

Partial Privatization and Strategic Budget Constraints in Unionized Mixed Duopoly with Firm Heterogeneity

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Abstract

With the incorporation of the firm heterogeneity and the budget constraint, this paper provides a fresh perspective toward the decision-making of privatization policy. Under the consideration of a public firm with lower productive efficiency than a private firm, if the budget constraint is not required, the best policy is full nationalization when the wage is decentralized bargained. If the wage is centralized bargained and the efficiency gap between the public firm and the private firm is small, partial privatization is the best policy with positive relation between the productivity difference and the privatization degree. If the efficiency gap is moderate, then the best policy is full privatization. If the efficiency gap is huge, then the best policy is again partial privatization with the inverse relation between the productivity difference and the privatization degree. Lastly, if the budget constraint is mandatory, then the neutrality of partial privatization holds.

Keywords: Firm heterogeneity; Budget constraint; Partial privatization

JEL classification: J31; J51; L13

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1. Introduction

It has been recognized that a public firm may not earn positive profit in the mixed oligopoly theory. Recently, the issue of budget constrain is being raised in different context of mixed market.¹ In Capuano and De Feo (2010) the budget constraint is introduced in the objective function of a public firm taking into account the distortionary effects of the taxes for covering public expenditures. Choi (2011) considers the budget-constraint problem where the government decides whether or not to impose a budget on a public firm assuming that the public firm to be less efficient than private firms. Choi (2011) finds that imposing budget constraints on a public firm is the preferred choice because of the welfare-improving effect, and the wage levels of the public firm can be lower or higher than those of private firms depending upon the degree of inefficiency. The above results differ from Ishida and Matsushima's (2009) findings that in a unionized mixed duopoly, tight budget constraints can enhance social welfare when the public firm is as efficient as private firms.

Instead of assuming cost difference between a public firm and a private firm, this paper incorporates "firm heterogeneity" into the privatization policy making to demonstrate the unique consideration concerning both budget constraint and productivity difference. The concept of "firm heterogeneity" is actually extended from the productivity difference among firms and therefore those two terms will be used interchangeably hereafter. The key concept of it is the difference of management or governance ability rather than exogenous asymmetric input characteristics (Bastos *et al.*, 2009). It is able to interpret the overall productivity improvement or deterioration with homogeneous input, which is particularly meaningful in the analyses of mixed oligopoly market with different productivity among firms. It would be more reasonable to reckon that the nature of manpower in any firm is close or even equal to another because what causes the difference of labor quality among firms is the structure or operating scheme of a company. It is quite often in many previous researches to assume that a public firm is less productive than a private firm but the rationale behind it is veiled. This paper tries to provide a more clear way to define the productivity difference between firms. Furthermore, it is not hard to see that performances of public firms in many countries are

¹ See De Fraja and Delbono (1989) for the specification of the public firm in mixed oligopoly and De Fraja and Delbono (1990) for the general review of mixed oligopoly models. For literature on union bargaining in mixed oligopoly, see De Fraja (1993), Willner (1999), Gronblom and Willner (2008), Ishida and Matsushima (2009) and Choi (2011).

subject to the rigidity generated by hierarchical bureaucracies. That is the reason why there has been a wave of privatization since 1970s from Western Europe to raise the autonomy such as personnel and finance of public firms to improve its performances. Therefore, it becomes very fair to assume that a public firm is less productive than a private firm and this is the reason why the concept of firm heterogeneity is well applied here. It then makes it reasonable to consider the scenario with less productive public firms only but not to deal with the cases of equally efficient and more efficient public firms. It is because there is no need to do so. When a public firm is as efficient as a private one, there is no need to privatize it owing to the reason why a public firm is needed to be privatized is resulting from its low efficiency, and since there is no such symptom, the pharmaceuticals is needless to be prescribed. Besides, public firms usually shoulder vital responsibilities that ordinary private firms simply won't bear. Therefore, discussing the privatization of an efficient public firm might be less worthwhile from social welfare point of view.

