The Stabilizing Effect of Government Spending in an Economy with Sectoral Externalities

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Abstract: This paper explores a policy rule of government purchases that can remove sunspot fluctuations in a two-sector model with sector-specific externalities. We show that government spending on the investment good can stabilize the economy against the sunspot fluctuations, while the government's purchases of the consumption good cannot sever a stabilization policy. Quantitatively, given an empirically plausible extent of the sectoral externality of 0.108, the government should purchase the investment good so that the ratio of government spending on the investment good to real GDP is higher than 8.15% in order to remove sunspot-driven fluctuations. This offers a new policy implication in the sense that, instead of the total amount of government spending, the composition of government spending is more crucial in terms of affecting macroeconomic stability.

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1 Introduction

An indeterminate steady state can be generated if aggregate returns to scale are sufficiently strong in a one-sector model (Benhabib and Farmer, 1994) or if sectoral externalities are high enough in a two-sector (the consumption and investment sectors) model (Benhabib and Farmer, 1996 and Harrison, 2001). Indeterminacy potentially indicates that agents' expectations concerning the future can be self-fulfilling and agents can coordinate with a continuum of paths that converge to the same steady-state equilibrium. As a result, the so-called "animal spirits" may generate belief-driven business cycle fluctuations without any change in economic fundamentals. The equilibrium indeterminacy creates room for Keynesian-type stabilization to insulate the economy from belief-driven fluctuations. In a one-sector model, Guo and Lansing (1998) and Christiano and Harrison (1999) propose that a progressive tax schedule operates like an automatic stabilizer that mitigates business cycle fluctuations, while a regressive tax destabilizes the economy.[1] Surprisingly, Guo and Harrison's (2001) numerical analysis finds that a progressive tax no longer serves as an automatic stabilizer and, by contrast, a regressive tax policy stabilizes the economy against sunspot-driven fluctuations in a two-sector model.

These notions put forward by the previous studies are not found to satisfy economists. On the one hand, the required regressive tax policy referred to by Guo and Harrison (2001) is neither empirically plausible nor observed in the actual data. On the other hand, the primary focus of the studies cited above has been on the tax policy regardless of whether in a one-sector model with an aggregate productive externality or in a two-sector model with a sector-specific externality. The possible stabilizing effect of government spending is entirely neglected in this strand of the literature.

By departing from their studies, our focus of attention rests solely on the government's expenditures. In an indeterminate two-sector real business cycle (RBC) model, we attempt to propose an alternative fiscal policy of government expenditure, rather than tax schedules, to stabilize the economy against sunspot fluctuations. We would like to answer the following questions: does the amount or the structure of government spending is matter in terms of governing macroeconomic stability? To gain the stabilizing effect for the government spending, should the government purchase either the investment good or the consumption good? Our analysis indicates that the different compositions of government spending will end up with very different stabilizing effects. The govern-

[1] In contrast to the assumption of a non-linear tax schedule (a continuously increasing marginal tax rate) in their studies, Dromel and Pintus (2007, 2008) specify a linearly progressive tax schedule and also confirm the stabilizing effect of progressive taxes.

ment's purchases of the consumption good have no stabilizing effect on the economy. By contrast, a government can stabilize the economy against sunspot fluctuations by purchasing the investment good. Given an empirically plausible extent of the sectoral externality of 0.108 (as estimated by Harrison 1997), the government should purchase the investment good and make the ratio of government expenditure of the investment good to real GDP to be higher than 8.15% in order to remove sunspot-driven fluctuations.

Why can the government stabilize the economy against sunspot fluctuations by purchasing the investment good, instead of the consumption good? The rationale is as follows. When agents anticipate a higher future rate of return on capital, they will reduce today's consumption for more investment, thereby increasing the future capital stock and output. Consequently, tomorrow's consumption will increase and the share of each factor used in the production of the consumption good will become higher. If there exist high enough sectoral externalities, as emphasized by Benhabib and Farmer (1996) and Harrison (2001), tomorrow's relative price of capital will increase as resources are shifted towards the consumption good sector, because of a *convex* production possibilities frontier (PPF).[2] Thus, agents' optimistic expectations become self-fulfilling and the sunspot fluctuations occur.

