Does A Stricter Environmental Policy Discourage Foreign Direct Investment?

Ya-Po Yang*
Institute of Business and Management, National University of Kaohsiung, Taiwan

Tsung-Hsiu Tsai
Department of Applied Economics, National University of Kaohsiung, Taiwan

Abstract
We set up a two-country model in which a foreign firm chooses FDI or export to enter the host market and plays Cournot competition with \( n \) host firms. Facing the emission tax policy of the host country, all firms located in the host country use identical emission abatement technology to save tax payments. The main findings in this paper are as follows. The impact of emission tax on the foreign firm’s optimal entry mode is determined by the emission tax effect, abatement effect, and tax avoidance effect. When the trade cost is low, a rise in the emission tax rate can only change the foreign firm’s entry mode from FDI to export. When the trade cost is high and the abatement technology is efficient, an increase in the emission rate tax may encourage the foreign firm to switch from export to FDI. Under certain circumstances, raising the emission tax, which changes the foreign firm’s entry mode from FDI to export, may result in a higher level of total emissions in the host country. Finally, if an increase in the emission tax rate changes the firm’s entry mode, then it may raise consumer surplus in the host country.

Keywords: emission tax, entry mode, emission abatement, foreign direct investment

* Corresponding author, Institute of Business and Management, National University of Kaohsiung, 700, Kaohsiung University Road, Kaohsiung 811, Taiwan. Email: yapo@nuk.edu.tw Tel: 886-7-5916587. Fax: 886-7-5919342
1. Introduction

Under the trend of globalization and WTO regulations, many countries are repealing import quotas and tariffs and starting to allow multinational enterprises to enter their markets. There are several entry modes can be taken up by multinational enterprises, such as export, foreign direct investment (FDI), acquisition, joint venture, international licensing, and so on. As foreign firms have multiple choices to enter a market, exploring the optimal entry mode has drawn much attention in the academia of economics, especially on the choice between export and FDI.

Along with the development of economies and the wave of international markets liberalization, many countries have set up environmental regulations such as emission taxes, emission standards, emission-abatement subsidies, and emission permits in order to maintain their environmental quality by taking the entry mode of foreign firms into account. Among these regulations, the imposition of an emission tax is the most popular method. Even so, countries still have various degrees of severity in their environmental regulations. As it is well known that trade liberalization may reduce traditional incentives such as ‘quota-jumping’ or ‘tariff-jumping’ for foreign firms to undertake FDI, many scholars have turned to explore whether the degree of severity in an environmental regulation is an important determinant for a foreign firm’s choice of entry mode. By taking firms’ emission abatement behavior into account, this paper explores how different levels of an emission tax imposed by the host country impacts a foreign firm’s optimal entry mode (FDI or export) and the associated welfare of the host country.

There are a lot of articles in the trade literature studying the entry choice of foreign firms such as Motta (1992), Horstmann and Markusen (1992), Ethier and Markusen (1996), Qiu and Tao (2001), Sinha (2010) and so on. There are also many studies in the literature discussing the impact of environmental policies on foreign firms’ entry decisions. Under a two-country and two-firm structure, Markusen et al. (1993) present the influence of a host country’s pollution tax on both foreign and host firms’ location decision and find that a small adjustment on the pollution tax may change firms’ location decision and the related welfare. In the papers of Markusen et al. (1995) and Hoel (1997), they both take firms’ plant locations into account and discuss two countries’ environmental regulation race. Markusen et al. (1995) find that when the marginal damage of pollution is high (low), both countries will set stricter (lax) environmental regulations. Hoel (1997) reaches a similar conclusion. Greaker (2003) builds a two-country and two-firm duopoly model to discuss the two governments’ decisions on environmental regulations, given that their domestic firms may
threaten to relocate. Kayalica and Lahiri (2005) build a three-country model to discuss how two countries (host country and domestic country) compete in environmental regulations in order to attract FDI, showing that when the number of foreign firms located in the host country is given, then the environmental regulation set by the host country will be stricter than that in the domestic country, but if the host country allows foreign firms to freely enter the market, then the result will be reversed. Dijkstra et al. (2011) discuss how the foreign firm’s entry modes affect two countries’ decisions on pollution taxes, finding that the foreign firm may undertake FDI to induce the host country to set a higher level of emission tax rate than its home country.

In the empirical literature, most articles focus on testing the ‘Pollution Heaven Hypothesis’ (PHH). The empirical studies of List et al. (2003) and Cole and Elliott (2005) support PHH, but Eskeland and Harrison (2003) and Javorcik and Wei (2004) note that there is no significant relationship between environmental regulations and firms’ location decisions. McConnell and Schwab (1990), Friedman et al. (1992), and Levinson (1996) find that FDI is sometimes positively related to environmental regulations. However, Dean et al. (2009) offer that an environmental regulation and FDI can be either positively or negatively related. By using state data in the U.S., Keller and Levinson (2002) examine if a low emission abatement cost can attract FDI. They take abatement cost as a proxy to represent the degree of severity of environmental regulations in different states, finding that for low polluting industries, the severity of the environmental regulation has no significant impact on FDI, but for highly polluting industries this relationship is negative. Xing and Kolstad (2002) gather 26 countries’ data and use the amount of emissions as a proxy to represent the severity of an environmental regulation. They find that for highly polluting industries, a stricter environmental regulation does affect a U.S. firm’s incentive for not undertaking FDI, but it is not significant in low polluting industries.

From the above literature review, the relationship among the entry mode of multinational enterprises and environmental policies is an important issue in both international trade and environmental economics. Although there are a lot of empirical papers examining the relationship between emission abatement and the entry mode of the foreign firm, however, to the best of our knowledge, there is no theoretical article studying this topic. Therefore, the purpose of this paper is to explore the impact of an environmental regulation on foreign firms’ entry choice, by taking firms’ emission abatement behavior into consideration. We build a two-country model with one foreign firm and $n$ host firms play Cournot Competition in the host country. Given this structure, we intend to find out how an
emission tax rate policy of the host country affects a foreign firm’s entry choice, as well as the associated environmental quality and consumer surplus in the host country.

There are two stages in our model. In the first stage, a foreign firm chooses FDI or export to enter the host market. In the second stage, the foreign firm and n host firms compete in a Cournot fashion. This paper presents some interesting findings. First of all, a rise in the emission tax rate in the host country creates three effects - emission tax effect, abatement effect, and tax avoidance effect - that determine the foreign firm’s entry mode. Second, when the trade cost is low, the increased emission tax rate can only induce the foreign firm to switch from FDI to export. Nevertheless, if the trade cost is high and the emission abatement technology is more efficient, then even a high emission tax rate can attract the foreign firm to adopt FDI. Moreover, when the number of host firms is no less than 3, if the trade cost is high enough and the emission abatement technology is very efficient, then it is possible that a newly enforced stricter emission tax policy of the host country may attract the foreign firm to adopt FDI. Third, when abatement technology is more efficient, if raising the emission tax rate induces the foreign firm to switch from FDI to export, then it may raise the total amount of emissions in the host country. Therefore, from the host country’s point of view, raising the emission tax to drive out the foreign firm may not be a good strategy to improve environmental quality. Our findings show that the entry choice of the foreign firm and firms’ emission abatement behavior both play key roles in control the host country’s environmental quality.

This paper is organized as follows. Section 2 introduces the model setting. Section 3 discusses market equilibrium and the impact of the emission tax on the foreign firm’s optimal entry choice. Section 4 discusses the impact of the emission tax adjustment on the associate welfare of the host country. Finally, section 5 concludes.

2. Model setting

We consider a two-country model. A foreign firm $f$ decides one way (export or FDI) to enter the host country and competes with the host firms. There are $n$ host firms in the host country, denoted as $h_i, i = 1, 2, ..., n$, respectively. All firms produce homogenous goods and compete in a Cournot fashion in the host market.  

The market demand in the host country is $P(Q) = a - Q$, where $Q = \sum q_{hi} + q_f$, $q_f$ and $q_{hi}$ are the output of the foreign firm $f$ and a representative host firm $h_i$, respectively.

---

1 The market structure is like that of Qiu and Tao(2001), Mukherjee and Sinha (2007), Sinha (2010), and Dijkstra et al. (2011).
Assume that all firms face a constant marginal production cost \( c \). Without loss of generality, each one output produced generates one unit of emission. In order to regulate the environment, the host government imposes a specific emission tax rate \( t \) on the pollution emitted in the host country, while the foreign country has no environment regulation policy.

Firms located in the host country can abate emissions to reduce their emission tax payment. Assume that each firm had the same abatement cost function \( \phi(b) = \frac{\alpha \cdot b^2}{2} \), where \( \alpha \) represents the abatement technological efficiency, a lower (high) value of \( \alpha \) implies a more(less) efficient abatement technology. Thereby, for any firm \( j \) located in the host country, if it produces \( q \) unit of product and abates \( b \) unit of pollution, then its total emission is \( (q - b) \) and total cost of producing \( q \) is:

\[
\psi'(q, b, t) = c \cdot q + t(q - b) + \frac{\alpha \cdot b^2}{2} \tag{1}
\]

where the first term is production cost, the second term is the emission tax payment, and the last term is the abatement cost. Differentiating \( \psi'(q, b, t) \) with respect to \( b \), we get \( \frac{\partial \psi'(q, b, t)}{\partial b} = \psi'_2 = -t + \alpha \cdot b \). By solving \( \psi'_2 = 0 \), firm \( j \)'s optimal emission abatement level that minimizes total cost for \( q \) is \( b^* = \frac{t}{\alpha} \). Because all firms located in the host country face the same emission tax rate and use the same abatement technology, hereafter we simplify \( b^* \) as \( b(t) = \frac{t}{\alpha} \) for all \( j \). As \( \frac{db}{dt} = \frac{1}{\alpha} > 0 \), a firm’s emission abatement increases with \( t \). Substituting \( b(t) = \frac{t}{\alpha} \) into \( \psi' \), firm \( j \)'s total cost function becomes:

\[
\psi'(q, b(t), t) = c \cdot q + t q - \frac{t^2}{2\alpha} \tag{2}
\]

where \( \psi'_1 = \frac{\partial \psi'}{\partial q} = c + t \) is firm \( j \)'s total marginal cost including marginal production cost and emission tax rate. \( \frac{d\psi'}{dt} = q - \frac{t}{\alpha} > 0 \) shows that a rise in the emission tax rate will raise the total cost of \( q \).\(^3\) This total cost function also reveals that \( \frac{t^2}{2\alpha} \) is emission tax payment saving from abatement.

