Portfolio Theory

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Stochastic Dominance

- Suppose that there are two risky securities. Under what conditions can we unambiguously say that an individual prefers one risky asset to another with certain information?
- To answer this question, we introduce two concepts of stochastic dominance.
  - First-order stochastic dominance.
  - Second-order stochastic dominance.

First-Order Stochastic Dominance

- If all individuals having utility functions in wealth that are increasing and continuous either prefer A to B or are indifferent between A and B, we say that risky asset A dominates risky asset B in the sense of first-order stochastic dominance, denoted by $A \underset{FSD}{\geq} B$.
- The necessary and sufficient condition for $A \underset{FSD}{\geq} B$: if the probability of asset A's rate of return exceeding any given level is not smaller than that of asset B's rate of return exceeding the same level, then any nonsatiable individual (utility function is strictly increasing) prefers A to B.

First-Order Stochastic Dominance

- In mathematics, the above necessary and sufficient condition is:

$$F_A(z) \leqq F_B(z), \forall y \in [0, 1].$$

- Assume that the rates of return on risky assets A and B lie in the $[0, 1]$ interval. Since $F_A(.)$ and $F_B(.)$ are CDFs, they are continuous from the right and $F_A(1) = F_B(1) = 1$. However, $F_A(0)$ may not be equal to $F_B(0)$ since $F_A(.)$ and $F_B(.)$ can have different masses at zero.

Second-Order Stochastic Dominance

- Suppose that the only information we have about an individual is that he/she is risk averse. We would like to know the conditions under which we can unambiguously say that he/she prefers risky asset A to risky asset B.
- We will say that asset A dominates asset B in the sense of second-order stochastic dominance, denoted by $A \underset{SSD}{\geq} B$, if and only if:

$$E[r_A] = E[r_B];$$

$$S(y) = \int_0^y [F_A(z) - F_B(z)]dz \leq 0, \; \forall y \in [0, 1].$$

The Mean-Variance Framework

- We would like to characterize those portfolios which have the minimum variance for various levels of expected rate of return.
- The motivation to do so is from the second-order stochastic dominance. When there are more than two assets and when portfolios can be formed without restrictions, if there exists a portfolio of assets that second-order stochastically dominates all the portfolios which have the same expected rate of return as it has, then this dominant portfolio must have the minimum variance among all the portfolios.

The Mean-Variance Framework

- The mean-variance model of asset choice has been used extensively in finance since its development by Markowitz (1952, Journal of Finance).
- A preference for expected return and an aversion to variance is implied by monotonicity and strict concavity of an individual's utility function.
- However, for arbitrary distributions and utility functions, expected utility cannot be defined over just the expected returns and variances. Nevertheless, the mean-variance model of asset choice is popular because of its analytical tractability of its rich empirical implications.

The Mean-Variance Framework

- Let's take a Taylor series around the expected end of period wealth of an individual's utility function. We have:

$$u(W) = u(E[W]) + u'(E[W])(W - E[W]) + \frac{1}{2}u''(E[W])(W - E[W])^2 + R_3.$$

- The expectation of the above equation would be:

$$E[u(W)] = u(E[W]) + \frac{1}{2}u''(E[W])\sigma(W)^2 + E[R_3].$$

- The above relationship indicates a preference for expected wealth and an aversion of variance of wealth based on an increasing and strictly concave utility function. However, remember we still have R_3.

How to Justify Mean-Variance Model?

- For arbitrary distributions, the mean-variance model can be motivated by assuming quadratic utility. Under quadratic utility, the third and higher order derivatives are zero; therefore, $E[R_3] = 0$ for arbitrary distributions.
- When expected rates of return and variances are finite, quadratic utility is sufficient for asset choice to be completely described in terms of mean and variance of expected returns.
- Unfortunately, quadratic utility displays the undesirable properties of satiation and increasing absolute risk aversion. Thus, economic conclusions are often counter intuitive.

How to Justify Mean-Variance Model?

- For arbitrary preferences, the mean-variance model can be motivated by assuming that rates of return on risky assets are multivariate normally distributed. The normal distribution is completely described by its mean and variance.
- Under normality, the third and higher order moments involved in $E[R_3]$ can be expressed as functions of the first two.
- Unfortunately, the normal distribution is unbounded from below, which is inconsistent with limited liability and with economic theory.

Mathematics of Portfolio Selection

- We would like to think about how an investor makes his/her optimal portfolio decision. Major assumptions are as follows.
  - There are $N \geqq 2$ risky assets traded in a frictionless economy where unlimited short selling is allowed.
  - The rates of return on these assets have finite variance and unequal expectations.
  - The random rate of return on any asset cannot be expressed as a linear combination of the rates of return on other assets.
  - Asset returns are assumed to be linearly independent, and their variance covariance matrix is nonsingular and positive definite.

Mathematics of Portfolio Selection

- A rational investor would invest his/her money in a risky portfolio on the so-called portfolio frontier.
- The case of all (N) risky assets: a portfolio p is a frontier portfolio if and only if W_p, the N-vector portfolio weights of p is the solution to the following quadratic program.

$$\min \frac{1}{2} W^T V W$$
$$\text{s.t. } W^T e = E[R_p] \text{ and } W^T \underline{1} = 1$$

- Note that short sales are permitted. Therefore, the range of expected returns on feasible portfolios is unbounded.

Mathematics of Portfolio Selection

- The Lagrangian is:

$$L = \frac{1}{2}W^T VW + \lambda(E[R_p] - W^T e) + \gamma(1 - W^T \underline{1}).$$

- The first-order conditions:

$$\frac{\partial L}{\partial W} = VW_p - \lambda e - \gamma \underline{1} = \underline{0};$$
$$\frac{\partial L}{\partial \lambda} = E[R_p] - W_p^T e = 0;$$
$$\frac{\partial L}{\partial \gamma} = 1 - W_p^T \underline{1} = 0.$$

Mathematics of Portfolio Selection

- Optimal portfolio weights are:

$$W_p = \lambda(V^{-1}e) + \gamma(V^{-1}\underline{1}).$$

- Solve for λ and γ as follows.

$$\lambda = \frac{CE[R_p] - A}{D} \qquad \gamma = \frac{B - AE[R_p]}{D}$$

$$A = \underline{1}^T V^{-1} e \qquad C = \underline{1}^T V^{-1} \underline{1}$$

$$B = e^T V^{-1} e \qquad D = BC - A^2$$

Portfolio Frontier

- Substituting for λ and γ in W_p gives the unique set of frontier portfolio weights having an expected rate of return of $E[R_p]$: $W_p = g + hE[R_p]$.

$$g = (1/D)[B(V^{-1}\underline{1}) - A(V^{-1}e)]$$
$$h = (1/D)[C(V^{-1}e) - A(V^{-1}\underline{1})]$$

- Therefore, any frontier portfolio can be represented by $W_p = g + hE[R_p]$. On the other hand, any portfolio that can be represented by $W_p = g + hE[R_p]$ is a frontier portfolio. The set of all frontier portfolios is called the portfolio frontier.

An Important Property of Frontier Portfolios

- The portfolio frontier can be generated by any two distinct frontier portfolios.
- First, we claim that the entire portfolio frontier can be generated by forming portfolios of the two frontier portfolios g and $g + h$. Let q be a frontier portfolio having an expected rate of return $E[R_q]$. We know that $W_q = g + hE[R_q]$.
- Consider the following portfolio weights on g and $g + h$: $\{1 - E[R_q], E[R_q]\}$, then we have:

$$(1 - E[R_q])g + E[R_q](g + h) = W_q.$$

An Important Property of Frontier Portfolios

- Since the portfolio q is arbitrarily chosen, we have shown that the entire portfolio frontier can be generated by the two frontier portfolios g and $g + h$.
- Now let p_1 and p_2 be two distinct frontier portfolios, and let q be any frontier portfolio. We want to show that q is a portfolio generated by p_1 and p_2.
- Since $E[R_{p1}]$ is not equal to $E[R_{p2}]$, there exists a unique number, a, such that $E[R_q] = aE[R_{p1}] + (1 - a)E[R_{p2}]$.

