Environmental Policies, Consumers Awareness, and Privatization in a Differentiated Oligopoly with Free Entry

Leonard F.S. Wang*
Department of Applied Economics, National University of Kaohsiung,
Chu-Chuan Hsu
Department of Marketing and Logistics Management, Yu Da University, Taiwan
and
Jen-Yao Lee
Department of International Business, National Kaohsiung University of Applied Sciences, Taiwan

Abstract. In this paper, we investigate the optimal environmental and privatization policies in a mixed oligopoly, wherein firms produce differentiated goods with environmental damage that is measured by government and observed by consumers. We find that in both mixed and pure oligopoly with no entry or free entry, the emission tax/output subsidy could be provided. Emission taxes are charged when the market power and consumer cognition of pollution are low, while the production pollution is large. However, when the consumers' cognition of pollution damage is increasing and the consumers are willing to pay less for the polluted good, the environmental policy for the government to choose would be a lower emission tax or even to provide output subsidy. In addition, we argue that even though more firms after privatization will increase the choice of product variety, the entry of private firms is excessive from the viewpoint of social optimum. Hence, the government should not privatize public firms.

Key words: Environmental Policies, Product Differentiation, Consumers Awareness

JEL Classification: H23, Q50

*Correspondence: Leonard F. S. Wang, Department of Applied Economics, National University of Kaohsiung, No. 700, Kaohsiung University Road, Nan-Tzu District 811, Kaohsiung, Taiwan, R.O.C. Tel:886-7-5919322, Fax:886-7-5919320, E-mail address: lfswang@nuk.edu.tw.

1. Introduction

Since the 1990s, concerns over environmental quality have been prominent in the public domain. Governments conduct environmental regulation by imposing taxes and use the proceeds to clean up firm-generated pollution. In some highly polluting industries, the inefficient management of public monopolies may overshadow the possible benefits of public ownership, motivating the entry of profit-maximizing private firms to compete against a welfare-maximizing public firm, and such entry increases the market competition but may harm the environment.

We have witnessed mixed markets exist in a broad range of industries such as oil, steel, electricity and telecommunications that were formerly dominated by public monopolies in many developing economies. Such mixed markets with competition between firms with different objectives deserve further study. In Taiwan, in the porcine industry, there are about six million live pigs cultivated per year creating an output value of about 2.3 billion US dollars[1] encompassing 6,800 cultivation entities. Among these cultivators, there is only one public firm (Taiwan Sugar Corporation, TSC), which is the biggest cultivator in Taiwan responsible for 300 thousand live pigs per year. In recent years, the Council of Agriculture Executive Yuan R.O.C. (Taiwan) provides traceable agriculture product certificates (TAP) to improve the safety and quality of agricultural products. Owing to this, consumers can trace the source of agricultural product, as the product differentiation has been acknowledged.

On early studies of environmental policies in an oligopolistic framework, Simpson (1995) derived the optimal pollution for a Cournot duopoly and found that if firms have different production costs, the optimal tax rate may exceed the marginal damage. Yin (2003) examined corrective taxes under oligopoly with inter-firm externalities and pollution abatement, and showed that when externalities are substantial and/or the number of polluters is large, effluent levies on these firms do not necessarily result in a deadweight loss. Lahiri and Ono (2007) showed that in a polluting oligopoly with homogenous goods, when the number of firms is fixed, (i) a relative emission standard is welfare-superior to an emission-equivalent emission tax, and (ii) an

[1] The exchange rate is USD/NTD 30, which is based on the exchange rate of 17th January, 2012.

emission tax is emission-superior to a welfare-equivalent relative emission standard. Under free entry and exit, the results are just the opposite when the inverse demand function is concave. Fujiwara (2009) constructed a model of polluting oligopoly with product differentiation, considering how product differentiation, together with the presence and absence of free entry, affects optimal pollution tax/subsidy policies. The sign of the short- and long-run optimal pollution taxes are highly sensitive to the parameters measuring product differentiation as well as the presence of free entry.

On the literature of environmental regulation under mixed markets, Beladi and Chao (2006) restricted the discussion to the case of a monopoly without considering pollution abatement and proved that privatization paradoxically exerts a negative effect upon the environment. Bárcena-Ruiz and Garzón (2006) explored how the decision on whether to privatize a public firm or not interacts with environmental policy, and showed that when the government sets a tax to protect the environment, the tax is lower in a mixed oligopoly than in a private one, and that the environmental damage is greater. Kato (2006, 2011) investigated the effects of tradable emission permits, and compared emission taxes with emission quotas in a mixed oligopoly. The above papers did not consider the role of product differentiation in a pure or mixed oligopoly, however. Wang and Wang (2009) re-examined whether privatization improves (or deteriorates) the environment in a mixed duopolistic framework with differentiated product and pollution abatement. They showed that when the product is highly substituted, the industry profit is increased because it softens the intensity of the product market, but the social welfare is deteriorated accompanied with the path of privatization. Matsumura and Kanda (2005), Fujiwara (2007), and Wang and Chen (2010) further explored the privatization policy at free entry mixed oligopoly.