This paper investigates how wage-bargaining process functions in a mixed duopoly in order to explicitly examine the budget constraints of a public firm, which enriches and broadens Choi (2011) because it considers full nationalization and full privatization only but fails to take partial privatization into account. This paper reveals that, under the consideration of a public firm with lower productive efficiency than a private firm, if the budget constraint is not required, the best policy is full nationalization when the wage is decentralized bargained. If the wage is centralized bargained and the efficiency gap between the public firm and the private firm is small, partial privatization is the best policy with positive relation between the productivity difference and the privatization degree. If the efficiency gap is moderate, then the best policy is full privatization. If the efficiency gap is huge, then the best policy is again partial privatization with the inverse relation between the productivity difference and the privatization degree. Lastly, if the budget constraint is mandatory, then the neutrality of partial privatization holds.

The remainder of the paper is organized as follows. Section 2 illustrates the model. Section 3 focuses on the scenario of decentralized bargaining, while Section 4 deals with the scenario of centralized bargaining. Section 5 concludes the paper.

2. The Basic Model

In a static model, consider a mixed duopoly setting where a public firm (firm 0) and a private firm (firm 1) produce a homogeneous product and compete in a quantity game. Assuming that the representative consumer's utility is a quadratic function given by $U = a(x_0 + x_1) - (x_0 + x_1)^2 / 2 + I$, where I is the composite good, the inverse demand function could be expressed as $p = a - X$, where $X = x_0 + x_1$ stands for the total demand and p is the market price. The output of the public firm is represented by x_0 while the output of the private firm is x_1 . It is assumed that labors are homogeneous and the public firm is less efficient than the private firm indicating that $x_0 = \phi(1-s)l_0$ and $x_1 = \phi(1+s)l_1$. The average productivity is represented by ϕ . The productivity difference between the public firm and the private firm is denoted by $0 < s < 1$ implying that to reach the same amount of output, the public firm needs to hire more labors. The cost function is $C(x_i) = w_i l_i + f$ with $i = 0, 1$ where fixed cost, amount of labor hired by each firm and the bargained wage by unions are denoted as f , l_i and w_i , respectively. Then the profit function is defined as

$$\pi_i = px_i - w_i l_i - f, \quad i = 0, 1 \quad (1)$$

There is one union in each firm making the market two unions in total but these two unions operate independently. The wages are determined by the bargain result between each union and its corresponding firm. Let the reserved wage be \bar{w} , then the utility functions of each union are

$$\text{Max}_{w_i} u_i = (w_i - \bar{w})^\theta L_i, \quad i = 0, 1 \quad (2)$$

The importance a union attaches to the wage is assigned as θ (See Leahy and Montagna, 2000, Lommerud *et al.*, 2003, Haucap and Wey, 2004, and Choi, 2011). To be able to focus on the purpose of this paper, the following assumptions are made: $\theta = 1$, $\bar{w} = 0$ and $f = 0$.

Social welfare is summation of consumer surplus, profits of firms and the utility of unions,

$$SW = CS + \sum_{i=0}^1 \pi_i + \sum_{i=0}^1 u_i \quad (3)$$

The consumer surplus is defined as $CS = X^2 / 2$. Following Matsumura (1998), the objective function of the public firm is associated to social welfare and the degree of privatization which is represented by α . The function is as

$$V = \alpha \pi_0 + (1 - \alpha) SW \quad (4)$$

A three-stage game is designed to explore the optimal degree of privatization under the consideration of the role of unions. In the first stage, the government decides the optimal degree of privatization with respect to the social welfare. The second stage deals with the wage bargained between unions and firms. Lastly, firms engage in Cournot competition. With SPNE, the equilibrium results will be derived.

3. The Optimal Degree of Privatization with Decentralized Bargaining

This section focuses on the decentralized negotiation between each union and its corresponding firm. The centralized bargain between the unions and firms will be analyzed in the section 4. In both sections 3 and 4, the scenario without budget constraint will be discussed first and it will be followed by the scenario with the budget constraint.