\(^2\) The abatement cost function is the same as in Jung et al. (1996), Lahiri and Ono (2007), and Ohori (2011).

\(^3\) We assume that \( t \) is not high enough that causes \( q - b = q - \frac{t}{\alpha} > 0 \), i.e. the emission is not complete eliminated.
In addition to the above setting, if the foreign firm $f$ chooses FDI to enter the market, then it needs to pay a fixed cost $G$ other than $\psi^f(q_f, b(t), t)$. If the foreign firm $f$ decides to export, then per unit of export needs to bear a trade cost $\tau$ (including transportation cost and tariff imposed by the host country) so that its total cost for producing $q_f$ is $(c + \tau)q_f$.

Based the above setting, if the foreign firm decides to adopt FDI, then its total profit function is:

$$\Pi^f = p(Q)q^f_f - \psi^f(q^f_f, b(t), t) - G,$$  \hspace{1cm} (3)

Alternatively, if the firm adopts export, then its objective function is:

$$\pi^E_f = p(Q)q^E_f - (c + \tau)q^E_f.$$  \hspace{1cm} (4)

For a representative host firm $hi$, its objective is to maximize:

$$\pi_{hi} = p(Q)q_{hi} - \psi_{hi}(q_{hi}, b(t), t).$$  \hspace{1cm} (5)

There are two stages in the game. In the first stage, the foreign firm decides its profit-maximizing entry mode between export and FDI, given the emission tax $t$ imposed by the host country. In the second stage, given $t$ and the foreign firm’s entry mode, all firms compete in a Cournot fashion in the host country’s market. The subgame perfect equilibrium of this model is solved by backward induction. Based on this setting, we first explore the foreign firm’s optimal entry strategy at different emission tax levels and then demonstrate the impacts from the adjustment of the emission tax rate on the entry mode of the foreign firm and the associated welfare of the host country.

3. Market equilibrium and optimal entry mode of the foreign firm

In this section, we first solves the market equilibrium under the two entry modes respectively, and then finds out the optimal entry mode of the foreign firm by backward induction.

3.1 Market equilibrium

3.1.1 FDI mode

If the foreign firm chooses FDI, then by substituting (2) into (3) and (5), we have the first order conditions of the firms’ profit maximization as follows:

\[4\] To simplify the analysis, we do not consider the possibility that the host country may use strategic regulations to deter foreign firms’ entry.
\[ \frac{\partial \pi_f^E}{\partial q_f^E} = a - Q^E - q_f^E - (c + t) = 0 \]  
\[ \frac{\partial \pi_{hi}^E}{\partial q_{hi}^E} = a - Q^E - q_{hi}^E - (c + t) = 0. \]

According to the symmetric property, we can solve the equilibrium outputs of all of firms as \( q_{hi}^E = q_h^E = q_f^E = \frac{(a-c-t)}{n+2} = \frac{A-t}{n+2} \), where \( A \equiv a - c \), and further get the equilibrium profits of firm \( f \) and a representative host firm as

\[ \Pi_f^E(t;\alpha,G) = \pi_f^E(t;\alpha) - G = \frac{(A-t)^2}{(n+2)^2} + \frac{t^2}{2\alpha} - G \quad \text{and} \quad \pi_{hi}^E(t,\alpha) = \frac{(A-t)^2}{(n+2)^2} + \frac{t^2}{2\alpha}, \]

respectively.

Because total pollution abated by every firm is \( \frac{t}{\alpha} \), thus total emission emitted by each firm is \( P_{hi}^E = P_f^E = \frac{A-t}{(n+2)} - \frac{t}{\alpha} \). The comparative-static analysis shows to us the following lemma.

**Lemma 1:** \( \frac{dq_f^E}{dt} < 0, \quad \frac{d\Pi_f^E}{dn} < 0, \quad \frac{d\Pi_f^E}{dt} < 0, \quad \frac{d\Pi_f^E}{dt} > 0. \)

The intuition of the first three results of Lemma 1 is so intuitive. The result \( \frac{d\Pi_f^E}{dt} = -2\frac{(A-t)}{(n+2)^2} + \frac{t}{\alpha} > 0 \) shows to us, the first term is negative, we call this the “emission tax effect”; the second term is positive, which we call the “abatement effect”. As these two effects are opposite, the impact of \( t \) on \( \pi^E \) is ambiguous. If the emission tax effect dominates (is dominated by) the abatement effect, then firms’ profit will decrease (increase) with \( t \).

### 3.1.2 Export mode

When the foreign firm adopts export as its entry mode, then according to (4) and (5), we can derive the following first-order conditions:

\[ \frac{\partial \pi_f^E}{\partial q_f^E} = a - Q^E - q_f^E - (c + \tau) = 0, \quad \text{(8)} \]

\[ \frac{\partial \pi_{hi}^E}{\partial q_{hi}^E} = a - Q^E - q_{hi}^E - (c + t) = 0. \quad \text{(9)} \]

Therefore, the equilibrium outputs can be solved as \( q_f^E = \frac{A + nt - (n+1)\tau}{n+2} \) and \( q_{hi}^E = \frac{A - 2t + \tau}{(n+2)^2} \), and the equilibrium profits are \( \pi_f^E(t;\tau) = \frac{(A + nt - (n+1)\tau)^2}{(n+2)^2} \) and \( \pi_{hi}^E(t;\alpha,\tau) = \frac{(A - 2t + \tau)^2}{(n+2)^2} + \frac{t^2}{2\alpha} \), respectively. Besides, total pollution emitted in the host
country by the each firm is \( P_h^E = \frac{A - 2t + \tau}{n + 2} - \frac{t}{\alpha} \) and \( P_f^E = 0 \).

A comparative-static analysis shows us the following results.

**Lemma 2:**
\[
\frac{d\pi_h^E}{dt} < 0, \quad \frac{dq_h^E}{dt} > 0, \quad \frac{dq_f^E}{dt} > 0, \quad \frac{dq_f^E}{d\tau} < 0, \quad \frac{d\pi_h^E}{dt} > 0, \quad \frac{d\pi_f^E}{dt} > 0, \quad \frac{d\pi_f^E}{d\tau} > 0, \quad \frac{d\pi_f^E}{d\tau} < 0.
\]

The reason for an uncertain sign of \( \frac{d\pi_h^E}{dt} \) is similar to that of \( \frac{d\pi_f^E}{dt} \) in Lemma 1, nevertheless, the increase in the emission tax can definitely enhance the foreign firm’s competition so that \( \frac{d\pi_f^E}{dt} > 0 \). The intuition of the other results of Lemma 2 is straightforward, it needs not to be mentioned here.

### 3.2 Optimal entry mode of the foreign firm

The market equilibriums reveal that the foreign firm’s profit in each entry mode depends on the emission tax rate \( t \) and parameters \( n, \tau, \) and \( \alpha \). To save the space, we only focus on the cases that all firms located in the host country do not completely eliminate their emissions and no firm exits the market no matter which entry mode firm \( f \) adopts, put it in another way, the emission tax rate is confined to be less than the rate whereby the emissions are completely eliminated and the output levels of all firms are positive.\(^5\) If we respectively denote \( t^F(\alpha) \) and \( t^E(\alpha) \) as the critical tax level a firm completely cleans up the pollution under the two modes, let \( P_h^E = P_f^E = 0 \) and \( P_f^E = 0 \), we can solve \( t^F(\alpha) = \frac{\alpha A}{n + 2 + \alpha} \) and \( t^E(\alpha) = \frac{\alpha (A + \tau)}{n + 2 + 2\alpha} \).\(^6\) Therefore, \( t \) is confined to be less than \( T(\alpha) \equiv \min(t^F(\alpha), t^E(\alpha)) \).

Given the emission tax rate, the foreign firm’s optimal entry mode depends on which mode making a higher profit. We define the firm’s profit difference under the two modes as:

\[
\Delta \Pi = \pi_f^E(t; \alpha) - G - \pi_f^E(t; \tau) = \frac{[A - t]^2}{(n + 2)^2} + \frac{t^2}{2\alpha} - G - \frac{[A + nt - (n + 1)\tau]^2}{(n + 2)^2}.
\]  

(10)

Based on (10), the decision rule of the foreign firm’s optimal entry mode is:

---

\(^5\) We assume that under any level of emission tax and trade cost, the firms’ equilibrium outputs must be positive.

