the item in the first stage. If $v_i < \tilde{v}$, it is optimal to refrain from placing BIN, and to compete in the second stage if $v_i \geq r$. The value of*

[8] The support of $f(\cdot)$ can be extended to be any $[v_L, v_H]$, $v_L < v_H$, with simple adaptation. We use $[0, \bar{v}]$ for the sake of convenience when we discuss the reserve price.

[9] eBay has changed rule for BIN frequently. At the time the paper was written, BIN disappears in the motors category only if the bid is greater than starting price by a certain amount. In other categories BIN disappears as soon as some bidder places bid.

$\tilde{v}$ is determined by

$$B = \tilde{v} - \left[\frac{n(1 - F(\tilde{v}))}{1 - F(\tilde{v})^n}\right] \int_r^{\bar{v}} F(x)^{n-1} dx. \tag{1}$$

Note that (1) also implies a strictly monotonic relationship between B and the buyout threshold $\tilde{v}$. Therefore, we can assume that the seller's decision variables are $\tilde{v}$ and r, instead of B and r. In that case we denote $B(\tilde{v}, r)$ as the value of the BIN price, when the starting price is r and the buyout threshold is $\tilde{v}$. It can be easily seen that the $B(\tilde{v}, r)$ function is exactly the right-hand side of (1). It can also be shown that when $\tilde{v} = \bar{v}$, then $B(\tilde{v}, r) = \bar{v} - \int_r^{\bar{v}} F(x)^{n-1} dx \equiv \bar{B}(r)$. That is, given the reserve price r, if the BIN price is set equal to or greater than $\bar{B}(r)$, then no bidder will be willing to buy out in the first stage, and the auction essentially becomes a second-price auction with reserve price r. We will identify the sellers who set the reserve price as r and the BIN prices greater or equal to $\bar{B}(r)$ as those who adopt the pure auction format. This is a reasonable classification, as the sellers are charged by eBay for posting BIN. If they find it optimal to set the BIN price greater than $\bar{B}$, they will not post BIN at all.

The following lemma shows that the restriction on B and r translates equivalently to that of $\tilde{v}$ and r:

Lemma 1. *If the values of B, $\tilde{v}$ and r are such that $B = B(\tilde{v}, r)$, then $B > r$ if and only if $\tilde{v} > r$, and $B = r$ if and only if $\tilde{v} = r$.*

Proof. From (1) we know that $B(\tilde{v}, r)$ is strictly increasing in $\tilde{v}$. If $B \geq r$ but $\tilde{v} < B$, then $B = B(\tilde{v}, r) < B(B, r) \leq B$, a contradiction. Therefore, $\tilde{v} \geq B \geq r$. Conversely, if $\tilde{v} \geq r$, then $B(\tilde{v}, r) \geq B(r, r) = r$. Therefore we have shown that $B \geq r$ if and only if $\tilde{v} \geq r$. Moreover, $B = r$ if and only if $B = B(B, r)$, which is equivalent to $B = \tilde{v}$, which in turn is equivalent to $\tilde{v} = r$. QED

Lemma 1 implies that, given the auction rule that B must be greater than r, the bidders who are willing to buy out in the first stage are exactly those who have valuations higher than the reserve price.

The expected revenue of the seller, given B and r, is

$$E\pi = n\int_{\tilde{v}}^{\bar{v}} B(\tilde{v},\, r)F(x)^{n-1}f(x)dx + \delta n\int_{r}^{\tilde{v}}\left(\int_{r}^{x} ydF(y)^{n-1} + \int_{0}^{r} rdF(y)^{n-1}\right)f(x)dx,$$

where the first-term is the seller's expected revenue if the auction ends with the first-stage BIN, and the second term is the expected revenue for the seller if it ends with competitive bidding in the second stage. The seller's optimization problem is to maximize $E\pi$ by choosing the values of B and r, subject to the constraint that $B \geq r$ which, by Lemma 1, is equivalent to $\tilde{v} \geq r$. The Lagrange optimization problem is then