3.1 Privatization policy without budget constraint

To utilize backward deduction, the third stage of the game needs to be figured out first. Differentiation of Eqs. (1) and (4) shows that

$$\begin{aligned}\frac{\partial V}{\partial x_0} &= a - (1 + \alpha)x_0 - x_1 - \frac{\alpha}{(1-s)\phi} w_0 = 0 \\ \frac{\partial \pi_1}{\partial x_1} &= a - x_0 - 2x_1 - \frac{1}{(1+s)\phi} w_1 = 0\end{aligned}\tag{5}$$

Then the equilibrium of the third stage can be obtained as

$$\begin{aligned}x_0^* &= \frac{\alpha(1-s^2)\phi a - 2(1+s)\alpha w_0 + (1-s)w_1}{(1-s^2)(1+2\alpha)\phi}, \\ x_1^* &= \frac{\alpha(1-s^2)\phi a + (1+s)\alpha w_0 - (1-s)(1+\alpha)w_1}{(1-s^2)(1+2\alpha)\phi}.\end{aligned}\tag{6}$$

Then in the stage two, with the knowledge of output that derived in the stage three, the utility functions of unions are

$$\begin{aligned}u_0 &= \frac{w_0[(1-s^2)\phi a - 2\alpha(1+s)w_0 + (1-s)w_1]}{(1-s)^2(1+s)(1+2\alpha)\phi^2}, \\ u_1 &= \frac{w_1[\alpha(1-s^2)\phi a + \alpha(1+s)w_0 - (1-s)(1+\alpha)w_1]}{(1-s)(1+s)^2(1+2\alpha)\phi^2}.\end{aligned}$$

The first order conditions of those utility functions with respect to the wages are

$$\begin{aligned}\frac{\partial u_0}{\partial w_0} &= \frac{(1-s^2)\phi a - 4\alpha(1+s)w_0 + (1-s)w_1}{(1-s)^2(1+s)(1+2\alpha)\phi^2} = 0, \\ \frac{\partial u_1}{\partial w_1} &= \frac{\alpha(1-s^2)\phi a + \alpha(1+s)w_0 - 2(1-s)(1+\alpha)w_1}{(1-s)(1+s)^2(1+2\alpha)\phi^2} = 0.\end{aligned}\tag{7}$$

Solutions to Eq. (7) show

$$\begin{aligned}w_0 &= \frac{(1-s)(2+3\alpha)\phi a}{\alpha(7+8\alpha)}, \\ w_1 &= \frac{(1+s)(1+4\alpha)\phi a}{7+8\alpha}.\end{aligned}\tag{8}$$

Then Lemma 1 is immediate.

Lemma 1 The impacts of privatization degree, the efficiency gap and the average productivity on the wage rates are

$$\begin{aligned}\frac{\partial w_0}{\partial \alpha} &= -\frac{2(1-s)(7+16\alpha+12\alpha^2)\phi a}{(7+8\alpha)^2\alpha^2} < 0, \quad \frac{\partial w_1}{\partial \alpha} = \frac{20(1+s)\phi a}{(7+8\alpha)^2} > 0, \quad \frac{\partial w_0}{\partial s} = -\frac{(2+3\alpha)\phi a}{(7+8\alpha)\alpha} < 0, \\ \frac{\partial w_1}{\partial s} &= \frac{(1+4\alpha)\phi a}{7+8\alpha} > 0, \quad \frac{\partial w_0}{\partial \phi} = \frac{(1-s)(2+3\alpha)a}{(7+8\alpha)\alpha} > 0, \quad \frac{\partial w_1}{\partial \phi} = \frac{(1+s)(1+4\alpha)a}{7+8\alpha} > 0.\end{aligned}$$

Lemma 1 illustrates that under decentralized bargaining and the existence of efficiency gap between the public and private firms, when the privatization degree is higher, the bargained public wage would decline with higher private bargained wage. However, if the productivity difference is enlarged, the wage of the private firm would be raised but lower the public wage. Lastly, if the productivity of firms is improved, both wages would be higher.

From Eq. (8), it can be seen that when $\alpha < \alpha_1$, ($\alpha_1 = (3\delta - 1 + \sqrt{9\delta^2 + 26\delta + 1})/8$, $\delta \equiv (1-s)/(1+s) < 1$), $w_0 > w_1$, and when $\alpha > \alpha_1$, $w_0 < w_1$. Accordingly, whether the public wage is higher than the private wage depends on the degree of privatization, which reflects the productivity difference of the labor between public and private firms. This result is different from Choi (2011).²

Back to the first stage, through maximization of social welfare, the optimal degree of privatization can be determined. Therefore,