An Important Property of Frontier Portfolios

- Consider a portfolio of p_1 and p_2 with weights $(a, 1-a)$, we have:

$$
\begin{aligned}
& aW_{p1} + (1-a)W_{p2} \\
& = a(g + hE[R_{p1}]) + (1-a)(g + hE[R_{p2}]) \\
& = g + h(aE[R_{p1}] + (1-a)E[R_{p2}]) \\
& = g + hE[R_q] = W_q.
\end{aligned}
$$

- We demonstrate that the portfolio frontier can be generated by any two distinct frontier portfolios.

The Covariance between Two Frontier Portfolios

- The covariance between the rates of return on any two frontier portfolios p and q is:

$$
\begin{aligned}
cov(R_p, R_q) &= W_p^T V W_q \\
&= \frac{C}{D}(E[R_p] - \frac{A}{C})(E[R_q] - \frac{A}{C}) + \frac{1}{C}.
\end{aligned}
$$

- Substituting portfolio p for portfolio q in the above equation gives the following.

$$
cov(R_p, R_p) = \sigma_p^2 = \frac{C}{D}(E[R_p] - \frac{A}{C})^2 + \frac{1}{C}
$$

$$
\Rightarrow \frac{\sigma_p^2}{1/C} - \frac{(E[R_p] - A/C)^2}{D/C^2} = 1
$$

The Covariance between Two Frontier Portfolios

- We can tell that the above equation in the previous slide is a hyperbola in the standard deviation-expected rate of return space with center $(0, A/C)$ and asymptotes $E[R_p] = (A/C) \pm [\sqrt{(D/C)}]\sigma(R_p)$.
- Equivalently,

$$\sigma_p^2 = \frac{1}{D}(C(E[R_p])^2 - 2AE[R_p] + B),$$

which is a parabola in variance-expected rate of return space with vertex $(1/C, A/C)$. The minimum variance portfolio is at $(\sqrt{(1/C)}, A/C)$ in the standard deviation-expected rate of return space.

A Property of The Minimum Variance Portfolio

- The covariance of the rate of return on the minimum variance portfolio (MVP) and that on any portfolio (not only those on the frontier) is always equal to the variance of the rate of return on the minimum variance portfolio: $cov(R_p, R_{MVP}) = var(R_{MVP})$.
- Let p be any portfolio. We consider a portfolio of p and MVP with weights a and $1 - a$ and with minimum variance. Since MVP is the minimum variance portfolio, $a = 0$ must satisfy the first order condition. Thus, the above property follows.

Efficient Portfolios

- Those frontier portfolios which have expected rate of returns strictly higher than that of the minimum variance portfolio are called efficient portfolios. Portfolios that are on the portfolio frontier but are neither efficient nor minimum variance are called inefficient portfolios.
- Let W_i, $i = 1, 2, \ldots, m$, be m frontier portfolios and a_i as weights. We can write:

$$\sum a_i W_i = \sum a_i (g + hE[r_i]) = g + h \sum a_i E[r_i],$$

indicating that any linear combination of frontier portfolios is on the frontier.

Zero Covariance Portfolios

- One important property of the portfolio frontier is that for any portfolio p, except for the MVP portfolio, there exists a unique frontier portfolio, denoted by $zc(p)$, which has a zero covariance with p. In other words, we have the following.

$$
\begin{aligned}
& cov(R_p, R_{zc(p)}) \\
& = \frac{C}{D}((E[R_p] - A/C)(E[R_{zc(p)}] - A/C) + D/C^2) = 0 \\
& \Rightarrow E[R_{zc(p)}] = \frac{A}{C} - \frac{D/C^2}{E[R_p] - A/C}
\end{aligned}
$$

Zero Covariance Portfolios

- We then differentiate totally the following.

$$\frac{\sigma_p^2}{1/C} - \frac{(E[R_p] - A/C)^2}{D/C^2} = 1$$

- We then obtain the slope of the portfolio frontier at $(\sigma_p, E[R_p])$.

$$\frac{dE[R_p]}{d\sigma_p} = \frac{\sigma_p D}{CE[R_p] - A}$$

$$E[R_p] - \frac{dE[R_p]}{d\sigma_p}\sigma_p = E[R_p] - \frac{\sigma_p D}{CE[R_p] - A}\sigma_p$$

$$= \frac{A}{C} - \frac{D/C^2}{E[R_p] - (A/C)}$$

$$= E[R_{zc(p)}]$$

Zero Covariance Portfolios

- It is easily seen that the line joining a frontier portfolio p and the MVP, in variance-expected rate of return space, can be expressed as a line that passes through two points: $(1/C, A/C)$ and $(\sigma_p^2, E[R_p])$.
- Suppose this line can be expressed as $E[R] = a + b\sigma^2(R)$. We have:

$$E[R_p] = a + b\sigma_p^2;$$
$$A/C = a + b(1/C).$$

Zero Covariance Portfolios

- Solving for a and b, we get the following.

$$b = \frac{E[R_p] - (A/C)}{\sigma_p^2 - (1/C)}$$

$$a = (A/C) - \frac{D/C^2}{E[R_p] - (A/C)}$$

$$E[R] = \frac{A}{C} - \frac{D/C^2}{E[R_p] - (A/C)} + \frac{E[R_p] - (A/C)}{\sigma_p^2 - (1/C)}\sigma^2(R)$$

Zero Covariance Portfolios

- Finally, let p be a portfolio which is not on the portfolio frontier. We claim that the intercept on the expected rate of return axis of the line joining p and MVP is equal to the expected rate of return on a portfolio, q, that has zero covariance with p and the minimum variance among all the zero covariance portfolios with p.
- To see this, we note that W_q is the solution to the following program.

$$\min \frac{1}{2} W_q^T V W_q$$

$$\text{s.t. } W_q^T V W_p = 0 \text{ and } W_q^T \underline{1} = 1$$

Zero Covariance Portfolios

- The Lagrangian is:

$$L = \frac{1}{2} W_q^T V W_q + l(0 - W_q^T V W_p) + g(1 - W_q^T \underline{1}).$$

- Optimal portfolio weights is as follows.

$$\begin{aligned} W_q &= \frac{1}{1 - C\sigma_p^2} W_p - \frac{C\sigma_p^2}{1 - C\sigma_p^2} \frac{V^{-1}\underline{1}}{\underline{1}^T V^{-1} \underline{1}} \\ &= \frac{1}{1 - C\sigma_p^2} W_p - \frac{C\sigma_p^2}{1 - C\sigma_p^2} W_{MVP} \\ W_q^T e &= \frac{E[R_p]}{1 - C\sigma_p^2} - \frac{C\sigma_p^2}{1 - C\sigma_p^2} (A/C) \\ &= \frac{E[R_p] - A\sigma_p^2}{1 - C\sigma_p^2} \end{aligned}$$

The Relation between any Portfolio and Frontier Portfolios

- Let p be a frontier portfolio other than the MVP, and let q be any portfolio. The covariance of p and q is:

$$\begin{aligned} cov(R_p, R_q) &= \lambda e^T V^{-1} V W_q + \gamma \underline{1}^T V^{-1} V W_q \\ &= \lambda e^T W_q + \gamma \underline{1}^T W_q = \lambda E[R_q] + \gamma. \end{aligned}$$

