Aging, Taxation and Population Policy in Developing Economies

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Abstract

This paper constructs a two-sector general equilibrium overlapping generations model with endogenous fertility and education choices to investigate the impacts of aging in a developing economy, optimal tax design and population policies. We focus on those developing economies with low fertility and rapid population aging. The existence of a large informal sector in developing economies is particularly characterized in the model. Thailand is taken as a representative of this type of economies for quantitative analysis. We find that if the extra cost caused by aging is high, consumption is the only feasible tax tool. In the scenario that capital and labor income taxes are feasible, using consumption tax tends to generate higher welfare improvement because it leads to less distortion on capital accumulation and labor allocation between the two sectors. Furthermore, policies aiming at increasing fertility by reducing child-rearing cost indeed improve age structure. However, these policies lower the levels of human capital and physical capital, and eventually will not alleviate the tax burden caused by population aging. In contrast, policies that subsidizing education investment will encourage the accumulation of human capital, enlarge the formal sector and alleviate the tax burden.

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1 Introduction

Population aging has drawn global attention. The increasing share of old population requires government to increase the expenditures on social welfare systems. This leads to an extra burden on government's budget. Moreover, a decline in fertility worsens the problem of aging. In contrast to the common impression that the decline in fertility is an issue in developed countries, it indeed happens in many developing countries with a much faster speed in the recent years, even in countries with much lower income levels. To explore the issue of rapid aging in developing economies, this paper develops a two-sector general-equilibrium overlapping-generations model to provide a structural analysis. Our framework incorporates endogenous choices on fertility, education investment and labor allocation between formal and informal sectors to capture the rapid demographic change and the existence of a large informal sector in developing countries.[1] We have the model calibrated to a real economy and quantitatively explore the impacts of aging, optimal taxation and population policies.

Longevity implies an increasing demand on medical care for old people. Based on the data in Japan, Taiwan, Thailand and the US during 2003-2007, Hsu, Huang and Yupho (2015) point out that the elderly significantly need more medical care than young people do – the annual medical expenditure of an old adult (age 64 and above) is two to six times more than that of a young individual. Thus, it is unavoidable for a country with population aging to face a significant increase in the expenditure of its health care system.

The problem could be worse if the increase in longevity comes with a sharp decline in fertility, as happened in many developing countries. Lee et al. (2014) report that the average TFR in middle-income developing countries was already below the replacement level during 2005-2010. Table 1 provides the TFRs for selected countries classified into three income groups.[2] As shown in the table, the TFRs of some low-income countries were even lower than that of the US. For example, Vietnam's TFR was 1.89, which was lower than the 2.06 in the US. Figure 1 plots the time series of TFRs in some developing countries and the US is used as a reference. The figure shows that the TFRs in these developing countries rapidly declined from around 6 to around 2 within 30-50 years. The sharp decrease in

[1]The informal sector is defined as an underground economy, in which economic activities are done by companies or agents not officially registered with the government. Incomes earned in the informal sector are not taxed by the government and workers are not protected by the labor law.

[2]The income group classification is based on the criteria of World Bank in 2014. There were 82 economies in the lower-income group, 53 economies in the upper-middle-income group and 80 economies in the high-income group.

Table 1: Total Fertility Rate during 2005-10

Lower income (<$4125)				
Group average	India	Indonesia	Philippines	Vietnam
4.03	2.66	2.50	3.27	1.89
Upper-middle income ($4125-12735)				
Group average	Brazil	China	Mexico	Thailand
2.09	1.90	1.63	2.37	1.49
High income (>$12735)				
Group average	Australia	Japan	UK	US
1.65	1.89	1.34	1.88	2.06

Source: Lee et al. (2014) and United Nations.

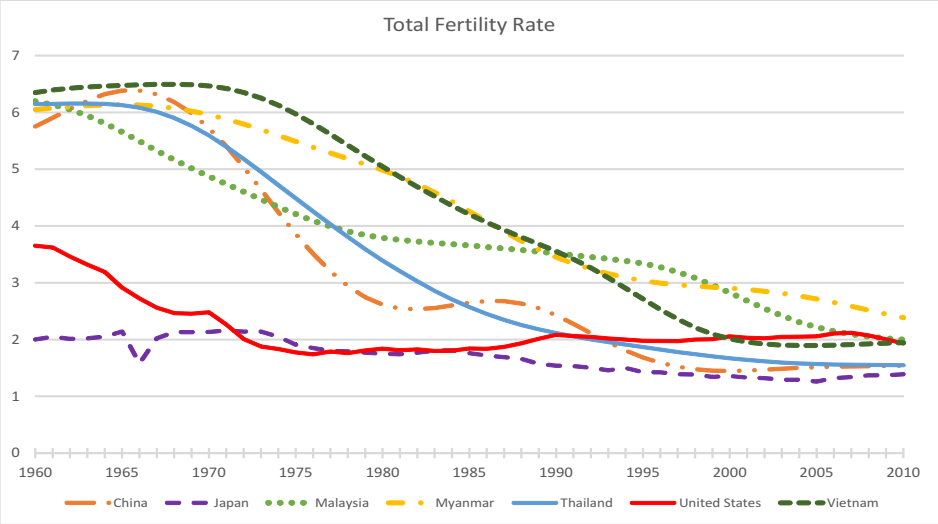
fertility implies that the percentage of labor force will shrink rapidly (and the share of the elderly will go up rapidly) in the near future.

In addition, different from developed countries, a common feature in developing countries is the existence of a large informal sector. This further constraints the government's ability on tax collection. Figure 2 presents the ratio of informal employment to total employment in non-agriculture sectors for selected developing countries. The ratio of informal employment was higher than 70% in several countries, such as India, Indonesia, Paraguay and Philippines. The informal employment share in Turkey, whose income level was the highest among the middle income countries, can be treated as the lower bound, at 30%. The ratio is even higher (generally higher than 50%) if the agricultural sector is included.

To study the issue of population aging in developing countries, we develop a two-sector general equilibrium overlapping generations model with endogenous fertility and education choices. Individuals also allocate labor between the formal and informal sectors. The life-cycle is characterized by three stages: childhood, young adulthood and old adulthood. Human capital is modeled in a discrete way that children will become skilled workers if parents invest in children' education. We assume that there are labor mobility constraints and institutional distortions in the labor market. The assumption allows us to have both voluntary and nonvoluntary informal workers in the equilibrium to be consistent with the empirical finding of higher wage inequality between formal and informal workers.

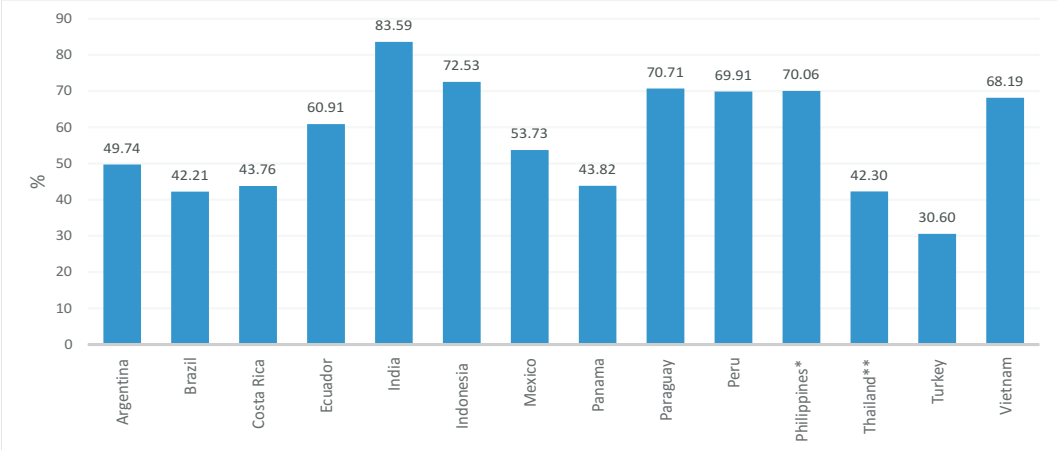
Quantitative analyses are provided to deliver clear policy implications. Our strategy is that the theoretical framework is calibrated to a representative real economy (a developing country) as a benchmark. Based on the benchmark economy, we first investigate the impacts of population aging in developing countries and discuss feasible arrangements of taxation

Figure 1: Total Fertility Rate 1960-2010



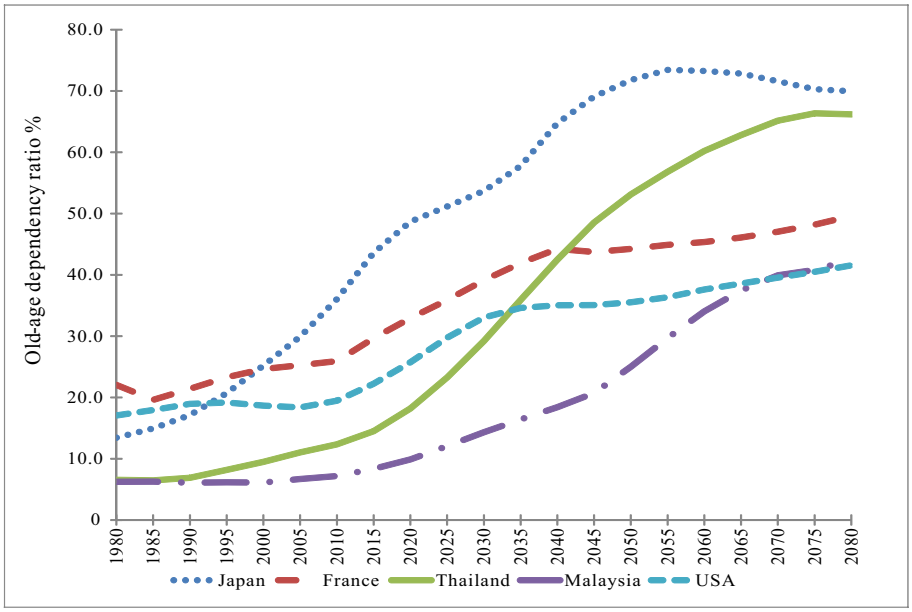
Source: WDI, World Bank.

Figure 2: Informal Employment Share (non-agriculture 2009)



Note: "*" denotes the data of 2008 and "**" is the data of 2010. Source: International Labor Organization.

Figure 3: Old-age Dependency Ratio



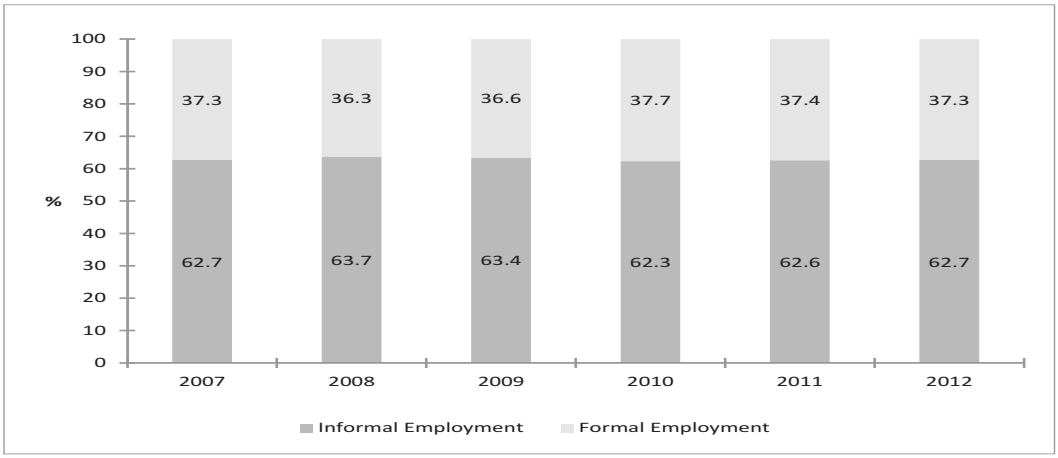
Source: United Nations.

for sustaining the fiscal balance. Alternative tax tools, including consumption tax, labor income tax and capital income tax, are particularly discussed in response to the conventional issues in the public finance and macroeconomic literature. Then, given the sharp decline in fertility in developing countries, alternative population policies in an aging economy are studied.