² See Choi's (2011) setting, the proposition 2: when $1 < \delta < 2$, $w_0 > w_1$, and when $\delta > 2$, $w_0 < w_1$.

$$\frac{\partial SW}{\partial \alpha} = -\frac{(2+3\alpha)(1+4\alpha)[33+8\alpha(13+11\alpha)]a^2}{(7+22\alpha+16\alpha^2)^3} < 0,$$

$$\frac{\partial^2 SW}{\partial \alpha^2} = \frac{359+4\alpha\{1859+8\alpha[1153+6\alpha(418+433\alpha+176\alpha^2)]\}}{(7+22\alpha+16\alpha^2)^4} a^2 > 0.$$

Hence, the optimal degree of privatization α^* is a corner solution.³

$$\alpha^* \rightarrow 0^+ \quad (9)$$

Then the following proposition is obtained:

Proposition 1 *Under decentralized union bargaining with firm heterogeneity, the optimal privatization policy is full nationalization.*

3.2 Privatization policy with budget constraint

With the consideration of budget constraint, the objective function of the public firm can be defined as

$$\begin{aligned} \text{Max}_{x_0} V &= \alpha\pi_0 + (1-\alpha)SW, \\ \text{s.t. } \pi_0 &= px_0 - w_0l_0 \geq 0 \end{aligned}$$

Denoting the multiplier of the budget constraint as λ , the Lagrangian equation can be written as

$$L = (\alpha + \lambda)\pi_0 + (1-\alpha)SW. \quad (10)$$

Taking w_1 as given, the first-order condition is given by

$$\frac{\partial L}{\partial x_0} = (1+\lambda)a - (1+\alpha+2\lambda)x_0 - (1+\lambda)x_1 - \frac{\alpha+\lambda}{(1-s)\phi} w_0 = 0, \quad (11)$$

$$\frac{\partial L}{\partial \lambda} = (a - x_0 - x_1 - \frac{w_0}{\phi(1-s)})x_0 = 0 \quad (12)$$

On the other hand, the first-order condition of the private firm's profit function is

$$\frac{\partial \pi_1}{\partial x_1} = a - x_0 - 2x_1 - \frac{1}{(1+s)\phi} w_1 = 0. \quad (13)$$

Simultaneously solving Eqs. (11), (12) and (13) reveals that

³ See from Eq. (8), $w_0 \rightarrow \infty$ when the public firm is nationalized.

$$x_0 = \frac{(1-s^2)\phi\alpha - 2(1+s)w_0 + (1-s)w_1}{(1-s^2)\phi},$$

$$x_1 = \frac{(1+s)w_0 - (1-s)w_1}{(1-s^2)\phi}, \quad (14)$$

$$\lambda = \frac{(1+s)(1+\alpha)w_0 - \alpha(1-s^2)\phi\alpha - \alpha(1-s)w_1}{(1-s^2)\phi\alpha - 2(1+s)w_0 + (1-s)w_1}.$$

In the stage two, similar to the Section 3, the utility of unions are

$$u_0 = \frac{w_0[(1-s^2)\phi\alpha - 2(1+s)w_0 + (1-s)w_1]}{(1-s)^2(1+s)\phi^2},$$

$$u_1 = \frac{w_1[(1+s)w_0 - (1-s)w_1]}{(1-s)(1+s)^2\phi^2}.$$

Then the first order conditions provided below help to find out the optimal bargained wage.

$$\frac{\partial u_0}{\partial w_0} = \frac{(1-s^2)\phi\alpha - 4(1+s)w_0 + (1-s)w_1}{(1-s)^2(1+s)\phi^2} = 0, \quad (15)$$

$$\frac{\partial u_1}{\partial w_1} = \frac{(1+s)w_0 - 2(1-s)w_1}{(1-s)(1+s)^2\phi^2} = 0$$

The optimal bargained wages are

$$w_0 = \frac{2(1-s)\phi\alpha}{7},$$

$$w_1 = \frac{(1+s)\phi\alpha}{7}, \quad (16)$$

From Eq. (16), Lemma 2 can be delivered.