$$\max_{\tilde{v},\, r} L = E\pi + \lambda(\tilde{v} - r); \tag{2}$$

where $\lambda \geq 0$ is the Lagrange multiplier. The first order conditions are

$$\begin{aligned}\frac{\partial L}{\partial \tilde{v}} =& n\delta rF(r)^{n-1}f(\tilde{v}) - nF(\tilde{v})^{n-1}f(\tilde{v})B + \left[1 - F(\tilde{v})^n\right]\frac{\partial B}{\partial \tilde{v}} \\ &+ nf(\tilde{v})\int_{r}^{\tilde{v}} \delta xdF(x)^{n-1} + \lambda = 0; \end{aligned} \tag{3}$$

$$\frac{\partial L}{\partial r} = n\delta F(r)^{n-1}\left[F(\tilde{v}) - F(r)\right] - n\delta rF(r)^{n-1}f(r) + \left[1 - F(\tilde{v})^n\right]\frac{\partial B}{\partial r} - \lambda = 0. \tag{4}$$

From (1) we know that

$$\frac{\partial B(\tilde{v}, r)}{\partial \tilde{v}} = \left[\frac{1 - nF(\tilde{v})^{n-1} + (n-1)F(\tilde{v})^n}{1 - F(\tilde{v})^n}\right]\left[1 + \frac{nf(\tilde{v})\int_r^{\tilde{v}} F(x)^{n-1}dx}{1 - F(\tilde{v})^n}\right]; \tag{5}$$

$$\frac{\partial B(\tilde{v}, r)}{\partial r} = \left[\frac{n(1 - F(\tilde{v}))}{1 - F(\tilde{v})^n}\right]F(r)^{n-1}, \tag{6}$$

Plugging (5) and (6) into (3) and (4) we obtain

$$1 - nF(\tilde{v})^{n-1} + (n-1)F(\tilde{v})^n - (1-\delta)nf(\tilde{v})\left[F(\tilde{v})^{n-1}\tilde{v} - \int_r^{\tilde{v}} F(x)^{n-1}dx\right] + \lambda = 0; \tag{7}$$

$$\delta nF(r)^{n-1}f(r)\left[-r + \frac{1 - F(r)}{f(r)} + \frac{1 - F(\tilde{v})}{f(r)}\left(\frac{1-\delta}{\delta}\right)\right] - \lambda = 0. \tag{8}$$

Let $\pi^*(\delta)$ be the maximum revenue for the seller with rate of time-impatience δ.

Lemma 2 in the follows shows that the optimal starting price must be strictly greater than 0.

Lemma 2. *For any $\delta \in (0, 1]$, $r > 0$.*

Proof. If $r = 0$, then $F(r) = 0$, and from (8) we know that $\lambda = 0$. Plugging $\lambda = 0$ into (7) we know that $\tilde{v} \neq 0$, which in turn implies that $\tilde{v} > r = 0$. Consider (8) again with $\lambda = 0$. Since $\tilde{v} > r$, we can increase the value of r without violating the constraint. Let $r = \varepsilon > 0$ be small enough, then the left-hand side of (8) is positive, meaning that the value of r characterized by the first-order condition cannot be a maximizer. QED

Lemma 2 shows that all types of seller will set a strictly positive starting bid. Since we will characterize the listing strategy as a function of the seller's type (i.e., value of δ), this also implies that the optimal starting price (and posted price in the fixed-price format) is strictly positive for any format of listing.

We have not characterized the sellers who will list the items with fixed price yet, but for the convenience of exposition that follows, we will first derive the optimal posted price for the fixed-price listing.

