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The combination of two tragedies: commons and anticommons tragedies

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Abstract Under a general constant elasticity of substitution (CES) function, this paper generalizes the substitution of concurrent possession of a property right—from perfect substitutes to partial substitutes to perfect complements—in the context of commons/anticommons tragedies. We demonstrate that when the possibility of substitution between property rights is relatively low, inefficiency arises in the underusage of a common resource (i.e. anticommons tragedy). When the possibility of substitution between property rights is relatively high, inefficiency arises in the overusage of a common resource (i.e. commons tragedy). When the possibility of substitution between property rights is moderate, the two tragedies (commons and anticommons) are combined and may achieve efficient usage of a common resource, which is the notion of two negatives making a positive.

Keywords Duopoly · Complementary monopoly · Commons · Anticommons · Cournot

JEL Classification L12 · L13 · H11

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1 Introduction

Chapter 7 in Cournot (1838) demonstrates the so-called Cournot's duopoly theory in which two firms selling identical products engage in quantity competition and each firm takes the other's output level as given in setting its own output level. Chapter 9 in Cournot (1838) presents what is known as the complementary monopoly theory, which examines pricing decisions by two monopoly sellers of complementary goods. In particular, Chapter 9 investigates how a sole supplier of zinc and a sole supplier of copper—the two essential constituents in the production of brass through their alloy—would independently set the prices of their respective goods to competitive brass producers. It is shown that in equilibrium the sum of the two prices will generally exceed the monopoly price that would be set by a single owner of both goods.[1]

The well-known tragedy of the commons, first proposed by Hardin (1968), refers to a situation covering the absence of incentives to prevent the overuse and depletion of a resource if it is non-partitionable and where multiple agents are assigned rights of usage. Standard examples include excessive mining, fishing, and hunting, depletion of oil and water, and the destruction of the quality of the earth's atmosphere. Inefficiency arises in the overusage of a common resource due to congestion externalities, in the sense that no one takes into account the effects of his actions on other users of the resources.

The tragedy of the anticommons, first introduced by Heller (1998) and Heller and Eisenberg (1998), arises when more than a single agent is assigned rights of exclusion. The anticommons tragedy results in underusage rather than overutilization as the conventional commons problem, because each excluder may prevent any potential user from gaining access for which the exercise of it creates interdependencies that remain outside the explicit considerations of the decision makers. Real world examples include stifled innovation and research and development (R&D) due to patent protection. A similar phenomenon is also responsible for inefficient idle buildings by reason of multiple gatekeepers of their property rights, each of whom can exercise a right of exclusion.

Buchanan and Yoon (2000) first provide a formal economic analysis of both the commons and anticommons problems. They show that if each agent is assigned a right to use a resource, then the interaction will converge at an equilibrium that is structurally analogous to that familiar in the "excessive" output of Cournot's (1838) duopoly. On the other hand, if each person is assigned a right to exclude the other from usage, then the interaction will converge at an equilibrium that is structurally analogous to the "underused" output of Cournot's (1838) complementary monopoly. Buchanan and Yoon (2000) also demonstrate the formal symmetry between a resource's overusage due to multiple accesses and a resource's underusage due to multiple exclusion rights, showing that the size of the value loss is an increasing function of the number of agents assigned simultaneous rights.

[1] Sonnenschein (1968) shows that there exists a duality between Cournot's duopoly theory and his complementary monopoly theory, in the sense that the two theories share identical strategic properties, and that we should be able to derive one theory from the other.

Our model is conducted in terms of a two-stage game involving two sole suppliers of each of the two essential factors in the production process, along with infinite competitive producers of final products. In the first stage, the two owners of the factors simultaneously and independently decide their factor prices to the downstream firms under a constant elasticity of substitution (CES) production function. In the second stage, all the firms producing final products engage in a quantity competition game.

We demonstrate that the equilibrium output of the final products is an increasing function of the elasticity of substitution. For a relatively low (high) elasticity of substitution, the equilibrium output is too low (high) from the two upstream firms' viewpoint. When the elasticity of substitution is moderate, the two upstream firms' profits are maximized. The interpretation behind this is straightforward. The profit a factor owner can realize depends on its factor's superiority over what it is meant to replace. When the elasticity of substitution is high, downstream firms can easily substitute between the two factors. In other words, the cheaper factor will be used as a substitute for the higher-priced factor. It follows that the higher the elasticity of substitution is, the lower the equilibrium factor price will be and the higher the equilibrium quantity of final products will be. The results in this model imply that suitable competition resulting from the appropriate substitution of factors turns out to be beneficial to the owners of the factors.