In the calibration, Thailand in the 2000s is selected as a representative because: (1) Thailand has experienced a dramatic demographic transition. As Figure 3 shows, the old-age dependency ratio of Thailand was below 10% in the 1980s but is expected to have a rapid increase to over 60%, which is close to the level of Japan and higher than that of France and the US, in the next few decades; (2) The ratios of workers in the informal sector of Thailand were large and stable. Figure 4 plots the ratios of informal employment in Thailand for 2007-2012. The ratio fluctuated between 62-64%.

Based on the calibration, quantitative impacts of population aging are explored. First, we consider a scenario as the baseline in which life expectancy increases from age 74.1 to age 80.1 that is projected for Thailand of 2055 and the ratio of total medical expenditure to GDP goes up from current 5.5% in Thailand to a forecasted value, 12%, due to the longer

Figure 4: Ratio of Informal Employment (Thailand)



Source: TBA

life expectancy. The government has to increase the tax revenue to finance the extra cost due to aging.

We find that individuals save more in response to longevity. This leads to the accumulation of physical capital. Besides, a longer life makes education investment more attractive and therefore human capital is accumulated. Because skilled agents tend to work in the formal sector, the accumulation of human capital enlarges the formal sector and contracts the informal sector. The negative relationship between human capital accumulation and the size of the informal sector is consistent with the findings in the literature.[3] Our result further suggest that only consumption tax is feasible in this scenario. The consumption tax rate has to increase from 12% to 20.5% in order to balance government budget. Labor and capital income taxes are not feasible because labor and physical capital can be reallocated to the informal sector if the corresponding tax rate is high. Therefore, consumption tax actually results in less distortion on the market.[4]

[3]In the literature, the informal sector is usually characterized by small firm sizes, unskilled jobs, low wages, and the loose regulations by the government. The experience of industrialization, skill upgrading, urbanization, and the shrinkage of the informal sector usually come together. See, for example, Marcouiller, de Castilla, and Woodruff (1997), Ranis and Stewart (1999), Gibson (2005), and Yuki (2007).

[4]In our calibration target, the consumption tax includes VAT, duties and other indirect taxes. It is largely contributed by the taxes on imported goods and energy, which are less avoidable. Therefore, we do not distinguish consumption in the informal sector from consumption in the formal sector. See Section 5 for the further

In a scenario with a mild increase in the ratio of total medical expenditure to GDP, the three tax tools are all feasible. However, in terms of welfare improvement, we still find that consumption tax is the best tool for an aging economy with a large informal sector. It has been discussed that the intergenerational redistribution effect of using consumption tax in replace of income taxes is preferred in an economy with a high old-age dependency ratio.[5] In this paper, an additional mechanism is that consumption tax has less distortion on human capital accumulation and the allocation of labor and capital between the two sectors.

Based on the baseline scenario of population aging with the consumption tax financing scheme, we investigate alternative population policies. We first study a child-rearing subsidy for encouraging fertility. Although it indeed improves the age structure, the policy distorts the relative price between skilled and unskilled children; thereby discouraging education investment. The accumulation of physical capital is also partly crowded out by the higher fertility. As a result, the formal sector shrinks and the aggregate output declines. The policy reduces the efficiency of resource allocation and eventually does not help to alleviate the tax burden caused by population aging. Alternatively, a policy that subsidizing children's educational cost improves the level of human capital and enlarges the formal sector. The aggregate output increases and the tax burden is reduced, thereby resulting in a welfare improvement. Our experiment suggests that the optimal subsidy rate on children's educational cost is about 26%.[6]

The theoretical framework in this paper builds on the literature that studies quantity-quality tradeoff of children, demographic change and economic growth. It is pioneered by Becker (1960) that introduces endogenous fertility to the theoretical model. Following up studies link fertility, demographic change and economic growth based on the mechanism of quantity-quality tradeoff, such as Becker, Murphy and Tamura (1990), Galor and Weil (1996), Doepke (2004), Doepke and Zilibotti (2005) and Liao (2011). These studies typically focus on the early stage of economic development with the demographic transition from high to low fertility. In contrast, our paper focuses on economies in a stage with a low fertility and extends the model to incorporate the informal sector and the fiscal structure in developing countries.

Our discussion on population policies is close to Lee et al. (2014). Given current age

discussion. In addition, we assume that the social security or pension system is fully funded. Therefore the government does not need to finance it by its tax revenue. It is probably not true for some countries, and hence the extra cost of aging in our analyses could be interpreted as a lower bound.

[5]Correia (2010) shows that consumption tax also has a preferred redistribution effect over income groups.

[6]Because of the calibration target, the education subsidy we discuss in this paper is on top of the current education subsidy.

structures and fiscal conditions, Lee et al. (2014) investigate optimal fertility rates in 40 countries. They suggest that the fertility rates should be lower than the current level in some countries, such as Brazil and Thailand, even though their current fertility rates are already below the replacement rate to improve living standards and fiscal burdens. However, fertility is exogenously determined in Lee et al. (2014). Our paper develops a richer framework that allows endogenous fertility choice and considers demographic dynamics. Therefore, our framework enables us to assess alternative population policies and examine the effects on fertility, human capital accumulation, long-run economic performance and welfare changes.

This paper also sheds light on understanding the implication of informal employment on optimal taxation and fiscal polices. However, most studies target developed countries. Only a few studies in the literature consider developing economies with a informal sector. For example, Penalosaa and Turnovsky (2005) study optimal taxation on labor and capital incomes in a standard growth model with an existence of the informal sector. Jung and Tran (2012) discuss the potential effects of the extension of social security to informal workers in developing countries to reduce the poverty problem for the elderly. As a complement to the literature on policy design with a informal sector, we further explore the implications of aging for fiscal and population policies in developing countries by allowing endogenous fertility and education choices.

The rest of this paper is organized as follows. In Section 2, we construct a two-sector general equilibrium overlapping generations model. The equilibrium features of the framework are explained. Section 3 describes the calibration for the current Thailand. In Section 4, we performs quantitative analysis and the results. Discussions on the model setting are provided in Section 5. Section 6 concludes this paper.

2 The Model

We develop a two-sector general equilibrium overlapping generations model with endogenous fertility and education choices. Individuals also choose labor allocation between formal and informal sectors. The life-cycle is characterized by three stages – childhood, young adulthood and old adulthood. A child relies on parents (young adults) without making any decisions. Parents care about children. They make decisions on the number of children, education investment on children, their own labor allocation and savings. Old adults are assumed to be retired. They consume their own savings and pay medical expenditures. Human capital is modeled in a discrete way that children will become skilled labor if par-

ents invest on education; otherwise, children will be unskilled labor. Labor and capital allocations between formal and informal sectors are characterized. Capital is freely mobile between the two sectors. In the labor market of developing countries, there exists a large wage gap between the two sectors and the informal employment share is relatively large. Thus, we consider frictions on the labor market to prevent completely free movement between formal and informal sectors. In the equilibrium, both voluntary and non-voluntary informal employments exist.

2.1 Features of the Labor Market in Developing Countries

To model the labor market in developing countries with a large informal sector, we take Thailand as an example. Skilled workers are defined as those whose education level is high school or above (schooling years >12). The main features of Thailand's labor employment are summarized as follows.[7]

1. About 70% of skilled workers are working in the formal sector, while more than 70% of unskilled workers are in the informal sector.

2. Skilled workers usually stay in the informal sector less than one year. The panel data of Household Social-Economic Survey (HSES) indicates that, for those skilled workers who are currently working in the informal sector, only about 10% of them will stay in the informal sector after one year. In contrast, the probability of unskilled workers to stay in the informal sector after one year is higher than 70%.[8] The fact suggests that, for skilled workers, having a job in the informal sector is more like a temporary arrangement. They tend to switch back to the formal sector quickly.

3. When a skilled worker has a job in the informal sector, his wage from the informal sector is very likely to be lower (with a probability of 90% to have a wage in the bottom 40% of total skilled workers). The pattern is not observed for unskilled workers.

4. The skill premium in the formal sector is around 2.4, while it is relatively smaller in the informal sector (about 1.4). The wage gap between formal and informal sectors also exists. For unskilled workers, the wage gap between the two sectors is about 1.7.

The facts mentioned above imply that skilled workers in the informal sector are more likely to be non-voluntary, seriously under-paid, and easy to be substituted by unskilled

[7] Data are from Household Social-Economic Survey (HSES), Thai National Statistics Office.

[8] See the employment transitions between formal and informal sectors in the appendix of Hsu, Huang and Yupho (2015).

workers. Although skilled workers in the informal sector are seriously under-paid, skill premium in the informal sector still exists but is much lower than that in the formal sector. To create the skill premium and wage gap between the two sectors, we introduce labor market frictions and distortions to our model. The detail setup is discussed in Section 2.5.

2.2 Demographics

Total population N at a time point consists of the population of children N^c, young adults N^y and old adults N^o:

$$N = N^c + N^y + N^o.$$

Population dynamics are determined by endogenous fertility choices and exogenous survival rates. Total number of children is determined by the fertility decisions of young adults. Denote n to be the average fertility per young adult. N^c is then given by:

$$N^c = nN^y.$$

By assumption, all children can survive to the young adulthood for sure. Thus, the population of young adult next period is $N^{y\prime} = N^c$. Furthermore, we assume that young adults survive to the old adulthood with the probability π^y. Therefore, the old population next period becomes $N^{o\prime} = \pi^y N^y$.

2.3 Education Investment and Human Capital Accumulation

In the framework, human capital is assumed to be discrete: skilled (s) and unskilled (u). Parents make decisions on children education level. Education investment costs e. If parents decide to do investment on children's education on top of the fundamental level, their children will become skilled workers. In contrast, children who do not receive higher education will become unskilled workers in the young adulthood. N_s^y is employed to denote the population of skilled young adults and N_u^y is the population of unskilled young adults. The population identity implies that $N^y = N_s^y + N_u^y$. According to the setting, human capital accumulation is represented by an increase in the proportion of skilled young adults to total young adults (N_s^y/N^y).

2.4 Government

The government taxes on consumption, labor income and capital income with the rate τ_C, τ_L and τ_K, respectively. The total tax revenue is the sum of consumption tax T_C, labor income

tax T_L and capital income tax T_K. By assumption, the government cannot monitor the labor income and the capital income in the informal sector. Thus, tax revenues of labor income and capital income are only collected from the formal sector.

The government runs a public medical care program for all old adults as a social welfare system. It covers a fraction ω of total medical expenditure M. Then, the public medical expenditure M_g is given by:

$$M_g = \omega M.$$

The government is required to maintain a balanced budget in each period. The budget constraint for the government is given by:

$$M_g + G = T_C + T_L + T_K, \tag{1}$$

where G is other government expenditures.