Lemma 3. *The optimal posted price, r_F, is unique in the fixed-price listing, and satisfies*

$$r_F = \frac{1 - F(r_F)^n}{nF(r_F)^{n-1} f(r_F)}. \tag{9}$$

Proof. The probability that the item is sold, when r_F is the starting bid in a fixed-price listing, is $1 - F(r_F)^n$. Therefore, the seller's expected profit is $(1 - F(r_F)^n)r_F$. Differentiate it with respect to r_F we get

$$\begin{aligned}
1 - &nF(r_F)^{n-1} f(r_F) r_F - F(r_F)^n \\
&= -nF(r_F)^{n-1} f(r_F) \left\{ r_F - \left[\frac{1 - F(r_F)}{f(r_F)}\right] \left[\frac{1 + F(r_F) + \cdots + F(r_F)^{n-1}}{nF(r_F)^{n-1}}\right] \right\} \\
&\equiv -nF(r_F)^{n-1} f(r_F) h(r_F).
\end{aligned}$$

Note that can place $F(r_F)$ in the denominator because Lemma 2 shows that $r_F > 0$ (and therefore $F(r_F) > 0$). Since $(1 - F(r_F))/f(r_F)$ is decreasing by the increasing hazard rate assumption and $(1 + F(r_F) + \cdots + F(r_F)^{n-1})/nF(r_F)^{n-1}$ is strictly decreasing, $h(r_F)$ is strictly increasing. Moreover, $\lim_{r_F \to 0^+} h(r_F) = -\infty$ and $\lim_{r_F \to \bar{v}} h(r_F) = \bar{v} > 0$. Therefore, the equation $h(r_F) = 0$ (or, equivalently, the first-order condition for r_F) has a unique solution, and can be easily seen to satisfy (9). QED

In order to characterize the seller's strategy we first need the following lemma.

Lemma 4. *There exist* $\hat{\delta} \equiv (1 - F(r_F))/(r_F f(r_F)) \in (0, 1)$ *such that any seller whose rate of time impatience* $\delta \in (\hat{\delta}, 1]$ *will never adopt the fixed-price format.*

Proof. We know that $r = \tilde{v} = r_F$ when the seller adopts the fixed-price format. Plugging this into (8) we have

$$\frac{1 - F(r_F)}{\delta f(r_F)} - r_F = \frac{\lambda}{\delta n F(r_F)^{n-1} f(r_F)} \geq 0,$$

which implies

$$\delta \leq \frac{1 - F(r_F)}{r_F f(r_F)} = \hat{\delta}.$$

In other words, only those sellers whose discount factors are less than $\hat{\delta}$ can possibly post fixed-price. Therefore, the sellers with discount factors greater than δ will never adopt fixed-price format. Finally, plugging the value of r_F into $\hat{\delta}$ we know that

$$\hat{\delta} = \frac{1 - F(r_F)}{\frac{1-F(r_F)^n}{nF(r_F)^{n-1}}} = \frac{nF(r_F)^{n-1}}{1 + F(r_F) + F(r_F)^2 + \cdots + F(r_F)^{n-1}} \in (0, 1). \qquad \text{QED}$$

Lemma 4 implies that the sellers who discount the future lightly will not adopt the fixed-price format. This is an intuitive result, as these sellers are willing to allow for the item, if not sold by BIN in the first stage, to enter the second stage to have it sold through competitive bids. Therefore, they never post fixed-price, as under the fixed-price format the item will remain unsold if no bidder exercises BIN in the first stage. More importantly, this also implies that $\tilde{v} > r$ for any seller with $\delta \in (\hat{\delta}, 1]$.

Lemma 5. $\partial \pi^*(\delta)/\partial \delta \geq 0$, *with the inequality strictly hold if* $\delta \in (\hat{\delta}, 1]$.