We propose that such a vertically related market model can be used to analyze a wide range of common property problems. In particular, the two upstream firms can be thought of as two agents that possess concurrent property rights on a common resource, and the amount of supplied final goods in the vertically related market model corresponds to the usage of a common resource. The CES production function is interpreted as a method of combining two property rights, and the "elasticity of substitution of factors" can now be analogously seen as the "substitution of concurrent possession of a property right," which means a partial right of usage of a resource (or a partial right of exclusion of a resource).[2]

According to the traditional concept of property, owners enjoy a complementary bundle of rights over their property, including the right to use the property and the right to exclude others from it. The tragedy of commons refers to perfect substitutability of concurrent possession of a property right. Conversely, the tragedy of anticommons indicates that no substitutability of concurrent possession of a property right is possible. In the real world, the concurrent possession of a property right may be a partial substitute.

To apply the context of commons/anticommons tragedies, our results imply that if anyone who wants to use a common resource must possess both of the two concurrent property rights, because exploitation rights can be granted to a third party only if every co-owner agrees to the transfer, then anticommons tragedies raise. The result confirms Buchanan and Yoon's (2000) anticommons tragedy, indicating the underutilization of a resource. This case corresponds to the severest of the anticommons tragedy.

From the other side, if anyone who wants to use the common resource must possess either one of the two concurrent property rights, because two co-owners are each capable of selling exploitation rights over the common resource, then commons tragedies

[2] We emphasize that the equivalence between the vertically related market framework and the commons/anticommons tragedies is by no means general and relies on specific assumptions about models.

raise. We thus can apply this setting to the commons tragedy as shown by Buchanan and Yoon (2000), implying the overutilization wastage of a resource. This case corresponds to the severest of the commons tragedy. When the concurrent possession of a property right is a partial substitute, the commons tragedies and anticommons tragedies are combined, and this is optimal in the usage of a common resource. In particular, for a moderate substitution of concurrent possession of a property right, the anticommons tragedy effect of underusage of a resource and the commons tragedy effect of overusage of a resource may be balanced and beneficial to the independent holders of the property right.

Regarding the commons and anticommons problems, the recent literature has offered evidence of symmetrical welfare effects from overuse and underuse of common property resources and their policy implications. Parisi et al. (2005) provide a dual model of property, where commons and anticommons problems are the consequence of a lack of conformity between use and exclusion rights. They formulate a hypothesis of legal rules for promoting unity in property. Vanneste et al. (2006) propose that underuse in anticommons properties may reflect a greater problem than overuse in commons properties, by showing that in two empirical studies the anticommons situation yields significantly higher asking prices than the amount of money taken in the commons situation. Hattori and Yoshikawa (2016) investigate the socially desirable free entry under co-opetition where firms compete in the product market while sharing common property resources that affect industry-wide demand.

Alvisi et al. (2011) discuss the tradeoff between the lack of competition and the anticommons tragedy in a vertically differentiated market in which more than one firm produces a complementary component. It is shown that the relative strength of the lack of competition and the anticommons tragedy is related to the type of quality leadership characterizing the market. Feinberg and Kamien (2001) analyze the hold-up problem in the context of a complementary monopoly in Cournot (1838) and suggest the endogenous creation of institutions among complementary monopolists that assure their customers are not being held up. Adachi and Ebina (2014) study unidirectional complementarity and mergers, finding that, in contrast to Cournot's (1838) traditional implication, a merger of firms producing strict complements may not make all consumers strictly better off.

This paper is also related to the literature on vertically related markets. Economides (1999) analyzes quality choice and vertical integration, showing that a sole integrated monopolist produces higher quality products and obtains higher consumer surplus and profits than independent vertically related monopolists. Economides and Salop (1992) generalize the Cournot duopoly complements model to the case in which there are multiple brands of compatible components. Vickers (1995) investigates competition and regulation in vertically related markets and finds that the overall welfare comparison between separation and integration is ambiguous. Wirl (2015) discusses the different terms of strategic interactions in downstream and upstream oligopolies—wholesale and retail pricing—between Bertrand competing retailers and an upstream oligopoly. This paper is discernible from the above papers by investigating how upstream firms' profit under different degrees of competitive pressure in factor markets is captured by the elasticity of substitution of factors. We show that profits of upstream firms are maximized for intermediate values of the elasticity of substitution.

We finally use the "parking example" proposed by Buchanan and Yoon (2000) to explain the substitution of concurrent possession of a property right. Suppose that two persons, person 1 and person 2, are jointly assigned the property right of a large vacant lot. Assume that person 1 possessing 10 green tickets is allowed to issue green permits and person 2 possessing 10 red tickets is allowed to issue red permits.