\(^6\) In the FDI mode, if \( t > t^F(\alpha) \) then both the tax effect and abatement effects will disappear. In the export mode, if \( t > t^E(\alpha) \), then the export competition advantage will disappear, therefore, the raise of tax rate will not affect the equilibrium output in the mode, to simplify the analysis, we do not pay attention to them.
If \( \Delta \Pi > (<) 0 \), then adopts FDI (export) \( (11) \)

In order to find the foreign firm’s optimal entry mode, we first observe the firm’s operation profit difference in the two modes and then take the fixed cost of FDI \( (G) \) into consideration. We define the difference of firm \( f \)'s operation profit in the two modes as:

\[
\Omega(t; \alpha, \tau) = \pi_f^F(t; \alpha) - \pi_f^E(t; \tau) = \frac{[A - t]^2}{(n + 2)^2} + \frac{t^2}{2\alpha} - \frac{[A + nt - (n + 1)\tau]^2}{(n + 2)^2}.
\]  \( (12) \)

\( \Omega(t; \alpha, \tau) \) can be treated as the incentive for firm \( f \) to adopt FDI without fixed cost. As we assume that the emissions are not fully eliminated and equilibrium output of each firm is positive, thus the domain of \( \Omega(t; \alpha, \tau) \) is on \( t \in [0, \tilde{t}(\alpha)] \), where \( \tilde{t}(\alpha) = \min(t^F(\alpha), t^E(\alpha)) \) and \( \tau < \frac{A}{n + 1} \) assures a positive output for firm \( f \) in the export mode when there is no emission tax policy in the host country.

Through (12), we have:

\[
\frac{\partial \Omega(t; \alpha, \tau)}{\partial t} = -\frac{2(A - t)}{(n + 2)^2} + \frac{t}{\alpha} - \frac{2n[A + nt - (n + 1)\tau]}{(n + 2)^2}. \tag{13}
\]

(13) tells us that effect of the raise of the tax rate on the incentive for firm \( f \) to adopt FDI is uncertain. The first two terms is the aforementioned negative emission tax effect and positive abatement effect, respectively, and the last term is the negative tax avoidance effect when firm \( f \) conducts export. Based on (11), (12), and (13), the property of \( \Omega(t; \alpha, \tau) \) on \( t \in [0, \tilde{t}(\alpha)] \) is presented in Lemma 3, 4, and 5.

**Lemma 3**: For any given \( n \) and \( \tau \), defining \( \alpha^\tau = \frac{(n + 2)\tau}{A - \tau} \) as the critical \( \alpha \) that make \( P_f^F = 0 \) at \( t = \tau \), then we have:

(i) for \( \alpha = \alpha^\tau \), \( \tilde{t}(\alpha) = t^F(\alpha) = t^E(\alpha) = \tau \); (ii) \( \forall \alpha \in (0, \alpha^\tau) \), \( \tilde{t}(\alpha) = t^F(\alpha) < \tau \); (iii) \( \forall \alpha \in (\alpha^\tau, \infty) \), \( \tilde{t}(\alpha) = t^E(\alpha) > \tau \).

Lemma 3 is about the domain of \( \Omega(t; \alpha, \tau) \). The intuition of Lemma 3 is as follows. Because the host firm always locates in the host country, we can illustrate the intuition of Lemma 3 from the viewpoint of the host firm. When \( \alpha = \alpha^\tau \), then by the definition of \( \alpha^\tau \), if firm \( f \) adopts FDI, then at \( t = \tau \), host firm (and firm \( f \) also) just completely abates its emission - that is, \( t^F(\alpha^\tau) = \tau \). At this moment, if firm \( f \) switches to the export mode, then the trade cost firm \( f \) faced is equal to the emission tax rate (i.e. \( \tau = t \)), thus, the total marginal cost of firm \( f \) is the same as he FDI mode, and the host firms produce the same level of output as in the FDI mode and clean up all of their emissions - this is to say,
If firm $f$ adopts FDI, then the host firm’s output level is the same as when $\alpha = \alpha^e$ and $t = \tau$, however, the abatement technology is less efficient than before, and so the host firm to completely eliminate its emission, it needs a higher tax rate - that is, $t^e(\alpha) > \tau$. Under $\alpha > \alpha^e$ and $t = \tau$, if the foreign firm switches to export, then the output of the host firm is the same as that in FDI mode, since the abatement technology is less efficient that before , it is obviously, for the host firm to fully abate its emission needs a higher tax rate which is higher than trade cost( i.e. $t^e(\alpha) > \tau$). Moreover, under $\alpha > \alpha^e$, if $t = t^F(\alpha) > \tau$, then the output level of the host firm in the FDI mode is greater than that in the export mode, since at $t = t^F(\alpha)$ the pollution (which is equal to output) generated by the host firm in FDI mode is fully abated, a smaller tax rate is needed to complete abate a smaller emission in the export mode. Therefore, we have $t^F(\alpha) > t^e(\alpha) > \tau$ when $\alpha > \alpha^e$. Alternatively, if $\alpha < \alpha^e$, then we can follow the same logic as above to find $t^F(\alpha) < t^e(\alpha) < \tau$.

**Lemma 4:** Given $n$ and $\tau \in (0, \frac{A}{(n+1)})$:

(i) If $\tau \in (0, \frac{A}{2(n+1)})$, then $\Omega(t;\alpha,\tau)$ is decreasing on $[0,\bar{t}(\alpha)]$ for all $\alpha > 0$.

(ii) If $\tau \in (\frac{A}{2(n+1)}, \frac{A}{(n+1)}]$, then defining $\alpha^H$ as the critical $\alpha$ that makes

$$\lim_{t \to \bar{t}(\alpha)} \frac{\partial \Omega}{\partial t} = 0,$$

we have $\alpha^H < \alpha^e$, and for $\alpha \in (0,\alpha^H)$, $\Omega(t;\alpha,\tau)$ has an interior minimum $\Omega_M > 0$ at $t^M(\alpha) \in (0,\bar{t}(\alpha))$; for $\alpha \in (\alpha^H, \infty)$, $\Omega(t;\alpha,\tau)$ is decreasing on $t \in (0,\bar{t}(\alpha))$.

The economic intuition of Lemma 4 is as follows. According to (12), when $t = 0$, $\Omega(0;\alpha,\tau) = \frac{A^2}{(n+2)^2} - \frac{[A-(n+1)\tau]^2}{(n+2)^2} > 0$, It shows to us that, without fixed cost FDI, firm $f$ will adopt FDI. Moreover, $\frac{\partial \Omega(0;\alpha,\tau)}{\partial t} < 0$ (by (13)) also shows that the incentive for firm $f$ to adopt FDI initially decreases by a rise in the emission tax rate, because when the tax rate is low, both the emission tax effect and the tax avoidance effect are great while the abatement effect is small, causing a totally negative effect on the incentive to adopt FDI. When the emission tax rate is continuously raised, the emission tax effect will decrease, but both the tax avoidance effect and abatement effect will increase. If the trade cost is relatively low (i.e. $\tau < \frac{A}{2(n+1)}$), then the aggregation of the two negative effects always dominates the positive abatement effect owing to a great tax avoidance effect. However, if the trade cost is high...
enough (i.e. \( \frac{A}{2(n+1)} < \tau < \frac{A}{(n+1)} \)) and abatement technology is more efficient (\( \alpha < \alpha^u \)), then the tax avoidance effect increases slowly, whereas the abatement effect increases rapidly with the tax rate. As long as the tax rate is high enough, the aggregation of the two negative effects will finally be dominated by the positive abatement effect, this is why the incentive to adopt FDI first decreases and then increases with \( t \) when \( \tau \) is high and \( \alpha \) is low.

In addition to the effect of the emission tax rate adjustment on the operation profit difference of the two entry modes. We further see the values of \( \Omega(t;\alpha,\tau) \) on the two end points (i.e. \( t = 0 \) and \( t = \bar{t}(\alpha) \)) and then combine them with the curvature of \( \Omega(t;\alpha,\tau) \) and fixed cost of FDI to observe the full incentive for the entry mode decision. Given \( n, \alpha, \tau \in (0,\frac{A}{(n+1)}) \), and \( t \in [0,\bar{t}(\alpha)] \), and denoting \( \Omega_o = \Omega(0;\alpha,\tau) \) and \( \Omega_c = \Omega(\bar{t}(\alpha);\alpha,\tau) \),\(^7\) we have Lemma 5.

**Lemma 5:** Given \( n \) and \( \tau \in (0,\frac{A}{(n+1)}) \):

(i) When \( n \leq 2 \), \( \Omega_o > \Omega_c \) for all \( \alpha \).

(ii) When \( n \geq 3 \), if \( \tau \in (0,\frac{(3n+2)A}{4(n+1)}) \) then \( \Omega_o > \Omega_c \) for all \( \alpha \); if \( \tau \in (\frac{(3n+2)A}{4n(n+1)},\frac{A}{(n+1)}) \), then defining \( \alpha^l \) as such makes \( \Omega_o(\alpha^l) = \Omega_c(\alpha^l) \), and then \( \alpha^l < \alpha^u < \alpha^r \) and \( \Omega_o < (>)\Omega_c \) for \( \alpha < (>)\alpha^l \).

(iii) Defining \( \alpha^u \) as such that makes \( \Omega(\bar{t}(\alpha^u);\alpha^u,\tau) = 0 \), then \( \alpha^u > \alpha^r \) and \( \Omega_c > (>)0 \) for \( \alpha < (>)\alpha^u \). (Please see the Appendix for the proof.)

Lemma 5 shows that when there is a smaller number of host firms (i.e. \( n \leq 2 \)), no matter what the trade cost and the abatement technology are, the incentive for firm \( f \) to adopt FDI at \( t = 0 \) (i.e. no emission tax policy) is greater than that at \( t = \bar{t}(\alpha) \), i.e. \( \Omega_o > \Omega_c \).