Lemma 2 The impacts of privatization degree, the efficiency gap and the average productivity on the bargained wages are

$$\frac{\partial w_0}{\partial \alpha} = 0, \quad \frac{\partial w_1}{\partial \alpha} = 0, \quad \frac{\partial w_0}{\partial s} = -\frac{2\phi\alpha}{7} < 0, \quad \frac{\partial w_1}{\partial s} = \frac{\phi\alpha}{7} > 0, \quad \frac{\partial w_0}{\partial \phi} = \frac{2(1-s)a}{7} > 0,$$

$$\frac{\partial w_1}{\partial \phi} = \frac{(1+s)a}{7} > 0.$$

Lemma 2 depicts that under decentralized bargaining and the existence of efficiency gap, the enlargement of the efficiency gap would deteriorate the public wage but make the private wages higher. The improvement of the productivity would raise both wages irrespective of the

privatization degree. Further, it can be understood that the privatization policy would neither alter the output nor affect the bargained wage indicating that the neutrality of the partial privatization still hold even with the consideration of the budget constraint.

In the stage one, the optimal social welfare can be found as

$$SW = \frac{45}{98} a^2. \quad (17)$$

It is clear that social welfare is not affected by the privatization policy meaning mathematically that α can be arbitrarily assigned between 0 and 1.

Proposition 2 *Under decentralized union bargaining with firm heterogeneity and the budget constraint, privatization policy is neutral.*

4. The Optimal Degree of Privatization with Centralized Bargaining

This section considers one centralized bargaining between one centralized union and all firms. Hence the utility of the union is revised as

$$\underset{w}{Max} U = (w - \bar{w})^\theta \sum_{i=0}^1 L_i. \quad (18)$$

In Eq. (18), w stands for the uniform bargained wage and all game proceeding follows the same process in the prior section.

4.1 Privatization policy without budget constraint

Once again, through the first order conditions that are stated in Eq. (19), the equilibrium output can be unfolded in Eq. (20) where c stands for ‘‘centralized union’’:

$$\begin{aligned} \frac{\partial V}{\partial x_0} &= a - (1 + \alpha)x_0 - x_1 - \frac{\alpha}{(1-s)\phi} w = 0, \\ \frac{\partial \pi_1}{\partial x_1} &= a - x_0 - 2x_1 - \frac{1}{(1+s)\phi} w = 0; \end{aligned} \quad (19)$$

$$\begin{aligned}
x_0^c &= \frac{\alpha(1-s^2)\phi a - [2(1+s)\alpha - (1-s)]w}{(1-s^2)(1+2\alpha)\phi}, \\
x_1^c &= \frac{\alpha(1-s^2)\phi a - [(1-s) - 2s\alpha]w}{(1-s^2)(1+2\alpha)\phi}.
\end{aligned} \tag{20}$$

The linkage between the stage two and stage three can be found in Eqs. (20) and (18). The utility of the union is

$$U = \frac{w\{(1-s^2)[(1+s) + \alpha(1-s)]\phi a - 2w[\alpha(1+s+2s^2) - s(1-s)]\}}{(1-s^2)^2(1+2\alpha)\phi^2}.$$

Similarly, through the first order and second order conditions, the bargained wage can be figured out in Eq. (21).⁴ Then Lemma 3 is around the corner.

$$\begin{aligned}
\frac{\partial U}{\partial w} &= \frac{(1-s^2)[(1+s) + \alpha(1-s)]\phi a - 4w[\alpha(1+s+2s^2) - s(1-s)]}{(1-s^2)^2(1+2\alpha)\phi^2} = 0, \\
\frac{\partial^2 U}{\partial w^2} &= -\frac{4[\alpha(1+s+2s^2) - s(1-s)]}{(1-s^2)^2(1+2\alpha)\phi^2} < 0, \\
w^c &= \frac{(1-s^2)[(1+s) + \alpha(1-s)]\phi a}{4[\alpha(1+s+2s^2) - s(1-s)]}.
\end{aligned} \tag{21}$$