However, agents' expectations will change in response to the presence of government spending. If the government purchases the investment good, households will be aware that today's private investment will be crowded out by the government's purchases of the investment good since the government's demand for the investment good will lower the relative price of capital under a *convex* PPF. As today's investment decreases, tomorrow's output and hence consumption will also decrease. Once the trend of shifting resources towards the consumption sector is reversed, households will expect tomorrow's relative price of capital to decrease, rather than increase. Consequently, government spending on the investment good prohibits agents' optimistic expectations from being self-fulfilling and stabilizes the economy against sunspot fluctuations. By contrast, the government's purchases of the consumption good are incapable of providing a stabilizing effect on the economy against the sunspot fluctuations. While the government's purchases of the consumption good may also crowd out private consumption, it has *no intertemporal* effect in terms of affecting capital accumulation in the private sector in the steady state and hence the equilibrium output-private capital ratio. As a result, it does not drive any wedge in terms of resource allocation between the consumption and investment sectors. Given that the relative price of the investment to the consumption

[2] As pointed out by Harrison (2001), due to the increasing returns to scale, a shift in the resources towards the production of the investment good will lower the relative price of the investment good.

good is expected to be inactive, the government spending has no stabilizing effect.

To the best of our knowledge, Grandmont (1986) and Raurich (2001) are rare exceptions which study how the macroeconomic stability is affected by government spending. Grandmont (1986) shows that money transfers and public expenditures may help reduce the endogenously cyclical property of a competitive economy. He focuses on the generations model that involves only one sector (the final good sector) and one asset (money, with the exclusion of physical capital), while we study aggregate (in)stability in a two-sector RBC model in which capital plays a crucial role in terms of affecting the equilibrium indeterminacy. In an endogenous growth model with both the *goods* and *education* sectors, Raurich (2001) shows that when the fraction of government revenues devoted to government expenditures is large, fiscal policies may cause, rather than suppress, belief-driven fluctuations if the government only makes purchases in the production sector. In contrast to their results, in an RBC model with both the *consumption* and *investment* sectors, we argue that instead of the total amount of government spending, the composition of government spending is more crucial in terms of affecting macroeconomic stability. It offers a new insight into the literature and an important implication for policy-makers.

The rest of the paper is arranged as follows. Section 2 presents our analytical framework. Section 3 analyzes the (in)determinacy properties and, accordingly, shows how the government spending by virtue of purchasing the investment good, rather than the consumption good, suppresses the aggregate instability arising from animal spirits. Section 4 concludes.

2 The Model

This study extends the two-sector model of Harrison (2001) and Guo and Harrison (2001) to include government spending. Consider an economy that consists of households, firms and a government. Households derive utility from consumption and leisure. On the production side, there are two sectors — the consumption good and investment good sectors. The firms in each sector produce with a constant-returns-to-scale technology and have access to sector-specific externalities. The government runs a balanced budget by levying a lump-sum tax to finance its expenditures. Under this environment, we examine whether the government's purchases can stabilize the economy against sunspot fluctuations.

2.1 Firms

Each of the consumption and investment goods is produced by a decentralized competitive sector and by using capital k_t and labor ℓ_t in competitive factor markets. Let $k_{c,t}$ and $\ell_{c,t}$ ($k_{i,t}$ and $\ell_{i,t}$) denote the capital and labor services in the consumption (investment) sector, respectively. Thus, the production technologies of a typical firm in the consumption good $y_{c,t}$ and investment good $y_{i,t}$ sectors are, respectively:

$$y_{c,t} = A_t \cdot k_{c,t}^a \ell_{c,t}^b \text{ and } y_{i,t} = B_t \cdot k_{i,t}^a \ell_{i,t}^b, \text{ with } a + b = 1, \tag{1}$$

where A_t and B_t stand for productive externalities in the consumption and investment sectors and take the following forms:

$$A_t = (\overline{k}_{c,t}^a \overline{\ell}_{c,t}^b)^{\theta_c} \text{ and } B_t = (\overline{k}_{i,t}^a \overline{\ell}_{i,t}^b)^{\theta_i}.$$

The relevant variables with a bar "–" denote the sector average levels and θ_c (θ_i) measures the extent of the sector-specific externalities in the consumption (investment) sector. To focus our point, we assume that the production technology exhibits constant returns-to-scale with respect to private capital and labor inputs and abstract the economy-wide externalities from the analysis for simplicity. To have a better understanding and to make our point more striking, we only focus on the case where the government expenditure is wasteful and is not valued by private agents.

Accordingly, the first-order conditions for the profit maximization of the consumption good producer are given by:

$$r_t = a\frac{y_{c,t}}{k_{c,t}} \text{ and } w_t = b\frac{y_{c,t}}{\ell_{c,t}}, \tag{2}$$

where r_t and w_t are the interest and wage rates, respectively.

Similarly, given that P_t is the relative price of the investment good to the consumption good, the first-order conditions for profit maximization of the investment good producers are:

$$r_t = a\frac{P_t y_{i,t}}{k_{i,t}} \text{ and } w_t = b\frac{P_t y_{i,t}}{\ell_{i,t}}. \tag{3}$$

Perfect mobility of factors will lead to "factor price equalization," indicating that the workforce (capital) will move around until the wage (interest) rates in both sectors are equal.