However, when there is a greater number of host firms (i.e. \( n \geq 3 \)), then as long as the trade cost is high enough (\( \frac{(3n+2)A}{4b(n+1)} < \tau < \frac{A}{(n+1)} \)) and the abatement technology is efficient enough (\( \alpha < \alpha^l \)), the ranking of the two incentives will reverse (i.e. \( \Omega_c > \Omega_o \)).

The economic intuitions of the above statements are as follows. As the number of host firms increases, the difference in operation profit between the two entry modes will be

\(^7\) By definition, \( \Omega_o \) is the difference of firm \( f \)'s operation profit between the two entry modes when the emission tax is zero; \( \Omega_c \) is that for when the emission tax rate is at the level where the firm located in the host country fully abates its emissions; and \( \Omega_M \) is the interior minimum of \( \Omega(t;\alpha,\tau) \) at \( t^M(\alpha) \in (0,\bar{t}(\alpha)) \).
reduced more at $t = 0$ than at $t = \bar{t}(\alpha)$, this is because at a given tax rate less than $\tau$, as $n$ increases, the operation profit of firm $f$ stolen by the entry of the host firm under the FDI mode is greater than that under the export mode, which causes a reduction of both $\Omega_o$ and $\Omega_c$.\(^8\) However, due to the total marginal cost difference of firm $f$ between export mode and FDI mode at $t = \bar{t}(\alpha)$ being less than that at $t = 0$, the magnitude of the reduction of $\Omega_o$ will be greater than that of $\Omega_c$, at this moment, if the abatement technology is more efficient, then the affected abatement effect is high,\(^9\) and $\Omega_c$ will turn out to be greater than $\Omega_o$. This has an important implication in that if the abatement technology is more efficient and the trade cost is high, then a high emission tax rate provides a stronger incentive for firm $f$ to adopt FDI than a zero tax rate. In addition to the above, Lemma 5(iii) says that if the abatement technology is very inefficient, then no matter what the trade cost is, the incentive for firm $f$ to adopt FDI even without a fixed cost is negative.

Based on Lemmas 3, 4, and 5, we first summarize the sign and ranking of $\Omega_o$, $\Omega_c$ and $\Omega_M$ (if exists) under different values of $n$, $\tau$, and $\alpha$ in Table 1 (Please see it in the Appendix). After knowing the properties of $\Omega$, we further incorporate the fixed cost $G$ of FDI to discuss firm’s optimal entry mode. According (11), a higher(lower) value of $G$ will incur a smaller(greater) value of $\Delta \Pi$, thus reducing (raising) the incentive for firm to conduct FDI. There are a lot of cases, however, many of them are so intuitive and similar to parts of what we will discuss, we only summarize the most interesting case of $n \geq 3$ and

$$(3n + 2)A \leq \tau \leq \frac{A}{4n(n+1)}$$

in Table 2. (Please see it in the final and the proof in the Appendix)

From Table 1 and Table 2, a very low (high) fixed cost of FDI causes FDI (export) is obvious, we summarize the cases of the switch of entry mode in Proposition 1.

**Proposition 1:**

(i) When the trade cost is low, raising the emission tax rate will discourage the foreign firm to adopt FDI.

(ii) When the trade cost is high and the abatement technology is efficient, if the fixed cost of FDI lies in some interval, then along with the increase in the emission tax rate, the foreign firm will change entry mode from FDI to export first and then switch back to FDI.

---

\(^8\) This can be seen by $\frac{\partial \Omega(0; \alpha, \tau)}{\partial n} = \frac{-2A\tau}{(n+2)^2} < 0$.

\(^9\) It’s obvious, the abatement effect $\frac{I}{\alpha}$ is not affected by $n$. 

11
(iii) When the number of host firms is greater than 3, the trade cost is high enough, and abatement technology is very efficient, then if the fixed cost of FDI lies in some interval, a newly enforced high emission tax rate will attract the foreign firm to conduct FDI.

We use Figure 1 to illustrate the economic intuition of Proposition 1. If the $\Delta \Pi$ curve is above (below) the horizontal axis, then the profit in the FDI mode is greater (less) than in the export mode so that FDI (export) will be chosen. Figure 1 represents the case of $n \geq 3$, $\frac{(3n+2)A}{4n(n+1)} \leq \tau \leq \frac{A}{(n+1)}$, and $0 < \alpha < \alpha^L$. Curve $\Delta \Pi(G = 0) = \Omega(\tau; \alpha, \tau)$ is the case without FDI fixed cost (i.e. $G = 0$), because it is above the horizontal axis, the foreign firm will always adopt FDI. If $G$ lies between $\Omega_u$ and $\Omega_o$ (curve $\Delta \Pi(\Omega_u < G < \Omega_o)$), then it is not so high that firm $f$ will adopt FDI initially at $t = 0$, when the tax rate rises, both the abatement effect and tax avoidance effect increase, but the emission tax effect decreases, as the tax rate exceeds $t^*_1$, then the negative incentive (summation of emission tax effect and tax avoidance effect) will dominate the positive incentive (the abatement effect) for adopting FDI, causing switch from FDI to export. If the tax rate continues to increase and exceeds $t^*_2$, then the abatement effect will finally dominate the other two effects, and firm $f$ will switch back to FDI again - that is, a higher emission tax rate in the host country still can attract FDI. If $G$ is even higher and lies between $\Omega_o$ and $\Omega_c$ (line $\Delta \Pi(\Omega_o < G < \Omega_c)$), then firm $f$ will adopt export at $t = 0$, and as $t$ exceeds $t^*$, firm $f$ will switch to FDI. This is because when the abatement technology is very efficient and the trade cost is high, a high tax rate create a great abatement effect but a relatively small tax avoidance effect, providing more incentive for firm $f$ to adopt FDI than zero tax rate. Furthermore, either the trade cost is high or the abatement technology is less efficient, the $\Delta \Pi$ curve is always downward sloping according to Table 1 and Table 2. This is because a rise in the emission tax rate causes the two negative effects always dominating the abatement effect. Hence, it can only change the entry mode of firm $f$ from FDI to export.

In addition to Proposition 1, there are some other interesting points. First of all, the case that firms have no abatement technology, but face an emission tax, is a special case of our model. This is the case of $\alpha^U < \alpha < \infty$ in Table 2, where $\alpha$ is approaching $\infty$. It is obvious that raising the emission tax rate can only drive the foreign firm out of the host country.

The second point can also be seen in Figure 1. When the fixed cost of FDI is $G_o$ (line $\Delta \Pi(G_o)$) and the emission tax rate is at $t_0$, the firm will choose export. However, if the host country raises the fixed cost to $G'_o$ and the emission tax to $t'_o$ simultaneously
(thick line $\Delta \Pi(G'_0)$), then the foreign firm will be attracted to conduct FDI. In other words, as long as the abatement effect is strong enough, raising the emission tax and FDI fixed cost simultaneously may also attract FDI.

The third point relates to the entry of the host firms. By (12), we have $\frac{d\Delta \Pi}{dn} < 0$ if $t < \tau$. This is because the entry of host firm will reduce the operation profit of firm $f$ under the two modes, if $t > (\leq \tau)$, the magnitude of the reduction of the operation profit in the FDI mode will be less (greater) than that in the export mode. Therefore, if the abatement technology is more efficient (i.e. $\alpha < \alpha^*$), then $\frac{d\Delta \Pi}{dn} < 0$ because of $t < \bar{\tau}(\alpha) < \tau$ by Lemma 3(ii), this is to say, entry of the host firm will drive firm $f$ out of the host country, however, if (i.e. $\alpha > \alpha^*$), the entry of host firm may attract firm $f$ to the host country, which can be seen in Figure 2. Figure 2 shows the impact of the increase of $n$ on firm $f$’s entry mode when the abatement technology is less efficient (i.e. $\alpha > \alpha^*$). As $\alpha > \alpha^*$, $\bar{\tau}(\alpha) > \tau$ and the $\Delta \Pi$ curve is always downward sloping by Lemmas 3 and 4. In the figure, curve $\Delta \Pi(n)$ is the initial curve when the number of host firms is $n$, it reveals that, if the emission tax rate is equal to the trade cost (i.e. $t = \tau$), then the foreign firm will adopt FDI, at this

\[
\frac{d\Delta \Pi}{dn} = \frac{2(t-\tau)[nA - (2n+1)t + (n+1)\tau]}{(n+2)^2} > 0 \quad \text{if} \quad t > \tau.
\]

Figure 1: The curve of $\Delta \Pi$ when $n \geq 3$, $\frac{3n+2)A}{4n(n+1)} \leq \tau \leq \frac{A}{n+1}$ and $0 < \alpha < \alpha^L$. The third point relates to the entry of the host firms. By (12), we have $\frac{d\Delta \Pi}{dn} < 0$ if $t > \tau$. This is because the entry of host firm will reduce the operation profit of firm $f$ under the two modes, if $t > (\leq \tau)$, the magnitude of the reduction of the operation profit in the FDI mode will be less (greater) than that in the export mode. Therefore, if the abatement technology is more efficient (i.e. $\alpha < \alpha^*$), then $\frac{d\Delta \Pi}{dn} < 0$ because of $t < \bar{\tau}(\alpha) < \tau$ by Lemma 3(ii), this is to say, entry of the host firm will drive firm $f$ out of the host country, however, if (i.e. $\alpha > \alpha^*$), the entry of host firm may attract firm $f$ to the host country, which can be seen in Figure 2. Figure 2 shows the impact of the increase of $n$ on firm $f$’s entry mode when the abatement technology is less efficient (i.e. $\alpha > \alpha^*$). As $\alpha > \alpha^*$, $\bar{\tau}(\alpha) > \tau$ and the $\Delta \Pi$ curve is always downward sloping by Lemmas 3 and 4. In the figure, curve $\Delta \Pi(n)$ is the initial curve when the number of host firms is $n$, it reveals that, if the emission tax rate is equal to the trade cost (i.e. $t = \tau$), then the foreign firm will adopt FDI, at this
moment, \( G < \frac{\tau^2}{2\alpha} \), and if the number of host firms increases (to \( n' \)), then line \( \Delta \Pi(n) \) shifts counter-clockwise on point \( a \) to \( \Delta \Pi(n') \). In this situation, if the tax rate (such as \( t_0 \)) lies between \( t^c \) and \( t^{c'} \), then the increase of host firms will attract the foreign firm to conduct FDI. Thus, we have Proposition 2.