Proof. By the Envelope Theorem,

$$
\begin{aligned}
\frac{\partial \pi^*(\delta)}{\partial \delta} = \frac{\partial L^*}{\partial \delta} &= n\int_r^{\tilde{v}} \left(\int_r^x yF(y)^{n-1} + \int_0^r ddF(y)^{n-1} \right) f(x)d(x) \\
&= n\int_r^{\tilde{v}} \left(xF(x)^{n-1} - \int_r^x F(y)^{n-1}dy \right) f(x)d(x) \\
&\geq 0,
\end{aligned}
$$

with equality holds if and only if $\tilde{v} = r$. However, by Leema 4 we know that $\tilde{v} > r$ for $\delta \in (\hat{\delta}, 1]$. Therefore $\frac{\partial \pi^*(\delta)}{\partial \delta} > 0$ for $\delta \in (\hat{\delta}, 1]$. QED

Lemma 6. *Let r be the starting price for a seller with rate of time-impatience $\delta \in [\hat{\delta}, 1]$. Then it must be that $r \leq r_F$, with the equality holds only if $\delta = \hat{\delta}$.*

Proof. From Lemma 4 we know that if $\delta \in (\hat{\delta}, 1]$, the optimal solution must be that $\tilde{v} > r$, implying $\lambda = 0$. If $\delta = \hat{\delta}$, then there are two possibilities: either $\tilde{v} > r$, or $\tilde{v} = r$. The former directly implies that $\lambda = 0$. Then the definition of $\hat{\delta}$ and (8) we know that λ also equals to 0. We therefore have, $\lambda = 0$ for all $\delta \in [\hat{\delta}, 1]$. Again, from (11),

$$
r = \frac{1-F(r)}{f(r)} + \frac{1-F(\tilde{v})}{f(r)}\left(\frac{1-\delta}{\delta}\right) \leq \frac{1-F(r)}{f(r)} + \frac{1-F(r)}{f(r)}\left(\frac{1-\delta}{\delta}\right) = \frac{1-F(r)}{\delta f(r)}. \tag{10}
$$

If, contrary to the claim of the lemma, $r_F < r$, then

$$
\frac{1-F(r)}{\delta f(r)} < \frac{1-F(r_F)}{\delta f(r_F)} \leq \frac{1-F(r_F)}{\hat{\delta} f(r_F)} = r_F; \tag{11}
$$

where the first inequality is from the increasing hazard rate assumption. Combine (11) with (10) we know that $r < r_F$, a contradiction. From (10) and (11) we can also easily see that $r = r_F$ if and only if $\delta = \hat{\delta}$, in which case it is also true that $\tilde{v} = r = r_F$. QED

The next proposition, one of the main results of this paper, shows under what conditions it is optimal for the seller to list the item as a pure auction and a BIN auction.

Proposition 2. *The seller with rate of time-impatience $\delta = 1$ will list the item as a pure auction, and the seller with rate of time-impatience $\delta \in (\hat{\delta}, 1)$ will list the item as a BIN auction.*

Proof. Lemma 6 has shown that $\lambda = 0$ for $\delta \in (\hat{\delta}, 1]$. If $\delta = 1$, (7) implies that

$$1 - nF(\tilde{v})^{n-1} + (n-1)F(\tilde{v})^n = 0. \tag{12}$$

Note that the left-hand side of (12) is strictly increasing in $F(\tilde{v})$. Moreover, it equals to 0 when $\tilde{v} = \bar{v}$. In other words, (12) holds only if $\tilde{v} = \bar{v}$, i.e., only if the item is listed as a pure auction.

When $\delta \in (\hat{\delta}, 1)$, the left-hand side of (7), when valued at $\bar{v}$, is

$$-(1-\delta)nf(\bar{v})\left[\bar{v} - \int_r^{\bar{v}} F(x)^{n-1}dx\right] < 0.$$

This means that the optimal value of $\tilde{v}$ must be strictly less than $\bar{v}$ when $\delta \in (\hat{\delta}, 1)$, and therefore the auction cannot be a pure auction. However, since the seller will not list the item in a fixed-price format when $\delta \in (\delta,\hat{} 1)$ by Lemma 4, we know that this item must be listed as a BIN auction. QED

The next proposition characterizes the types of seller who will adopt the fixed-price format. Unlike the cases for pure and BIN auctions, more restrict conditions are required in order to ensure the seller's optimal listing is indeed the fixed-price format.