2.5 Production and Labor Market

There exists one final goods but two production sectors: formal and informal sectors. We use superscript f to denote variables in the formal sector and superscript x for the informal sector. In both sectors, there are perfectly competitive firms using physical capital (K), skilled workers (L_s) and unskilled workers (L_u) as inputs to produce the final goods. The two sectors employ a standard constant return to scale production technology. The production functions are summarized as follows:

$$Y^f = A^f (K^f)^{\alpha_1} (L_s^f)^{\alpha_2} (L_u^f)^{\alpha_3}; \tag{2}$$

$$Y^x = A^x (K^x)^{\gamma_1} (\eta L_s^x + L_u^x)^{\gamma_2}, \tag{3}$$

where Y^f and Y^x are output in the formal and informal sector, respectively. A^f and A^x are the total factor productivity (TFP) in the formal and informal sector, respectively. α_1, α_2 and α_3 are income shares of physical capital, skilled labor and unskilled labor in the formal sector. γ_1 and γ_2 are income shares of physical capital and total labor in the informal sector. The total output Y of the economy is then given by:

$$Y = Y^f + Y^x. \tag{4}$$

According to the features of labor market described in Section 2.1, the wage of skilled workers in the informal sector tends to be lower. For skilled workers, having a job in the informal sector is more likely as a temporary arrangement. Therefore, we assume skilled

and unskilled workers are substitutable in the informal sector. The labor efficiency η is included to capture the existence of skill premium in the informal sector.

Without distortion, firms' profit maximization in the two sectors imply that:

$$w_s^f = \alpha_2 A^f (K^f)^{\alpha_1} (L_s^f)^{\alpha_2 - 1} (L_u^f)^{\alpha_3}; \tag{5}$$

$$w_u^f = \alpha_3 A^f (K^f)^{\alpha_1} (L_s^f)^{\alpha_2} (L_u^f)^{\alpha_3 - 1}; \tag{6}$$

$$w_s^x = \eta \gamma_2 A^x (K^x)^{\gamma_1} (\eta L_s^x + L_u^x)^{\gamma_2 - 1}; \tag{7}$$

$$w_u^x = \gamma_2 A^x (K^x)^{\gamma_1} (\eta L_s^x + L_u^x)^{\gamma_2 - 1}. \tag{8}$$

However, as we describe in Section 2.1, there exists a wage gap between formal and informal sectors. It could be due to the institutional distortions on the labor market. For example, the minimum wage regulation and the requirement on employer-sponsored insurance coverage for employees in the formal sector. To capture the main spirit, we assume there are distortions on wage in the labor market and the distortions mainly affect unskilled workers. Therefore, the wage gap of unskilled workers is created by the distortion X_u such that the wage gap of unskilled workers between formal and informal sectors is given by:

$$\frac{(1-\tau_L) w_u^f}{w_u^x} = X_u. \tag{9}$$

2.6 Individual's Problem

A young adult with skill level $i = \{s, u\}$ cares about his current consumption c_i^y, his consumption at old age $c_i^{o\prime}$ and how his children will be in the next period. Therefore, a young adult allocates his disposable income on his current consumption, asset holdings in formal and informal sectors ($a_i^{f\prime}$ and $a_i^{x\prime}$) and total education investment on his children.

Each young adult is assumed to be endowed with one unit of time. Raising a child costs a fraction ϕ of a young adult's time. Then, he can supply the rest of time to the labor market. Assume that he allocates the fraction θ_i of his rest time to the formal sector. The time he works in the informal sector is $1 - \theta_i$ of his rest time. Therefore, a young adult's income consists of labor income from both sectors. When a young adult survives to the old adulthood, he consumes his own savings and capital income from both sectors. At old age, he has to pay the proportion of medical expenditure m' that is not covered by public medical care program.

To summarize, a young adult chooses current consumption, savings, the number of children n_{ij}, education investment and the proportion of labor supply in the formal sector to

maximize his lifetime utility. j represents children's skill level, $j = \{s,u\}$. The maximization problem can be expressed as follows:

$$V_i = \max_{\{c_i^y, a_i^{f\prime}, a_i^{x\prime}, n_{ij}, e_i, \theta_i\}} \left\{ u(c_i^y) + \beta \pi^y u(c_i^{o\prime}) + \psi n_{ij}^{-\varepsilon} [n_{ij} V_j'] \right\}, \tag{10}$$

subject to

$$(1+\tau_C)c_i^y + \pi^y(a_i^{f\prime} + a_i^{x\prime}) + e_i n_{ij} = (1-\phi n_{ij})[\theta_i(1-\tau_L)w_i^f + (1-\theta_i)w_i^x]; \tag{11}$$

$$(1+\tau_C)c_i^{o\prime} = [1+(1-\tau_K)r^{f\prime}]a_i^{f\prime} + [1+r^{x\prime}]a_i^{x\prime} - (1-\omega)m'; \tag{12}$$

$$j = s, \text{ if } e_i = \bar{e};\ j = u, \text{ if } e_i = 0;$$

$$0 \le \theta_i \le \bar{\theta_i};$$

where $r^{f\prime}$ and $r^{x\prime}$ are capital returns in the formal and the informal sectors, respectively. We assume there is a perfect annuity market in each sector. A young adult, who holds $\pi^y a_i^{f\prime}$ annuity (as savings) in the formal sector, will receive $(1+(1-\tau_K)r^{f\prime})a_i^{f\prime}$ next period if he survives. Similarly, the asset holdings $\pi^y a_i^{x\prime}$ in the informal sector will deliver a payment of $(1+r^{x\prime})a_i^{x\prime}$ next period if he survives. Furthermore, education cost is discrete, $e_i = \{0, \bar{e}\}$. If parents pay the cost $\bar{e}$, the child will become skilled worker next period. Finally, to capture the features of non-voluntary informal workers, we assume there exists a labor mobility constraint $\bar{\theta_i}$ to represent the employment capacity of sector i. When the labor mobility constraint is not binding, a young adult can freely allocate his working time between formal and informal sectors. Otherwise, he will be forced to work as a non-voluntary worker in the informal sector.

2.7 Equilibrium Features

2.7.1 Individual Optimal Decision Rules

- Savings ($a_i^{f\prime}$ and $a_i^{x\prime}$):
  Asset holdings can be used in both formal and informal sectors. Intuitively, an individual will hold the asset that gains a higher rate of return. The optimal asset holdings are determined by the following equations:

$$u_{c_i^y} = \beta(1+(1-\tau_K)r^{f\prime})u_{c_i^{o\prime}}, \quad \text{if } (1-\tau_K)r^{f\prime} \ge r^{x\prime}; \tag{13}$$

$$u_{c_i^y} = \beta(1+r^{x\prime})u_{c_i^{o\prime}}, \quad \text{if } r^{x\prime} \ge (1-\tau_K)r^{f\prime}; \tag{14}$$

  where $u_{c_i^y}$ and $u_{c_i^{o\prime}}$ are marginal utility of consumption at young adulthood and at old adulthood, respectively. Because there are no distortions and no mobility constraints

in the capital market, in equilibrium after-tax capital gains from the two sectors must be the same, $(1-\tau_K)r^{f\prime} = r^{x\prime}$.[9] Thus, an individual is indifferent between the two assets.

- Labor allocation (θ_i):
  Intuitively, an individual will supply labor to a sector with a higher return. The marginal return of one additional labor supply in the informal sector is w_i^x. The after-tax marginal return of labor is $(1-\tau_L)w_i^f$ in the formal sector. Therefore, in equilibrium, $(1-\tau_L)w_i^f = w_i^x$ and $0 < \theta < 1$ if there is no mobility constraint or the constraint is not binding in the labor market. If the constraint is binding, $\theta_i = \bar{\theta_i}$.

  As we observed in the data, when a skilled worker having a job in the informal sector, his wage is very likely to be lower. This implies that the mobility constraint on skilled workers must be binding. Otherwise, in equilibrium, $(1-\tau_L)w_s^f = w_s^x$, which contradicts to the data. Therefore, in the quantitative analysis that we provide later, we require the mobility constraint on skilled workers is binding, $\theta_s = \bar{\theta_s}$. Besides, there are distortions $X_u > 1$ on the unskilled labor market such that the wage gap between formal and informal sector for unskilled workers exists in equilibrium, as shown in equation (9).

- Fertility (n_{ij}):
  The first order condition with respect to n_{ij} implies the following equation:

$$\psi(1-\varepsilon)(n_{ij})^{-\varepsilon}V_j' = \left(\frac{u_{c_i^y}}{1+\tau_C}\right)\{\phi[\theta_i(1-\tau_L)w_i^f + (1-\theta_i)w_i^x] + e_i\}. \qquad (15)$$

  The left-hand side of equation (15) is the marginal benefit of having an additional child. On the right-hand side is the marginal cost of having an additional child. A young adult chooses the number of children until the marginal benefit is equal to the marginal cost.

- Education investment on children (e_i):
  Within a family, children are indifferent except education investment. Thus, children in the same family should be all skilled or all unskilled. Skilled and unskilled children will not live in the same family. Besides, in our quantitative analysis, we focus on an equilibrium that both skilled and unskilled workers exist in the economy. It can be shown that the only feasible equilibrium is that skilled parents always invest on

[9] The data for capital gains in the informal sector is not available. To focus on the features of the labor market in developing countries, we simply assume there is no distortion on the capital market.

children's education, while some unskilled parents make education investment and others do not. Thus, in equilibrium, only unskilled parents are indifferent between investing and not investing on children's education. It is impossible for both types of parents to satisfy the indifference condition. The details are provided in Appendix A.

2.7.2 Markets Clearing Conditions

- Market of skilled workers:
  The mobility constraint on skilled workers is binding. Thus, the proportion of skilled workers working in the formal sector is given by:

$$\frac{L_s^f}{L_s} = \bar{\theta_s}. \tag{16}$$

- Market of unskilled workers:
  By assumption, there exists a distortion X_u on the unskilled labor market to create the wage gap between formal and informal sectors. Substituting marginal products of labor to the definition of distortion in equation (9) and using the facts that $L_s^f = \bar{\theta_s} L_s$, $L_s^x = (1-\bar{\theta_s})L_s$, $L_u^f = \theta_u L_u$ and $L_u^x = (1-\theta_u)L_u$, the proportion of unskilled workers working in the formal sector is then given by:

$$\theta_u = \frac{\eta(1-\bar{\theta_s})l_s + l_u}{(1+B)l_u}, \tag{17}$$

  where $B = \frac{X_u \eta \gamma_2 y^x}{(1-\tau_L)\alpha_3 y^f}$, $y^f = \frac{Y^f}{N^y}$, $y^x = \frac{Y^x}{N^y}$, $l_s = \frac{L_s}{N^y}$ and $l_u = \frac{L_u}{N^y}$.

- Market of physical capital:
  Because there is no mobility constraint on capital market, capital market clearing implies a no-arbitrage condition: $(1-\tau_K)r^f = r^x$. Therefore, the optimal capital allocation between the two sectors is given by:

$$\frac{K^f}{K^x} = (1-\tau_K)\frac{\alpha_1 A^f (K^f)^{\alpha_1} (L_s^f)^{\alpha_2} (L_u^f)^{\alpha_3}}{\gamma_1 A^x (K^x)^{\gamma_1} (\eta L_s^x + L_u^x)^{\gamma_2}}. \tag{18}$$

3 Calibration and the Benchmark Economy

As mentioned in the introduction, Thailand has a relatively low fertility but a large share of informal employment. Therefore, Thailand is selected as a representative economy for our quantitative analysis. The model described in Section 2 is calibrated to match data from Thailand during 2000-2012. Then the calibrated economy is used as a benchmark

economy. Based on the benchmark economy, quantitative analysis and policy experiments are performed in the next section.