**Proposition 2:** If the emission tax rate is higher than the trade cost and the tax saving of emission abatement exceeds the fixed cost of FDI, then increasing the number of host firms may induce the foreign firm to switch from export to FDI.

To sum up, when taking emission abatement into consideration, we find same interesting cases of the impact of an emission tax rate adjustment on the foreign firm’s entry mode which are not found in the previous literature.

4. The impact of the emission tax adjustment on the associate welfare

This section explores the influence of the emission tax rate on total emission and consumer surplus in the host country by considering the shift of the foreign firm’s entry mode.

4.1 Total emission in the host country

According to market equilibrium in section 3, we can obtain the total emission in the host country under the entry modes of FDI and export as

\[
P^E(t) = (n + 1)\left[\frac{(a - c - t)}{n + 2} - \frac{t}{\alpha}\right] \quad \text{and} \quad P^E(t) = n\left[\frac{A - 2t + \tau}{(n + 2)} - \frac{t}{\alpha}\right],
\]

respectively, and now have Lemma 6.

\[\text{As} \quad \Pi_f^E(\tau; \alpha, G) = \left[\frac{A - \tau^2}{(n + 2)^2}\right] + \frac{\tau^2}{2\alpha} - G, \quad \Pi_f^E(\tau; \tau) = \left[\frac{A - \tau^2}{(n + 2)^2}\right], \quad \text{therefore} \quad \frac{\tau^2}{2\alpha} > G \text{ assures} \]

\[\Pi_f^E(\tau; \alpha, G) > \Pi_n^E(\tau; \tau).\]
Lemma 6:

(i) $P^E(0) < P^F(0)$.

(ii) $\forall \alpha \in (0, \alpha^*)$, and there exists $t^e(\alpha) = \frac{\alpha(A-n\tau)}{(n+2)-(n-1)\alpha}$ such that $P^E(t^e(\alpha)) = P^F(t^e(\alpha))$

and $P^E(t) \leq P^F(t)$ if $t^e(\alpha) < t^\alpha(\alpha) < t^\mu(\alpha) < t^F(\alpha)$.

(iii) $\forall \alpha \in (\alpha^*, \infty)$, $P^E(t) < P^F(t)$, and $\forall t \in [0, \bar{t}(\alpha)]$. (Please see the Appendix for the proof.)

The economic intuition of Lemma 6 is as follows. (i) When the emission tax rate is zero, all firms located in the host countries do not abate emissions, therefore, total emission is equal to total output produced in the host country. Because total output produced in the host country in the FDI mode is greater than that in the export mode, as is the total emission. (ii) We can treat total pollution emitted in the host country as total emission generated subtracting total emission abated in the host country. For a given $t < \tau$, both the total output produced and total emissions generated in the host country in the FDI mode is greater than in the export mode. When the emission tax rate is low or the abatement technology is less efficient, although there is one more firm abating emission in the FDI mode than in the export mode, the magnitude of more emissions abated under FDI still does not surpass that of more emissions generated under FDI, leading to a greater total emissions in the FDI mode than in the export mode, however, when the emission tax rate is high and abatement technology is more efficient, it is the reverse, resulting in more pollution emitted in the export mode than that in the FDI mode.

Based on Lemma 6 and the optimal entry mode of firm $f$, we summarize main results of the influence of the emission tax rate on the host country’s total emission in Proposition 3.

Proposition 3:

(i) If raising the emission tax rate does not alter the foreign firm’s entry mode, then total emission in the host country will be reduced.

(ii) If raising the emission tax rate attracts the foreign firm to conduct FDI, then total emission in the host country will be reduced.

(iii) If raising the emission tax rate drives the FDI foreign firm out of the host country, then the total emission in the host country may be raised.

The intuition of (i) is straightforward, as no matter which entry mode the foreign firm chooses, raising the emission tax rate will definitely reduce total output but raise total emission abated, thus total emission will be reduced. As for (ii) and (iii), they can be
elaborated in Figure 3. Figure 3 depicts the total emission curve for the case of $\Delta \Pi(\Omega_m < G < \Omega_o)$ in Figure 1 (i.e. $0 < \alpha < \alpha^*$ and $\frac{(3n+2)A}{4n(n+1)} \leq \tau \leq \frac{A}{(n+1)}$). If there are two critical rates at which firm $f$ changes its entry mode (ex. $\Omega_m < G < \Omega_o$), then as $t^M > t^p$ from Lemma 6(ii), $t'_2$ will always be greater than $t^p$, but $t'_1$ may be higher or lower than $t^p$ depending on the magnitude of $G$. We know from Figure 1, when $G$ (lying between $\Omega_m$ and $\Omega_o$) is relatively low, both $t'_1$ and $t'_2$ will be closer to $t^M$, if $G$ increases gradually, both $t^M$ and $t^p$ will not be affected, but both $t'_1$ and $t'_2$ will move farther away from $t^M$. The solid line $kabcdt^F$ is the total emission curve of a relatively low $G$, therefore, when $t < t'_1$, the emission curve is $ka$ (in the FDI mode), when $t'_1 < t < t'_2$, it is $bc$ (in the export mode), likewise, when $t > t'_2$, the total emission is $dt^F$ (in the FDI mode), it represents a jump from point $a$ to $b$ at $t'_1$. If the initial $t$ lies between $t'$ and $t'_1$, then raising $t$ to be greater than $t'_1$ will alter the entry mode from FDI to export and thereby total emission may rise. Putting it in another way, when the emission tax rate lies between $t'_1$ and $t''$, if the host country wants to reduce emission, it can do so by reducing the tax rate to attract FDI except that raising the tax rate. Figure 3 also shows that if raising the tax rate attracts firm $f$ to conduct FDI, then total emission will be reduced. The impact of $G$ on the total emission can also be seen in Figure 3. If $G$ increases, the two initial critical tax rates $t'_1$ and $t'_2$ become $t''_1$ and $t''_2$, respectively. The total emission curve $(kabcdt^F)$ becomes the solid dash line $ka'b'c'd't^F$. If the tax rate lies between $t'_2$ and $t''_2$, or is between $t^p$ and $t'_1$, then as long as the entry mode switches from FDI to export due to a rise of fixed cost, total emission in the host country will rise.

When $\alpha^H < \alpha < \alpha^*$, the two total emission curves in this situation are similar to those in Figure 3. Under this circumstance, according to Table 2, if $G$ lies between $G_o$ and $G_C$, then raising the tax rate can only change the entry mode from FDI to export. As long as $G$ is relatively low, the critical tax rate such as $t'_1$ will be greater than $t^p$, and the total emission curve is $kab^E$. Thus, raising the tax rate from less than $t'_1$ to greater than it may in fact raise the total amount of emission. Alternatively, if $G$ is relatively high, then the critical tax rate such as $t''_1$ will be less than $t^p$. Thus, the total emission line becomes $ka'b't^E$, and raising the tax rate will always reduce the total amount of emission.

When $\alpha > \alpha^*$, then by Lemma 6(iii), line $P^F$ is always above line $P^E$. According to Lemma 4(ii), raising the emission tax rate can only drive firm $f$ out of the host country,

12 Based on Lemma 6(ii), this critical tax rate $t^c$ exists when $\Omega_C < G < \Omega_o$. The smaller the value of $G$ is, the higher the value will be of the critical $t^c$ - that is, when $G$ is relatively low, the more likely it will happen.
total emission curve will jump down from $P^F$ to $P^E$, causing a reduction in total emission.

![Figure 3. Total emission curve when $0 \leq \alpha \leq \alpha^2$.](image)

4.2 Consumer surplus in the host country

We next examine the influence of an emission tax rate adjustment on consumer surplus (CS) in the host country. According to market equilibrium, the host country’s consumer surplus in the two entry modes are

$$CS^F = \frac{(nq^F_k + q^F_f)^2}{2} = \frac{[(n+1)(A-t)]^2}{2(n+2)^2}$$

and

$$CS^E = \frac{(nq^E_k + q^E_f)^2}{2} = \frac{[(n+1)A - nt - t\tau]^2}{2(n+2)^2},$$

respectively. Thus, we have the following lemma.

**Lemma 7:**

(i) $\frac{dCS^F}{dt} < 0$ and $\frac{dCS^E}{dt} < 0$. (ii) Given the same $t$, $CS^E - CS^F = \frac{(Q^E + Q^F)(t-\tau)}{2(n+2)^2} > 0$, if $t > \tau$.

Lemma 7 is more obvious, it is well known that, under Cournot competition, consumer surplus is solely dependent on the market’s total output, which is negatively related to the summation of total marginal cost of all firms. Based on previous analysis and Lemma 7, we have the main results in Proposition 4.