A1. $f(\cdot)$ is differentiable, and that $F(\cdot)$ is a convex function.[10]

A2. $\frac{2f(r)+rf'(r)}{F(r)} \geq \frac{f(r_F)}{1+F(r_F)}$ for all $r \in (0, r_F)$.

Assumption A2 is satisfied by a broad class of density functions, including the uniform density and, more generally, density functions of the form

$$f(v) = \left(\frac{v - v_L}{\bar{v} - v_L}\right)^\alpha, \quad \alpha \geq 1;$$

where v_L is the lower bound of the density function's support (which is 0 in our model).

Proposition 3. *Assume A1 and A2, then all sellers with rate of time-impatience $\delta \in (0, \hat{\delta}]$ will adopt the fixed-price format.*

[10] This is a strong assumption which is also used to show that the first-order condition approach for the principal-agent problem is valid. (Jewett XX).

Proof. See Appendix.

Propositions 2 and 3 completely characterize the optimal listing strategy as a function of the seller's rate of time-impatience. The perfectly patience seller, with $\delta = 1$, will seek for the highest expected revenue by forcing all bidders to compete in the second stage. The most impatient sellers, with rate of discount smaller than $\hat{\delta}$, aim to minimize the chance that the auction enters into the second stage. Therefore, they choose the fixed-price format, which has the highest feasible reserve price ($r = B$) to make sure that if there is a sale, it occurs at the first stage. Only the sellers with medium rates of time-impatience, will balance price and time-impatience considerations to allow for sale to occur in the first and second stages both with positive probabilities.

Finally, we will compare the relative value of reserve price in the three types of format. This is summarized in the following proposition. Before going into the details, it need to be emphasized that since only seller with $\delta = 1$ will adopt pure auction, its optimal reserve price, r_A, is unique and independent of δ. Moreover, although all sellers with rates of time-impatience $\delta \in (0, \hat{\delta}]$ will adopt their optimal values are identical, and is given by (9). Therefore, only sellers who adopt BIN auction can have different value of optimal reserve price, $r_B(\delta)$, which function is of rate of time-impatience, and is defined only on $(\hat{\delta}, 1)$.

Proposition 4. *Assuming A1. Denote the optimal reserve price in the pure and BIN auctions as r_A and $r_B(\delta)$ respectively. Then for any $\delta \in (\hat{\delta}, 1)$, $r_A < r_B(\delta) < r_F$.*

Proof. The fact that $r_F > r_B(\delta)$ and r_A has already be proved in Lemma 6. To prove Proposition 4, it suffices to show that $r_A < r_B(\delta)$ for $\delta \in (\hat{\delta}, 1)$. For a seller in the pure auction, $\delta = 1$ and $\lambda = 0$. Therefore, from (8) we know that

$$r_A = \frac{1 - F(r_A)}{f(r_A)}. \tag{13}$$

For a seller in a BIN auction, $\delta < 1$ and $\lambda = 0$. Again, from (8) we have

$$r_B = \frac{1 - F(r_B)}{f(r_B)} + \frac{1 - F(\tilde{v})}{f(r_B)} \frac{1 - \delta}{\delta} > \frac{1 - F(r_B)}{f(r_B)} \tag{14}$$

Comparing (13) and (14), and using the increasing hazard rate assumption, we can easily show that $r_B(\delta) > r_A$. QED

Proposition 4 is actually stronger than it must have seem. As the sellers with different rates of time-impatience will choose different reserve prices, a natural question is, when we compare the reserve price of a BIN auction with that of another listing format, which seller's reserve price are we using as basis of comparison. The result in Proposition 4 is reassuring: posted price in a fixed-price listing is always greater than the reserve price of the BIN auction, which in turn is greater than that of a pure auction, regardless of the rate of time-impatience of the seller.