Let the variables Y_t, $K_t(= k_{c,t} + k_{i,t})$, and $L_t(= \ell_{c,t} + \ell_{i,t})$ represent total output, the aggregate stock of capital, and aggregate labor hours, respectively. Thus, by defining $\mu_{k,t}$ and $\mu_{\ell,t}$ as the fractions of capital K_t and labor L_t used in the consumption good industry, the relative factor intensities are: $\mu_{k,t} = k_{c,t}/K_t$ and $\mu_{\ell,t} = \ell_{c,t}/L_t$. Since firms use identical technologies and face equal factor prices across the two sectors, factor intensities are also identical across these two sectors, i.e., $\mu_t = \mu_{k,t} = \mu_{\ell,t}$. Thus, the production possibilities frontier can be expressed as:

$$Y_t = y_{c,t} + P_t y_{i,t} = A_t \cdot K_t^a L_t^b. \tag{4}$$

It is evident from (2)-(4) that the relative price $P_t = A_t/B_t = \mu_t^{\theta_c}(1-\mu_t)^{-\theta_i}(K_t^a L_t^b)^{(\theta_c-\theta_i)}$ is the slope of the PPF. If there are no sector-specific externalities ($\theta_c = \theta_i = 0$), the relative price turns out to be constant and hence the PPF is linear. However, in the presence of the specific-sector externality the PPF is convex, as shown in Figure 1.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each (representative) household acts to maximize the following discounted present value of utility function which is separable in consumption and leisure:

$$\sum_{t=0}^{\infty}\beta^t(\ln c_t - \frac{L_t^{1+\chi}}{1+\chi}) \tag{5}$$

where $\beta(> 0)$ is the discount factor and χ is the inverse of the labor supply elasticity.

By denoting T_t as a lump-sum tax, the budget constraint facing the representative household is given by:

$$w_t L_t + r_t K_t - T_t = c_t + P_t \cdot i_t.$$

This indicates that the household's disposable income is equal to its purchases of the consumption and investment goods. Given that the law of motion for capital accumulation is $K_{t+1} = i_t + (1-\delta)K_t$, where δ is the depreciation rate, the intertemporal budget constraint can be re-expressed as follows:

$$K_{t+1} = \frac{1}{P_t}[w_t L_t + r_t K_t - c_t - T_t] + (1-\delta)K_t. \tag{6}$$

The household chooses consumption and working hours in order to maximize the discounted sum of utility defined in (5), subject to the budget constraint (6). The optimal conditions necessary for this optimization problem are given by:

$$\frac{1}{c_t} = \frac{\lambda_t}{P_t} \tag{7}$$

$$L_t^{\chi} = \frac{\lambda_t w_t}{P_t} \tag{8}$$

$$\frac{c_{t+1}}{c_t} = \beta[\frac{r_{t+1} + (1-\delta)P_{t+1}}{P_t}] \tag{9}$$

together with the budget constraint (6) and the transversality condition $\lim_{t\to\infty}\beta^t(K_{t+1}/c_t) = 0$. The term λ_t is the co-state variable which can be interpreted as the shadow value of the capital stock, measured in terms of utility. Equations (7) and (8) refer to the optimal conditions for consumption and labor, respectively. Equation (9) is the consumption Euler equation of the two-sector model.

2.3 Government

The government purchases consumption good and/or investment good in each period. For analytical convenience, we assume that the government's purchases of the consumption (investment) good is set as a fraction g_c (g_i) of the output of the consumption (investment) good, i.e., $G_{c,t} = g_c \cdot y_{c,t}$ ($G_{i,t} = g_i \cdot y_{i,t}$). To finance this spending, the government levies a non-distortionary lump-sum tax on the households. The government budget constraint can then be expressed as:

$$G_{c,t} + P_t \cdot G_{i,t} = g_c \cdot y_{c,t} + P_t(g_i \cdot y_{i,t}) = T_t. \tag{10}$$

By substituting (10) into (6), we further obatin the economy-wide resource constraint as follows:

$$c_t + P_t \cdot i_t + G_{c,t} + P_t \cdot G_{i,t} = Y_t.$$

3 Equilibrium Indeterminacy and Stabilization Policy

By putting the firm's optimal conditions (2) and (3), the government's budget constraint (10), and the household's budget constraint (6) together, we have the following market-clearing conditions for the consumption good and investment good markets:

$$c_t = y_{c,t} - G_{c,t} = (1 - g_c)y_{c,t} \tag{11}$$

$$i_t = y_{i,t} - G_{i,t} = (1 - g_i)y_{i,t} \tag{12}$$

With aggregate consistency (i.e., $k_{c,t} = \bar{k}_{c,t}$, $k_{i,t} = \bar{k}_{i,t}$, $\ell_{c,t} = \bar{\ell}_{c,t}$, and $\ell_{i,t} = \bar{\ell}_{i,t}$), this model economy defines a macroeconomic (competitive) equilibrium by a tuple of paths for quantities $\{c_t, K_{t+1}, L_t\}_{t=0}^{\infty}$, prices $\{P_t, r_t, w_t\}_{t=0}^{\infty}$, and policy variables $\{G_{c,t}, G_{i,t}, T_t\}_{t=0}^{\infty}$, that satisfy:

(i) The firm's profit maximization conditions: (2) and (3);

(ii) The household's utility maximization conditions: (7)-(9) and (6);

(iii) The government's budget constraint: (10);

(iv) The market-clearing conditions: (11) and (12) (hence, the aggregate resource constraint).

3.1 Equilibrium Dynamics

With the *equal* relative factor intensity, we have $y_{c,t} = \mu_t Y_t$. Putting (1), (4) and (11) together leads to the following relationship for the relative factor intensity:

$$\mu_t = \left[\frac{c_t}{(1-g_c)K_t^{a(1+\theta_c)}L_t^{b(1+\theta_c)}}\right]^{\frac{1}{1+\theta_c}}. \tag{13}$$

Given (13), we can further obtain the relationship for labor from (1), (2), (7), and (8):

$$L_t = \left[\frac{b^{1+\theta_c}K_t^{a(1+\theta_c)}}{(1-g_c)^{\theta_c}c_t}\right]^{\frac{1}{(1+\chi-b)(1+\theta_c)}}. \tag{14}$$

In addition, the market-clearing condition for the investment good ($i_t = (1-g_i)y_{i,t}$) with the capital evolution equation ($K_{t+1} = i_t + (1-\delta)K_t$) and the production technology (1) allow us to derive:

$$K_{t+1} = (1-g_i)y_{i,t} + (1-\delta)K_t = (1-g_i)(1-\mu_t)^{1+\theta_i}K_t^{a(1+\theta_i)}L_t^{b(1+\theta_i)} + (1-\delta)K_t, \tag{15}$$

By combining (2) with (9), we rewrite the consumption Euler equation as:

$$\frac{c_{t+1}}{c_t} = \beta[\frac{r_{t+1}}{P_t} + \frac{(1-\delta)P_{t+1}}{P_t}] = \beta(1-\mu_t)^{\theta_i}\mu_t^{-\theta_c}K_t^{a(\theta_i-\theta_c)}L_t^{b(\theta_i-\theta_c)}.\Gamma \tag{16}$$

where $\Gamma = a\mu_{t+1}^{\theta_c}K_{t+1}^{a(1+\theta_c)-1}L_{t+1}^{b(1+\theta_c)} + (1-\delta)\mu_{t+1}^{\theta_c}(1-\mu_{t+1})^{-\theta_i}K_t^{a(\theta_c-\theta_i)}L_t^{b(\theta_c-\theta_i)}$. By substituting (13) and (14) into (15) and (16), the dynamical system in our model economy can be reduced to a 2×2 one in terms of K_t and c_t.

Let the superscript "$\tilde{}$" denote the steady-state value for the relevant variables. Given an initial value of capital $\widehat{K}_0$, taking log-linear approximations to the dynamic system in the neighborhood of the steady state yields:

$$\begin{pmatrix} \widehat{K}_{t+1} \\ \widehat{c}_{t+1} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} \widehat{K}_t \\ \widehat{c}_t \end{pmatrix} \tag{17}$$

where the hat variables denote percentage deviations from their steady-state values and the J_{ij} are elements of Jacobian matrix J, specifically,

$$J_{11} = \frac{a\delta(1+\theta_i)(1+\chi)}{(1-\tilde{\mu})(a+\chi)} + 1 - \delta,$$

$$J_{12} = -\frac{\delta(1+\theta_i)[b(1-\tilde{\mu}) + (1+\chi)\tilde{\mu}]}{(1-\tilde{\mu})(a+\chi)(1+\theta_c)},$$

$$J_{21} = \frac{1}{\Psi_1}\left\{a(\theta_i-\theta_c) + \frac{a\left[(1+\chi)(\theta_c + \frac{\theta_i\tilde{\mu}}{1-\tilde{\mu}}) + b(\theta_i-\theta_c)\right]}{(a+\chi)} + \Psi_2\cdot J_{11}\right\},$$

$$J_{22} = \frac{1}{\Psi_1}\left\{1 - \frac{(1+\chi)(\theta_c + \frac{\theta_i\tilde{\mu}}{1-\tilde{\mu}}) + b(\theta_i - \theta_c)}{(a+\chi)(1+\theta_c)} + \Psi_2 \cdot J_{12}\right\},$$