In this paper, we extend the analysis by Fujiwara (2009) considering a mixed oligopoly rather than a private oligopoly at free entry, wherein firms produce the differentiated goods with environmental damage that is measured by the government and observed by the consumers. Furthermore, we re-examine the environmental taxes or output subsidies that are levied or provided at no entry and free entry showing how privatization affects environmental damage and social welfare. Similar to Fujiwara

(2009), we find that in both mixed and pure oligopolies with no entry or free entry, the emission tax/output subsidy could be provided but with somewhat different reasons. The emission taxes are charged when the market power and consumer cognition of pollution are low, while the production pollution is large. However, when the consumers' cognition of pollution damage is increasing and the consumers are willing to pay less for the polluted good, *the environmental policy for the government to choose would be a lower emission tax or even to provide output subsidy*. In addition, we argue that even though more firms after privatization will increase the choice of product variety, the entry of private firms is excessive from the viewpoint of social optimum. Accordingly, the government should not privatize public firms.

This paper is organized as follows. Section 2 provides the basic model. Section 3 provides the results of the environmental policies in a differentiated mixed oligopoly with differentiated goods with no entry and at free entry. Section 4 does a similar analysis in pure oligopoly at no entry and free entry, and compares two scenarios to see whether the privatization neutrality theorem holds. Section 5 presents concluding remarks.

2. The model

We consider an economy in which there exists one public firm (indexed by 0) and $n-1$ private firms (indexed by j, $j=1,...,n-1$). All firms produce heterogeneous goods and there is a numeraire good. The utility function of the representative consumer specified by Ottaviano *et al.* (2002) and Fujiwara (2009) is generalized to allow for $n-1$ private firms and one public firm with the environmental awareness of an individual,

$$U=\alpha\sum_{j=0}^{n-1}q_j-\frac{\beta-\gamma}{2}\sum_{j=0}^{n-1}q_j^{\,2}-\frac{\gamma}{2}(\sum_{j=0}^{n-1}q_j)^2+I-\frac{\phi}{2}Q^2-\frac{s}{2}Z^2, \tag{1}$$

$$\alpha,\beta,\gamma,\phi>0, \text{ and } \beta>\gamma$$

where $Q=q_0+\sum_{j=1}^{n-1}q_j$, and q_j, respectively, denote the aggregate output and the quantity of each differentiated good. The parameter γ denotes product variety. The term "strong love of variety" is used when γ is much smaller than β, and I is the composite homogeneous good. The environmental awareness of an individual is described by his sensitivity to environmental friendliness in consuming the good, i.e., environmental quality of the goods (Yakita and Yamauchi, 2010). The consumers realize not only their own consumption but also other consumers' consumption will damage the environment. Hence, the total consumption Q will reduce consumers' utility level from cognizable $\frac{\phi}{2}Q^2$ that will affect the WTP for the concerned goods. Analogous to Fujiwara (2009), $\frac{s}{2}Z^2$ is not cognized by consumers. The total negative effect in the utility function is $\frac{\phi+s}{2}Q^2$ with $Z=Q$.

The utility function (1) generates the inverse demand functions as follows

$$p_0 = \alpha - (\beta - \gamma)q_0 - (\gamma + \phi)\sum_{j=0}^{n-1} q_j , \tag{2}$$

$$p_j = \alpha - (\beta - \gamma)q_j - (\gamma + \phi)\sum_{i=0}^{n-1} q_i , \text{ for } j = 1,...,n-1, \; i = 1,...,n-1. \tag{3}$$

Note that in Eqs. (2) and (3), what the consumer is willing to pay for the polluted good is affected by the consumers' awareness, which is not considered by Lahiri and Ono (2007) and Fujiwara (2009).

Each firm has to pay an environmental tax t per unit of pollutant; $T = t\sum_j e_j$ is the total taxes collected by the government. The cost function is measured by a quadratic form, $q_0^2/2$ and $q_j^2/2$ for public and private firms[2] respectively, and the profit functions are,

$$\pi_0 = p_0 q_0 - \frac{q_0^{\,2}}{2} - te_0 - f , \tag{4}$$

[2] In Fujiwara (2009), the constant marginal cost function is adopted. Notice that the linearly increasing marginal cost function is more general, which allows for possible interactions between environmental and production variables. See De Fraja and Delbono (1989), Bárcena-Ruiz and Garzón (2006), Kato (2006) and Wang and Wang (2009) for the specification of linearly increasing marginal cost function under mixed oligopoly.

$$\pi_j = p_j q_j - \frac{q_j^{\ 2}}{2} - te_j - f\ , \tag{5}$$

where f is the fixed cost.