3.1 Parameters

The model economy is calibrated as a steady state. The model period is 30 years. Table 2 summarizes the parameters for calibration. According to the estimate in World Development Indicators (WDI), the life expectancy at birth for Thailand during 2010-2012 is 74.1. In our model, children survive to the young adulthood for sure. Therefore, the survival rate for young adults π^y is 0.47 such that the life expectancy at birth is matched. Medical expenditure per old adult m is set at 0.077 to match the ratio of total medical expenditure to GDP 5.5%, which is reported by WDI for Thailand during 2007-2010.

There are four preference parameters: β, σ, ε and ψ. The annual discount factor is 0.951 to match annual capital-output ratio 1.9 in 2012. ψ is set to be 0.227 so that the total fertility rate 1.54 is matched. σ and ε are jointly calibrated. They are discussed later.

There are two costs associated with children: child-rearing time cost ϕ and education cost $\overline{e}$. In the calibration, skilled worker is defined as a worker with an education level equal to or above high (secondary) school. The report from the National Statistical Office of Thailand suggested that the cost of raising a child until age 24 (net of the labor income earned by children) was about 1.156 million Thai baht in 2004. To be consistent with our model, we use 0.867 million baht ($1.156 \times 18/24$) to represent the cost until graduation from high school. It is about 24.3% of the average earnings (30 years). Therefore, ϕ is set at 0.243. The education cost is assume to be proportional to parents' income in the calibration. The proportion ϕ_s is chosen to be 0.089 to match the ratio of skilled workers to total workers 17% (average of 2010-12). This implies that the current educational subsidy on skilled children in Thailand has been included in ϕ_s. Policy experiments on educational subsidy in Section 4.2 refers to the educational subsidy on top of the current level.

The government collects three taxes: τ_C, τ_K and τ_L. They are set so as to match the ratios of total revenues collected from consumption tax to GDP, capital income tax to GDP and labor income tax to GDP to be 9%, 4% and 2%, respectively. Here, consumption tax refers to VAT, duties and sales taxes; capital income tax is cooperate tax; labor income tax is individual income tax. Thus, $\tau_C = 12.1\%$, $\tau_K = 11.4\%$ and $\tau_L = 10.9\%$. The government runs a public medical care program for all old adults as a social welfare system. It covers the fraction ω of total medical expenditure. According to the report of WDI, the average ratio of public health expenditure to total health expenditure is about 85% in Thailand during 2007-2010. Thus, we set $\omega = 85\%$.

There are two parameters for the distortion and the capacity constraint in the labor market. The distortion on unskilled labor market X_u is chosen to be 1.514 in order to match the wage gap of unskilled workers between the formal and informal sectors $w_u^f/w_u^x = 1.7$. The capacity constraint on skilled workers $\bar{\theta}_s$ is 0.7 from data.

The rest parameters in the production side are two TFP (A^f and A^x), three income shares in the formal sector (α_1, α_2 and α_3), two income shares in the informal sector (γ_1 and γ_2) and the labor efficiency in the informal sector (η). The TFP in the formal sector is normalized to be 10. Then, the TFP in the informal sector is set at 4.283 such that the relative output ratio (Y^x/Y^f) is 0.52. The income share of physical capital in the formal sector α_1 is set at 0.67. With the skill premium and the ratio of skilled workers in the formal sector, we can compute α_3. We know that the skill premium in the formal sector is 2.4. Furthermore, the ratio of skilled workers to total workers (L_s/L) is 17%. We believe that the fraction of skilled workers in the formal sector, $L_s^f/(L_s^f + L_u^f)$, should be higher than that in the informal sector. If the fraction is 0.2, the income share of unskilled labor in the formal sector should be equal to 0.20. If the fraction is 0.25, α_3 is 0.183. If the fraction is 0.3, α_3 is 0.1627. Here we set $\alpha_3 = 0.182$ to obtain a better calibrated results. Therefore, $\alpha_2 = 0.148$. The labor efficiency in the informal sector η is set at 1.4 to match $w_s^x/w_u^x = 1.4$. The last two parameters are γ_1 and γ_2 in the informal sector. In the calibration, γ_1, σ and ε are jointly calibrated to match the ratio of skilled parents' TFR to unskilled parents' TFR 0.5, the proportion of unskilled workers working in the formal sector $L_u^f/L_u = 0.3$ and the skill premium in the formal sector $w_s^f/w_u^f = 2.4$. Thus, $\gamma_1 = 0.616$, $\sigma = 0.525$ and $\varepsilon = 0.51$. Finally, γ_2 is 0.384. All targets and data moments are summarized in Table 3.[10]

3.2 Benchmark Economy

The benchmark economy is calibrated to the data from Thailand during 2000-2012. Table 4 provides the macroeconomic features of the benchmark economy. In the benchmark economy, the total fertility rate is mainly contributed by unskilled parents with unskilled children. Other parents prefer less than one child. In addition, the relative TFR implies that the number of children that skilled parents want is only a half of the amount that unskilled parents choose.

On the production side, the output from the informal sector is about a half of that produced in the formal sector. Besides, about 63.2% of workers work in the informal sector, while only 36.8% of workers in the formal sector. The result is close to the ratio of the informal employment presented in Figure 4. Our benchmark economy captures the main

[10] Data sources will be added later.

Table 2: Parameters

Parameters	Value	Source / Target
Survival rate and medical expenditure		
π^y	0.470	match life expectancy= 74.1
m	0.077	match $M/Y = 5.5\%$
Preference		
β	0.951	match $K/Y = 1.9$
σ	0.525	jointly calibrated
ε	0.510	jointly calibrated
ψ	0.227	match TFR= 1.54
Labor markets and productions		
X_u	1.514	match $w_u^f/w_u^x = 1.7$
$\bar{\theta}_s$	0.700	computed from data
$\bar{\theta}_u$	0.700	preset
A^f	10	normalization
A^x	4.283	match $Y^x/Y^f = 0.52$
α_1	0.670	computed from data
α_2	0.148	computed by $1-\alpha_1-\alpha_3$
α_3	0.182	computed from data
γ_1	0.616	jointly calibrated
γ_2	0.384	computed by $1-\gamma_1$
η	1.400	match $w_s^x/w_u^x = 1.4$
Costs of children		
ϕ_s	0.089	match $L_s/L = 0.17$
ϕ	0.243	computed from data
Government		
τ_C	12.1%	match $T_C/Y = 9\%$
τ_L	10.9%	match $T_L/Y = 2\%$
τ_K	11.4%	match $T_K/Y = 4\%$
ω	85%	computed from data

Table 3: Calibration Targets for Benchmark Economy

Target moment	Model	Data
Life expectancy	74.1	74.1
Ratio of total medical exp to output (M/Y)	0.055	0.055
Average TFR	1.540	1.54
Relative TFR (skilled/unskilled)	0.500	0.5
Capital-output ratio (K/Y)	1.900	1.9
Relative output (Y^x/Y^f)	0.520	0.52
Share of unskilled in the formal sector $(\theta_u = L_u^f/L_u)$	0.300	0.3
Share of skilled labor to total labor (L_s/L)	0.170	0.17
Skill premium in the formal sector (w_s^f/w_u^f)	2.396	2.4
Wage gap for unskilled labor (w_u^f/w_u^x)	1.700	1.7
Skill premium in the informal sector (w_s^x/w_u^x)	1.400	1.4
Ratio of consumption tax to output (T_C/Y)	0.090	0.09
Ratio of capital tax to output (T_K/Y)	0.040	0.04
Ratio of labor tax to output (T_L/Y)	0.020	0.02

features that there exists a large informal sector in developing countries. Furthermore, the level of human capital, the fraction of skilled workers as a percentage of total workers, is 17% in the benchmark economy. Since we assume that the labor mobility constraint on skilled workers is binding, only 70% of skilled workers can be in the formal sector.

Compared with unskilled workers, the wage gap between the formal and informal sectors for skilled workers is relatively large, about 2.91. The wage gap for unskilled workers is 1.70. This captures the feature in the data that skilled workers tend to obtain a lower wage when they work in the informal sector. In addition, the skill premium in the formal sector (2.396) is also larger than that in the informal sector (1.400). These results imply that skilled workers working in the informal sector more like to be non-voluntary. Once they have a chance, skilled workers will switch back to the formal sector.

Other government expenditure is solved so that the government maintains balanced budget every period. In the benchmark economy, the ratio of other government expenditure to output is about 10.3%. In the next section, we will keep this ratio and assume it is unchanged when the experiments are solved. Finally, based on WDI data, the ratio of total medical expenditure to output is 5.5%. The public medical care program for old adults covers 85% of total medical expenditure. Thus, the ratio of public medical expenditure to output is roughly equal to 4.7%.

Table 4: Calibrated Results for the Benchmark Economy

Description	Notation	Value
Life Expectancy and fertility		
Life expectancy		74.1*
Fertility of type *ss*	n_{ss}	0.834
Fertility of type *us*	n_{us}	0.804
Fertility of type *uu*	n_{uu}	1.832
Average (weighted) TFR		1.540*
Relative TFR (skilled/unskilled)		0.500*
Production		
Capital-output ratio	K/Y	1.900*
Relative output	Y^x/Y^f	0.520*
Allocation of workers		
Share of workers in the formal sector	$(L_s^f + L_u^f)/L$	0.368
Share of skilled workers in the formal sector	L_s^f/L_s	0.700**
Share of unskilled workers in the formal sector	L_u^f/L_u	0.300*
Share of skilled workers to total workers	L_s/L	0.170*
Relative wages		
Wage gap for skilled workers	w_s^f/w_s^x	2.910
Wage gap for unskilled workers	w_u^f/w_u^x	1.700*
Skill premium in the formal sector	w_s^f/w_u^f	2.396*
Skill premium in the informal sector	w_s^x/w_u^x	1.400*
Government		
Ratio of consumption tax to output	T_C/Y	0.090*
Ratio of capital tax to output	T_K/Y	0.040*
Ratio of labor tax to output	T_L/Y	0.020*
Ratio of other Gov't exp to output	G/Y	0.103
Medical expenditure		
Ratio of public medical exp to output	M_g/Y	0.047
Ratio of total medical exp to output	M/Y	0.055*

Note: "*" represents the moment is our calibration target. "**" denotes the moment is preset.

4 Policy Experiments

Based on the benchmark economy, we start with the scenario of population aging. Three financial tools, consumption tax, capital income tax and labor income tax, are considered to finance the increase in medical expenditure due to population aging. Then, based on the results of population aging, two population policies are discussed: a subsidy on child-rearing cost and a subsidy on education cost.