with $\Psi_1{=}1{+}\frac{b[(1+\theta_c)-\beta(1-\delta)(1+\theta_i)]-(\theta_c+\frac{\beta(1-\delta)\theta_i\tilde{\mu}}{1-\tilde{\mu}})(1+\chi)}{(a+\chi)(1+\theta_c)}$, $\Psi_2{=}\eta{+}\frac{a\{b[(1+\theta_c)-\beta(1-\delta)(1+\theta_i)]-(\theta_c+\frac{\beta(1-\delta)\theta_i\tilde{\mu}}{1-\tilde{\mu}})(1+\chi)\}}{(a+\chi)}$, and $\eta = a(1+\theta_c) + \beta(1-\delta)[1-a(1+\theta_i)] - 1$. Notice that in addition to the sectoral externalities, government spending also plays a role in terms of governing the equilibrium dynamics via the steady-state factor intensity $\tilde{\mu}$. The model consists of a control (jump) variable λ_t and a predetermined (non-jump) variable K_t. Thus, local determinacy requires the system to exhibit saddle path stability in which one eigenvalue of J lies inside, and the other one outside, the unit circle. By contrast, the steady state exhibits indeterminacy (sink) if both eigenvalues are inside the unit circle. When both eigenvalues are outside the unit circle, the steady state becomes an unstable source.

Analytically, we can establish the following proposition:

Proposition 1: (Government Purchases and Equilibrium Dynamics) *In the presence of sectoral externalities (θ_i and θ_c), the policy of government spending can govern the equilibrium dynamics by purchasing the investment good, while the government purchases of the consumption good play no role in terms of affecting the equilibrium dynamics.*

Proof. The equilibrium of the steady state is characterized by $K_t = K_{t+1} = \tilde{K}$, $c_t = c_{t+1} = \tilde{c}$, and $P_t = P_{t+1} = \tilde{P}$. Accordingly, from (15) and (16) with $\tilde{k}_i = (1-\tilde{\mu})\tilde{K}$, we have:

$$1 = (1-g_i)\frac{\tilde{y}_i}{(1-\tilde{\mu})\tilde{k}_i} + (1-\delta), \tag{18}$$

$$1 = \beta[\frac{\tilde{y}_i}{\tilde{k}_i} + (1-\delta)]. \tag{19}$$

A simple manipulation on the above equations yields the following steady-state factor intensity:

$$\tilde{\mu} = 1 - \frac{a\delta}{(1-g_i)(1/\beta - 1 + \delta)}.$$

Given this, we can learn from (17) that g_i enters the steady-state factor intensity and plays a role in terms of governing the equilibrium dynamics. By contrast, g_c is incapable of governing the equilibrium dynamics since it does not affect $\tilde{\mu}$ and appear in (17). ■

The consumption Euler equation (19) indicates that the output-capital ratio of the investment sector $\frac{\tilde{y}_i}{\tilde{k}_i}$ must be constant in the steady-state equilibrium, given that the depreciation and discount rates are constant. Since under an *equal* relative factor intensity $P_t \cdot y_{i,t} = (1-\mu_t)Y_t$ and $k_{i,t} = (1-\mu_t)K_t$, this implies that the aggregate output-private capital ratio $\frac{\tilde{Y}}{\tilde{P}\tilde{K}}$ must also be constant in the steady state. As the government purchases more of the investment good (increases g_i), the private investment $\tilde{\imath}$ is crowded out and the private capital stock thereby decreases. Given a *convex* PPF, (18) shows that the resource of the economy has to be shifted to the investment sector (i.e., $(1 - \tilde{\mu})$ increases) in order to increase private capital such that an equilibrium ratio of output to private capital $\frac{\tilde{Y}}{\tilde{P}\tilde{K}}$ can be maintained. Since g_i can affect the resource allocation between the consumption and investment sectors, i.e., the factor intensity $\tilde{\mu}$, as we will see later, it plays a crucial role in terms of governing the economy's equilibrium dynamics.

By contrast, while the government's purchases of the consumption good g_c may crowd out private consumption, it has *no intertemporal* effect in terms of affecting capital accumulation in the private sector and hence the equilibrium output-private capital ratio. In other words, the economy responds to a rise in g_c by the adjustment within the consumption sector, specifically on the one hand decreasing consumption and, on the other hand, increasing the output of the consumption goods. Since the consumption goods output $\tilde{y}_c$ and aggregate output $\tilde{Y}$ increase proportionally, there is no impact on the equilibrium output-private capital ratio $\frac{\tilde{Y}}{\tilde{P}\tilde{K}}$, so no wedge is driven in terms of resource allocation between the consumption and investment sectors. Due to the fact that the steady-state factor intensity $\tilde{\mu}$ is not affected by g_c, it is evident from (16) that the government's purchases of the consumption good will have no stabilizing effect on the economy.