(1) If anyone who wants to park in the vacant lot must secure 10 tickets, no matter whether their color is green or red, then the concurrent possession of a property right is a perfect substitute. In this case, the large vacant lot will be overused for parking. (2) If anyone who wants to park in the vacant lot must secure 5 green tickets and 5 red tickets, then there is no substitution of concurrent possession of a property right. In this case, the large vacant lot will be underused for parking. (3) If anyone who wants to park in the vacant lot must secure, say, either 6 green tickets and 4 red tickets, or 4 green tickets and 6 red tickets, then the concurrent possession of a property right is a partial substitute. In this case, it may achieve efficient usage of the large vacant lot for parking.

2 The model

Suppose the production technology of final products is represented by the constant elasticity of substitution (CES) production function, proposed by Arrow et al. (1961), with constant returns to scale:

$$Q = f(x_1, x_2) = \left(x_1^{\frac{r}{r-1}} + x_2^{\frac{r}{r-1}} \right)^{\frac{r-1}{r}}, \text{ for } r < 1, \tag{1}$$

where Q denotes the unit of final products, and x_1 and x_2 denote the units of factor 1 and factor 2, respectively.

The elasticity of substitution, s, between the two factors, referring to the rate of substitution between the two factors with regards to the ratio of their own price, is constant:

$$s = -\frac{d \ln \left(\frac{x_1}{x_2} \right)}{d \ln \left(\frac{w_1}{w_2} \right)} = 1 - r, \tag{2}$$

where w_1 and w_2 denote the prices of the two factors, respectively. When the elasticity of substitution s is less than one, the two factors are gross complements. Conversely, when the elasticity of substitution s is greater than one, the two factors are gross substitutes.

The parameter r can be varied to encompass a wide range of possible production functions. First, if $r \to 1 (s \to 0)$, then the production function is given by $Q = \min \{x_1, x_2\}$, which is a perfect complementary Leontief production function. Second, if $r \to -\infty (s \to \infty)$, then the production function is given by $Q = x_1 + x_2$, which is a perfect substitute linear production function. Third, if $r \to 0 (s \to 1)$, then the production function is given by $Q = x_1^{0.5} x_2^{0.5}$, which is an imperfect substitute Cobb–Douglas production function.[3]

Suppose the two factors have no other use than that of being jointly employed to form a final product, and in supplying one unit of factor for firm 1 and firm 2, each firm incurs a constant cost, which is normalized to zero, without loss of generality.[4] We analyze a two-stage game with two individual owners of factors, a sole supplier of factor 1 and a sole supplier of factor 2, and an infinite number of identical producers of final products.

The game structure is as follows. In the first stage, each upstream firm quotes a per-unit price, w_i for $i = 1, 2$, of its factor to each downstream firm. Given these decisions, product market competition takes place in the second stage, with all firms producing final products that face a linear market demand function, $p = 1 - Q$, where p denotes product price and Q denotes industry output, and compete in quantity, q_i. The equilibrium concept is a subgame perfect Nash equilibrium, which can be derived using backward induction. Here, we concentrate only on pure strategies.

3 The analysis

Before discussing the equilibrium outcome in the model, let us review some remarks on it. From the two upstream firms' viewpoint, whatever the elasticity of substitution between the two factors is, optimal outcomes are obtained when $Q = 1/2 (p = 1/2)$, and the sum of the two upstream firms' profits is given by $\pi_1 + \pi_2 = 1/4$, which is independent of r.[5] We emphasize that $Q = 1/2$ $(p = 1/2)$ is the collusive outcome; in the sense that collusion allows for the optimal outcome to be achieved. If $Q^* > 1/2$ $(p^* < 1/2)$, then a commons tragedy emerges in equilibrium, in the sense that the equilibrium quantity of final products is too high. If $Q^* < 1/2$ $(p^* > 1/2)$, then an anticommons tragedy emerges in equilibrium, in the sense that the equilibrium quantity of final products is too low.

3.1 Perfect complements

We first discuss the extreme case of production processes in which the two factors are perfect complements and they cannot be substituted for each other, meaning that

[3] We note that when $r = 0$, the CES production function is not defined, due to division by zero. However, as r approaches zero, the marginal rate of technical substitution (MRTS) tends to a limit of $-x_2/x_1$, which is simply the MRTS for the Cobb–Douglas production function $Q = x_1^{0.5} x_2^{0.5}$. It follows that as r approaches zero, the isoquant of the CES production function is the same as the isoquant of the Cobb–Douglas production function.

[4] The factor may also be some unique facility such as a right-of-way that is not subject to congestion, or a lower-cost source of a natural resource that is not subject to exhaustion.

[5] Note that in the interfirm rivalry context, total surplus, defined as the sum of producers' surplus and consumers' surplus ($TS = PS + CS = p^* \cdot Q^* + Q^{*2}/2$), is maximized when $r \to -\infty$ $(s \to \infty)$. In this case, the product price is equal to marginal cost (zero). On the other hand, in our commons/anticommons application, optimality is obtained when the two upstream firms' profits are maximized, which is equivalent to the maximal profit a single owner of both resources can acquire.

factor 1 and factor 2 must be used in absolutely fixed proportions. Obvious examples include computer operating systems and the software that run together on a personal computer. The production function of mowing a lawn is also characterized by fixed proportions—each worker must have one lawn mower in order to produce any output.