4.1 Impacts of Population Aging and Feasible Taxation Arrangement

4.1.1 Steady-State Comparison

To study the impacts of aging, we mainly consider changing two parameters together in the benchmark economy: increases in life expectancy and the ratio of total medical expenditure to GDP (M/Y). Other parameters remain unchanged. Two scenarios are discussed. First, the life expectancy increases from 74.1 to 80.1, as forecasted by WDI in 2055 and the ratio of total medical expenditure to GDP mildly increases from 5.5% to 6.3%.[11] Second, the life expectancy goes up to 80.1 with a rapid increase in the ratio of total medical expenditure to GDP, from 5.5% to a forecasted value 12%. Figure 5 plots the ratio of total medical expenditure to GDP in Thailand during 1995-2014. Based on the time series data, the ratio is forecasted by a simple linear regression. Then the forecasted value is 12%.[12]

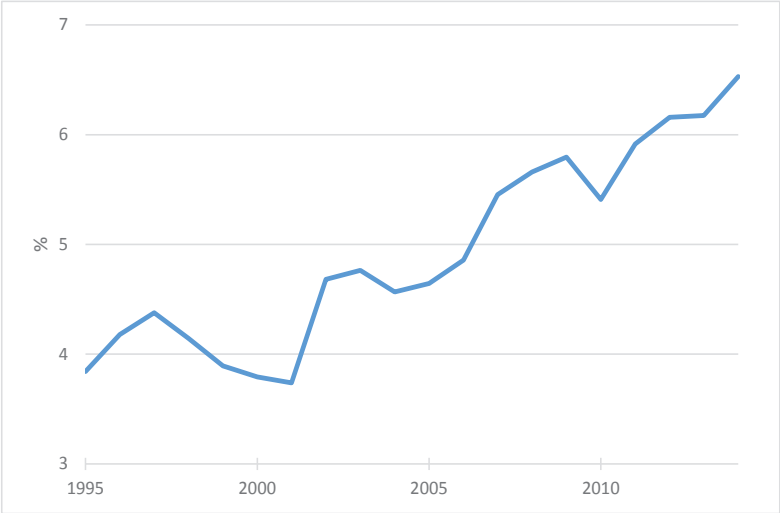
Each scenario is solved as a new steady state. Using the ratio of other government expenditure to output in the benchmark economy, we first use labor income tax as the financing tool to ensure balanced budget in the aging economy. Alternative tax tools, capital income tax and consumption tax, are also investigated for comparison.

Table 5 provides the simulated results for the first scenario: the life expectancy goes up to 80.1 and the M/Y ratio increases to 6.3%. The main effects of population aging are summarized as follows. First, population aging implies that people live longer and face higher medical expenditure. Therefore, individuals require more savings for their retirement life. As a result, the capital-output ratio goes up from 1.9 in the benchmark economy to

[11] When the life expectancy reached 80 in Japan of 1995, the ratio of total medical expenditure to GDP was 6.9%. However, if we consider 6.9% for the experiments, there is no equilibrium because the tax rate cannot sustain (financed by labor income tax and capital income tax). Therefore, we consider 6.3% in the first scenario.

[12] Thailand implemented an universal health insurance (UHI) system in 2001. When the UHI system is implemented, the pattern of medical expenditure could be different. To consider this, we also use the data during 2002-2014 to forecast the ratio. The forecasted ratio of total medical expenditure to GDP becomes 13%. Since the average life expectancy of OECD countries was 80 and the average M/Y ratio in OECD countries was 12.3% in 2014, we choose 12% to be the second scenario.

Figure 5: Ratio of Medical Expenditure to GDP in Thailand



Source: The Thai National Statistical office

around 2.2 in all cases. Second, increased savings crowd out children. The average TFR declines from 1.54 to around 1.47-1.50. Third, longer life and the higher requirement of savings make the education investment in children more valuable. Thus, an increase in human capital stock is observed. The share of skilled workers to total workers increases from 17% in the benchmark to 18.4%-20.1%. Using labor income tax as the financing tool will reduce the return of education because the majority of skilled workers is allocated in the formal sector. Therefore, compared with other financing tools, the share of skilled workers to total workers increases not much (to 18.4%) when population aging is financed by the labor income tax. Financing by capital income tax does not distort labor allocation directly, but it reduces the capital return in the formal sector. This would indirectly decrease the wage rates in the formal sector because of changes in capital-labor ratio. Financing by consumption tax results in a highest level of human capital stock since it does not distort the labor allocation between the two sectors. Fourth, the increases in both physical and human capital stocks enlarge the capacity of the formal sector. The labor employment in the formal sector increases from 36.8% to 42.5% under the labor tax financing scheme. It goes up to around 48.5% under the consumption tax financing scheme. Finally, the capacity of the formal sector increases, so the informal sector shrinks. The output ratio between informal and formal sectors becomes smaller, from 52% in the benchmark to 31% when

using consumption tax as the financing tool.

Table 5 also provides changes in welfare, measured by consumption equivalent variation (CEV). Compared with the case that financing by labor income tax, using the other two tax tools are both better off. This is because labor income tax is only collected from young adults who are working in the formal sector. This also distorts labor allocation. In contrast, capital income tax is collected from old adults whose assets are invested in the formal sector. The distortion on labor allocation for young adults is indirect and smaller. Thus, welfare is slightly improved. Since consumption tax is collected from all adults and there is no distortion on the labor market, the welfare improvement is the largest one.

Although we start with scenario 1, it is not likely to be the situation that Thailand will face in 2055 because Thailand's total medical expenditure to GDP ratio has gone up rapidly and reached over 6% since 2012. Scenario 2 is exactly what happened in OECD countries and will be more likely to occur in Thailand. Because the ratio of total medical expenditure to GDP goes up a lot, a significant increase in tax revenue is required in this scenario. Table 6 summarizes the results. Unfortunately, both labor income tax and capital income tax fail to sustain an equilibrium. Because labor and capital may switch to the informal sector to avoid high taxes, the marginal tax revenue by increasing one unit of tax rate will be negative when it reaches some critical point. As a result, no equilibrium is found. In scenario 2, consumption tax is the only feasible financing tool. It increase from 12% in the benchmark economy to 20.5%. Other impacts of population aging on the economy in scenario 2 are similar to that in scenario 1.

4.1.2 Transition Path for Aging

The effects of aging could be different between generations. Therefore, this subsection conducts a transition path for the second scenario discussed in the last subsection. The transition starts with the steady state of the benchmark economy. In period 0, the shock of aging occurs: the life expectancy jumps to age 80.1 and the ratio of total medical expenditure to GDP increases to 12% permanently. The extra cost caused by aging is financed by consumption tax. Others remain unchanged. Then, the transition path converges to a new steady state. Figure 6 plots the transition path for the skilled-worker share, the formal-worker share, and the relative output ratio (informal to formal sector). Table 7 reports the transition path two periods before and three periods after the shock of aging. Here we do not report the welfare improvement in comparison with labor income tax because the labor income tax is not sustainable when the total medical expenditure to GDP ratio is 12%.

The model is perfectly foresight. Therefore, individuals response to aging before the

Table 5: Financing Population Aging: Scenario 1

		Financing tools		
	Benchmark	Labor tax	Capital tax	Cons. tax
Life expectancy	74.1	80.1	80.1	80.1
Medical exp/GDP	5.5%	6.3%	6.3%	6.3%
Labor tax τ_L	10.9%	13.1%	10.9%	10.9%
Capital tax τ_K	11.4%	11.4%	12.6%	11.4%
Consumption tax τ_C	12.1%	12.1%	12.1%	12.2%
Average TFR	1.540	1.500	1.490	1.466
Skilled-worker share	17.0%	18.4%	19.1%	20.1%
Formal-worker share	36.8%	42.5%	44.2%	48.5%
Capital-output ratio	1.900	2.216	2.229	2.233
Y^x/Y^f ratio	52.0%	39.1%	37.3%	31.1%
Skill premium (w_s^f/w_u^f)	2.396	2.630	2.650	2.804
Δ welfare (skilled, CEV_s)	–	Baseline	2.9%	6.9%
Δ welfare (unskilled, CEV_u)	–	Baseline	3.1%	4.2%
Δ welfare (average, CEV)	–	Baseline	3.0%	4.6%

Table 6: Financing Population Aging: Scenario 2

		Financing tools	
	Benchmark	Labor/Capital taxes	Cons. tax
Life expectancy	74.1	80.1	80.1
Medical exp/GDP	5.5%	12%	12%
Labor tax τ_L	10.9%	–	10.9%
Capital tax τ_K	11.4%	–	11.4%
Consumption tax τ_C	12.1%	–	20.5%
Average TFR	1.54	**No equilibrium**	1.451
Skilled-worker share	17.0%	–	20.3%
Formal-worker share	36.8%	–	49.4%
Capital-output ratio	1.900	–	2.256
Y^x/Y^f ratio	52.0%	–	29.9%
Skill premium (w_s^f/w_u^f)	2.396	–	2.832

Table 7: Transition Path for Aging

Transition Period	-2	-1	0	1	2	3
Life expectancy	74.1	74.1	80.1	80.1	80.1	80.1
Medical exp/GDP	5.5%	5.5%	12%	12%	12%	12%
Consumption tax τ_C	12.1%	12.1%	22.0%	21.0%	21.2%	20.8%
Average TFR	1.531	1.551	1.606	1.578	1.528	1.509
Skilled-worker share	17.0%	17.0%	16.8%	20.5%	19.0%	20.2%
Formal-worker share	36.9%	36.9%	36.6%	45.5%	43.6%	47.0%
Capital-output ratio	1.900	1.904	1.912	1.975	2.078	2.118
Y^x/Y^f ratio	51.9%	51.7%	52.3%	36.2%	38.4%	33.5%
Skill premium (w_s^f/w_u^f)	2.396	2.402	2.411	2.493	2.615	2.665

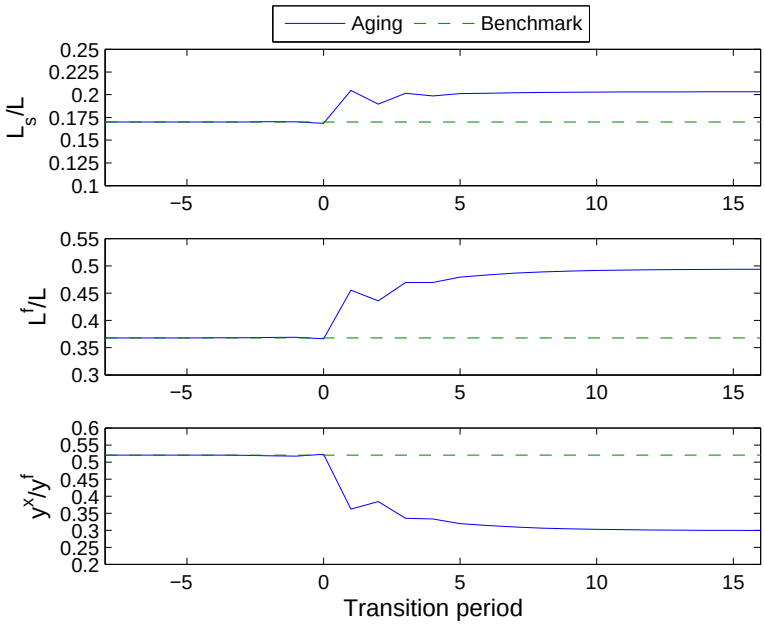
Note: The shock of aging occurs at period 0 permanently. The extra cost caused by aging is financed by consumption tax. This table does not report the welfare improvement in comparison with labor income tax because the labor income tax is not sustainable when the total medical expenditure to GDP ratio is 12%.

period that the shock of aging occurs. As Table 7 shows, the young generation living at period -1 responses to aging by increasing their savings. Thus, the capital-output ratio at period 0 goes up. Besides, this generation also chooses to have slightly more children in response to aging. However, it is interesting to note that the young generation living at period -1 does not response to aging by investing in children's education, as the skilled-worker share slightly declines at period 0.

In addition to higher savings and more children, the young generation living at period 0 responses to aging by investing in children's education. Therefore, the share of skilled workers increases to 20.5% in period 1. Although this generation enjoys the benefit of a higher capital-output ratio contributed by their parents, they do not benefit from the accumulation of human capital and the expansion of the formal sector. Thus, they face a higher consumption tax rate (22.0%).