3.2 Numerical Analysis

This section will perform a simple numerical analysis in order to gain more policy implications for the policy of government spending. Since the government purchases of the consumption good play no role in terms of affecting the equilibrium dynamics, we will restrict our attention to the stabilizing effect of the government's purchases of the investment good when there exist distinct sectoral externalities in the consumption and investment sectors.

The Benchmark parameterization Most parameters presented in Table 1 are taken from Harrison (2001). By following Harrison (2001), we set $a = 0.3$, $b = 0.7$, $\delta = 0.025$, $\chi = 0$, and $\beta = 1/1.01$. These values are common in the related literature, as in, for example, Benhabib and Farmer (1996) and Guo and Harrison (2001). In line with Harrison's (1997) empirical evidence, we set $\theta_c = \theta_i = 0.108$ which is located within the plausible range of the evidence from the U.S. With these parameters, (11) and (12) allow us to calibrate $g_c = 0.13$ and $g_i = 0.3$. These parameterizations imply that $\tilde{c}/\tilde{Y} = 0.604$, $\tilde{P} \cdot \tilde{\imath}/\tilde{Y} = 0.214$, and $\tilde{G}/\tilde{Y} = 0.182$, all of which are located within the empirically relevant range of the actual data and well within the range that is common in the literature. In the meantime, we also have $\tilde{G}_c/\tilde{G} = \tilde{P} \cdot \tilde{G}_i/\tilde{G} = 0.5$, which satisfies the observation in the OECD countries during 2001-2010.[3] All benchmark parameterizations are summarized in Table 1.

The stabilizing effect of the government's purchases of the investment good We first discuss the relationship between the government's purchases of the investment good (g_i) and the externality in the investment sector (θ_i). Figure 2 refers to the region of determinacy, indeterminacy and source in the (g_i, θ_i) space. Based on Figure 2, we establish:

Result 1: (Stabilizing Effect of Government Purchases of the Investment Good) *Increasing the government's purchases of the investment good can remove the indeterminacy caused by sectoral externalities.*

Result 1 indicates that the steady state is more likely to be determinate if the government raises the proportion of g_i. That is, the government can remove the indeterminacy caused by sectoral externalities by purchasing more of the investment good. To be more specific, if the ratio of the government's purchases of the investment good to the output of the investment good is substantially high, i.e., $g_i > 0.2757$ in our calibrated model, the government spending will stabilize the economy against sunspot fluctuations when the sector-specific externality in the investment sector is equal to 0.108, as estimated by Harrison (1997) using U.S. data. This implies that government spending will give rise to a stabilizing effect on the economy, provided that the ratio of government spending on the investment good to real GDP (PG_i/Y) is higher than 8.15%.

Figure 3 shows that if $\theta_i = 0.108$, the equilibrium determinacy occurs if the ratio of the government spending on the investment good to real GDP is higher than $g_i = 0.2757$. We also learn from Figure 3 that the equilibrium dynamics is independent of the extent of the sector-specific externality in the consumption sector θ_c. This result, on the one hand, confirms the stabilizing effect of the government's purchases of the investment good and, on the other hand, recovers Harrison's (2001) result whereby the specific externality of the consumption sector does not cause the equilibrium indeterminacy if we calibrate the U.S. economy. When we shut down the specific externality of the investment sector, i.e., $\theta_i = 0$, the steady state then turns out to be locally determinate.

[3] By following the classification of Kneller *et al.* (1999), we aggregate the OECD's functional classifications of fiscal data (OECD stat.) into two main categories, namely government investment and government consumption.

Interpretation In order to glean the intuition for the stabilizing effect of the government purchases of the investment good, we rewrite the consumption Euler equation as:

$$\frac{c_{t+1}}{c_t} = \beta \cdot NRK_{t+1} = \beta \left[\frac{r_{t+1} + (1-\delta)P_{t+1}}{P_t} \right], \tag{20}$$

where r_{t+1} is the marginal product of capital in period $t+1$, and hence the net rate of return on capital is $NRK_{t+1} = \frac{r_{t+1}+(1-\delta)P_{t+1}}{P_t}$. The economy starts from the steady-state equilibrium in period t. Suppose that agents become optimistic about the future returns on capital, say, next period's return on capital r_{t+1}. In acting upon this belief, the household will sacrifice consumption today c_t for more investment i_t. Since today investment results in the accumulation of more capital stock K_{t+1}, the levels of the future output Y_{t+1} and the future consumption c_{t+1} increase. A lower c_t and a higher c_{t+1} indicate that the value of the LHS of (20) increases because of households' optimistic expectations. In order to remain in equilibrium, the RHS of (20) must also increase.