The consumer surplus is defined as

$$CS = \alpha(q_0 + \sum_{j=1}^{n-1} q_j) - \frac{\beta-\gamma}{2}(q_0^{\ 2} + \sum_{j=1}^{n-1} q_j^{\ 2}) - \frac{\gamma+\phi+s}{2}(q_0 + \sum_{j=1}^{n-1} q_j)^2 - p_0 q_0 - \sum_{j=1}^{n-1} p_j q_j\ ,$$

in which the private cognition of environmental damage is deducted from consumer surplus. As in Bárcena-Ruiz and Garzón (2006), the social welfare is the sum of the consumer surplus, the firm's profit, and the total tax revenue measured by the environmental authority,

$$W = CS + \pi_0 + \sum_{j=1}^{n-1} \pi_j + T\ . \tag{6}$$

Following Ulph (1996), the production of this good by both public and private firms will lead to pollution $e_j = \sqrt{\theta} q_j$, $\sqrt{\theta} \le 1$. Environmental damage is measured by the quadratic form $GED = (\sum_j e_j)^2 / 2 = (\sum_j \sqrt{\theta} q_j)^2 / 2 = \theta Q^2 / 2 = (\phi + s) Q^2 / 2$ which is the government measure and $\theta = \phi + s$.

3. Environmental policies in mixed oligopoly

3.1 Environmental policy with no entry

In the 1st stage, the government chooses an environmental policy to maximize social welfare. In the 2nd stage, Cournot-Nash competition with a quantity-setting game is postulated. The public firm is assumed to maximize social welfare, and the private firms are maximizing their profits. (Superscript M represents the equilibrium of mixed oligopoly)

In the short run, the number of private firms is fixed. In the market stage, the public firm maximizes social welfare, $\partial W / \partial q_0 = 0$, and the private firms maximize their profits, $\partial \pi_j / \partial q_j = 0$. From the first-order conditions, we have

$$q_0 = \frac{\alpha[1+\phi+2\beta-\gamma-(n-1)s]+(n-1)t\sqrt{\phi+s}(\gamma+\phi+s)}{H_1}, \tag{7}$$

$$q_j = \frac{\alpha(1+s+\beta-\gamma)-t\sqrt{\phi+s}(1+\beta+\phi+s)}{H_1}, \tag{8}$$

where $H_1 = (1+\beta+\phi+s)[1+2\beta+(n-2)\gamma+n\phi]-(n-1)(\gamma+\phi)(\gamma+\phi+s)$.

Differentiating Eqs. (7) and (8) with respect to t, we obtain

$$\frac{\partial q_0}{\partial t} = \frac{(n-1)\sqrt{\phi+s}(\gamma+\phi+s)}{H_1} > 0,$$

$$\frac{\partial q_j}{\partial t} = \frac{-\sqrt{\phi+s}(1+\beta+\phi+s)}{H_1} < 0,$$

$$\frac{\partial Q}{\partial t} = \frac{\partial q_0}{\partial t} + (n-1)\frac{\partial q_j}{\partial t} = \frac{-(n-1)\sqrt{\phi+s}(1+\beta-\gamma)}{H_1} < 0.$$

Lemma 1. *In the short run, the output of public firm is increasing in* t*, but the output of private firms and total market supply are decreasing in* t.

Given that the output of the private firms decreases with the tax (since it is a cost for the firms), the output of the public firm increases with the tax to compensate the reduction in production of the private firms (since the public firm considers consumer surplus).[3] The higher tax raises the marginal cost inclusive of tax, and hence decreases the outputs of the private firms and increases the output of the public firm due to the benevolence of social welfare. The market supply is decreasing with a higher tax.

We see that in the environmental policy stage, using the results in the market stage, the government sets the environmental tax to maximize social welfare and solves the first-order condition that gives the short-run optimal environmental tax given the satisfaction of second-order condition:

$$t^M = \frac{\alpha[ns-(\beta+\phi)]}{\sqrt{\phi+s}[1+\beta-\gamma+n(\gamma+\phi+s)]}. \tag{9}$$

Lemma 2. *In mixed oligopoly with no entry, in equilibrium, the optimal emission tax,*

[3] We appreciate one referee providing the intuition of the result.

the output level of each firm, the total output, and the market price, the profit of each firm, the environmental damage, and the social welfare are, respectively:

$$t^M = \frac{\alpha[ns-(\beta+\phi)]}{\sqrt{\phi+s}[1+\beta-\gamma+n(\gamma+\phi+s)]}, \quad q_j^M = q_0^M = \frac{\alpha}{1+\beta-\gamma+n(\gamma+\phi+s)},$$

$$Q^M = \frac{\alpha n}{1+\beta-\gamma+n(\gamma+\phi+s)}, \quad p_j^M = p_0^M = \frac{\alpha[1+ns]}{1+\beta-\gamma+n(\gamma+\phi+s)},$$

$$\pi_j^M = \pi_0^M = \frac{\alpha^2[1+2(\beta+\phi)]}{2[1+\beta-\gamma+n(\gamma+\phi+s)]^2} - f, \quad GED^M = \frac{\alpha^2 n^2(\phi+s)}{2[1+\beta-\gamma+n(\gamma+\phi+s)]^2},$$

$$W^M = n\{\frac{\alpha^2}{2[1+\beta-\gamma+n(\gamma+\phi+s)]} - f\}.$$

From Lemma 2, we can see that when the marginal cost of production is linearly increasing, the differentiated goods produced by both public and private firms will lead to the same pollution. The reasoning is: firstly, the objective function of the public firms is welfare maximization, and due to the self correction mechanism, it leads to a reduction of its output to the social optimum; and secondly, the optimal emission tax will ameliorate the distortion reflecting an asymmetric objective function between the public and the private firms, resulting in the same output and price[4].