For a Leontief production function $Q = \min\{x_1, x_2\}$, each downstream firm produces the final output at constant average cost $w_1 + w_2$, and the equilibrium product price in the second stage is given by $p^* = w_1 + w_2$. Turning to the game's first stage, the two upstream firms engage in factor price competition for the business of downstream firms, and Cournot's (1838) complementary monopoly theory is replicated.

Proposition 1 *If* $r \to 1 (s \to 0)$, *then the equilibrium outcomes are given by* $w_1^* = w_2^* = 1/3$, $Q^*(w_1^*, w_2^*) = 1/3$, *and* $p^*(w_1^*, w_2^*) = 2/3$. *In this case, the anticommons tragedy is the severest.*

The quantity of final products is too low from the two upstream firms' viewpoint. The reason is that when suppliers of complementary factors are pricing independently, they will not take into account the positive effect of a drop in the price of their factor on the sales of the other factor.

3.2 Perfect substitutes

We then discuss the extreme case in which the two factors are perfect substitutes for $r \to -\infty$ $(s \to \infty)$. This production function might represent, for example, either potatoes from Maine or potatoes from Idaho, both of which can be used to produce potato salad or french fries. Another example of perfect substitutability is found in a farmer's output of wheat. As farmers gain access to better farm equipment, they discover it is very possible to substitute capital for labor while continuing to harvest about the same amount (i.e. a very capital-intensive technique).

For a special case of the linear production function ($Q = x_1 + x_2$), each factor of a downstream firm supplied by an upstream firm is a perfect substitute. The Bertrand paradox shows that the equilibrium factor prices are both zero, $w_1^* = w_2^* = 0$, and each upstream firm's profit is zero, $\pi_1^* = \pi_2^* = 0$.

Proposition 2 *If* $r \to -\infty (s \to \infty)$, *then the equilibrium outcomes are given by* $w_1^* = w_2^* = 0$, $Q^*(w_1^*, w_2^*) = 1$, *and* $p^*(w_1^*, w_2^*) = 0$. *In this case, the commons tragedy is the severest.*

The upstream firm that quotes a lower price supplies the whole factor market, while the other upstream firm that quotes a higher price sells zero. It follows that the two upstream firms have an incentive to undercut each other continually until prices are driven down to zero. The non-cooperative equilibrium shows that the two upstream firms' profits are both zero if their factors are perfect substitutes.

We next illustrate the connection of this vertically related market model and the standard Cournot duopoly theory. We note that if the production technology of final products is common knowledge and if we allow the two owners of the factors to have the option of either selling them to the downstream firms or producing final products by themselves, then the two upstream firms will cooperate in refusing to sell their factors. It follows that only the two upstream firms have factors available, and there is a duopolistic product market competition in the second stage. Under the assumption that firms compete in quantity in the final goods market, no matter which firm produces the final goods, Cournot's (1838) duopoly theory is replicated. Applied to the commons/anticommons situation, the quantity of final products is also too high from the two upstream firms' viewpoint, and Buchanan and Yoon's (2000) commons tragedy is replicated.

Proposition 3 *If the production technology of final products is common knowledge and the two upstream firms can cooperate in refusing to sell their factors and produce final products by themselves, then the results are given by* $q_1^* = q_2^* = 1/3$, $Q^* = 2/3$, *and* $p^* = 1/3$.

We propose that the situation where two upstream firms produce final products by themselves corresponds to vertically integrated firms. To apply the context of commons/anticommons tragedies, the situation where two upstream firms produce final products by themselves corresponds to the situation where the two agents who jointly assigned the property right of the resource use the common resource by themselves.

We note that cooperation in refusing to sell their factors yields higher profits. However, either firm does have incentives to deviate in this one-period game. We propose that if the analysis is extended to an infinitely repeated game, then cooperation—that is, both firms refuse to sell their factors—can occur in every stage of a subgame-perfect outcome for both firms to adopt a trigger strategy, provided a discount factor is high enough.[6] The basic logic obviously comes from Friedman's (1971) Theorem (sometimes also called the Folk Theorem). We emphasize that Proposition 3 is the collusive outcome in which the two upstream firms collude to produce final products by themselves and compete in quantity in the final goods market. We only discuss the implication of Proposition 3 and do not further deal with the issues of collusion sustainability, which is out of the scope of this current study.