Let us first explain why indeterminacy arises in the model with sectoral externalities (as shown in Harrison, 2001) when there is no government spending ($g_c = g_i = 0$), and then explain why the government spending on the investment good ($g_i > 0$) can give rise to a stabilizing effect against sunspot fluctuations. As indicated in Benhabib and Farmer (1996) and Harrison (2001), there exist specific externalities in the investment good θ_i such that the PPF in terms of the consumption and investment goods is convex, as shown in Figure 1. A convex PPF implies that a shift in the resources towards the production of a good will lower the relative price of that good. It turns out that today's relative price of capital (P_t) will decrease as today's consumption (c_t) decreases, and tomorrow's relative price of capital (P_{t+1}) will increase as resources are shifted towards the consumption good sector (c_{t+1}). Thus, the RHS of (20) also increases. Agents' optimistic expectations then become self-fulfilling and the sunspot fluctuations occur. In other words, if there is high enough sectoral externality θ_i, local indeterminacy can occur even though the interest rate r_{t+1} decreases with a higher capital stock K_{t+1}.

Result 1 reveals that to gain the stabilizing effect for the government's purchases, we need a substantially high fraction of the government's purchases of the investment good (g_i) to remove indeterminacy. That is to say, agents' expectations will change in response to the government's purchases of the investment good. In the presence of government spending, households will be aware that today's private investment will be crowded out by the government's purchases of the investment good. As today's investment decreases, tomorrow's output and hence consumption (c_{t+1}) will decrease accordingly. Once the trend of shifting resources towards the consumption sector is reversed, households will anticipate that tomorrow's relative price of capital (P_{t+1}) will decrease, rather than increase. Thus, (20) indicates that the decrease in NRK_{t+1} contradicts the intertemporal Euler equation and this contradiction invalidates the initial rise in the expected return on capital. As a result, government spending on the investment good prohibits agents' optimistic expectations from being self-fulfilling and stabilizes the economy against sunspot fluctuations.

Before ending this section, we emphasize that the stabilizing effect of the government's purchases of the investment good is still valid even though there exists an aggregate production externality. Our numerical exercise shows that government spending can stabilize the economy against sunspot fluctuations as long as g_i is higher than 0.3148 (or equivalently, $PG_i/Y > 9.84\%$), if we set the aggregate production externality as 0.03 and sector-specific externalities as $\theta_c = \theta_i = 0.108$.[4]

4 Concluding Remarks

The primary focus in the existing literature on stabilization has been on the tax policy, regardless of whether in a one-sector model with an aggregate productive externality or in a two-sector model with a sector-specific externality. The possible stabilizing effect of government purchases has been mostly ignored. In the presence of sectoral externalities, this paper has explored a policy rule of government purchases that can remove sunspot fluctuations. We have shown that government spending on the investment good can stabilize the economy against the sunspot fluctuations, but the government's purchases of the consumption good cannot sever a stabilization policy. Specifically, given an empirically plausible extent of the sectoral externality 0.108, the government should purchase the investment good so that the ratio of government spending to real GDP is higher than 8.15% in order to gain the stabilizing effect on the economy. This offers a new policy implication in the sense that, instead of the total amount of government spending, the composition of government spending is crucial in terms of affecting macroeconomic stability.

It may be natural to extend our analysis to include productive public services in the sense that government expenditure on infrastructure has a positive external effect on private production in both sectors. Under such circumstances, it is expected that government purchases of both consumption and investment goods may alter the allocation of resources between the two sectors and, as a result, affect the economy's equilibrium dynamics. Further investigation of this issue will be on our research agenda in the near future.

[4] According to the estimation of Basu and Fernald (1997), for the U.S. private business economy the empirically plausible degree of increasing returns-to-scale in production is between 1.03 and 1.18.

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Benchmark values			
a	0.3	χ	0
b	0.7	δ	0.025
θ_c	0.108	g_c	0.12
θ_i	0.108	g_i	0.3
β	1/1.01		