From Eq. (9) and comparative static analysis, we obtain Corollary 1.

Corollary 1. *In mixed oligopoly with no entry:*

(i) $\frac{\partial t^M}{\partial \gamma} < 0, \frac{\partial t^M}{\partial \phi} < 0,$ when $n > \tilde{n} \equiv (\beta+\phi)/s,$

(ii) $\frac{\partial t^M}{\partial s} > 0, \ \frac{\partial t^M}{\partial n} > 0.$

Proof: *See Appendix 1.*

From Corollary 1(i), on consumption side, because more product variety is better

[4] In Wang and Wang (2009), the modeling of mixed duopoly with differentiated good and pollution abatement have a different result, mainly because of different specifications of the objective function for the public firm. This case is being called CSPS maximizer in Kato (2011) who compared equilibrium outcomes under four environmental regulations-uniform emission taxes and quota, and differentiated emission taxes and quotas. We appreciate one referee pointing out the possible implications of CSPS maximizer in our generalized framework, which deserves further consideration in free entry when the private firms are allowed to enter the product market.

for the consumers, the emission tax could be lower, and in addition to that, the consumers who have private knowledge of the pollution damage will adopt self-protection measures; accordingly, the emission tax can be lower. From Corollary 1(ii), on the production side, when there are more firms, the less the environmental damage is cognized by the consumer(s), and the environment will be damaged more with higher emission tax then being needed.

From Eq. (9) and Corollary 1, we obtain the following Proposition 1.

Proposition 1. *In mixed oligopoly with no entry, the optimal emission tax is positive under* $n > \tilde{n}$ *; otherwise, the output subsidy is provided.*

With the consideration of production pollution and consumer awareness of pollution damage, the short-run optimal emission tax could either be positive or negative. The intuition behind Proposition 1 is stated in Corollary 1.

What is left for us to analyze is that when ϕ and θ are not equal to zero; if γ is lower, each firm's market power increases and higher emission taxes are levied. We close this section by mentioning the case with $\phi = 0$ and $\theta = s$, when there is no environmental cognition by the consumers, it follows that $t^M = \alpha[ns - \beta]/\sqrt{s}[1 + \beta - \gamma + n(\gamma + s)]$, the short-run emission tax could either be positive or negative which is shown in Fujiwara (2009) without having the public firm compete with the private firms.

3.2 Environmental policy at free entry

In this section, we examine the design of environmental policy at free entry equilibrium. In the 1st stage, the government chooses an environmental policy to maximize social welfare. In the 2nd stage, private firms decide on entry or not. In the 3rd state, Cournot-Nash competition with a quantity-setting game is postulated.

Consider the case where entry occurs in the market. The net profits of the $j-th$ firm, $j = 1, \ldots, n-1$ are

$$\pi_j = \frac{[1 + 2(\beta + \phi)][\alpha(1 + \beta - \gamma + s) - t\sqrt{\phi + s}(1 + \beta + \phi + s)]^2}{2H_1^2} - f \,. \quad (10)$$

Postulating that all j firms' profits are driven to zero, we attempt to explore the long run equilibrium wherein free entry and exit prevail; that is, the number of private firms $n-1$ is endogenously determined.

$$n^{ME}=\frac{\sqrt{2(1+2\phi+2\beta)}[\alpha(1+\beta-\gamma+s)-t\sqrt{\phi+s}(1+\beta+\phi+s)]-2\sqrt{f}[1+\phi+\phi^2+3\beta+2\phi\beta+2\beta^2+\gamma^2-2\gamma(1+\beta)+(1+\phi+2\beta-\gamma)s]}{2\sqrt{f}(\phi+\gamma)(1+\beta-\gamma)}, \tag{11}$$

where the superscript ME denotes the free entry equilibrium in mixed oligopoly. Substituting Eq. (11) into Eqs. (7) and (8), we obtain

$$q_0^{ME}=\frac{\sqrt{2(1+2\phi+2\beta)}[t\sqrt{\phi+s}(\gamma+\phi+s)-\alpha s]+2\sqrt{f}(1+\phi+2\beta-\gamma)(\gamma+\phi+s)}{\sqrt{2(1+2\phi+2\beta)}(\phi+\gamma)(1+\beta-\gamma)}, \tag{12}$$

$$q_j^{ME}=\sqrt{\frac{2f}{1+2\phi+2\beta}}. \tag{13}$$

Under free entry and exit of firms, Lahiri and Ono (2007) found that there is an effect that works via a change in the number of firms. The sign of this effect depends on the convexity/concavity of the inverse demand function. In our linear demand case, we derive that the output of the private firms does not depend on the environmental policy, which is the same as that obtained in Fujiwara (2009).

Differentiating Eqs. (11), (12) and (3) with respect to t, we obtain

Corollary 2. *In mixed oligopoly at free entry,*

$$\frac{\partial n^{ME}}{\partial t}=-\frac{\sqrt{2(1+2\phi+2\beta)}(1+\beta+\phi+s)\sqrt{\phi+s}}{2\sqrt{f}(\phi+\gamma)(1+\beta-\gamma)}<0,\quad \frac{\partial q_0^{ME}}{\partial t}=\frac{(\gamma+\phi+s)\sqrt{\phi+s}}{(\phi+\gamma)(1+\beta-\gamma)}>0,\quad \frac{\partial q_j^{ME}}{\partial t}=0.$$

Corollary 2 indicates that the number of private firms at free entry is decreasing in emission tax, and the output of the private firms remains unchanged, but the public firm needs to produce more to comprehend the reduced consumer's surplus.