- Substituting the solutions of λ and γ into the above equation:

$$E[R_q] = \frac{AE[R_p] - B}{CE[R_p] - A} + cov(R_p, R_q) \frac{D}{CE[R_p] - A}.$$

The Relation between any Portfolio and Frontier Portfolios

- Following the previous slide, we get the following.

$$
\begin{aligned}
E[R_q] = & \frac{A}{C} - \frac{D/C^2}{E[R_p] - A/C} + \\
& \frac{cov(R_p, R_q)}{\sigma^2(R_p)}[\frac{1}{C} + \frac{(E[R_p] - (A/C))^2}{D/C}]\frac{D}{CE(R_p) - A} \\
= & E[R_{zc(p)}] + \beta_{qp}[E[R_p] - \frac{A}{C} + \frac{D/C^2}{E[R_p] - (A/C)}] \\
= & E[R_{zc(p)}] + \beta_{qp}(E[R_p] - E[R_{zc(p)}]) \\
= & (1 - \beta_{qp})E[R_{zc(p)}] + \beta_{qp}E[R_p]
\end{aligned}
$$

The Relation between any Portfolio and Frontier Portfolios

- This result says that the expected rate of return on any portfolio q can be written as a linear combination of the expected rates of return on p and its zero covariance portfolio.
- Note that since $zc(zc(p)) = p$, for any portfolio p other than the MVP, we can also write the following.

$$E[R_q] = (1 - \beta_{qzc(p)})E[R_p] + \beta_{qzc(p)}E[R_{zc(p)}]$$
$$\Rightarrow E[R_q] = \beta_{qp}E[R_p] + \beta_{qzc(p)}E[R_{zc(p)}]$$

Adding a Riskless Asset

- Let p be a frontier portfolio of all $N+1$ assets, and let W_p denotes the N-vector portfolio weights of p on risky assets. Then W_p is the solution to the following quadratic program.

$$\min \frac{1}{2} W^T V W$$
$$\text{s.t. } W^T e + (1 - W^T \underline{1}) R_f = E[R_p]$$

- The Lagrangian is:

$$L = \frac{1}{2} W^T V W + \lambda (E[R_p] - W^T e - (1 - W^T \underline{1}) R_f).$$

Adding a Riskless Asset

- The first-order conditions:

$$\frac{\partial L}{\partial W} = VW_p - \lambda(e - \underline{1}R_f) = 0;$$
$$\frac{\partial L}{\partial \lambda} = E[R_p] - W_p^T(e - \underline{1}R_f) - R_f = 0.$$

- The optimal weights are:

$$W_p = V^{-1}(e - R_f\underline{1})\frac{E[R_p] - R_f}{H},$$

where $H = (e - R_f\underline{1})^T V^{-1}(e - R_f\underline{1}) = B - 2AR_f + CR_f^2$.

The Portfolio Frontier when a Riskless Asset Exists

- The variance of the rate of return on portfolio p is as follows.

$$\sigma_p^2 = W_p^T V W_p = \frac{(E[R_p] - R_f)^2}{H}$$

- Equivalently, we can write as follows.

$$\sigma_p = \begin{cases} \frac{(E[R_p] - R_f)}{\sqrt{H}} \\ -\frac{(E[R_p] - R_f)}{\sqrt{H}} \end{cases}$$

- The portfolio frontier of all assets is composed of two half-lines emanating from the point $(0, R_f)$ in the standard deviation-expected rate of return space.

Definition of Two Fund Separation

- We now know that all portfolios on the portfolio frontier can be generated by any two distinct frontier portfolios. Thus, if individuals prefer frontier portfolios, they can simply hold a linear combination of two frontier portfolios or mutual funds.
- Two (mutual) fund separation: in that case, given any feasible portfolio, there exists a portfolio of two mutual funds such that individuals prefer at least as much as the original portfolio.

Definition of Two Fund Separation

- A vector of asset rate of returns is said to exhibit two fund separation if there exist two mutual funds, α_1 and α_2, such that for any portfolio q, there exists a scalar λ such that $E[u(\lambda r_{\alpha_1} + (1-\lambda) r_{\alpha_2})] \geqq E[u(r_q)]$ for all concave utility functions.
- Assume all assets are risky, until specified otherwise. Moreover, the variance-covariance matrix of asset returns exists and is positive definite. Then portfolio frontier exists, and every frontier portfolio is uniquely determined in that there is a unique set of portfolio weights associated with each frontier portfolio.

Two Fund Separation and Second-Order Stochastic Dominance

- Suppose that the vector of asset rates of return, r , exhibits two fund separation. We first claim that the separating mutual funds α_1 and α_2 must be frontier portfolios.
- To see this, we note that by the definition of two fund separation, for any portfolio q there must exist a scalar λ such that $E[u(\lambda r_{\alpha_1} + (1-\lambda) r_{\alpha_2})] \geqq E[u(r_q)]$ for all concave utility functions. This is equivalent to $\lambda\alpha_1 + (1-\lambda)\alpha_2 \underset{SSD}{\geq} q$.

Two Fund Separation and Second-Order Stochastic Dominance

- According to the equivalent statements of second-order stochastic dominance, we must then have $E[\lambda r_{\alpha_1} + (1-\lambda) r_{\alpha_2}] = E[r_q]$ and $Var[\lambda r_{\alpha_1} + (1-\lambda) r_{\alpha_2}] \leqq Var[r_q]$.
- Suppose, for example, that α_2 is not a frontier portfolio. Then there must exist a portfolio α^* that has a variance strictly smaller than that of any portfolio formed by α_1 and α_2.

Two Fund Separation and Second-Order Stochastic Dominance

- This contradicts the hypothesis that α_1 and α_2 are separating portfolios. Hence, α_1 and α_2 must be on the portfolio frontier.
- Since α_1 and α_2 are frontier portfolios, any linear combination of them is also on the frontier. Hence, for any portfolio q, the dominating portfolio formed from two separating portfolios is the frontier portfolio that has the same expected rate of return as q.

Two Fund Separation and Second-Order Stochastic Dominance

- As the portfolio weights of any frontier portfolio are uniquely determined and any two distinct frontier portfolios span the whole portfolio frontier, whenever two fund separation obtains, it must be that any two distinct frontier portfolios can be separating portfolios.
- In particular, we can pick any frontier portfolio, $p \neq$ MVP, and its zero covariance portfolio, $z(p)$, to be the separating portfolios.

Mean-Variance Spanning and Intersection Tests

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What is Mean-Variance Spanning?

- In portfolio analysis, if an investor chooses portfolios based on mean and variance, he/she is interested in whether adding a new set of risky assets can enlarge the mean-variance frontier of a given set of assets.
- Conditional on $N + K$ assets, can investors maximise his/her utility by holding just a smaller set of K assets?
- Conditional on K assets, can investors benefit from investing a new set of N assets?

Tests of Mean-Variance Spanning

- Huberman and Kandel (1987, Journal of Finance) are the first to address this issue. They propose a multivariate test of the hypothesis that the mean-variance frontier of a set of K benchmark assets is the same as the mean-variance frontier of the K assets plus a set of N test assets.
- Recent development of mean-variance spanning includes:
  - Kan and Zhou (2012, Annals of Economics and Finance);
  - De Roon and Nijman (2001, Journal of Empirical Finance).