Lemma 3 The impacts of privatization degree, the efficiency gap and the average productivity on the bargained wage are

$$\begin{aligned}
\frac{\partial w^c}{\partial \alpha} &= -\frac{(1+s^2)(1-s^2)(1+3s)\phi a}{4[\alpha(1+s+2s^2) - s(1-s)]^2} < 0, \\
\frac{\partial w^c}{\partial s} &= \frac{\{(1+s)(1-s)^3 + \alpha[1-8s - (2+s)^2s^2] - 2\alpha^2(1-s)(1+4s+2s^2+s^3)\}\phi a}{4[\alpha(1+s+2s^2) - s(1-s)]^2}, \\
\frac{\partial w^c}{\partial \phi} &= \frac{(1-s^2)[(1+s) + \alpha(1-s)]a}{4[\alpha(1+s+2s^2) - s(1-s)]} > 0.
\end{aligned}$$

Lemma 3 points out that with centralized bargaining and a public firm with relative lower productivity, higher privatization degree indicating lower bargained wage, and higher productivity implies higher bargained wage. Nonetheless, how does the changes of efficiency gap affects the bargained wage is vague.

⁴ The inner solutions requires that $\alpha > s(1-s)/(2s^2 + s + 1) > 0$. It indicates under the centralized bargaining, the public firm cannot be fully nationalized.

In the first stage, the optimal degree of privatization can be solved through the following expression:⁵

$$\frac{\partial SW}{\partial \alpha} = \frac{\alpha[(1-s) + 2s^2 + \alpha(3+s+4\alpha^2)]Ha^2}{4(1+2\alpha)^3[\alpha(1+s+2s^2) - s(1-s)]^3} = 0$$

where $H = [s(1+6\alpha+2\alpha^2) + 3s^3(1+2\alpha) - s^2(2+4\alpha+9\alpha^2) - 2s^4(1+2\alpha)^2 - \alpha^2]$. The optimal privatization degree is

$$\alpha^c = \frac{s(1-s)(3+s+4s^2) + \sqrt{s(1-s)(1+3s)^2(1+s^2)}}{1-2s+9s^2+8s^3} \quad (22)$$

The Eq. (22) can be depicted as Fig. (1)

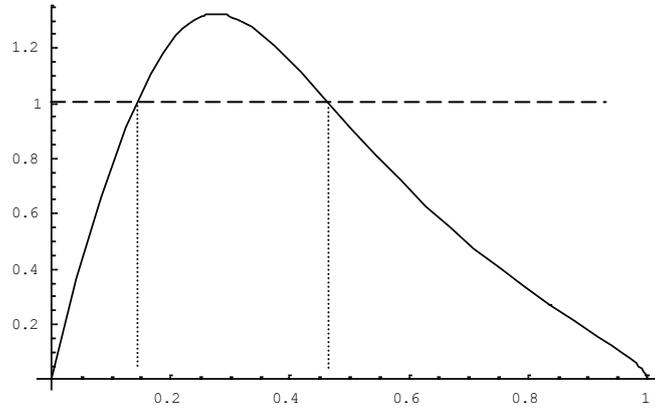


Figure 1 The relation between the efficiency gap and the privatization degree

Figure 1 states that if the efficiency gap between the public and private firms is small ($0 < s < s_1$, $s_1 \cong 0.143$), the best privatization policy is partial privatization. However, if the gap becomes larger, the privatization degree should be raised. If the efficiency gap is moderate, the best policy becomes full privatization ($s_1 \leq s \leq s_2$, $s_2 \cong 0.466$). If the efficiency gap is large ($s > s_2$), the best policy turns out to be partial privatization. However, the optimal privatization degree should be reduced if the efficiency gap becomes bigger. Thus Proposition 3 is obtained.

Proposition 3 *Under centralized bargaining with firm heterogeneity, optimal privatization policy depends upon the magnitude of productivity difference:*

⁵ The second order condition is $\partial^2 SW / \partial \alpha^2 < 0$.

- (i) *When the difference of labor productivity is small, optimal privatization policy is partial privatization, and the degree of privatization is positively related to the technical efficiency;*
- (ii) *When the difference of labor productivity is moderate, optimal privatization policy is complete privatization;*
- (iii) *When the difference of labor productivity is wide, optimal privatization policy is partial privatization, but the degree of privatization is inversely related to the technical difference.*

4.2 Privatization policy with budget constraint

Similar to Subsection 3.2, the objective function of the public firm is as follows:

$$\begin{aligned} \underset{x_0}{\text{Max}} \quad V &= \alpha\pi_0 + (1-\alpha)SW \\ \text{s.t.} \quad \pi_0 &= px_0 - wl_0 \geq 0. \end{aligned}$$