Since period 1, young generations enjoy the benefits of higher savings and investment in human capital. A higher skilled-worker share results in an expansion in the formal sector and a shrinkage of the informal sector. Therefore, the consumption tax rate can be lower and the tax burden caused by aging is released.

Figure 6: Transition Path for Aging



4.2 Population Policies – Improving Labor Quantity or Quality?

Based on the second scenario that using the consumption tax as the financing tool, we further explore if additional population policies are able to improve the welfare in an aging society. Two population policies are considered. First, we aim at an increase in TFR to represent an improvement on population quantity. Alternatives, education subsidy is discussed to provide an experiment on improving the quality of population.

4.2.1 Improving Labor Quantity

We first study a subsidy on the child-rearing cost to encourage fertility. The results are presented in Table 8. The baseline is the scenario with population aging and using consumption tax as a financing tool, which is reported in Table 6. In the baseline, life expectancy is 80.1 and the ratio of total medical expenditure to GDP is 12%. The TFR in the baseline is 1.54. Based on the baseline, we conduct experiments with the assumption that the TFR increases to a higher level, from 1.6 to 2.5. The government provides subsidies on child-rearing cost to encourage fertility to the target level. Others remain unchanged and are fixed at the baseline level.

The results suggest that the encouragement to TFR with an subsidy on child-rearing cost indeed increases the amount of labor force and improves the age structure. The old-young ratio sharply declines as the TFR goes up. However, the policy lowers the level of human capital stock in the economy. It is because, first, the subsidy on child-rearing cost makes education investment relatively more expensive. Second, an subsidy on child-rearing cost mainly encourages the fertility of unskilled parents having unskilled children. Therefore, the share of skilled workers to total workers declines from 20.3% to 13.2%. Furthermore, the increase in fertility also crowds out the accumulation of physical capital. The capital-output ratio declines from 2.256 to 1.846 as TFR increases. The share of workers in the informal sector goes up accordingly. Therefore, the relative output (informal to formal) increases to 78%.

In summary, under the policy of aiming at TFR, although the amount of labor force and age structure are improved, the policy eventually hurts the economy. Because the subsidy requires more tax revenue, the government has to raise the consumption tax to a higher level. Then education and physical investment are both crowded out. The formal sector is shrinking and the government collects fewer tax revenues from the formal sector. Thus, the policy does not help to alleviate the tax burden caused by population aging. Everyone in the economy is worse off with the implementation of the policy.

The literature has discussed the effect of family policies on encouraging fertility. Governmental family policies usually aim at two categories: reducing the direct cost of children and reducing the opportunity cost of employed women for child-rearing. The former policy refers to cash transfer, such as family allowance and tax credit. Some empirical findings show that transfers to families with children have significantly positive effects on fertility.[13] In contrast, Gauthier and Hatzius (1997) suggest that cash benefits are positively related to fertility but the effect is of a limited magnitude. Kalwij (2010) finds that family allowance has no significant effect on birth probabilities. He further suggests that the positive relationship in previous empirical studies may be spurious because of the lack of appropriate controls. The latter family policy attempts to lower the opportunity cost of women who are withdrawn from the labor market for child-bearing, such as benefits of maternity and parental leave and childcare services. Empirical conclusions on the effect of lower employed women's opportunity cost are controversial. For example, Zhang et al. (1994) find insignificant effects of maternity leave for Canada. Castles (2003) suggests that fertility is positively related to formal childcare provisions but negatively associated with publicly

[13] See, for example, Ermisch (1988), Whittington (1992), Whittington, Alm and Peters (1990), and Zhang, Quan and van Meerbergen (1994).

funded childcare provisions. Kalwij (2010) shows that the benefits of maternity and parental leave (childcare services) have a significantly positive effect on the probability of first birth (subsequent births), but not significant effect on the probability of subsequent births (first birth).

Our results in Table 8 suggest that the subsidy on child-rearing should be roughly equal to 15% of child-rearing cost in order to increase the total fertility rate to the replacement level. Kalwij (2010) provides the monetary benefits of family allowance, maternity and parental leave, and childcare services for 16 western European countries of 2003. This allows us to evaluate the child-rearing subsidy in our experiments. Using GDP per capita as an approximate of individual wage income and considering the model period, the current subsidies of family allowance for the 16 western European countries are relatively small, around 3.1% of child-rearing cost on average. In contrast, the subsidies of maternity and parental leave and childcare services are relatively larger, roughly equal to 5.9% of child-rearing cost. In sum, the average total benefits of family policies are about 9% of child-rearing cost.[14] The unweighed total fertility rate of the 16 western European countries in 2003 was 1.6. The above information suggests that our experiments on child-rearing subsidy are reasonable. To reach the replacement rate, the child-rearing subsidy should be much larger.

4.2.2 Improving Labor Quality

The alternative policy is that the government provides subsidies on education cost of children. Based on the baseline, experiments with 10% to 30% subsidy of education cost are conducted. Each experiment is solved as a steady state. The educational subsidy here refers to the subsidy on top of the current educational subsidy in Thailand. Other parameters are fixed at the baseline level. The results are summarized in Table 9.

In contrast to the subsidy on child-rearing cost, the subsidy on education cost does not improve the age structure of the economy. The old-young ratio increases as the subsidy on education cost increases. However, due to the educational subsidy, education investment becomes relatively cheaper. More unskilled parents start to invest education on their children. Therefore, the share of skilled workers to total workers goes up from 20.3% in

[14]The percentage of subsidy for each country in each category is computed as follows. First, we assume that the subsidy for family allowance is up to 12 years old, the subsidy for maternity and parental leave is a one-time benefit, and the childcare service is up to 3 years old. Besides, we double GDP per capita to be the annual family income. Then the monetary benefits are divided by 24.3% of annual family income and the model period (30 years) is adjusted.

Table 8: Improving Labor Quantity (TFR) – Subsidy on Child-rearing Cost

	Baseline	Policy of child-rearing subsidy			
Subsidy	0%	5%	10%	15%	20%
TFR	1.5	1.6	1.9	2.1	2.5
Old-young ratio	92.3%	81.5%	71.5%	62.4%	54.2%
Consumption tax	20.5%	27.5%	36.9%	50.3%	70.0%
Capital-output ratio	2.256	2.144	2.037	1.938	1.846
Skilled-worker share	20.3%	18.5%	16.6%	14.8%	13.2%
Formal-worker share	49.4%	43.3%	37.6%	32.5%	28.0%
Y^x/Y^f ratio	29.9%	38.5%	49.1%	62.3%	78.1%
Δ welfare (skilled, CEV_s)	–	-15.1%	-28.9%	-41.5%	-53.0%
Δ welfare (unskilled, CEV_u)	–	-13.5%	-25.9%	-37.5%	-48.4%
Δ welfare (average, CEV)	–	-13.8%	-26.5%	-38.3%	-49.3%

Table 9: Improving Labor Quality – Subsidy on Education Cost

	Baseline	Policy of education subsidy				
Subsidy	0%	10%	20%	25%	26%	30%
TFR	1.45	1.42	1.39	1.38	1.38	1.37
Old-young ratio	92.3%	94.2%	96.2%	97.2%	97.3%	97.5%
Consumption tax	20.5%	17.8%	15.1%	13.8%	13.8%	14.0%
Capital-output ratio	2.256	2.249	2.241	2.237	2.236	2.226
Skilled-worker share	20.3%	23.3%	26.8%	28.7%	29.1%	30.2%
Formal-worker share	49.4%	56.6%	64.9%	69.6%	70.0%	70.0%
Y^x/Y^f ratio	29.9%	22.8%	16.5%	13.6%	13.3%	13.3%
Δ welfare (skilled, CEV_s)	–	2.1%	4.2%	5.3%	4.8%	1.2%
Δ welfare (unskilled, CEV_u)	–	5.7%	12.6%	16.5%	16.7%	15.8%
Δ welfare (average, CEV)	–	5.0%	11.0%	14.4%	14.5%	13.1%

Figure 7: Tax Burden and Policy of Education Subsidy

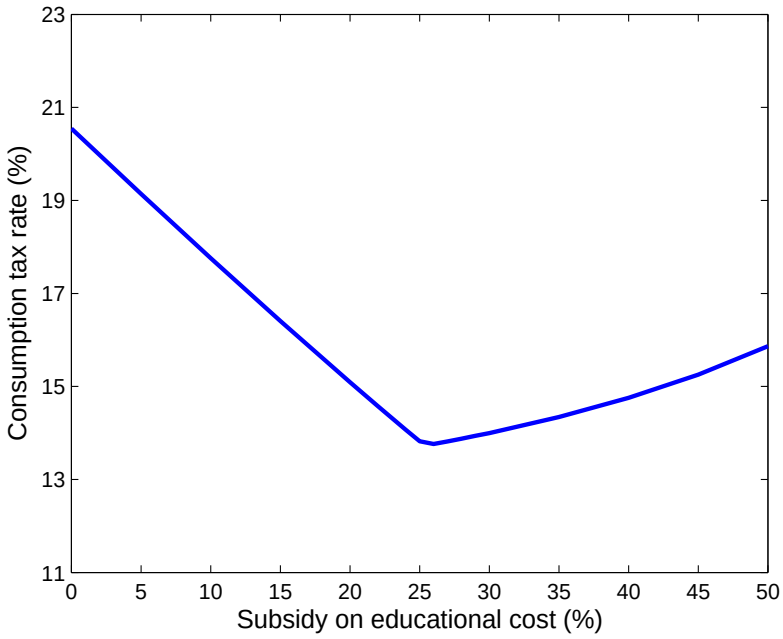
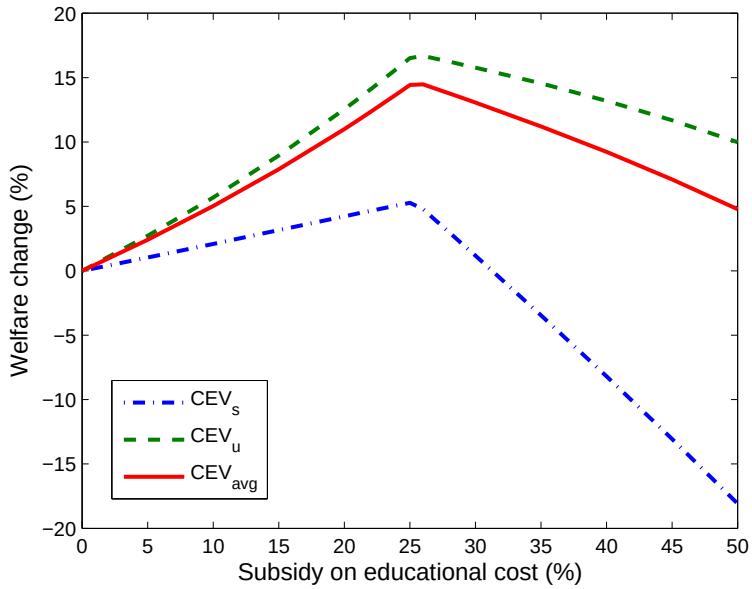


Figure 8: Welfare Changes and Policy of Education Subsidy



the baseline to 30.2% in the experiment with 30% subsidy. Furthermore, parents adjust the number of children they want in response to the educational subsidy (quantity-quality trade-off), so the crowding out effect on physical capital is not large. Capital-output ratio slightly declines from 2.256 to 2.226. With more skilled workers, the labor employment in the formal sector goes up, from 49.4% to 70%.[15] The relative output (informal to formal) becomes lower.