Tests of Mean-Variance Spanning

- Consider $K(R_{1t})$ basis assets and $N(R_{2t})$ test assets:

  $$R_t = [R_{1t}', R_{2t}']'; \; \mu = E[R_t] \equiv \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix};$$

  $$V = Var[R_t] \equiv \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix},$$ where V is nonsingular.

- Projecting R_{2t} on R_{1t}, we have:

  $$R_{2t} = \alpha + \beta R_{1t} + \xi_t; \; E[\xi_t] = 0_N; \; E[\xi_t R_{1t}'] = O_{N\times K}.$$

- We also know that:

  $$\alpha = \mu_2 - \beta\mu_1; \; \beta = V_{21}V_{11}^{-1}; \; \sum = V_{22} - V_{21}V_{11}^{-1}V_{12}.$$

Tests of Mean-Variance Spanning

- In Huberman and Kandel (1987), the null hypothesis of "spanning" is:

$$H_0 : \alpha = 0_N, \quad \delta = 1_N - \beta 1_K = 0_N.$$

- Intuitively, think β as weights, if the null hypothesis holds, we can find a portfolio of K basis assets that has the same mean as every test asset, but with a smaller variance, since $E[\xi_t R'_{1t}] = O_{N\times K}$ and $Var[\xi_t]$ is positive definite.

Tests of Mean-Variance Spanning

- Consider two portfolios from $N + K$ assets on mean-variance frontier.
  - Tangency portfolio from the origin.

$$\omega_1 = \frac{V^{-1}\mu_{N+K}}{1'_{N+K}V^{-1}\mu}$$

  - Global minimum-variance portfolio.

$$\omega_2 = \frac{V^{-1}1_{N+K}}{1'_{N+K}V^{-1}1_{N+K}}$$

Tests of Mean-Variance Spanning

- Let $Q = [O_{N\times K}, I_N]$, using partitioned matrix inverse formula, the weights of N test assets in these two portfolios are:

$$
\begin{aligned}
Q\omega_1 &= \frac{QV^{-1}\mu}{1'_{N+K}V^{-1}\mu} = \frac{[-\sum^{-1}\beta, \sum^{-1}]\mu}{1'_{N+K}V^{-1}\mu} \\
&= \frac{\sum^{-1}(\mu_2 - \beta\mu_1)}{1'_{N+K}V^{-1}\mu} = \frac{\sum^{-1}\alpha}{1'_{N+K}V^{-1}\mu}; \\
Q\omega_2 &= \frac{QV^{-1}1_{N+K}}{1'_{N+K}V^{-1}1_{N+K}} = \frac{[-\sum^{-1}\beta, \sum^{-1}]1_{N+K}}{1'_{N+K}V^{-1}1_{N+K}} \\
&= \frac{\sum^{-1}(1_N - \beta 1_K)}{1'_{N+K}V^{-1}1_{N+K}} = \frac{\sum^{-1}\delta}{1'_{N+K}V^{-1}1_{N+K}}.
\end{aligned}
$$

Tests of Mean-Variance Spanning

- Huberman and Kandel (1987) test is actually the test of whether N test assets have zero weight in these two portfolios that lie on the mean-variance frontier.
- By two-fund separation, if the above holds, every portfolio on the mean-variance frontier will have zeero weight on N test assets, i.e., no expansion by adding N test assets from K basis assets.

Asymptotic M-V Spanning Tests

- The mean-variance spanning tests follow a regression based setting.

$$R_{2t} = \alpha + \beta R_{1t} + \xi, \;\; t = 1, 2, ..., T$$
$$R = XB + E \text{ (in matrix notation)}$$

$$\hat{\beta} \equiv [\hat{\alpha}, \hat{\beta}]' = (X'X)^{-1}(X'R)$$
$$\hat{\sum} = \frac{1}{T}(R - X\hat{\beta})'(R - X\hat{\beta})$$
$$\xi_t \sim N(0, \sum)$$
$$vec(\hat{\beta}') \sim N(vec(\beta'), (X'X)^{-1} \otimes \sum)$$

Asymptotic M-V Spanning Tests

- Define the following mean-variance coefficients.

$$\hat{a}_1 = \hat{\mu}_1' \hat{V}_{11}^{-1} \hat{\mu}_1 \qquad \hat{b}_1 = \hat{\mu}_1' \hat{V}_{11}^{-1} 1_K$$
$$\hat{c}_1 = 1_K' \hat{V}_{11}^{-1} 1_K \qquad \hat{d}_1 = \hat{a}_1 \hat{c}_1 - \hat{b}_1^2$$

$$\hat{a} = \hat{\mu}' \hat{V}^{-1} \hat{\mu} \qquad \hat{b} = \hat{\mu}' \hat{V}^{-1} 1_{N+K}$$
$$\hat{c} = 1_{N+K}' \hat{V}^{-1} 1_{N+K} \qquad \hat{d} = \hat{a}\hat{c} - \hat{b}^2$$

Asymptotic M-V Spanning Tests

- The null hypothesis of "spanning" can be written as follows.

$$\Theta = [\alpha, \delta]' = O_{2\times N} = AB - C$$

$$A = \begin{bmatrix} 1 & 0'_k \\ 0 & -1'_K \end{bmatrix} \quad C = \begin{bmatrix} 0'_N \\ -1'_N \end{bmatrix}$$

$$vec(\hat{\Theta}') \sim N(vec(\Theta'), A(X'X)^{-1}A' \otimes \sum)$$

- Define the following.

$$\hat{G} = TA(X'X)^{-1}A' = \begin{bmatrix} 1 + \hat{\mu}'_1 \hat{V}_{11}^{-1} \hat{\mu}_1 & \hat{\mu}'_1 \hat{V}_{11}^{-1} 1_K \\ \hat{\mu}'_1 \hat{V}_{11}^{-1} 1_K & 1'_K \hat{V}_{11}^{-1} 1_K \end{bmatrix}$$

Asymptotic M-V Spanning Tests

- The likelihood ratio test is:

$$LR = T\ln(\frac{1}{U}) \overset{A}{\sim} \chi^2_{2N}.$$

- From Seber (1984, Book), we know $\tilde{\sum} - \hat{\sum} = \hat{\Theta}'\hat{G}^{-1}\hat{\Theta}$, therefore we get the following.

$$\frac{1}{U} = \frac{|\tilde{\sum}|}{|\hat{\sum}|} = |\hat{\sum}^{-1}(\hat{\sum} + \hat{\Theta}'\hat{G}^{-1}\hat{\Theta})|$$

$$= |I_N + \hat{\sum}^{-1}\hat{\Theta}'\hat{G}^{-1}\hat{\Theta}| = |I_2 + \hat{H}\hat{G}^{-1}|$$

$$\hat{H} = \hat{\Theta}\hat{\sum}^{-1}\hat{\Theta}' = \begin{bmatrix} \hat{\alpha}'\hat{\sum}^{-1}\hat{\alpha} & \hat{\alpha}'\hat{\sum}^{-1}\hat{\delta} \\ \hat{\alpha}'\hat{\sum}^{-1}\hat{\delta} & \hat{\delta}'\hat{\sum}^{-1}\hat{\delta} \end{bmatrix}$$

Asymptotic M-V Spanning Tests

- Denote λ_1 and λ_2 are two eigenvalues of $\hat{H}\hat{G}^{-1}$, so we have the likelihood ratio test:

$$\frac{1}{U} = (1+\lambda_1)(1+\lambda_2);\ LR = T\sum_{i=1}^{2} \ln(1+\lambda_i) \overset{A}{\sim} \chi^2_{2N}.$$