Mathews and Katzman (2006) have proved an opposite result, namely, the optimal reserve price in a BIN auction is smaller than that in a pure auction. Their comparison, however, is problematic, as their comparison is made on the seller with identical rate of time-impatience.[11] As we have shown, a seller, given his rate of time-impatience, have two different types of listing formats as optimal simultaneously, and therefore is inappropriate as a guide for empirical prediction.

3 Conclusion

This paper adopts the time-impatience framework of the BIN auction to characterize the optimal listings for the sellers. It is shown that only the seller who are perfectly patient will adopt the pure auction. If the sellers discount the future, then those who discount the future relatively lightly will adopt the BIN auction, while the more impatient among them will adopt the fixed-price listing. We also show that the posted price in the fixed-price is higher than the optimal reserve price for the BIN auction, which in turn is greater than the reserve price for the pure auction. Not only is the characterization simple and intuitive, it also offers clear empirical implication regarding the relatively value of reserve prices for the three listing formats.

[11] That is, they show that, for a seller who adopts pure (BIN) auction, the reserve price he sets will be higher (lower) than the reserve price if he switches to a BIN (pure) auction.

As mentioned in the Introduction, another strand of literature rationalizes BIN by showing its function in reducing price risk for either the bidders or the sellers. It will be a worthwhile endeavour to characterize the seller's optimal listing strategy as a function of his degree of risk aversion. Note that there are two types of risk in auctions: sales risk and price risk. While the pure auction has the lowest sales risk, it has the highest price risk. On the contrary, the fixed-price format has the lowest price risk but the highest sales risk. In other words, we cannot expect a monotic ranking of the three formats in term of their riskiness. Therefore the characterization will likely be more difficult along the line of risk consideration than the time-impatience consideration. However, such characterization can enhance our understanding of the sellers' strategic behavior in online auction.

Appendix: Proof of Proposition 3

We will first show that

$$\frac{(n-1)\hat{\delta} f(r_F) r_F}{1-\hat{\delta}} \geq 1 + F(r_F). \tag{15}$$

Using (9) and the definition of $\hat{\delta}$, (15) is equivalent to

$$J(F(r_F)) \equiv (n-2) - nF(r_F) + nF(r_F)^{n-1} - (n-2)F(r_F)^n \geq 0. \tag{16}$$

First note that $J(F(\bar{v})) = J(1) = 0$. Therefore, in order to prove (16), we only need to show that $J(x)$ is a decreasing function for all $x \in [0,1]$. For this purpose, first note that $J'(F(\bar{v})) = J'(1) = 0$. Second, $J''(x)$ can be easily seen to be positive, meaning that $J'(x)$ is an increasing function. Combine this with the fact that $J'(1) = 0$, we know that $J'(x) \leq 0$, which shows that $J(x)$ is indeed a decreasing function, and (16) is proved.

Now we turn to our main proof. First note that the expected revenue from a fixed-price auction is independent of δ. Moreover, by Lemma 5 we know that the seller's expected revenue is an increasing function of δ, and is strictly increasing for $\delta \in (\hat{\delta}, 1]$. Therefore, if we can prove that the optimal listing of the seller with time-impatience $\hat{\delta}$ is the fixed-price format, then we complete the proof. The reason is simple: If any seller with $\delta_0 \in (0, \hat{\delta})$ is not using the fixed-price format, then it must be that, for this seller, $\tilde{v}(\delta_0) > r(\delta_0)$. Then from the proof of Lemma 5 we know that the seller's revenue is strictly increasing. If the optimal listing strategy of the seller with $\hat{\delta}$ is the fixed-price format, then his revenue must be strictly higher than the δ_0-seller. But since the expected revenue from the fixed-price format is independent of δ, we know that δ_0-seller can improve his revenue by switching to the fixed-price format.