3.3 Imperfect substitutes

For an imperfect substitute production function, routine manipulation yields each downstream firm i's conditional factor demand function:

$$x_1(w_1, w_2, q_i) = q_i \cdot x_1(w_1, w_2, 1) = q_i \cdot w_1^{r-1} \left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}. \tag{3}$$

$$x_2(w_1, w_2, q_i) = q_i \cdot x_2(w_1, w_2, 1) = q_i \cdot w_2^{r-1} \left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}. \tag{4}$$

We calculate each downstream firm i's cost function as:

$$c(w_1, w_2, q_i) = q_i \cdot c(w_1, w_2, 1) = q_i \left(w_1^r + w_2^r\right)^{\frac{1}{r}}. \tag{5}$$

[6] In particular, the trigger strategy means that each firm begins the infinitely repeated game by cooperating and then cooperates in each subsequent stage game if and only if both firms have cooperated in every previous stage. We thank an anonymous referee for pointing out the issues of collusion sustainability in this case.

In the second stage, the equilibrium quantities are solved as $q_i^* = (1-c)/(n+1)$ and $Q^* = n(1-c)/(n+1)$, and the equilibrium price and profits are solved as $p^* = (1+nc)/(n+1)$ and $\pi_i^* = (1-c)^2/(n+1)^2$, where $c = c(w_1, w_2, 1) = \left(w_1^r + w_2^r\right)^{\frac{1}{r}}$ and n denotes the number of downstream firms. It follows that $\lim_{n\to\infty} p^* = c = \left(w_1^r + w_2^r\right)^{\frac{1}{r}}$, $\lim_{n\to\infty} Q^* = 1 - c = 1 - \left(w_1^r + w_2^r\right)^{\frac{1}{r}}$, and $\lim_{n\to\infty} \pi_i^* = 0$.

Turning to the game's first stage, we calculate the two upstream firms' profits as:

$$\begin{aligned}
\pi_1(w_1, w_2) &= w_1 \cdot x_1(w_1, w_2, Q^*) \\
&= w_1\left(Q^* \cdot w_1^{r-1}\left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}\right) \\
&= w_1\left(\left(1 - \left(w_1^r + w_2^r\right)^{\frac{1}{r}}\right) \cdot w_1^{r-1}\left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}\right). \qquad (6)\\
\pi_2(w_1, w_2) &= w_2 \cdot x_2(w_1, w_2, Q^*) \\
&= w_2\left(Q^* \cdot w_2^{r-1}\left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}\right) \\
&= w_2\left(\left(1 - \left(w_1^r + w_2^r\right)^{\frac{1}{r}}\right) \cdot w_2^{r-1}\left(w_1^r + w_2^r\right)^{\frac{1-r}{r}}\right). \qquad (7)
\end{aligned}$$

Solving for $\partial\pi_1(w_1, w_2)/\partial w_1 = 0$ and $\partial\pi_2(w_1, w_2)/\partial w_2 = 0$ simultaneously yields:

$$\begin{aligned}
&(w_1^* = 0, w_2^* = 1/2),\ (w_1^* = 1/2, w_2^* = 0),\\
&\left(w_1^* = e^{\left(-\ln 2 + r\ln\left(\frac{1+r}{2+r}\right)\right)\cdot r^{-1}}, w_2^* = e^{\left(-\ln 2 + r\ln\left(\frac{1+r}{2+r}\right)\right)\cdot r^{-1}}\right). \qquad (8)
\end{aligned}$$

Since for a given $w_2^* = 1/2$ ($w_1^* = 1/2$, respectively), firm 1 (firm 2, respectively) can guarantee positive demand and profit by quoting a slightly higher price than zero, the first two candidate solutions in Eq. (8), $(w_1^* = 0, w_2^* = 1/2)$ and $(w_1^* = 1/2, w_2^* = 0)$, are not the equilibrium factor prices. Substituting the third candidate solution in Eq. (8), $\left(w_1^* = e^{\left(-\ln 2 + r\ln\left(\frac{1+r}{2+r}\right)\right)\cdot r^{-1}}, w_2^* = e^{\left(-\ln 2 + r\ln\left(\frac{1+r}{2+r}\right)\right)\cdot r^{-1}}\right)$, into $\partial^2\pi_1(w_1, w_2)/\partial w_1^2$ and $\partial^2\pi_2(w_1, w_2)/\partial w_2^2$, we obtain:

$$\begin{aligned}
\frac{\partial^2\pi_1(w_1^*, w_2^*)}{\partial w_1^2} &= \frac{\partial^2\pi_2(w_1^*, w_2^*)}{\partial w_2^2} \\
&= \frac{-\left((2)^{\frac{1-r}{r}}\left(1 - r + \left((2)^{\frac{1+2r}{r}} r w^*\right)\right)\right)}{4w^*} < 0,\\
&\quad \text{for } r > -1 \text{ or } r < -2. \qquad (9)
\end{aligned}$$