- From Berndt and Savin (1977, Econometrica), Breusch (1979, Econometrica) and Muirhead (1982, Book), we can also obtain Wald and Lagrange multiplier tests:

$$W = T(\lambda_1+\lambda_2) \overset{A}{\sim} \chi^2_{2N};\ LM = T\sum_{i=1}^{2} \frac{\lambda_i}{1+\lambda_i} \overset{A}{\sim} \chi^2_{2N}.$$

Finite Sample M-V Spanning Tests

- Huberman and Kandel (1987) show that the exact finite sample distribution of the likelihood ratio test under the null hypothesis is:

$$\left(\frac{1}{U^{\frac{1}{2}}}-1\right)\left(\frac{T-K-N}{N}\right) \sim F_{2N,2(T-K-N)} \text{ for } N \geqq 2;$$
$$\left(\frac{1}{U}-1\right)\left(\frac{T-K-1}{2}\right) \sim F_{2,2(T-K-1)} \text{ for } N = 1.$$

Finite Sample M-V Spanning Tests

- To discuss the geometry and spanning tests, let us recall:
$$\hat{G} = \begin{bmatrix} 1+\hat{a}_1 & \hat{b}_1 \\ \hat{b}_1 & \hat{c}_1 \end{bmatrix};\ \hat{H} = \begin{bmatrix} \hat{a}-\hat{a}_1 & \hat{b}-\hat{b}_1 \\ \hat{b}-\hat{b}_1 & \hat{c}-\hat{c}_1 \end{bmatrix}$$
(using partitioned matrix inverse formula to prove).
- We then have the following.

$$\frac{1}{U} = |I_2 + \hat{H}\hat{G}^{-1}| = \frac{|\hat{H}+\hat{G}|}{|\hat{G}|} = \frac{(1+\hat{a})\hat{c}-\hat{b}^2}{(1+\hat{a}_1)\hat{c}_1-\hat{b}_1^2}$$
$$= \frac{\hat{c}+\hat{d}}{\hat{c}_1+\hat{d}_1} = \left(\frac{\hat{c}}{\hat{c}_1}\right)\left(\frac{1+\hat{d}/\hat{c}}{1+\hat{d}_1/\hat{c}_1}\right)$$

Finite Sample M-V Spanning Tests

$$\left(\frac{T-K-N}{N}\right)\left(\left(\frac{\sqrt{\hat{c}}}{\sqrt{\hat{c}_1}}\right)\left(\frac{\sqrt{1+\hat{d}/\hat{c}}}{\sqrt{1+\hat{d}_1/\hat{c}_1}}\right)-1\right)\sim F_{2N,2(T-K-N)}$$

for $N \geqq 2$

$$\left(\frac{T-K-1}{2}\right)\left(\left(\frac{\hat{c}}{\hat{c}_1}\right)\left(\frac{1+\hat{d}/\hat{c}}{1+\hat{d}_1/\hat{c}_1}\right)-1\right)\sim F_{2,2(T-K-1)}$$

for $N = 1$

Finite Sample M-V Spanning Tests

- From the figure, we have g_1 as K asset global mean-variance portfolio and g as $N+K$ asset global mean-variance portfolio. They have standard deviation of $1/\sqrt{\hat{c}_1}$ and $1/\sqrt{\hat{c}}$ respectively. The ratio of $\sqrt{\hat{c}}/\sqrt{\hat{c}_1}$ should be $\geqq 1$.
- The absolute value of the asymptotic slope is $\sqrt{\hat{d}_1}/\sqrt{\hat{c}_1}$ for K asset and $\sqrt{\hat{d}}/\sqrt{\hat{c}}$ for $N+K$ asset. $\sqrt{1+\hat{d}_1/\hat{c}_1}$ is the distance of asymptotic line of K asset hyperbola from $\hat{\sigma}=0$ to $\hat{\sigma}=1$. $\sqrt{1+\hat{d}/\hat{c}}$ is the same distance but for $N+K$ asset.

Finite Sample M-V Spanning Tests

- When the two hyperbolae are similar, the two ratios should be close to 1, which make F statistic towards zero; when they are different, we get large F and reject the null hypothesis.

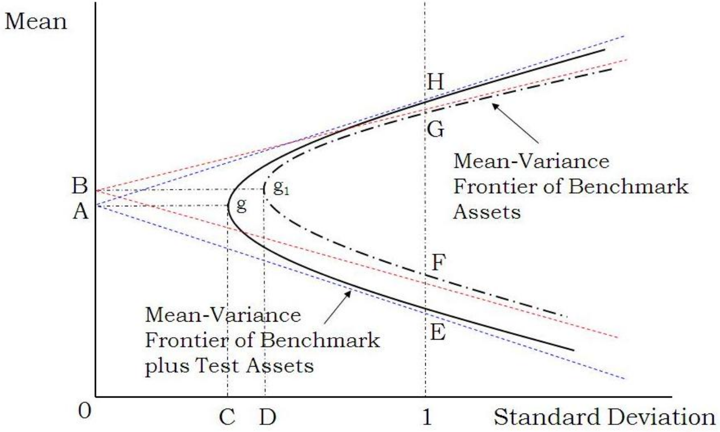
$$\left(\frac{T-K-N}{N}\right)\left(\left(\frac{OD}{OC}\right)\left(\frac{AH}{BF}\right)-1\right)$$

$$\sim F_{2N,2(T-K-N)} \text{ for } N \geqq 2$$

$$\left(\frac{T-K-1}{2}\right)\left(\left(\frac{OD}{OC}\right)^2\left(\frac{AH}{BF}\right)^2-1\right)$$

$$\sim F_{2,(T-K-1)} \text{ for } N = 1$$

Geometry of M-V Spanning Test



Step-Down Tests

- By studying the power of the regression based M-V spanning tests, Kan and Zhou (2012) find that the tests have very good power for assets that could improve the variance of the global minimum-variance portfolio, but they have little power against assets that could only improve the tangency portfolio. Hence, they suggest a step-down procedure to separate the two kinds of spanning.
- The advantage is that we could know the cause of rejection.

Step-Down Tests

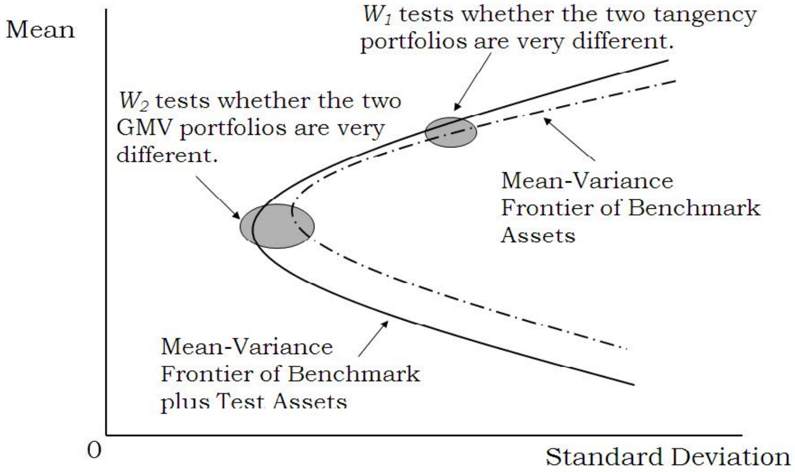
- First, we test $\alpha = O_N$, then we test $\delta = O_N$ conditional on $\alpha = O_N$. The two tests are shown as:

$$
\begin{aligned}
F_1 &= \Big(\frac{T-K-N}{N}\Big)\Big(\frac{|\overline{\sum}|}{|\hat{\sum}|} - 1\Big) \\
&= \Big(\frac{T-K-N}{N}\Big)\Big(\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}\Big) \sim F_{N,(T-K-N)}; \\
F_2 &= \Big(\frac{T-K-N+1}{N}\Big)\Big(\frac{|\tilde{\sum}|}{|\overline{\sum}|} - 1\Big) \\
&= \Big(\frac{T-K-N+1}{N}\Big)\Big(\frac{1+\hat{a}_1}{1+\hat{a}} \times \frac{\hat{c}+\hat{d}}{\hat{c}_1+\hat{d}_1} - 1\Big) \\
&\sim F_{N,(T-K-N+1)}.
\end{aligned}
$$

Step-Down Tests

- If the rejection is due to the first test, we know that it is because the two tangency portfolios are very different.
- If the rejection is due to the second test, we know that it is because the two global min. variance portfolios are very different.