**Proposition 4:**

(i) If raising the emission tax rate does not change the entry mode of the foreign firm, then the consumer surplus in the host country is reduced.

(ii) If raising the emission tax rate does change the entry mode of the foreign firm, then it may raise the consumer surplus of the host country, if the following situations happen:

(a) The abatement technology is efficient and raising the emission tax rate attracts the foreign firm to conduct FDI.

(b) The abatement technology is less efficient and the emission tax rate is greater than
the trade cost, whereby raising the emission tax rate drives the FDI foreign firm out of the host country to conduct export.

We use Figure 4 to illustrate the economic intuition of Proposition 4. Figure 4a shows the case of an efficient abatement technology (\( \alpha < \alpha^H \)) with two critical tax rates where the foreign firm changes entry mode.\(^{13}\) The CS curve of this case is line \( labcdg \). Under this circumstance, if \( t < t^*_1 \) initially, then raising the tax rate to drive the foreign firm out of the host country will reduce the consumer surplus. If \( t' < t < t^*_2 \), then raising the tax rate to attract FDI may raise or reduce consumer surplus. This is because if the firm switches to FDI, then the summation of all firms’ total marginal cost will change from \( nt + \tau + (n + 1)c \) to \( nt_1 + t_1 + (n + 1)c \), \( nt_1 > nt \) but \( t_1 < \tau \), thus, summation of all firms’ total marginal cost may decrease and lead to an increase of consumer surplus. In other words, although a higher tax rate attracts FDI, the consumer surplus may not be sacrificed.

Figure 4b shows the case when the abatement technology is less efficient (i.e. \( \alpha > \alpha^* \)). We have already known from Proposition 1, if raising the emission tax does change the firm’s entry model, then it can only push the firm to switch from FDI to export. In the figure, we can see, if the critical tax rate where the foreign firm changes entry mode is less than the trade cost (i.e. \( t^e < \tau \)), then the CS curve is line \( la'b'g \). In this situation, if raising the emission tax drives the foreign firm out of the host country, then consumer surplus will be reduced. Alternatively, if the critical tax rate (such as \( t^c \) in the figure) is higher than the trade cost, then the CS curve is line \( labg \). In this situation, if the tax rate is in the range of \( t^c < t < t^e \), then the rise in emission tax driving out the FDI foreign firm may cause an increase or decrease of the host country’s consumer surplus.

\(^{13}\) This is because \( \alpha < \alpha^H \) implies \( t^F(\alpha) < \tau \) by Lemma 3.
5. Concluding remarks

We set up a two-country model in which a foreign firm chooses FDI or export to enter the host market and plays Cournot competition with $n$ host firms. The main findings in this paper are as follows. The foreign firm’s optimal entry mode is determined by the emission tax effect, abatement effect, and tax avoidance tax. When the trade cost is low, a rise in the emission tax rate discourages the foreign firm to FDI. When the trade cost is high and the abatement technology is efficient, an increase in the emission rate tax may encourage foreign firm to conduct FDI. When the abatement technology is efficient and trade cost is high, raising the tax rate to drive FDI foreign firm out of the host country may result in an increase of the total emission. Furthermore, if raising the emission tax rate changes the foreign firm’s entry mode, then it may raise the consumer surplus in the host country.

Previous theoretical literature studying the effects of an emission tax policy on the entry mode of the foreign firm has always overlooked the emission abatement behavior of the firms. The findings of this paper show that abatement behavior with trade cost plays key roles in the effects of the emission tax policy on the entry mode of the foreign firm. A stricter emission tax policy may encourage the foreign firm to conduct FDI because of a great abatement effect induced by a higher tax rate. We also find that imposing a higher emission tax rate to drive the foreign firm out of the host country may be detrimental to the environmental quality and consumer surplus at the same time. These findings points out to regulators that both the entry mode choice and abatement behavior of the foreign firm are important factors when they are looking for using an emission tax policy to control the environmental quality.

This paper does not discuss the optimal emission tax rate of the host country’s welfare maximization for two reasons. The first one is that, one of the main purposes of emission tax policy is to control the environmental quality, especially, the vogue of environmental protection calls for a stricter environmental policy recently, and the second one is that, to solve the optimal tax rate will make the model more complex but cannot gain more inside. Furthermore, as for the assumption of no policy in the foreign country, the exposition is as follows. $\tau$ can be treated as trade cost plus emission tax rate of the foreign country, in this circumstance, the foreign firm adopting export will abate emissions, it creates an abatement effect in export mode, however, as long as the emission tax rate of the foreign is kept constant, the results of this paper still hold.
Table 1  The sign and ranking of \( \Omega_o, \Omega_c, \) and \( \Omega_M \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \tau )</th>
<th>( \alpha )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \leq 2 )</td>
<td>( 0 &lt; \tau &lt; \dfrac{A}{2(n+1)} )</td>
<td>( 0 &lt; \alpha &lt; \alpha^U )</td>
<td>( \Omega_o &gt; \Omega_c &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^U &lt; \alpha &lt; \infty )</td>
<td>( \Omega_o &gt; \Omega_c )</td>
</tr>
<tr>
<td></td>
<td>( \dfrac{A}{2(n+1)} &lt; \tau &lt; \dfrac{A}{(n+1)} )</td>
<td>( 0 &lt; \alpha &lt; \alpha^H )</td>
<td>( \Omega_o &gt; \Omega_c &gt; \Omega_M &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^H &lt; \alpha &lt; \alpha^U )</td>
<td>( \Omega_o &gt; \Omega_c &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^U &lt; \alpha &lt; \infty )</td>
<td>( \Omega_o &gt; \Omega_c )</td>
</tr>
<tr>
<td>( n \geq 3 )</td>
<td>( 0 &lt; \tau &lt; \dfrac{A}{2(n+1)} )</td>
<td>The same as ( n \leq 2 )</td>
<td>( \Omega_o &gt; \Omega_c &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>( \dfrac{A}{2(n+1)} &lt; \tau &lt; \dfrac{(3n+2)A}{4n(n+1)} )</td>
<td>The same as ( n \leq 2 )</td>
<td>( \Omega_o &gt; \Omega_c &gt; \Omega_M &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>( \dfrac{(3n+2)A}{4n(n+1)} &lt; \tau &lt; \dfrac{A}{(n+1)} )</td>
<td>( 0 \leq \alpha &lt; \alpha^l )</td>
<td>( \Omega_c &gt; \Omega_o &gt; \Omega_M &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^l \leq \alpha &lt; \alpha^H )</td>
<td>( \Omega_o &gt; \Omega_c &gt; \Omega_M &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^H &lt; \alpha &lt; \alpha^U )</td>
<td>( \Omega_o &gt; \Omega_c &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha^U &lt; \alpha &lt; \infty )</td>
<td>( \Omega_o &gt; \Omega_c )</td>
</tr>
</tbody>
</table>

Table 2.  Entry mode of firm \( f \) when \( n \geq 3 \) and \( \dfrac{(3n+2)A}{4n(n+1)} \leq \tau < \dfrac{A}{(n+1)} \)

<table>
<thead>
<tr>
<th>( G )</th>
<th>( G &lt; \Omega_M )</th>
<th>( \Omega_M \leq G \leq \Omega_o )</th>
<th>( \Omega_o \leq G \leq \Omega_c )</th>
<th>( G &gt; \Omega_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>All ( t )</td>
<td>( 0 &lt; t &lt; t^*_1 )</td>
<td>( t^<em>_1 &lt; t &lt; t^</em>_2 )</td>
<td>( t &gt; t^*_2 )</td>
</tr>
<tr>
<td>( m )</td>
<td>F</td>
<td>F</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>( t )</td>
<td>All ( t )</td>
<td>( 0 &lt; t &lt; t^c )</td>
<td>( t^<em>_1 &lt; t &lt; t^</em>_2 )</td>
<td>( t &gt; t^*_2 )</td>
</tr>
<tr>
<td>( m )</td>
<td>F</td>
<td>F</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>( t )</td>
<td>All ( t )</td>
<td>( 0 &lt; t &lt; t^c )</td>
<td>( t^<em>_1 &lt; t &lt; t^</em>_2 )</td>
<td>( t &gt; t^*_2 )</td>
</tr>
<tr>
<td>( m )</td>
<td>F</td>
<td>F</td>
<td>E</td>
<td>F</td>
</tr>
</tbody>
</table>

※ \( G \) : fixed cost of FDI; \( t \) : emission tax rate; \( m \) : optimal entry mode; F: FDI ; E: export.
Appendix

Proof of Lemma 3:
(i) As $t^E(\alpha) = \frac{\alpha A}{n + 2 + \alpha}$ and $t^F(\alpha) = \frac{\alpha (A + \tau)}{n + 2 + 2\alpha}$, we have:

$$
t^E(\alpha) - t^E(\alpha) = \frac{\alpha [\alpha (A - \tau) - (n + 2)\tau]}{(n + 2 + \alpha)(n + 2 + 2\alpha)}, \quad t^F(\alpha) - \tau = \frac{\alpha A}{n + 2 + \alpha} - \tau = \frac{[\alpha (A - \tau) - (n + 2)\tau]}{(n + 2 + \alpha)},
$$

thus, denoting $\frac{(n + 2)\tau}{A - \tau} \equiv \alpha'$, we have that, if $\alpha \leq \alpha'$, then $t^E \leq t^F \leq \tau$ i.e., if $\alpha < (>) \alpha'$, then $\bar{t}(\alpha) = t^F(t^E)$.