Because of the subsidy on education cost, the scale of the formal sector becomes larger. It becomes easier for the government to collect tax revenues from the formal sector. Therefore, even in the experiment with the subsidy of 30%, the consumption tax rate (14%) is still lower than the baseline (20.5%). To conclude, we find that the policy of education subsidy can help to alleviate the tax burden caused by population aging.

The subsidy itself requires an additional tax revenue to be collected. We find that there exists a critical level. Beyond the critical level, it might not be worth to provide a higher level of subsidy. Figure 7 shows a V-shape relationship between level of education subsidy and the consumption tax rate. The lowest tax rate is about 13.8% with an education subsidy

[15]The formal-worker share stops at 70% because we set both $\bar{\theta}_s$ and $\bar{\theta}_u$ at 70%.

Table 10: Transition Path for 25% Education Subsidy

Transition Period	-2	-1	0	1	2	3
Life expectancy	80.1	80.1	80.1	80.1	80.1	80.1
Medical exp/GDP	12%	12%	12%	12%	12%	12%
Consumption tax τ_c	20.6%	20.6%	22.2%	16.6%	16.5%	15.5%
Average TFR	1.454	1.477	1.430	1.431	1.416	1.406
Skilled-worker share	20.3%	20.3%	19.2%	31.2%	27.7%	28.7%
Formal-worker share	49.4%	49.3%	46.9%	70.0%	65.8%	68.2%
Capital-output ratio	2.256	2.258	2.262	2.108	2.170	2.177
Y^x/Y^f ratio	29.9%	30.0%	33.0%	13.6%	16.0%	14.5%
Skill premium (w_s^f/w_u^f)	2.832	2.834	2.837	2.584	2.738	2.747
Δ welfare (skilled, CEV_s)	0.0%	0.1%	0.9%	-11.8%	-3.6%	-2.2%
Δ welfare (unskilled, CEV_u)	0.0%	0.1%	1.1%	5.0%	7.5%	9.9%
Δ welfare (average, CEV)	0.0%	0.1%	1.1%	1.9%	5.5%	7.7%

Note: The shock of 25% education subsidy occurs at period 0 permanently. The extra cost due to education subsidy is financed by consumption tax. The welfare changes are compared with the baseline reported in Table 9.

of 26%. Beyond the level of 26% subsidy, the consumption tax rate goes up again. In terms of welfare changes, Figure 8 presents the relationship between the level of education subsidy and welfare changes for skilled workers (CEV_s), unskilled workers (CEV_u) and the whole economy (social average, CEV_{avg}). The figure suggests that the welfare improvement reaches the highest point with the 26% subsidy. The trade off determines that the optimal subsidy on education could be at the 26% subsidy in our experiments.[16]

To study the effects of education subsidy on different generations, a transition path with 25% education subsidy is conducted. The transition path starts with the steady state with aging and 0% education subsidy (the baseline in Table 9). In period 0, 25% education subsidy is permanently implemented. The extra cost due to the subsidy is financed by

[16]Abbott, Gallipoli, Meghir and Violante (2016) construct a general equilibrium life-cycle model with heterogenous agents to study current education policy in the United State. They show that the current education financial aid system in the US is welfare improving. In their framework, college tuition is funded by parental transfers, government grants and loans, private loans and labor supply during college. They further find that expanding current financial aid on education has limited influence because of the crowding-out effect between various means of funding college education. In our framework, only education subsidies and education investment made by parents are available. Therefore, additional education subsidies directly lower parents' education investment.

Figure 9: Transition Path for 25% Education Subsidy

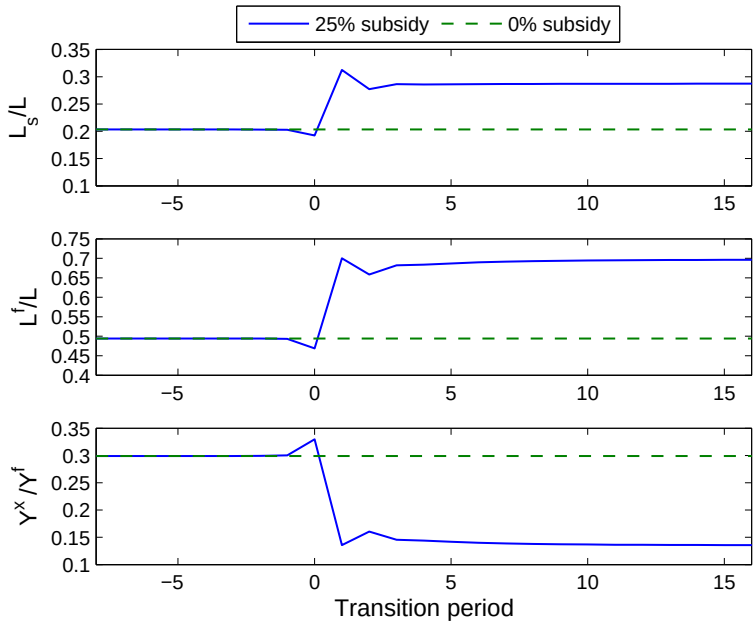
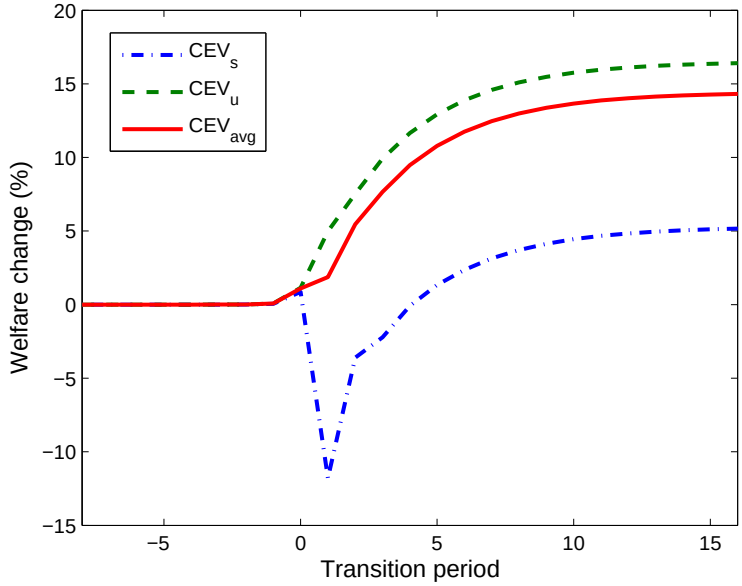


Figure 10: Welfare Changes for 25% Education Subsidy



consumption tax. Others remain unchanged. The transition path then converges to the steady state reported in Table 9. Table 10 summarizes the transition path two periods before and three periods after the education subsidy. Figure 9 plots the path for the skilled-worker share, the formal-worker share and the informal to formal output ratio. Welfare changes are provided in Figure 10.

The education subsidy changes the relative price of skilled to unskilled children. Therefore, more parents provide education to their children at period 0. This partially crowds out the number of children, thus the total fertility rate at period 0 declines. At period 1, those children with education investment become skilled workers; thereby increasing the share of skilled worker to 31%, increasing the share of formal worker to the limit, 70%, and depressing skill premium. As Figure 10 shows, because of the lower skill premium at period 1, skilled young workers living at period 1 are actually hurt by the education subsidy, while unskilled young workers are better off. Then parents living at period 1 and thereafter adjust their resource allocation between quantity and quality of children in response to the lower skill premium. Therefore, skill premium gradually goes up and the welfare of skilled young workers are improved. At the new steady state (25% education subsidy in Table 9), everyone enjoys the benefit of education subsidy.

We find that the education subsidy has different welfare effects across generations and skill groups. All unskilled young workers enjoy welfare improvement, no matter which generation they are. In contrast, skilled young workers living in the first few periods of introducing the education subsidy are actually hurt. This is mainly due to the redistribution effect. Eventually, their offsprings are better off.

5 Discussions

5.1 Old-age Workers

Population aging leads to higher savings and the education investment becomes more valuable. The effects on savings and education investment could be crowded out if old adults do not retire and still work to obtain wage income, especially working in the informal sector. The magnitude depends on how many old adults stay in the labor market and how much they earns. If the proportion of old adults staying in the labor market is not large and their wage incomes are relatively lower, the crowding-out effect would not be large.

Fujioka and Thangphet (2009) report that the labor force participation rate for old adults (defined as age 60 and above in the report) in Thailand was 33.6% in 2000 and 38.8% in 2005. There exists a significant difference between urban and rural areas. In 2005 (2000),

the labor force participation rate in urban areas was 28.2% (22.6%), while it was 42.9% (36.2%) in rural area. This is partly due to the legal retirement age is 60 years for the public sector but there is no legal retirement age in the private sector in Thailand.[17] In addition, the share of informal employment in total employment for old adults in 2005 was 90.3%. In other words, most old workers are in the informal sector. The above information suggests that those who still work in the old adulthood tend to be low-income workers when they were young.[18]

In addition, Fujioka and Thangphet (2009) report the wage of all employed persons and the income of older persons. In 2005, the income of older persons is about 65% of the wage of all employed persons. The report also indicates that data on all age groups are the average of all the employed persons, whereas the data for older persons are the average of government, state enterprise and private employees. Therefore, it is possible that the relative income of older persons to all employed persons could be even lower than 65%. Therefore, allowing old-age working in our framework might lower the effects of aging on savings and education investment but the crowding-out effect is limited.

5.2 Formal and Informal Consumption

In the theoretical framework, we assume that every unit of consumption is taxed. We do not differentiate consumption in the formal sector from that in the informal sector. First of all, in reality, consumption in the informal sector is not exactly with 0% tax rate. The raw materials and intermediate goods used to produce final consumption goods in the informal sector are actually taxed when they are imported. To capture this, in our calibration, the consumption tax refers to not only sales taxes but also VAT and duties. Second, because consumption in the informal sector is actually still taxed, the distortion in the scenario with aging and the extra cost financed by consumption tax will not be as large as that of labor income tax or capital income tax. Third, the tax burden of consumption tax is shared by both young and old adults, while that of labor income tax is only taken by formal young workers and that of capital income tax is only taken old agents having assets in the formal

[17] Although there is no legal retirement age in the private sector of Thailand, it is typically specified in the employment contracts.

[18] Fujioka and Thangphet (2009) also indicates that, in 2005, the majority of employment status for old workers are own account workers (62.4%) and unpaid family workers (17.8%). According to the report, "an own account worker refers to a person operating an enterprise on his/her own or jointly with others in the form of a partnership for profit or dividends but without engaging any employees. An unpaid family worker is a person who works without pay on a farm or in a business enterprise owned or operated by the household head or any other family member".

sector. Therefore, differentiating informal consumption from formal consumption would not change our main results.

5.3 The Role of the Informal Sector

This paper includes an informal sector to capture the existence of a large informal employment in many developing countries. To examine the role of the informal sector, we consider an alternative framework with the formal sector only and re-do the quantitative analysis. First, given all parameters unchanged, the benchmark economy with the formal sector only is solved as a steady state. Second, based on the benchmark economy with the formal sector only, a scenario with population aging (life expectancy to 80.1 and total medical expenditure to GDP ratio to 12%) is computed to be the baseline. The extra cost due to aging is financed by consumption tax. Third, we also consider the scenario with aging and 25% education subsidy in the framework with the formal sector only. Finally, the percentage changes relative to the baseline are computed in comparison with the economy with two sectors. The results are summarized in Table 11.