Here, $w^* = w_1^* = w_2^* = e^{\left(-\ln 2 + r\ln\left(\frac{1+r}{2+r}\right)\right)\cdot r^{-1}}$. It can be shown that $\lim_{r\to-1} p^*(w_1^*, w_2^*) = 0$, $\lim_{r\to-1} Q^*(w_1^*, w_2^*) = 1$, and $Q^*(w_1^*, w_2^*) = 1 - \left((2)^{\frac{1}{r}} w^*\right) > 0$ only

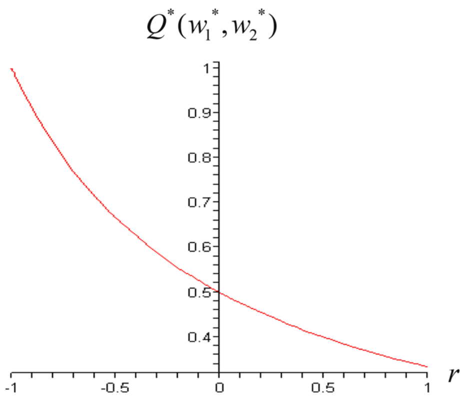

Fig. 1 The relationship between $Q^*(w_1^*, w_2^*)$ and r

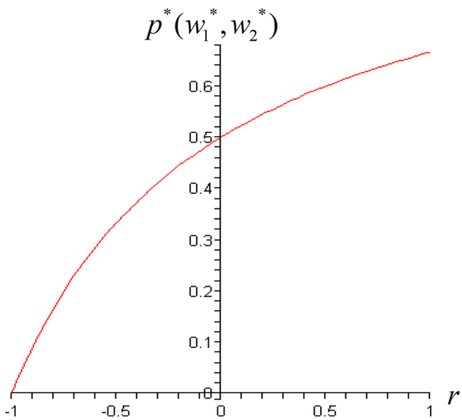

Fig. 2 The relationship between $p^*(w_1^*, w_2^*)$ and r

when $r > -1$. In this sense, the equilibrium factor prices are given by $\left(w_1^* = e^{\left(-\ln 2 + r \ln\left(\frac{1+r}{2+r}\right)\right) \cdot r^{-1}}, w_2^* = e^{\left(-\ln 2 + r \ln\left(\frac{1+r}{2+r}\right)\right) \cdot r^{-1}}\right)$ for $-1 < r < 1$ and $\partial w_1^*/\partial r = \partial w_2^*/\partial r > 0$.

Substituting $\left(w_1^* = e^{\left(-\ln 2 + r \ln\left(\frac{1+r}{2+r}\right)\right) \cdot r^{-1}}, w_2^* = e^{\left(-\ln 2 + r \ln\left(\frac{1+r}{2+r}\right)\right) \cdot r^{-1}}\right)$ into $Q^*(w_1, w_2)$ and $\pi_1(w_1, w_2) + \pi_2(w_1, w_2)$, we obtain $\lim_{r\to 1} Q^*(w_1^*, w_2^*) = 1/3$, $\lim_{r\to 0} Q^*(w_1^*, w_2^*) = 1/2$, $\lim_{r\to 1} \pi_1(w_1^*, w_2^*) + \pi_2(w_1^*, w_2^*) = 2/9$, $\lim_{r\to 0} \pi_1(w_1^*, w_2^*) + \pi_2(w_1^*, w_2^*) = 1/4$, and $\partial Q^*(w_1^*, w_2^*)/\partial r < 0$.

Figures 1 and 2 illustrate the relationship between the equilibrium quantity $Q^*(w_1^*, w_2^*)$ and the parameter r, and the relationship between the equilibrium price $p^*(w_1^*, w_2^*)$ and the parameter r, respectively.

Figure 3 illustrates the relationship between the sum of the two upstream firms' profits $\pi_1(w_1^*, w_2^*) + \pi_2(w_1^*, w_2^*)$ and the parameter r.

Suppose the two upstream firms can cooperate in factor price setting, and denote $(w_1^o = w^o, w_2^o = w^o)$ as the total-profits-maximized factor prices. It can be shown

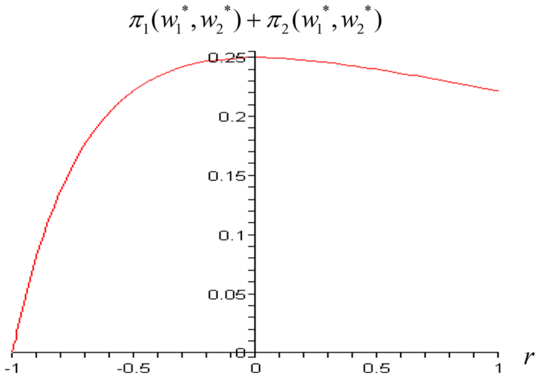

Fig. 3 The relationship between $\pi_1(w_1^*, w_2^*) + \pi_2(w_1^*, w_2^*)$ and r