Table 1. Parameter Values in the Benchmark

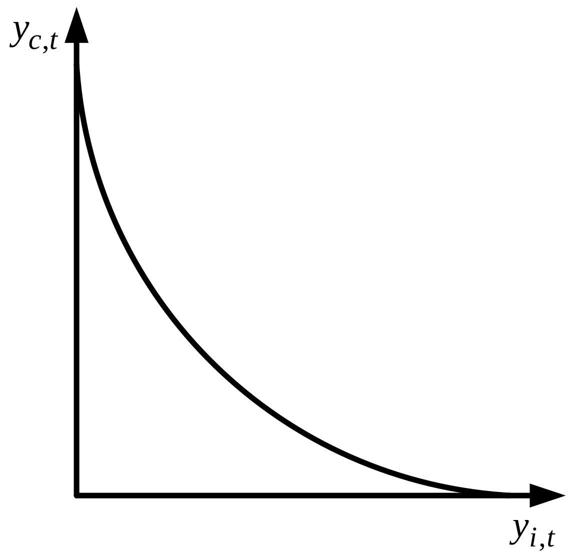

Figure 1. The PPF with Sectoral Externalities

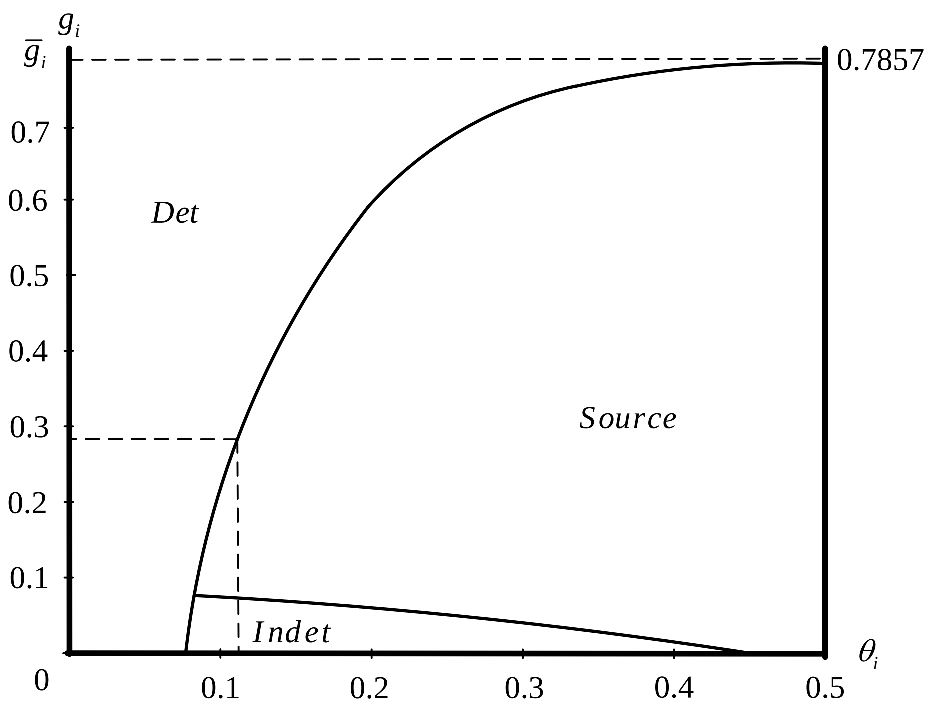

Figure 2: Determinacy in (g_i, θ_i) space

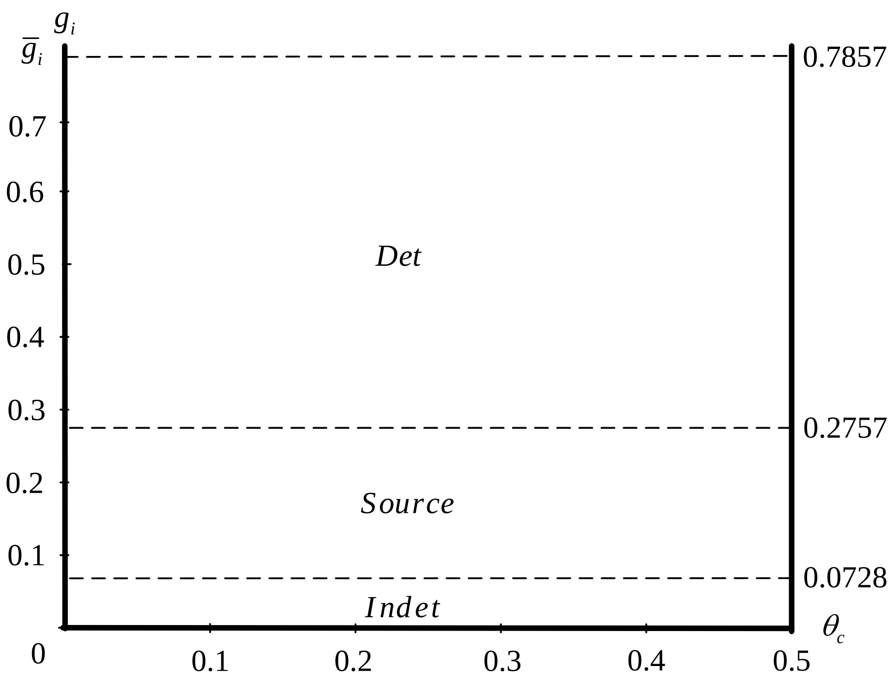

Figure 3: Determinacy in (g_i, θ_c) space ($\theta_i = 0.108$)