Proof of Lemma 4:
(i) Given $n$ and $\tau$, then reorganizing (13), we have:

$$
\frac{\partial \Omega(t; \alpha, \tau)}{\partial t} = \frac{(n + 2)^2 - 2\alpha (n^2 - 1)\tau - 2\alpha (n + 1)(A - n \tau)}{(n + 2)^2} \quad \text{(A1)}
$$

Because $\frac{\partial \Omega(0; \alpha, \tau)}{\partial t} = -\frac{2\alpha (n + 1)(A - n \tau)}{(n + 2)^2} < 0$, when $\Omega(t; \alpha, \tau)$ is concave, then $\frac{\partial \Omega(t; \alpha, \tau)}{\partial t} < 0$ for all $t \in [0, \bar{t}(\alpha)]$; when $\Omega(t; \alpha, \tau)$ is convex, then as long as $\frac{\partial \Omega(t; \alpha, \tau)}{\partial t} \big|_{t=\bar{t}(\alpha)} < 0$, $\Omega(t; \alpha, \tau)$ also decreases on $t \in [0, \bar{t}(\alpha)]$; otherwise, if $\frac{\partial \Omega(t; \alpha, \tau)}{\partial t} \big|_{t=\bar{t}(\alpha)} > 0$, then $\Omega(t; \alpha, \tau)$ has an interior minimum. To find out the conditions for $\Omega(t; \alpha, \tau)$ to have an interior minimum, we should observe the partial differentiation of $\Omega(t; \alpha, \tau)$ at the left limit of $\bar{t}(\alpha)$ (i.e. $t \rightarrow \bar{t}(\alpha)^-$). According to Lemma 3(i), if $\alpha < (>) \alpha'$, then $\bar{t}(\alpha) = t^F(\alpha(t^E(\alpha)))$, thus, we have the cases of $\alpha < \alpha'$ and $\alpha > \alpha'$.

(a) $\alpha < \alpha'$

Substituting $t = \bar{t}(\alpha) = t^E(\alpha) = \frac{\alpha A}{n + 2 + \alpha}$ into (A1), we have:

$$
\lim_{t \rightarrow \bar{t}(\alpha)^-} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} = \frac{n[(n + 2)[2(n + 1)\tau - A] - 2\alpha (n + 1)(A - \tau)]}{(n + 2 + \alpha)(n + 2)^3}. \quad \text{(A2)}
$$

Because $A - \tau > 0$, from (A2), we know that:

(1) If $2(n + 1)\tau - A < 0$, i.e. $\tau < \frac{A}{2(n + 1)}$, then $\lim_{t \rightarrow \bar{t}(\alpha)^-} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} < 0$, for all $\alpha > 0$.

(2) If $2(n + 1)\tau - A > 0$, i.e. $\frac{A}{2(n + 1)} < \tau < \frac{A}{(n + 1)}$, as long as $\alpha$ is small(great) enough, then $\lim_{t \rightarrow \bar{t}(\alpha)^-} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} > (>) 0$. By setting the numerator of (A2) to be zero, we get the critical $\alpha'' = \frac{(n + 2)[2(n + 1)\tau - A]}{2(n + 1)(A - \tau)}$. Moreover, $\alpha'' - \alpha'' = \frac{(n + 2)A}{2(n + 1)(A - \tau)} > 0$, so we conclude, if $\tau \in \left(\frac{A}{2(n + 1)}, \frac{A}{(n + 1)}\right)$, then as long as $0 < \alpha < \alpha''$, $\Omega(t; \alpha, \tau)$ has an
interior minimum on \([0, \bar{t}(\alpha)]\); otherwise, as \(\alpha^H < \alpha < \alpha^\prime\), then \(\Omega(t; \alpha, \tau)\) is decreasing on \(t \in [0, \bar{t}(\alpha)]\). Furthermore, by setting (A1) to be zero, we can solve

\[
t^M(\alpha) = \frac{2\alpha(n+1)(A-n\tau)}{(n+2)^2 - 2\alpha(n^2-1)}
\]

at which \(\Omega(t; \alpha, \tau)\) has an interior minimum \(\Omega_M\).

Based on the above, we can summarize the situation of \(\alpha < \alpha^\prime\) as follows.

1. If \(\tau < \frac{A}{2(n+1)}\), then \(\Omega(t; \alpha, \tau)\) is decreasing on \(t \in [0, \bar{t}(\alpha)]\).

2. If \(\frac{A}{2(n+1)} < \tau < \frac{A}{n+1}\), then for \(0 < \alpha < \alpha^H\), \(\Omega(t; \alpha, \tau)\) has an interior minimum \(\Omega_M = \Omega(t^M(\alpha); \alpha, \tau)\); for \(\alpha^H < \alpha < \alpha^\prime\), \(\Omega(t; \alpha, \tau)\) is decreasing on \(t \in [0, \bar{t}(\alpha)]\).

(b) \(\alpha > \alpha^\prime\)

According to Lemma 3(i), if \(\alpha > \alpha^\prime\), then \(\bar{t}(\alpha) = t^E(\alpha) = \frac{\alpha(A + \tau)}{n^2 + 2\alpha}\), thus,

\[
\lim_{t \to t^E(\alpha)} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} = -\frac{(n+2)[nA - (n^2 + 3n + 2)\tau] + 2\alpha(n+1)^2(A - \tau)}{(n+2)^2(n+2\alpha)}.
\]

(B3)

Because \(\alpha > \alpha^\prime\), rearranging (B3),

\[
\lim_{t \to t^E(\alpha)} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} \leq -\frac{2\alpha(n+1)^2(A - \tau)}{(n+2)^2(n+2\alpha)} = -\frac{(n+2)(nA - \tau)}{(n+2)^2(n+2\alpha)} < 0,
\]

therefore, \(\forall \alpha > \alpha^\prime\), \(\Omega(t; \alpha, \tau)\) is decreasing on \(t \in [0, \bar{t}(\alpha)]\).

To summarize (a) and (b), we have Lemma 4.

Proof of Lemma 5:

If \(\alpha \in (0, \alpha^H)\), then \(\lim_{t \to \bar{t}(\alpha)} \frac{\partial \Omega(t; \alpha, \tau)}{\partial t} > 0\) by Lemma 4(ii), and \(\Omega_c\) may be greater than \(\Omega_o\). From (12), the difference between \(\Omega(t; \alpha, \tau)\) and \(\Omega_o\) is:

\[
\Omega(t; \alpha, \tau) - \Omega(0; \alpha, \tau) = -\frac{t[4\alpha(n+1)(A-n\tau) - t[(n+2)^2 - 2\alpha(n^2-1)]]}{2\alpha(n+2)^2}.
\]

(A4)

If \(\alpha \in (0, \alpha^H)\), then \(\alpha < \alpha^\prime < \alpha^H\), thus, \(\bar{t}(\alpha) = t^F(\alpha) = \frac{\alpha A}{n + 2 + \alpha}\) by Lemma 3(i), substituting \(t = \bar{t}(\alpha) = t^F(\alpha) = \frac{\alpha A}{n + 2 + \alpha}\) into (A4), we have:

\[
\Omega_c - \Omega_o = -\frac{t^F[(n+2)[(3n+2)A - 4n(n+1)\tau] + 2\alpha(n+1)[(n+1)A - 2n\tau]]}{2(n+2)^2(n+2\alpha)}.
\]

(A5)

Because, \((n+1)A - 2n\tau\) is positive, from (A5), we have:

1. if \(3n + 2)A - 4n(n+1)\tau > 0\), i.e. \(\tau < \frac{(3n+2)A}{4n(n+1)}\), then \(\Omega_c < \Omega_o\).

2. if \(3n + 2)A - 4n(n+1)\tau < 0\), i.e. \(\tau > \frac{(3n+2)A}{4n(n+1)}\), then a small (large) value of \(\alpha\) causes a positive (negative) numerator, thereby, \(\Omega_c > (<)\Omega_o\).
Besides, when \( n \leq 2 \), because \( \tau < \frac{A}{(n+1)} \) assures \( \tau < \frac{(3n+2)A}{4n(n+1)} \) due to 
\[
\frac{A}{(n+1)} < \frac{(3n+2)A}{4n(n+1)} , \text{ thus } \Omega_C < \Omega_o \text{ from (1). When } n \geq 3, \text{ because of } \frac{(3n+2)A}{4n(n+1)} < \frac{A}{(n+1)},
\]
therefore, if \( \frac{(3n+2)A}{4n(n+1)} < \tau < \frac{A}{(n+1)} \), then by setting (A5) to be zero, we can solve the critical 
\[
\alpha^l = \frac{(n+2)[4(n+1)\tau - (3n+2)A]}{2(\tau + 2n\tau)} \quad \text{such that if } \alpha < (>) \alpha^l, \quad \Omega_C > (<) \Omega_o .
\]
In addition, 
\[
\alpha^l - \alpha^H = \frac{-2(n+1)(A-n\tau)}{2(n+1)[(n+1)A-2n\tau][A-\tau]} < 0 , \text{ thus, (i) and (ii) are proven.}
\]
As for (iii), substituting \( \tau^l(\alpha^l) = \tau \) into (12), we have:
\[
\Omega(t^E(\alpha^l);\alpha^l,\tau) = \Omega(t^E(\alpha^l);\alpha^l,\tau) = \frac{\tau^2}{2\alpha} > 0 ; \quad \text{substituting } t = \lim_{\alpha \to \infty} t^E(\alpha) = \frac{(A+\tau)}{2} \text{ into (12), we have:}
\]
\[
\Omega(t^E(\alpha);\alpha,\tau) = \frac{(n+1)(n+3)(A-\tau)^2}{4(n+2)^2} < 0 ; \text{ moreover,}
\]
\[
\frac{d\Omega(t^E(\alpha);\alpha,\tau)}{d\alpha} - \frac{2\Omega \partial t^E}{\partial \alpha} < 0 . \text{ These conditions assure there exist a } \alpha^U \in (\alpha^l,\infty) \text{ that makes } \Omega(t^U(\alpha^U);\alpha^U,\tau) = 0 \text{ by Intermediate Value Theorem, and if } \alpha \in (\alpha^l,\alpha^U) , \text{ then} \Omega_C = \Omega(t^U(\alpha);\alpha,\tau) > 0 ; \text{ if } \alpha^l \in (\alpha^l,\infty) , \text{ then} \Omega_C = \Omega(t^U(\alpha);\alpha,\tau) < 0 .
\]