that the collusive factor price is solved as $w^o = e^{(-\ln 2 - r \ln 2)\cdot r^{-1}}$, which is dependent on r. In particular, $w^o = 1/4$ for $r \to 1$ (Leontief production function); $w^o = 1/2$ for $r \to -\infty$ (linear production function); and $w^o = 1/4$ for a Cobb–Douglas production function $Q = x_1^{0.5} x_2^{0.5}$ $(r \to 0)$.[7] However, optimal outcomes are obtained when $Q = 1/2$ $(p = 1/2)$, whatever the elasticity of substitution between the two factors is. The intuition is that the sum of the two upstream firms' profits is identical to the total cost of all downstream firms, which is the same as the total revenue of all the downstream firms (i.e. $p^* \cdot Q^*$). Moreover, $p^* = c(w_1, w_2, 1)$ and $Q^* = 1 - c(w_1, w_2, 1)$ in a perfectly competitive market. Therefore, from the two upstream firms' viewpoint, choosing w_1 and w_2 to maximize the sum of the two upstream firms' profits is equivalent to choosing p and Q to maximize the total revenue of all the downstream firms. The optimal outcomes are obtained when $Q = 1/2$ $(p = 1/2)$, and the sum of the two upstream firms' profits is given by $1/4$. We summarize the results in the following proposition.

Proposition 4 *If* $-1 < r < 1$, *then (1) there exists an anticommons tragedy in the case of gross complements* $r > 0$ $(s < 1)$, *and the equilibrium quantity is given by* $Q^*(w_1^*, w_2^*) < 1/2$*; (2) there exists a commons tragedy in the case of gross substitutes* $r < 0$ $(s > 1)$, *and the equilibrium quantity is given by* $Q^*(w_1^*, w_2^*) > 1/2$*; (3) when* $r \to 0$ $(s \to 1)$, *the equilibrium outcomes are optimal (Pareto efficient) and are given by* $Q^*(w_1^*, w_2^*) = 1/2$ *and* $\pi_1(w_1^*, w_2^*) + \pi_2(w_1^*, w_2^*) = 1/4$.

For the application of a vertically related market context, our results imply that when the elasticity of substitution of factors is relatively high $(s > 1)$, the price competition in the factor markets is too fierce. It follows that the equilibrium prices of factors are too low and the equilibrium outputs are too high from the two upstream firms' viewpoint. Somewhat surprisingly, when the elasticity of substitution of factors is relatively low $(s < 1)$, the two upstream firms do not face sufficiently strong competitive pressure in the factor markets, which is also harmful to the upstream firms. In this sense, the equilibrium prices of factors are too high and the equilibrium outputs are too low from the two upstream firms' viewpoint. Finally, profits of upstream firms are maximized

[7] We note that $w^o = e^{(-\ln 2 - r \ln 2)\cdot r^{-1}}$ is undefined if $r = 0$.

for intermediate values of the elasticity of substitution of factors ($s \rightarrow 1$). The message of our analysis is that suitable competition resulting from appropriate substitution of factors turns out to be beneficial to the owners of factors.

The intuition behind Proposition 4 is as follows. A larger magnitude of the elasticity of substitution implies a more likely substitute. In this sense, the higher the elasticity of substitution is, the fiercer the price competition in the factor markets is. Therefore, the equilibrium prices of factors and final products are decreasing in the elasticity of substitution, and the equilibrium quantity of final products is increasing in the elasticity of substitution.

We hence identify an upstream firm's incentive to charge a higher price in order to enhance its profits as an "anticommons tragedy effect" and an upstream firm's incentive to charge a lower price (obtain a higher quantity) in order to enhance its profits as a "commons tragedy effect." The anticommons tragedy effect dominates the commons tragedy effect when the elasticity of substitution is relatively low ($s < 1$). In contrast, the anticommons tragedy effect is more than offset by the commons tragedy effect if the elasticity of substitution is relatively high ($s > 1$). The equilibrium outcomes in this model are the balance between said commons tragedy effect and anticommons tragedy effect.

With the elasticity of substitution $s \rightarrow 0$ ($r \rightarrow 1$), the wastage of value from the anticommons tragedy effect is the severest and the equilibrium quantity is the lowest. From a lower s, the anticommons tragedy effect is decreasing and the commons tragedy effect is increasing in s. The two upstream firms' profit is inverse U-shaped with respect to the elasticity of substitution s, increasing from a lower s and then declining to a relatively high s. It is thus not surprising that the anticommons tragedy effect and the commons tragedy effect may balance out in equilibrium and the equilibrium outcomes are optimal (Pareto efficient) for a moderate elasticity of substitution ($s \rightarrow 1$).