In the scenario with education subsidy, the relative price of skilled to unskilled children is lower. Parents are more willing to provide education to their children. Thus, skill premium decreases. We find that the magnitude of the decline in skill premium in the model with two sectors is smaller than that in the framework with the formal sector only. This is because the informal sector absorbs skilled workers. Then, the share of skilled workers greatly increases (41.3%) and the total fertility rate is partially crowded out (-5.0%). In contrast, in the framework with the formal sector only, the total fertility rate goes up (1.9%).

With a lower total fertility rate, per capita output in the model with two sectors increases by 22.9%, while it only slightly increases by 2.1% in the framework with the formal sector only. Furthermore, the higher fertility in the framework with the formal sector only does not alleviate the tax burden. The result suggests that consumption tax can be lowered to 13.8% in the model with two sectors, but it is as high as 21.4% in the framework with the formal sector only. In terms of welfare changes, we find that, in the framework with the formal sector only, skilled workers are hurt by the education subsidy. This is mainly due to the large decline in skill premium. We conclude that the results could be misleading if the informal sector is not included in the framework when we study developing countries with large informal employment.

Table 11: The Role of the Informal Sector

	Aging		Aging with education subsidy	
	Two sectors	Formal only	Two sectors	Formal only
Life expectancy	80.1	80.1	80.1	80.1
Medical exp/GDP	12%	12%	12%	12%
Education subsidy	0%	0%	25%	25%
Consumption tax τ_c	20.5%	21.9%	13.8%	21.4%
			(-32.7%)	(-2.5%)
Average TFR	1.451	1.419	1.379	1.446
			(-5.0%)	(1.9%)
Skilled-worker share	20.3%	22.9%	28.7%	26.6%
			(41.3%)	(16.3%)
Capital-output ratio	2.256	2.358	2.237	2.338
			(-0.8%)	(-0.9%)
Output per capita	0.521	0.854	0.641	0.872
			(22.9%)	(2.1%)
Skill premium	2.832	2.755	2.820	2.253
			(-0.4%)	(-18.2%)
Δ welfare (skilled, CEV_s)	baseline	baseline	5.3%	-10.8%
Δ welfare (unskilled, CEV_u)	baseline	baseline	16.5%	8.3%
Δ welfare (average, CEV)	baseline	baseline	14.4%	4.3%

Note: Percentage changes relative to the baseline are reported in the parentheses. Output per capita in the model with two sectors refers to the sum of output in the formal and informal sectors and is divided by total population.

6 Conclusions

This paper explores the feasible taxation arrangement and population policies in a developing economy with a trend of population aging. The facts of low fertility and rapid aging in many middle-income developing economies are taken into consideration. Furthermore, a distinct feature in developing economies is included: the existence of a large informal sector. A three-period overlapping generations model is constructed to capture the above features.

The quantitative analysis suggests that the existence of a large informal sector constraints the ability of labor income tax and capital income tax to finance the extra cost caused by aging in developing countries in the future. This is mainly because factor inputs may switch from the formal to the informal sector if the tax rate is high. Consumption tax is less avoidable and brings less distortion on resource allocation. Thus, in terms of welfare, consumption tax is a better tool. We also find that population aging makes education investment more attractive and thus leads to a higher level of human capital stock. The capacity of the formal sector also increases and the informal sector shrinks. Two alternative population policies are also discussed. The findings show that the policy that subsidizing on educational cost improves the level of human capital and enlarges the formal sector. It could alleviate the tax burden.

Population aging is a global trend and has drawn economists' attention. However, the market structures in developing countries are different from those in developed countries in many perspectives. Thus, the features of developing countries should be considered when we discuss the issue of aging. This paper incorporates one of the features: a large informal sector. Other features, such as immature financial markets and high income inequality in the society, are left for future research.

References

Abbott, B., Gallipoli, G., Meghir, C., and Violante, G.L. (2016). Education Policy and Intergenerational Transfers in Equilibrium. *Cowles Foundation Discussion Paper* No. 1887R.

Becker, G.S. (1960). An Economic Analysis of Fertility. in Demographic and Economic Change in Developed Countries, NBER Conference Volume, New Jersey: Princeton University Press.

Becker, G.S., Murphy, K.M., and Tamura, R. (1990). Human Capital, Fertility, and Economic Growth. *Journal of Political Economy*, 98(5), 12-37.

Correia, I. (2010). Consumption Taxes and Redistribution. *American Economic Review*, 100(4), 1673-1694.

Doepke, M. (2004). Accounting for Fertility Decline during the Transition to Growth. *Journal of Economic Growth*, 9(3), 347-83.

Doepke, M. and Zilibotti, F. (2005). The Macroeconomics of Child Labor Regulation. *American Economic Review*, 95(5), 1492-1527.

Ermisch, J. (1988). Econometric Analysis of Birth Rate Dynamics in Britain. *Journal of Human Resources*, 23, 563-576.

Fujioka, R. and Thangphet, S. (2009). Decent Work for Older Persons in Thailand. *ILO Asia-Pacific Working Paper Series*, ILO Regional Office for Asian and the Pacific.

Galor, O. and Weil, D.N. (1996). The Gender Gap, Fertility, and Growth. *American Economic Review*, 86(3), 374-87.

Gauthier, A.H. and Hatzius, J. (1997). Family Benefits and Fertility: An Econometric Analysis. *Population Studies*, 51(3), 295-306.

Gibson, B. (2005). The Transition to a Globalized Economy: Poverty, Human Capital and the Informal Sector in a Structuralist CGE Model. *Journal of Development Economics*, 78, 60-94.

Hsu, M., Huang, X. and Yupho, S. (2015). The Development of Universal Health Insurance Coverage in Thailand: Challenges of Population Aging and Informal Economy. *Social Science and Medicine*, 145, 227-236.

Jung, J. and Tran, C. (2012). The Extension of Social Security Coverage in Developing Countries. *Journal of Development Economics*, 22(2), 439-458.

Kalwij, A. (2010). The Impact of Family Policy Expenditure on Fertility in Western Europe. *Demography*, 47(2), 503-519.

Lee, R., A. Mason and members of the NTA Network. (2014). Is Low Fertility Really a Problem? Population Aging, Dependency, and Consumption. *Science*, 346, 229-234.

Liao, P. (2011), Does Demographic Change Matter for Growth?. *European Economic Review*, 55(5), 659-677.

Marcouiller, D., de Castilla, V.R., and Woodruff, C. (1997). Formal Measures of the Informal-sector wage gap in Mexico, EI Salvador, and Peru. *Economic Development and Cultural Change*, 45(2), 367-392.

Penalosaa and Turnovsky. (2005). Second-Best Optimal Taxation of Capital and Labor in a Developing Economy. *Journal of Public Economics*, 89, 1045-1074.

Ranis, G. and Stewart, F. (1999). V-goods and the Role of the Urban Informal Sector in Development. *Economic Development and Cultural Change*, 47(2), 259-288.

United Nations, Department of Economic and Social Affairs, Population Division. (2013). *World Population Ageing 2013*. ST/ESA/SER.A/348.

Whittington, L.A. (1992). Taxes and the Family: The Impact of the Tax Exemption for Dependents on Marital Fertility. *Demography*, 29, 215-226.

Whittington, L.A., Alm, J., and Peters, H.E. (1990). The Personal Exemption and Fertility in the United States. *American Economic Review*, 80, 545-556.

Yuki, K. (2007). Urbanization, Informal Sector, and Development. *Journal of Development Economics*, 84, 76-103.

Zhang, J., Quan, J. and van Meerbergen, P. (1994). The Effect of Tax-Transfer Policies on Fertility in Canada. *Journal of Human Resources*, 29, 181-201.

Appendix A: Proof for The Indifference Condition

A young adult's maximization problem is given by:

$$\max \quad \left\{ \frac{c_i^{y\,1-\sigma}}{1-\sigma} + \beta \pi_y \frac{c_i^{o'\,1-\sigma}}{1-\sigma} + \psi \left(\frac{E_i}{P_{ij}} \right)^{1-\varepsilon} V_j \right\}$$

subject to

$$\begin{aligned} (1+\tau_C)c_i^y &= [\theta_i(1-\tau_L)w_i^f + (1-\theta_i)w_i^x] - E_i - \pi^y[a_i^{f'} + a_i^{x'}] \\ (1+\tau_C)c_i^{o'} &= [1+(1-\tau_K)r^{f'}]a_i^{f'} + (1+r^{x'})a_i^{x'} - (1-\omega)m' \end{aligned}$$

where $E_i = P_{ij}n_{ij}$.

A young adult is indifferent between having skilled or unskilled children if the following condition holds:

$$\psi \left(\frac{E_i}{P_{is}} \right)^{1-\varepsilon} V_s = \psi \left(\frac{E_i}{P_{iu}} \right)^{1-\varepsilon} V_u.$$

Rewrite to obtain the following equation:

$$\frac{V_s}{V_u} = \left(\frac{P_{is}}{P_{iu}} \right)^{1-\varepsilon} \tag{19}$$

where

$$\begin{aligned} P_{is} &= \phi[\theta_i(1-\tau_L)w_i^f + (1-\theta_i)w_i^x] + \bar{e} \\ P_{iu} &= \phi[\theta_i(1-\tau_L)w_i^f + (1-\theta_i)w_i^x]. \end{aligned}$$

On the right-hand side, the relative price for a skilled young adult is given by:

$$\frac{P_{ss}}{P_{su}} = \frac{\phi[\theta_s(1-\tau_L)w_s^f + (1-\theta_s)w_s^x] + \bar{e}}{\phi[\theta_s(1-\tau_L)w_s^f + (1-\theta_s)w_s^x]} = 1 + \frac{\bar{e}}{\phi[\theta_s(1-\tau_L)w_s^f + (1-\theta_s)w_s^x]};$$

the relative price for an unskilled young adult is given by:

$$\frac{P_{us}}{P_{uu}} = \frac{\phi[\theta_u(1-\tau_L)w_u^f + (1-\theta_u)w_u^x] + \bar{e}}{\phi[\theta_u(1-\tau_L)w_u^f + (1-\theta_u)w_u^x]} = 1 + \frac{\bar{e}}{\phi[\theta_u(1-\tau_L)w_u^f + (1-\theta_u)w_u^x]}.$$

The condition $\frac{P_{ss}}{P_{su}} < \frac{P_{us}}{P_{uu}}$ holds if and only if

$$\theta_s(1-\tau_L)w_s^f + (1-\theta_s)w_s^x > \theta_u(1-\tau_L)w_u^f + (1-\theta_u)w_u^x. \tag{20}$$

Note that,

$$\begin{aligned} & \theta_s(1-\tau_L)w_s^f + (1-\theta_s)w_s^x \\ > \quad & \theta_s(1-\tau_L)w_u^f + (1-\theta_s)w_u^x \\ > \quad & \theta_u(1-\tau_L)w_u^f + (1-\theta_u)w_u^x. \end{aligned}$$

With the assumptions that $w_s^f > w_u^f$, $(1-\tau_L)w_u^f > w_u^x$ and $w_s^x > w_u^x$, the last inequality holds if $\theta_s > \theta_u$. Therefore, if $\theta_s > \theta_u$, equation (20) holds. A skilled child is relatively cheaper for skilled parents than for unskilled parents, $\frac{P_{ss}}{P_{su}} < \frac{P_{us}}{P_{uu}}$. Only one type of parents will be indifferent between having skilled or unskilled children.