Now we show that the $\hat{\delta}$-seller's optimal listing strategy is the fixed-price format. That is, we need to show that $\tilde{v}(\hat{\delta}) = r(\hat{\delta})$ is the unique solution for (2). The necessity part is easy to show: simply plugging in $\tilde{v} = r = r_F$ into (7) and (8) to check that they satisfy the first-order conditions.

To show that $\tilde{v} = r$ is the unique solution, we first note that by the proof of Lemma 6 we have $\lambda = 0$. Therefore,

$$F(\tilde{v}) = \frac{1}{1-\hat{\delta}} - \frac{\hat{\delta}}{1-\hat{\delta}}(rf(r)) + F(r). \tag{17}$$

Denote the right-hand side of (17) as $H(r)$. Then since $f(v) > 0$ for all v, we cam write $\tilde{v} = F^{-1}(H(r))$. Moreover,

$$\frac{d\tilde{v}}{dr} = F^{-1}(H(r))' \cdot H'(r) = \frac{H'(r)}{f(\tilde{v})}. \tag{18}$$

The optimal revenue of the seller, when viewed as a function of r, satisfies

$$\frac{\partial \pi}{\partial r} = \left(\frac{\partial \pi}{\partial \tilde{v}}\frac{d\tilde{v}}{dr} + \frac{\partial \pi}{\partial r}\right)_{\tilde{v}=F^{-1}(H(r))} = \frac{\partial \pi}{\partial \tilde{v}}\Big|_{\tilde{v}=F^{-1}(H(r))} \cdot \frac{H(r)}{f(\tilde{v})}, \tag{19}$$

where the last equality comes from (18).

By (A-2) we know that

$$H'(r) = -\frac{\hat{\delta}}{1-\delta}\big(2f(r) + rf'(r)\big) < 0.$$

If we can show that $\partial\pi/\partial\tilde{v}\big|_{\tilde{v}=F^{-1}(H(r))}$ is strictly increasing, then from (19) we can know that the seller's first-order for r, $\partial\pi/\partial r = 0$, has a unique solution which from the necessity part, must be $r = r_F$. Straightforward calculation shows that

$$\frac{\partial \pi}{\partial \tilde{v}}\Big|_{\tilde{v}=F^{-1}(H(r))} = 1 - nH(r)^{n-1} + (n-1)H(r)^n - (1-\hat{\delta})nf(\tilde{v})\left[F(\tilde{v})^{n-1}\tilde{v} - \int_r^{\tilde{v}} F(x)^{n-1}dx\right],$$

who derivative against r is

$$A(r) + n(1-\hat{\delta})F(\tilde{v})B(r), \tag{20}$$

where

$$A(r) = -nH'(r)\Big[(n-1)F(\tilde{v})^{n-2}(1-F(\tilde{v})) + (1-\delta)\frac{f'(\tilde{v})}{f(\tilde{v})}I(r)\Big],$$

$$B(r) = \frac{\hat{\delta}}{1-\hat{\delta}}(2f(r) + rf'(r))(n-1)\tilde{v}F(\tilde{v})^{n-2} - F(r)^{n-1},$$

and $I(r) = F(\tilde{v})^{n-1}\tilde{v} - \int_r^{\tilde{v}} F(x)^{n-1}dx$. To show that (20) is strictly positive, it suffices to show that $B'(r) \geq 0$. Again, by straightforward calculation, we have

$$
\begin{aligned}
B(r) =& F(\tilde{v})^{n-2}\left[\frac{(n-1)\hat{\delta} f(r_F) r_F}{1-\hat{\delta}} \cdot \frac{2f(r)+f'(r)}{f(r_F)} \cdot \frac{\tilde{v}}{r_F} - F(r)\big(\frac{F(r)}{F(\tilde{r})}\big)^{n-2}\right] \\
&\geq F(\tilde{v})^{n-2}\left[(1+F(r_F)) \cdot \frac{2f(r)+f'(r)}{f(r_F)} - F(r)\right] \geq 0,
\end{aligned}
$$

where the first inequality comes from (17), and the second inequality comes from assumption A2.

QED

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