3.4 Applications

We have shown that if the elasticity of substitution is less than one, then the equilibrium quantity is too low. If the elasticity of substitution is larger than one, then the equilibrium quantity is too high. If the elasticity of substitution is moderate, then the equilibrium outcome may be optimal.

If we denote Q as the usage and p as the average value of a resource, then the above analysis applies to the commons/anticommons situation.[8] In particular, to apply the context of the commons/anticommons tragedies, the two upstream firms can be thought of as two agents that possess concurrent property rights in a common resource. The two concurrent property rights must be combined in order to use the common resource, and the CES production function is interpreted as a method for combining the two property rights. The "elasticity of substitution of factors" can now be analogously seen as the "substitution of concurrent possession of a property right."

To further apply the context of commons/anticommons tragedies, our results imply that if anyone who wants to use the common resource must possess both the two

[8] We note that this is the standard interpretation in the related literature of the common-property problem.

concurrent property rights (corresponding to the Leontief function ($s \to 0$)), because each excluder may prevent any potential user from gaining access, then anticommons tragedies arise. If anyone who wants to use the common resource must possess either one of the two concurrent property rights (corresponding to the linear function ($s \to \infty$)), because both agents are assigned rights of usage, then commons tragedies arise. If anyone who wants to use the common resource must possess a partial right to use or be excluded from the two concurrent property rights, because the concurrent possession of a property right is a partial substitute, then an optimal outcome may be obtained.

We have used the "parking example" proposed by Buchanan and Yoon (2000) to explain the substitution of concurrent possession of a property right in the Introduction. In this section we propose that the "substitution of concurrent possession of a property right" can also analogously be used when thinking about the "number (percentage) of agreements of independent holders of a property right." Consider another example of a multi-unit condominium in which the community association would like to use money from its public housing funds to buy a piece of art for placement in a common public area. Suppose the decision of whether or not to buy a piece of art is determined by voting.

(1) If buying is permitted whenever the community association can obtain the agreement of just only one condominium owner, then the concurrent possession of a property right is a perfect substitute and the piece of art may very likely be bought. (2) If buying is permitted whenever the community association can obtain the agreement of all condominium owners, then there is no substitution of concurrent possession of a property right and the piece of art may very likely not be bought. (3) If buying is permitted whenever the community association can obtain the agreement of, say, half of condominium owners (50% of votes), then the substitution of concurrent possession of a property right is moderate and an optimal outcome may be obtained.

Proposition 4 implies that the socially optimal outcome can be obtained by embedding the basic game proposed above into an extended game where a social planner first decides the method of combining the two property rights (r) in the basic game. In this case, the concurrent possession of a property right is a partial substitute (i.e. $r \to 0$ ($s \to 1$)), meaning a partial right of usage of a resource (or a partial right of exclusion of a resource). The policy implication to eliminate or at least mitigate the inefficiency of the usage of commons/anticommons is that the optimal outcome is obtained if the usage of a resource is determined by collective management with majoritarian decision-making institutions, and a social planner can set an optimal threshold of the number (percentage) of agreements of independent holders of the property right.

4 Conclusion

Under a general constant elasticity of substitution (CES) production function, this paper generalizes the elasticity of substitution between factors in Cournot's (1838) complementary monopoly theory. We assume there are two essential factors, each controlled by a single owner involved in the production of a final product, and there is a competitive final product market, in which the firms produce a homogeneous product. We analyze a two-stage game where the two upstream firms simultaneously and independently decide their factor prices to the downstream firms in the first stage. Given these decisions, product market competition takes place in the second stage.

We show that the equilibrium quantity of final products is an increasing function of the elasticity of substitution. In one extreme case of the Leontief production function (i.e. the two factors are perfect complements), our model replicates Cournot's (1838) complementary monopoly theory. In this case, the equilibrium factor prices are the highest and the equilibrium quantity of final products is the lowest. In the other extreme case of the linear production function (i.e. the two factors are perfect substitutes), the equilibrium quantity of final products is the highest and the equilibrium factor prices and profits of both upstream firms are zero. We finally show that for a moderate elasticity of substitution of a Cobb–Douglas production function, the equilibrium outcome in this model may be optimal (Pareto efficient).

We propose that such a vertically related market model is a particularly convenient and powerful tool for analyzing the commons/anticommons tragedies and demonstrate commons and anticommons are logically related to one another and can be framed within a unified concept of property. The concept of the tragedy of the commons describes situations where multiple individuals are endowed with the privilege to use a given resource. The concept of the tragedy of the anticommons describes situations where multiple owners hold rights to exclude others from a resource. When the concurrent possession of a property right is a partial substitute, meaning a partial right of usage of a resource (or a partial right of exclusion of a resource), the commons tragedies and anticommons tragedies are combined, and this may be optimal in the usage of a common resource, which is the notion of two negatives making a positive.

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