Geometry of the Spanning Test Again



Mean-Variance Intersection Tests

- The intersection occurs when the original mean-variance frontier and the new mean-variance frontier have only one point in common.
- Using the same regression model as the spanning test, the null hypothesis for the intersection test is as follows.

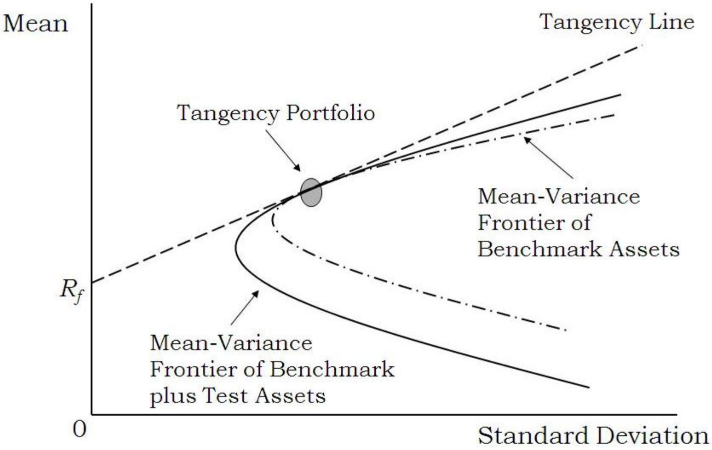
$$\mathrm{H}_0 : \alpha - \eta(1 - \beta 1_K) = 0 \ (\eta \text{ is the risk-free rate})$$

Mean-Variance Intersection Tests

- DeRoon and Nijman (2001, Journal of Empirical Finance): the intersection test statistic can be related to the difference in the squared maximal Sharpe ratios attainable for K benchmark assets and $K+N$ assets, respectively.

$$W_I = T\left(\frac{\hat{\theta}_R(\eta)^2 - \hat{\theta}_{R1}(\eta)^2}{1+\hat{\theta}_{R1}(\eta)^2}\right)$$

Geometry of the Intersection Test



CEO Overconfidence and Corproate Cash Holdings

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Motivation: Motives of Holding Cash

- There has been a surge of research on corporate cash holdings in the recent decade.
- Trade-off theory weights the costs and benefits of holding cash, suggesting that firms set optimal cash levels to maximize shareholder wealth.
  - Market imperfection makes external financing costly, so that firms have **transaction motive** to accumulate cash.
  - The information asymmetry argument of Myers and Majluf (1984) points out that firms have **precautionary motive** to preserve internal funds in order to better cope with adverse shocks when access to capital markets is costly.
- Jensen (1986) argues that entrenched managers would rather retain cash for poor investment opportunities than increase payouts. Such **agency motive** of holding cash may harm shareholder wealth.

Motivation: Cash Holdings Literature

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- A substantial literature has discussed the association between firm characteristics and corporate cash holdings.
- **Determinants of cash holdings**. Firms with strong growth opportunity and higher cash flow volatility hold more cash, while firms with better access to capital markets hold less cash.
  - Opler, Pinkowitz, Stulz, and Williamson (1999); Bates, Kahle, and Stulz (2009).
- **Value of cash**. The value of cash is higher for firms with financial constraint; poorly governed firms have lower value of cash.
  - Faulkender and Wang (2006); Dittmar and Mahrt-Smith (2007); Denis and Sibilkov (2010).
- **Cash saving behavior**. Firms face greater difficulties in raising external funds save larger portions of cash flow as cash, and firms increasingly issue shares for the purpose of cash savings.
  - Almeida, Campello, and Weisbach (2004); McLean (2011).

Motivation: CEO Overconfidence Literature

- Recent studies on managerial overconfidence provide empirical evidence that overconfident CEOs affect the **investment decisions** and **financing decisions** of the firms.
- Investment decisions.
  - Malmendier and Tate (2005) argue that managerial overondifence leads to overestimation of future returns and causes corporate investment distortions. Overconfident CEOs overinvest when the firm has abundant internal funds and underinvest when the firm requires external funds.
  - Malmendier and Tate (2008) show that overconfident CEOs are more likely to undertake value destroying mergers.
  - Hirshleifer, Low, and Teoh (2012) suggest that overconfident CEOs invest more in innovation and obtain more patents and patent citations.

Motivation: CEO Overconfidence Literature

- Financing decisions.
  - Malmendier, Tate, and Yan (2011) argue that overconfident CEOs believe that the firms are undervalued and view external financing as overpriced, especially equity financing. They thus issue less equity than the non-overconfident counterparts.
  - Ben-David, Graham, and Harvey (2013) conduct survey on CFOs. They find that overconfident managers tend to underestimate the volatility of future cash flows and follow more aggressive corporate policies: investing more and using more debt financing.

Main Research Questions

- Based on the past literature, managerial characteristics, such as CEO overconfidence, are highly associated with corporate investment decision. Since cash plays a crucial role in investment strategy, it is intuitive to argue that managerial characteristics affect corporate cash policy.
- To the best of our knowledge, there is very few study regarding CEO overconfidence and corporate cash holdings. We thus contribute the literature by exploring the following three research questions.
  - Do firms with overconfident CEOs hold more cash or less cash?
  - Which motive could better explain the level of cash holdings for firms with overconfident CEOs?
  - What are the potential sources of cash savings if the firms are managed by overconfident CEOs?

- We find that firms with overconfident CEOs tend to have higher cash holdings as well as higher marginal value of cash than those without. This finding verifies the costly external financing hypothesis in the CEO overconfidence literature and is consistent with the **trade-off theory** of cash holdings.
- The above effect is lower for firms with higher growth opportunity, because firms with higher growth opportunity have better accessibility and lower costs to external financing, reducing the **transaction motive** of cash holdings.
- We further show that firms with overconfident CEOs would save more cash than those without, especially from external sources.
- Overall, our empirical results indicate that in addition to the demand of capital and the economic condition, the **manager's belief** also demonstrates a strong impact on corporate cash policy.

- Overconfident CEOs tend to overestimate future returns or underestimate the likelihood of failure on their investment projects. As a result, there is difference in opinion regarding the firm value between overconfident CEOs and capital markets, which makes overconfident CEOs view external financing as costly. (Malmendier and Tate, 2005; Malmendier et al., 2011; Ben-David et al., 2013)
- Overconfident CEOs are better innovators, which leads their firms involving in more severe information asymmetry. (Hirshleifer et al., 2012)
- Overconfident CEOs, compared with the non-overconfident counterparts, have higher incentive of preserving funds.

Hypothesis 1.
The cash holdings are higher for firms with overconfident CEOs than those without.