Substituting Eq. (11) into Eq. (6), the long-run emission tax, environmental damage and social welfare are obtained as:

$$t^{ME}=\frac{\alpha s\sqrt{2(1+2\phi+2\beta)}-\sqrt{f}[2(1+\phi+2\beta-\gamma)s+(\beta-\gamma)(\phi+\gamma)]}{(\gamma+\phi+s)\sqrt{2(1+2\phi+2\beta)(\phi+s)}}, \tag{14}$$

$$GED^{ME}=\frac{[\alpha\sqrt{2(1+2\phi+2\beta)}-\sqrt{f}(2+2\phi+3\beta-\gamma)]^2(\phi+s)}{4(1+2\phi+2\beta)(\gamma+\phi+s)^2}, \tag{15}$$

$$W^{ME} = \frac{1}{2(\gamma+\phi+s)}\{\alpha^2 - \alpha\sqrt{f}\,\frac{\sqrt{2}(2+2\phi+3\beta-\gamma)}{\sqrt{1+2\phi+2\beta}} + \frac{f}{2(1+2\phi+2\beta)(1+\beta-\gamma)}\times$$
$$[4+4\phi^3+9\beta^3+\beta^2(21-14\gamma+s)+4\phi^2(1+2\beta+\gamma+s)+\phi(8+20\beta+13\beta^2-12\gamma-10\beta\gamma+9\gamma^2+4(\beta+\gamma)s)$$
$$+\gamma(\gamma(5+s)-8)+\beta(16+\gamma(9\gamma+2s-22))]\}. \qquad (16)$$

From Eq. (14), we have $t^{ME} > 0$ if $s > \tilde{s}$ at free entry of private firms, where

$$\tilde{s} \equiv \frac{(\beta-\gamma)(\phi+\gamma)}{\alpha\sqrt{2+4\phi+4\beta} - 2\sqrt{f}(1+\phi+2\beta-\gamma)}.$$

In long-run equilibrium, the government imposes an emission tax when $s > \tilde{s}$; however, subsidization is required when $s < \tilde{s}$.[5] The same reasoning can be found as we provided for Corollary 1 and Proposition 1. We obtain the following Proposition 2.

Proposition 2. *In a free-entry mixed oligopoly, the optimal emission tax is positive under* $s > \tilde{s}$*; otherwise, the output subsidy is provided.*

Next, we discuss how the change in γ will affect the optimal environmental policy in the long-run.

Corollary 3. Emission tax is increasing in γ if and only if

$$\Gamma(s,\phi) \equiv \sqrt{2f}[(\phi+\gamma)^2 + (2+5\phi+3\beta+2\gamma)s + 2s^2] - 2\alpha s\sqrt{(1+2\phi+2\beta)} > 0.$$

Proof. Setting $t^{ME} > 0$ in Eq. (14), condition $s > \tilde{s}$ is needed. Differentiating Eq. (14) with respect to γ, we have

$$\frac{dt^{ME}}{d\gamma} = \frac{\Gamma(s,\phi)}{2\sqrt{(1+2\phi+2\beta)(\phi+s)}(\gamma+\phi+s)^2} > 0,$$

and we see that $dt^{ME}/d\gamma > 0$ is equivalent to $\Gamma(s,\phi) > 0$.

Setting $t^{ME} > 0$ in Eq. (14), the condition for $s > \tilde{s}$ is equivalent to $\gamma > \tilde{\gamma}$, where

$$\tilde{\gamma} \equiv \frac{(\beta-\phi-2s) + \sqrt{(\phi+\beta-2s)^2 + 8(1+\phi+2\beta)s + 4\beta\phi - 4\alpha s\sqrt{2(1+2\phi+2\beta)/f}}}{2}.$$

[5] Similarly, Fujiwara (2009) showed that when $s > F(\gamma) = \gamma(\beta-\gamma)\big/2[(\alpha-c)\sqrt{\beta/f^2} - 2\beta+\gamma]$, $t^E > 0$.

Differentiating $\Gamma(s,\phi)$ with respect to γ, we have $d\Gamma(s,\phi)/d\gamma \equiv 2\sqrt{2f}(\gamma+\phi+s) > 0$. When γ is sufficiently large, $t^{ME} > 0$. Meanwhile, when $\Gamma(s,\phi) > 0$, $dt^{ME}/d\gamma > 0$. That is, the tax rates become lower when products are more differentiated. It is reasonable for the government to levy the emission taxes because the environmental damages are sufficiently high.

In Figure 1, the pair of (γ,ϕ) which makes $t^{ME} = 0$ is depicted as an inverted-U locus as L_1, when $\phi = 0.05$. In the area below L_1 (region I), we have $t^{ME} < 0$, while in the area above L_1 (region II and III), we have $t^{ME} > 0$. As we stated above, the emission taxes are affected by three effects: pollution reduction, variety reduction and cognition reduction.