Denoting the multiplier of the budget constraint as λ , the Lagrangian equation can be written as

$$L = (\alpha + \lambda)\pi_0 + (1-\alpha)SW$$

Taking w_1 as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = (1 + \lambda)a - (1 + \alpha + 2\lambda)x_0 - (1 + \lambda)x_1 - \frac{\alpha + \lambda}{(1-s)\phi} w = 0 \quad (23)$$

$$\frac{\partial L}{\partial \lambda} = (a - x_0 - x_1 - \frac{w}{\phi(1-s)})x_0 = 0 \quad (24)$$

On the other hand, the first-order condition of the private firm's objective function is

$$\frac{\partial \pi_1}{\partial x_1} = a - x_0 - 2x_1 - \frac{1}{(1+s)\phi} w = 0 \quad (25)$$

Solving the first-order conditions simultaneously that are represented by Eqs. (23), (24) and (25) gives that

$$x_0 = a - \frac{(1+3s)w}{(1-s^2)\phi}$$

$$x_1 = \frac{2sw}{(1-s^2)\phi} \quad (26)$$

$$\lambda = \frac{(1+s+2s\alpha)w_0 - \alpha(1-s^2)\phi a}{(1-s^2)\phi a - (1+3s)w}$$

In the stage two, the utility of the union is determined as

$$U = \frac{w[(1-s^2)(1+s)\phi a - (1+2s+5s^2)w]}{(1-s^2)^2 \phi^2}.$$

The first and second order conditions are stated below and the bargained wage can be discovered in Eq. (27). Lemma 4 then can be unveiled.

$$\frac{\partial U}{\partial w} = \frac{(1-s^2)(1+s)\phi a - 2(1+2s+5s^2)w}{(1-s^2)^2 \phi^2} = 0,$$

$$\frac{\partial^2 U}{\partial w^2} = -\frac{2(1+2s+5s^2)}{(1-s^2)^2 \phi^2} < 0,$$

$$w^c = \frac{(1-s^2)(1+s)\phi a}{2(1+2s+5s^2)}. \quad (27)$$

Lemma 4 The impacts of privatization degree, the efficiency gap and the average productivity on the bargained wage are

$$\frac{\partial w^c}{\partial \alpha} = 0, \quad \frac{\partial w^c}{\partial s} = -\frac{[(1-s^2)+s(1+5s^2)](1+s)\phi a}{2(1+2s+5s^2)^2} < 0, \quad \frac{\partial w^c}{\partial \phi} = \frac{(1-s^2)(1+s)a}{2(1+2s+5s^2)} > 0.$$

Lemma 4 deliberates that with the budget constraint and lower productive public firm under centralized bargaining, larger efficiency gap would reduce the bargained wage⁶. If the productivity of firms is enhanced, the wage would be improved. Furthermore, the privatization degree is not influential to the bargained wage. Moreover, it shows that the neutrality of partial privatization holds under the consideration of the budget constraint.

The social welfare within this scenario is decided in stage one, which is

$$SW^c = \frac{(1+2s+9s^2)(3+6s+11s^2)}{8(1+2s+5s^2)} a^2 \quad (28)$$

There is no doubt that the optimal social welfare is independent to the privatization degree

⁶ Reproduction of the results in Bastos *et al.* (2009.)

meaning that α can be any rational number between 0 and 1. That is

Proposition 4 *With a lower productive public firm and the budget constraint, the output and the bargained wage are not affected by the privatization policy. The neutrality of partial privatization holds.*

5. Conclusions

This paper incorporates “firm heterogeneity” into the privatization policy with the consideration of budget constraint to propose different interpretation of the efficiency gap between a public firm and a private firm and the decisions of the policy.

In the scenario of no budget constraint, if the wage is decentralized bargained, the best policy of privatization is full nationalization. However, if the wage is centralized bargained, then partial privatization would be the best policy when the efficiency gap is small with positive relation between the productivity difference and the privatization degree. However, if the productivity difference is moderate, full privatization policy would be the best, while partial privatization is the best if the productivity difference is large with inverse relation between productivity difference and the privatization degree.

Lastly, if the budget constraint is compulsory to the public firm, the neutrality of the partial privatization holds.

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