**Proof of Table 1:**

Based on Lemmas 3, 4, and 5, it is easy to find out ranking of \( \Omega_o, \Omega_C \) and \( \Omega_M \) in Table 1. To save the space, we only prove the case of when \( n \geq 3 \),
\[
\frac{(3n+2)A}{4n(n+1)} < \tau < \frac{A}{(n+1)} , \text{ and } 0 < \alpha < \alpha^l . \quad \text{As } \alpha < \alpha^l < \alpha^H , \text{ thus } \alpha < \alpha^l \text{ and}
\]
\[
\frac{(3n+2)A}{4n(n+1)} < \tau < \frac{A}{(n+1)} \quad \text{assure that } \Omega(t;\alpha,\tau) \text{ has an interior minimum } \Omega_M \quad (\text{i.e. } \Omega(t;\alpha,\tau) \text{ is “U” shape} \text{ and } \Omega_C > \Omega_o \text{ by Lemma 4(ii) and Lemma 5(ii). Moreover, because } \alpha < \alpha^l < \alpha^H < \alpha^l , \text{ it implies } t^H(\alpha) < \tilde{t}(\alpha) = t^E(\alpha) < \tau \text{ by Lemma 3(i), and thus, we have } \Omega_M > 0 \text{ from (12). To summarize, we have } \Omega_C > \Omega_o > \Omega_M > 0 .
\]

**Proof of Table 2**

Following the above, since \( \Omega_C > \Omega_o > \Omega_M > 0 \), to subtract \( G \) from all of them will not change the ranking, and \( \Delta \Pi(t;\alpha,\tau,G) = \Omega(t;\alpha,\tau) - G \) is also “U” shape, thus, we have the following results.

(i) \( G < \Omega_M : \) It implies the minimum of \( \Delta \Pi(t;\alpha,\tau,G) = \Omega_M - G > 0 \). Thus, 
\[
\Delta \Pi(t;\alpha,\tau,G) > 0 \text{ for all } t \in [0,\tilde{t}(\alpha)], \text{ i.e. firm } f \text{ will adopt FDI for all } t \in [0,\tilde{t}(\alpha)].
\]

(ii) \( \Omega_M < G < \Omega_o : \) It implies 
\[
\Delta \Pi(t^M(\alpha);\alpha,\tau,G) = \Omega_M - G < 0 , \quad \Delta \Pi(0;\alpha,\tau,G) = \Omega_o - G > 0 , \quad \text{and } \Delta \Pi(\tilde{t}(\alpha);\alpha,\tau,G) = \Omega_C - G > \Omega_o - G > 0 . \quad \text{Because } \Delta \Pi(t;\alpha,\tau,G) \text{ is “U” shape on } t \in [0,\tilde{t}(\alpha)], \text{ then by Intermediate Value Theorem, there exists } t^i_1 \in (0,t^M(\alpha)) \text{ and } t^i_2 \in (t^M(\alpha),\tilde{t}(\alpha)) \text{ such that } \Delta \Pi(t^i_1;\alpha,\tau,G) = \Delta \Pi(t^i_2;\alpha,\tau,G) = 0 , \quad \Delta \Pi(t;\alpha,\tau,G) > 0 \text{ for } 0 < t < t^i_1 ,
\]
\( \Delta \Pi(t; \alpha, \tau, G) < 0 \) for \( t_1^c < t < t_2^c(\alpha) \), and \( \Delta \Pi(t; \alpha, \tau, G) > 0 \) for \( t_2^c < t < \bar{t}(\alpha) \).

(iii) \( \Omega_o < G < \Omega_c : \) Because \( \Omega(t; \alpha, \tau) \) is decreasing on \( t \in [0, t^M(\alpha)] \),
\( \Delta \Pi(0; \alpha, \tau, G) = \Delta \Pi(t; \alpha, \tau, G) < 0 \) for all \( t \in [0, t^M(\alpha)] \).
\( \Delta \Pi(t^M(\alpha); \alpha, \tau, G) = \Delta \Pi(t; \alpha, \tau, G) < 0 \) and
\( \Delta \Pi(\bar{t}(\alpha); \alpha, \tau, G) = \Omega_c - G > 0 \) reveal that there exists a unique \( t^c \) such that
\( \Delta \Pi(t^c; \alpha, \tau, G) = 0 \) and \( \Delta \Pi(t; \alpha, \tau, G) < 0 \) for \( t < t^c \).

(iv) \( G > \Omega_c : \) Because \( \Omega_c > \Omega_o, \Omega_c \) is the maximum of \( \Omega(t; \alpha, \tau) \) on \( t \in [0, \bar{t}(\alpha)] \), so is
\( \Omega_c - G \) of \( \Delta \Pi(t; \alpha, \tau, G) \). Thus, the maximum \( \Delta \Pi(\bar{t}(\alpha); \alpha, \tau, G) = \Omega_c - G < 0 \) assures \( \Delta \Pi(t; \alpha, \tau, G) < 0 \) for all \( t \in [0, \bar{t}(\alpha)] \), i.e. firm \( f \) will always adopt export.

The other cases in Table 2 can also be proven in the same way.

Proof of Lemma 6:

(i) As \( P^F(0) - P^C(0) = \frac{(n+1)A - n(A+\tau)}{(n+2)} = \frac{A-n\tau}{n+2} > 0 \), therefore \( P^F(0) > P^C(0) \).

(ii) \( P^C(t) - P^C(t) = \frac{\alpha(n\tau - A) + [(n+2)-(n+\alpha)]t}{(n+2)\alpha} \), as \( n\tau - A < 0 \), we have:

(a) if \( (n+2)-(n+\alpha) < 0 \), then \( P^C(t) < P^C(t) \) for all \( t \).

(b) if \( (n+2)-(n+\alpha) > 0 \), then \( P^C(t) > P^C(t) \) for \( t > t^C(\alpha) \).

Defining \( \bar{\alpha} \equiv \frac{(n+2)}{(n+1)} \) as such that makes \( (n+2)-(n+\alpha) = 0 \), then if \( \alpha > \bar{\alpha} \),
\( (n+2)-(n+\alpha) > 0 \). Moreover, because of \( \bar{\alpha} - \alpha = \frac{(n+2)(A-n\tau)}{A-\tau} > 0 \), we have the following results:

(1) if \( \alpha > \bar{\alpha} \), it assures \( (n+2)-(n+\alpha) < 0 \), then \( P^C(t) < P^C(t) \) for all \( t \) from (a);

(2) if \( \alpha < \alpha < \bar{\alpha} \), it assures \( (n+2)-(n+\alpha) > 0 \), at this moment \( \bar{t}(\alpha) = t^C(\alpha) = \frac{\alpha(A+\tau)}{(n+2+\bar{\alpha})} \)
by Lemma 3(i), and \( t^C(\alpha) - t^C(\alpha) = \frac{-\alpha(n+1)[A(\tau - \alpha - (n+\alpha)]}{(n+2+\alpha)(n+\alpha)} < 0 \), therefore, we
have \( t^C(\alpha) < t^C(\alpha) \), and \( t < t^C(\alpha) \) implies \( t < t^C(\alpha) \), it assures \( P^C(t) < P^C(t) \) from (b), that is, if \( \alpha > \alpha < \bar{\alpha} \), then \( P^C(t) < P^C(t) \) for all \( t \).

(3) if \( \alpha < \bar{\alpha} \), \( (n+2)-(n+\alpha) > 0 \), and then according to Lemma 3,
\( \bar{t}(\alpha) = t^C(\alpha) = \frac{\alpha A}{(n+2+\alpha)} \), and \( t^C(\alpha) - t^C(\alpha) = \frac{-\alpha n[A(\tau - (n+\alpha)]}{(n+2+\alpha)(n+\alpha)} > 0 \).

Therefore, we have \( t^C(\alpha) > t^C(\alpha) \), and thus \( P^C(t) < P^C(t) \) for \( t < t^C(\alpha) \) and
\( P^C(t) > P^C(t) \) for \( t^C(\alpha) < t < \bar{t}(\alpha) = t^C(\alpha) \) from (b).

Furthermore, if \( \alpha < \alpha^* \), then by Lemma 4, \( \Omega(t; \alpha, \tau) \) has a minimum at \( t^M(\alpha) \), as
\( t^C(\alpha) - t^M(\alpha) = \alpha(A-n\tau)\left[1 - \frac{1}{n+2-(n+1)\alpha} - \frac{1}{3(n+1)^2-(n+1)\alpha}\right] < 0 \) due to \( n+2 > \frac{(n+2)^2}{2(n+1)} \) - that is, \( t^C(\alpha) < t^M(\alpha) \).
Reference