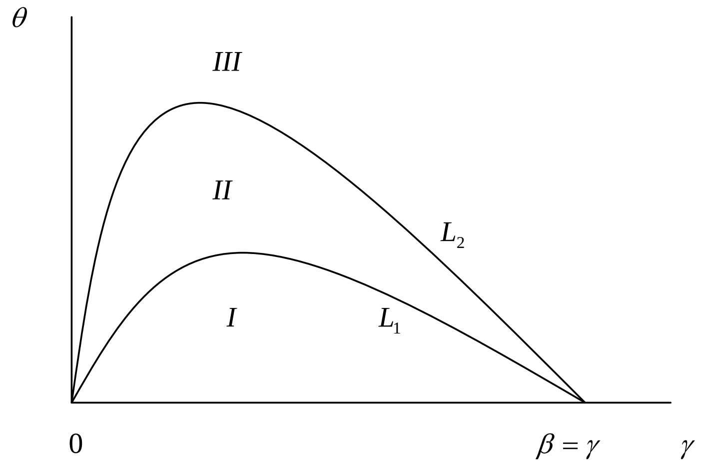

Figure 1. The region for t^{ME}

Figure 1 provides us with a few possibilities for the sign pattern of t^{ME} and $dt^{ME}/d\gamma$. In the case where $s > \tilde{s}$ and $\Gamma(s,\phi) > 0$, which is mostly possible if the consumer is sufficiently environmentally conscious, taxation is optimal and its response to γ is positive. It implies that the more differentiated the products are, the lower the tax rate will be, which is because the government has a stronger incentive to raise the consumer's gains from product variety by levying a lower environmental tax.

When γ is close to β, products are less substitutable, and it is optimal to impose a very high environmental tax to restrict entry since there is no gain from product variety. This result corresponds to the case that Katsoulacos and Xepapadeas (1995) and Fujiwara (2009) addressed, in which the economy suffers from two market distortions and the correcting tax exceeds marginal environmental damage.

When γ is close to 0, each firm's market power is strongest and so is market distortion from imperfect competition. It is in the government's interest to levy a high environmental tax for the purpose that is not only to reduce pollution damage but to restrict the number of firms as well. Figure 1 identifies these findings, where subsidization becomes optimal only if products are apparently less differentiated.

Now we turn to discuss the effect of the pollution cognition of the consumers. In Figure 1, the pair of (γ,ϕ), which makes $t^{ME}=0$ is depicted as an inverted-U locus that will increase from L_1 to L_2, when ϕ is increasing from 0.05 to 0.06. In the area below L_2 (region I and II), we have $t^{ME}<0$ while in the area above L_2 (region III), we have $t^{ME}>0$. The tax rates become lower as consumer cognition is rising. It is reasonable to argue that the higher the consumer cognition, the lower the tax rates become, because the government has a stronger incentive to raise the consumer's gains from product variety by levying a lower tax or even providing subsidies.

4. Environmental policy in private oligopoly

4.1 Environmental policy with no entry

When the public firm is privatized, there are n private firms competing in the market by choosing output. (Superscript P represents the equilibrium of mixed oligopoly)

In the market stage, from the first-order conditions, we have

$$q_j = \frac{\alpha - t\sqrt{\phi + s}}{1 + (1+n)\phi + 2\beta + (n-1)\gamma}. \tag{17}$$

From Eq. (17), the outputs of the private firms are decreasing in the environmental tax.

In the policy stage, substituting Eq. (17) into Eq. (6), we then take the first-order derivative with respect to t, which gives the following optimal emission tax:

$$t^P = \frac{\alpha[ns - (\beta + \phi)]}{\sqrt{\phi + s}[1 + \beta - \gamma + n(\phi + \gamma + s)]} \tag{18}$$

Note that when $\phi = 0$ and $\theta = s$, the similar result is found in Fujiwara (2009) irrespective of the specification of cost function. [6]

In the scenario of pure oligopoly with no entry, the following Lemma is derived for the purpose of comparison.

Lemma 3. *In pure oligopoly with no entry, in equilibrium, the optimal emission tax, the output level of each firm, the total output, and the market price, the profit of each firm, the environmental damage, and the social welfare are, respectively*:

$$t^P = \frac{\alpha[ns - (\beta + \phi)]}{\sqrt{\phi + s}[1 + \beta - \gamma + n(\gamma + \phi + s)]}, \quad q_j^{\,P} = \frac{\alpha}{1 + \beta - \gamma + n(\gamma + \phi + s)}, \quad Q^P = \frac{\alpha n}{1 + \beta - \gamma + n(\gamma + \phi + s)},$$

$$p_j^{\,P} = \frac{\alpha(1 + ns)}{1 + \beta - \gamma + n(\gamma + \phi + s)}, \quad \pi_j^{\,P} = \frac{\alpha^2(1 + 2\phi + 2\beta)}{2[1 + \beta - \gamma + n(\gamma + \phi + s)]^2} - f,$$

$$GED^P = \frac{\alpha^2 n^2(\phi + s)}{2[1 + \beta - \gamma + n(\gamma + \phi + s)]^2}, \quad W^P = n\{\frac{\alpha^2}{2[1 + \beta - \gamma + n(\gamma + \phi + s)]} - f\}.$$