- Shin and Stulz (2000) document that there is a negative relation between Tobin's q and firm total risk. Thus, firms with higher growth opportunities are safer to the capital markets; they are more likely to access to external funds with lower costs.
- Pilotte (1992) shows that markets react positively to external financing activities of growth firms. The participants in capital markets recognize such value, potentially reducing the difference in opinion on firm value between overconfident CEOs and capital markets. This could also make external financing less costly.
- The benefit of growth opportunity lowers the transaction motive of holding cash for overconfident CEOs.

Hypothesis 2.
The postive association between CEO overconfidence and cash holdings is weaker for firms with higher growth opportunities.

- If overconfident CEOs hold cash for precautionary motive, they are less likely to curtail investments that are beneficial for their firms. Even if they believe that external funds are costly, they can still invest in beneficial projects with internal funds.
- If overconfident CEOs hold cash for agency motive, they are more likely to invest internal funds in value destroying projects.

Hypothesis 3.
With precautionary motive, increasing in cash holdings for firms with overconfident CEOs enhances firm value.

Hypothesis 4.
With agency motive, increasing in cash holdings for firms with overconfident CEOs reduces firm value.

Measure of CEO Overconfidence

- Since CEO overconfidence cannot be observed directly, the previous literature often measures CEO overconfidence based on the actions taken by CEOs.
- Malmendier and Tate (2005, 2008), Campbell, Gallmeyer, Johnson, Rutherford, and Stanley (2011), and Hirshleifer et al. (2012) measure CEO overconfidence based on CEOs' exercising decisions of their executive stock options.
- We follow Hirshleifer et al. (2012) by identifying CEOs as overconfident if they hold options that are at least 67% in-the-money. Once CEOs are classified as overconfident CEOs, they retain the same classification forward in the sample period. However, such classification does not go backward during a CEO's tenure.

Data Source

- The financial data and executive compensation data are from Compustat and ExecuComp, respectively.
- The sample consists of U.S. firms from 1992 to 2014.
- To be included in our sample, firms are required to have positive assets and sales in a given year.
- Financial firms (SIC code 6000–6999) and utilities firms (SIC code 4900–4999) are excluded from the sample, because these firms are often regulated and have different industrial traits.
- Our sample construction process yields a firm-year panel of 25,979 observations for 2,387 firms.

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Variable	Mean	Standard deviation	25th percentile	Median	75th percentile	Observation
Overconfidence	0.618	0.486	0.000	1.000	1.000	25,979
Cash	0.151	0.172	0.025	0.083	0.219	25,979
Market to book	2.078	1.769	1.234	1.614	2.303	25,967
Real size	7.487	1.608	6.358	7.367	8.519	25,979
Cash flow	0.077	0.191	0.054	0.086	0.120	24,314
Cash flow volatility	0.232	0.278	0.063	0.125	0.259	25,979
Net working capital	0.070	0.240	−0.018	0.067	0.165	25,319
Leverage	0.222	0.184	0.059	0.206	0.332	25,883
Capital expenditure	0.058	0.056	0.022	0.041	0.073	25,834
Acquisition	0.030	0.064	0.000	0.000	0.027	24,405
R&D	0.093	0.812	0.000	0.004	0.052	25,952
Dividend	0.494	0.500	0.000	0.000	1.000	25,940

- We follow Bates et al. (2009) to winsorize extreme observations at the 1st and 99th percentiles, in order to mitigate the effect of outliers to our empirical results.
- The proportion of firms that are managed by overconfident CEOs is about 62%; similar percentage is documented by Hirshleifer et al. (2012).

Univariate Comparison by CEO Overconfidence

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	Overconfident CEO			Non-overconfident CEO				
Variable	Mean	Median	Observation	Mean	Median	Observation	*t*-statistic	Wilcoxon *Z*
Cash	0.164	0.093	16,049	0.131	0.070	9,930	14.97***	13.81***
Market to book	2.345	1.793	16,042	1.647	1.404	9,925	31.52***	42.36***
Real size	7.425	7.307	16,049	7.587	7.487	9,930	−7.90***	−8.25***
Cash flow	0.086	0.093	14,927	0.062	0.074	9,387	9.25***	28.94***
Cash flow volatility	0.237	0.131	16,049	0.225	0.106	9,930	3.52***	9.45***
Net working capital	0.068	0.065	15,623	0.073	0.069	9,696	−1.75*	−1.18
Leverage	0.211	0.193	15,998	0.240	0.229	9,885	−12.15***	−13.34***
Capital expenditure	0.061	0.043	15,969	0.053	0.038	9,865	11.11***	9.01***
Acquisition	0.033	0.001	15,124	0.025	0.000	9,281	9.73***	9.39***
R&D	0.100	0.003	16,033	0.081	0.004	9,919	1.88*	2.74**
Dividend	0.452	0.000	16,021	0.561	1.000	9,919	−17.02***	−16.93***

- Consistent with Hypothesis 1, firms managed by overconfident CEOs tend to hold higher levels of cash.
- There are also significant difference in other firm characteristics.

Propensity Score Matching

- Whether firms are managed by overconfident CEOs or by non-overconfident CEOs might not be exogenous. Firms may choose to hire overconfident or non-overconfident CEOs for reasons correlated with investment or financing decisions. Those decisions are likely to affect cash holdings.
- To deal with potential sample selection biases, we implement an one-to-one nearest neighbor propensity score matching with replacement.
- To compute the propensity score, we estimate a logistic regression model with a set of firm characteristics.
- We match each overconfident firm-year observation with a non-overconfident firm-year observation that minimizes the absolute value of the difference between propensity scores.

Univariate Comparison by CEO Overconfidence (Propensity Score Matching)

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	Overconfident CEO		Non-overconfident CEO		
Variable	Mean	Observation	Mean	Observation	*t*-statistic
Cash	0.195	7,629	0.155	7,629	8.28***
Market to book	2.451	7,629	1.733	7,629	17.94***
Real size	7.340	7,629	7.443	7,629	−1.43*
Cash flow	0.075	7,629	0.063	7,629	2.28***
Cash flow volatility	0.304	7,629	0.304	7,629	0.00
Net working capital	0.066	7,629	0.086	7,629	−3.61***
Leverage	0.201	7,629	0.202	7,629	−0.35
Capital expenditure	0.048	7,629	0.048	7,629	−0.32
Acquisition	0.029	7,629	0.033	7,629	−2.89***
R&D	0.120	7,629	0.112	7,629	0.38
Dividend	0.449	7,629	0.509	7,629	−5.12***

- The difference in the cash ratio between firms managed by overconfident CEOs and firms managed by non-overconfident CEOs is still statistically significant.

Cash Holdings and CEO Overconfidence

- We examine the relation between cash holdings and CEO overconfidence based on the model in Bates et al. (2009). The CEO overconfidence indicator and its interaction term with market-to-book ratio are included in the model.
- The coefficients on overconfidence are significantly positive, indicating that firms managed by overconfident CEOs hold higher levels of cash in comparison with firms managed by non-overconfident CEOs. This is consistent with **Hypothesis 1**.
- The coefficients on the interaction term are significantly negative, implying that firms managed by overconfident CEOs have less incentive to hold cash when their firms have higher growth opportunities. This is consistent with **Hypothesis 2**.
- The coefficients on other control variables are consistent with the findings in Bates et al. (2009).