From Lemmas 2 and 3, we see that the equilibrium outcomes under mixed oligopoly and pure oligopoly are identical. The reasoning is that there are two distortions in the model setting with the same production efficiency between public and private firms, and due to that, the market distortion and pollution distortion are dependent, so only one policy instrument is needed. Given that the second-best taxation is implemented, the output behavior of the public and private firms will not

[6] In the case of constant marginal cost as specified in Fujiwara (2009), $t^P = \dfrac{(\alpha - c)[ns - (\beta + \phi)]}{\beta - \gamma + n(\gamma + \phi + s)}$.

deviate from the social optimum.

From Eq. (18) and comparative static analysis, we obtain Corollary 4.

Corollary 4. *In pure oligopoly with no entry:*

(i) $\frac{\partial t^P}{\partial \gamma} < 0, \frac{\partial t^P}{\partial \phi} < 0$, when $n > \tilde{n}$,

(ii) $\frac{\partial t^P}{\partial s} > 0, \frac{\partial t^P}{\partial n} > 0$.

The results shown in Corollary 4 are similar to Corollary 1; the same explanation can be found as we provided for Corollary 1. We immediately have the following Proposition 3.

Proposition 3. *In pure oligopoly with no entry, the optimal emission tax, environmental damage, and social welfare are the same as the one in a mixed oligopoly; the privatization neutrality theorem holds.*

The emission taxes in pure and mixed oligopolies are the same, but this result is different from Wang and Wang (2009). In Wang and Wang (2009), the objective function of the public firm is defined as the sum of consumer surplus and the firm's profit, and the modeling of mixed duopoly with differentiated good and pollution abatement leads to a different result, mainly because of the different specification of the objective function for the public firm. From Lemma 2, we see that when the marginal cost of production is linearly increasing, the differentiated goods produced by both public and private firms lead to the same pollution, and the optimal emission tax will ameliorate the distortion reflecting an asymmetric objective function between the public and the private firms, resulting in the same output and price.

4.2 Environmental policy at free entry

Similar to Fujiwara (2009), the long-run emission tax and social welfare assuming quadratic production cost function in our model are obtained as:

$$t^{PE} = \frac{\alpha s\sqrt{2(1+2\phi+2\beta)} - \sqrt{f}[2(1+\phi+2\beta-\gamma)s + (\beta-\gamma)(\phi+\gamma)]}{(\gamma+\phi+s)\sqrt{2(1+2\phi+2\beta)(\phi+s)}}, \tag{19}$$

$$GED^{PE} = \frac{[\alpha\sqrt{2(1+2\phi+2\beta)} - f(2+2\phi+3\beta-\gamma)]^2(\phi+s)}{4(1+2\phi+2\beta)(\gamma+\phi+s)^2}, \tag{20}$$

$$W^{PE} = \frac{[\alpha\sqrt{2(1+2\phi+2\beta)} - f(2+2\phi+3\beta-\gamma)]^2}{4(1+2\phi+2\beta)(\gamma+\phi+s)}. \tag{21}$$

From Eq. (19), we find the $t^{PE} > 0$ if $s > \tilde{s}$ in the long-run equilibrium. We obtain the following Proposition 4.

Proposition 4. *The optimal emission tax and environmental damage in pure and mixed oligopolies are the same at free entry equilibrium.*

The reasoning for Proposition 4 was partly provided in Propositions 2 and 3. We see that at free entry equilibrium, the socially optimum output is the same under pure and mixed oligopolies through the market mechanism under which the number of private firms and the distribution of total output among the public and private firms are adjusted in pure and mixed markets.

Furthermore, we find that even though the optimal emission taxes and environmental damage in pure and mixed oligopolies are equal at free entry equilibrium, the social welfare comparison exhibits the implication of privatization with optimal environmental policy. We compare the welfare differences before and after the public firm are privatized, and obtain

$$W^{ME} - W^{PE} = \frac{f(2\phi+\beta+\gamma)^2}{4(1+2\phi+2\beta)(1+\beta-\gamma)} > 0. \tag{22}$$

From Eq. (22), we have the next Proposition 5.

Proposition 5. *The social welfare with free entry in mixed oligopoly is larger than pure oligopoly. The government should not privatize the public firm.*

To understand the reasoning behind this proposition, we need to compare the numbers of the private firms in mixed and pure oligopolies which are, respectively,

$$n^{ME} = \frac{1}{2}\left[\frac{\alpha\sqrt{2+4\phi+4\beta} - \sqrt{f}(2+2\phi+3\beta-\gamma)}{\sqrt{f}(\gamma+\phi+s)} - \frac{2\phi+\beta+\gamma}{1+\beta-\gamma}\right],$$

$$n^{PE} = \frac{1}{2}\left[\frac{\alpha\sqrt{2+4\phi+4\beta} - \sqrt{f}(2+2\phi+3\beta-\gamma)}{\sqrt{f}(\gamma+\phi+s)}\right].$$

We have

$$n^{ME} - n^{PE} = -\frac{2\phi+\beta+\gamma}{2(1+\beta-\gamma)} < 0.$$

We argue that the number of the private firms at differentiated free-entry oligopoly after the privatization of the public firm exceeds the scenario with the public firm mainly because the consumers have more choice of product varieties. Nevertheless, even though more firms after privatization will increase the choice of product variety, the entry of private firms is excessive from the viewpoint of social optimum. Hence, the government should not privatize the public firm.

5. Concluding Remarks

In this paper, we investigated the optimal environmental and privatization policies in a mixed oligopoly, wherein firms produce the differentiated goods with environmental damage that is measured by the government and considered by the consumers. Furthermore, we re-examined the emission taxes or output subsidies that are levied or provided at no entry and free entry showing how privatization affects environment damage and social welfare.

We found that in both mixed and pure oligopolies with no entry or free entry, the emission tax/output subsidy could be provided. The emission taxes are charged when the market power and consumer cognition of pollution are low, while the production pollution is large. However, when the consumers' cognition of pollution damage is increasing and the consumers are willing to pay less for the polluted good, *the environmental policy for the government to choose would be a lower emission tax or even to provide output subsidy*. In addition, we argue that even though more firms after privatization will increase the choice of product variety, the entry of private firms is excessive from the viewpoint of social optimum. Hence, the government should not privatize the public firm.

References

Bárcena-Ruiz, J. C., Garzón, M. B., 2006. Mixed oligopoly and environmental policy. Spanish Economic Review 8, 139-160.

Beladi, H., Chao, C.-C., 2006. Does privatization improve the environment? Economics Letters 93, 343-347.

De Fraja, G., Delbone, F., 1989. Alternative strategies of a public enterprise in oligopoly, Oxford Economic Papers 41, 302-311.

Fujiwara, K., 2009. Environmental policies in a differentiated oligopoly revisited. Resource and Energy Economics 31, 239-247.

Fujiwara, K., 2007. Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92, 51–65.

Kato, K., 2011. Emission quota versus emission tax in a mixed duopoly. Environmental Economics and Policy Studies 13, 43-63.

Kato, K., 2006. Can allowing to trade permits enhance welfare in mixed oligopoly? Journal of Economics 88, 263-283.

Katsoulacos, Y., Xepapadeas, A., 1995. Environmental policy under oligopoly with endogenous market structure. Scandinavian Journal of Economics 97, 411-420.

Lahiri, S., Ono, Y., 2007. Relative emission standard versus tax under oligopoly: the role of free entry. Journal of Economics 91, 107-128.

Matsumura, T., Kanda, O., 2005. Mixed oligopoly at free entry markets. Journal of Economics 84, 27-48.

Ottaviano, G.I.P., Tabuchi, T., Thisse, J., 2002. Agglomeration and trade revisited. International Economic Review 43, 409-436.

Simpson, R.D., 1995. Optimal pollution taxation in a Cournot duopoly. Environmental and Resource Economics 6, 359-369.

Ulph, A., 1996. Environmental policy and international trade when governments and producers act strategically. Journal of Environmental Economics and Management 30, 265-281.

Wang, L.F.S., Chen, T-L., 2010. Do cost efficiency gap and foreign competitors matter concerning optimal privatization policy at the free entry market? Journal of Economics 100, 33-49.

Wang, L.F.S., Wang, J., 2009. Environmental taxes in a differentiated mixed duopoly. Economics Systems 33, 389-396.

Yakita, A., Yamauchi, H., 2010. Environmental awareness and environmental r&d spillovers in differentiated duopoly. Research in Economics (forthcoming).

Yin, X., 2003. Corrective taxes under oligopoly with inter-firm externalities. Environmental and Resource Economics 26, 269-277.

Appendix 1

(i) $\frac{\partial t^M}{\partial \gamma} = \frac{-\alpha(n-1)[ns-(\beta+\phi)]}{\sqrt{\phi+s}[1+\beta-\gamma+n(\gamma+\phi+s)]^2} < 0$, when $n > \tilde{n}$,

$$\frac{\partial t^M}{\partial \phi} = -\frac{\alpha \hat{H}_1}{(\phi+s)[1+\beta-\gamma+n(\gamma+\phi+s)]^2}$$, where

$$\hat{H}_1 \equiv 2(\phi+s)[1+\beta-\gamma+n(\gamma+\phi+s)]+[ns-(\beta+\phi)][1+\beta-\gamma+n(\gamma+3\phi+3s)]$$

The sign of $\frac{\partial t^M}{\partial \phi}$ depends on the sign of $\hat{H}_1$. When $n > \tilde{n}$, $\hat{H}_1 > 0$. we obtain

$\frac{\partial t^M}{\partial \phi} < 0$, when $n > \tilde{n}$.

(ii) $\frac{\partial t^M}{\partial s} > 0$.

$$\frac{\partial t^M}{\partial n} = \frac{\alpha[(1+\phi+2\beta-\gamma)s+(\beta+\phi)(\phi+\gamma)]}{\sqrt{\phi+s}[1+\beta-\gamma+n(\gamma+\phi+s)]^2} > 0.$$