Cash Holdings and CEO Overconfidence

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Dependent Variable	Cash/Assets				Log(Cash/Non-cash Assets)			
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Overconfidence	0.007*	0.007**	0.027***	0.023***	0.049	0.028	0.337***	0.248***
	(1.87)	(2.12)	(2.74)	(2.61)	(1.34)	(0.85)	(3.84)	(3.11)
Market to book	0.019***	0.017***	0.029***	0.025***	0.166***	0.151***	0.309***	0.259***
	(10.24)	(9.36)	(5.51)	(5.00)	(10.64)	(10.40)	(7.17)	(6.32)
Market to book × Overconfidence			−0.012**	−0.009*			−0.167***	−0.127***
			(−2.05)	(−1.80)			(−3.55)	(−2.95)
Real size	−0.019***	−0.021***	−0.019***	−0.021***	−0.118***	−0.120***	−0.118***	−0.120***
	(−11.05)	(−11.54)	(−11.08)	(−11.62)	(−7.02)	(−7.44)	(−7.06)	(−7.53)
Cash flow	0.123***	0.135***	0.126***	0.138***	1.175***	1.199***	1.215***	1.239***
	(4.15)	(5.50)	(4.20)	(5.43)	(5.33)	(6.92)	(5.77)	(7.06)
Cash flow volatility	0.063***	0.009	0.062***	0.009	0.665***	−0.021	0.651***	−0.021
	(8.00)	(1.53)	(8.05)	(1.53)	(9.12)	(−0.32)	(9.14)	(−0.31)
Net working capital	−0.140***	−0.143***	−0.142***	−0.146***	−1.316***	−1.255***	−1.357***	−1.296***
	(−3.74)	(−3.38)	(−3.98)	(−3.57)	(−4.00)	(−3.55)	(−4.41)	(−3.85)
Leverage	−0.238***	−0.201***	−0.240***	−0.202***	−2.880***	−2.344***	−2.901***	−2.367***
	(−15.72)	(−12.96)	(−15.89)	(−13.08)	(−20.10)	(−17.06)	(−20.55)	(−17.26)
Capital expenditure	−0.614***	−0.519***	−0.621***	−0.528***	−6.549***	−4.378***	−6.642***	−4.497***
	(−17.46)	(−14.04)	(−17.73)	(−14.32)	(−16.09)	(−11.28)	(−16.51)	(−11.67)
Acquisition	−0.333***	−0.323***	−0.335***	−0.324***	−3.082***	−2.866***	−3.104***	−2.881***
	(−18.77)	(−19.46)	(−18.78)	(−19.45)	(−16.77)	(−17.53)	(−16.87)	(−17.59)
R&D	0.035***	0.032***	0.035***	0.032***	0.231***	0.210***	0.229***	0.209***
	(3.63)	(3.84)	(3.67)	(3.87)	(3.15)	(3.35)	(3.22)	(3.41)
Dividend	−0.060***	−0.050***	−0.061***	−0.051***	−0.603***	−0.467***	−0.613***	−0.476***
	(−12.97)	(−10.89)	(−13.01)	(−10.95)	(−12.53)	(−10.27)	(−12.71)	(−10.42)
Intercept	0.357***		0.343***		−0.763***		−0.976***	
	(20.80)		(16.53)		(−5.02)		(−5.42)	
Year fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
Industry fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
Number of observations	22,083	22,083	22,083	22,083	21,995	21,995	21,995	21,995
Adjusted R-squared	0.399	0.458	0.401	0.459	0.365	0.450	0.368	0.451

Cash Holdings and CEO Overconfidence (Propensity Score Matching)

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Dependent Variable	Cash/Assets				Log(Cash/Non-cash Assets)			
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Overconfidence	0.012*	0.013**	0.052***	0.050***	0.067	0.073	0.548***	0.497***
	(1.90)	(2.17)	(3.56)	(3.48)	(1.11)	(1.34)	(4.29)	(3.92)
Market to book	0.019***	0.017***	0.037***	0.034***	0.156***	0.151***	0.377***	0.348***
	(8.32)	(7.99)	(5.50)	(5.16)	(8.37)	(8.43)	(7.52)	(6.61)
Market to book×Overconfidence			−0.021***	−0.020***			−0.262***	−0.231***
			(−2.97)	(−2.81)			(−4.77)	(−4.15)
Real size	−0.020***	−0.022***	−0.020***	−0.022***	−0.089***	−0.096***	−0.090***	−0.098***
	(−6.58)	(−7.36)	(−6.76)	(−7.60)	(−3.45)	(−4.04)	(−3.63)	(−4.28)
Cash flow	0.147***	0.159***	0.144***	0.158***	1.252***	1.320***	1.220***	1.305***
	(4.42)	(4.81)	(4.53)	(4.91)	(4.83)	(4.94)	(5.37)	(5.30)
Cash flow volatility	0.019	−0.004	0.018	−0.003	0.236*	−0.246**	0.224*	−0.227**
	(1.31)	(−0.42)	(1.30)	(−0.27)	(1.66)	(−2.33)	(1.66)	(−2.23)
Net working capital	−0.153***	−0.161***	−0.155***	−0.164***	−1.284***	−1.304***	−1.314***	−1.339***
	(−3.15)	(−2.90)	(−3.37)	(−3.09)	(−3.31)	(−2.98)	(−3.70)	(−3.28)
Leverage	−0.276***	−0.257***	−0.280***	−0.262***	−3.001***	−2.617***	−3.049***	−2.674***
	(−10.15)	(−9.41)	(−10.40)	(−9.61)	(−13.57)	(−12.62)	(−14.22)	(−13.09)
Capital expenditure	−0.777***	−0.779***	−0.792***	−0.812***	−8.135***	−6.568***	−8.325***	−6.959***
	(−11.98)	(−9.88)	(−12.24)	(−10.29)	(−11.95)	(−8.44)	(−12.60)	(−9.17)
Acquisition	−0.333***	−0.338***	−0.338***	−0.343***	−2.821***	−2.890***	−2.888***	−2.940***
	(−10.42)	(−10.45)	(−10.61)	(−10.70)	(−7.28)	(−8.16)	(−7.60)	(−8.45)
R&D	0.047***	0.044***	0.045***	0.042***	0.333***	0.305***	0.304**	0.280**
	(2.85)	(2.84)	(2.67)	(2.65)	(2.82)	(2.76)	(2.50)	(2.47)
Dividend	−0.084***	−0.076***	−0.085***	−0.077***	−0.779***	−0.673***	−0.798***	−0.692***
	(−11.24)	(−10.03)	(−11.42)	(−10.23)	(−10.90)	(−9.48)	(−11.22)	(−9.83)
Intercept	0.410***		0.383***		−0.590**		−0.924***	
	(14.44)		(12.02)		(−2.55)		(−3.56)	
Year fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
Industry fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
Number of observations	15,258	15,258	15,258	15,258	15,217	15,217	15,217	15,217
Adjusted R-squared	0.402	0.443	0.406	0.447	0.378	0.440	0.387	0.447

- We examine the effect of CEO overconfidence on the value of cash following the model in Faulkender and Wang (2006). The CEO overconfidence indicator and its interaction term with change in cash are included in the model.
- The coefficients on overconfidence are significantly negative, suggesting that CEO overconfidence is value decreasing for firms.
- The coefficients on the interaction term are significantly positive, indicating that market participants view the increase in cash holdings to be more valuable for firms with overconfident CEOs. This is consistent with **Hypothesis 3** rather than **Hypothesis 4**.
- The coefficients on most of the other variables are consistent with the findings in Faulkender and Wang (2006).

Financial Constraint Criteria				Payout Ratio	Size	Bond Ratings	Paper Ratings
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Change in cash	1.282***	1.483***	0.664***	0.677**	0.531*	0.704***	0.699**
	(9.49)	(8.22)	(2.75)	(2.25)	(1.69)	(2.86)	(2.41)
Change in non-cash assets	0.224***	0.232***	0.090**	0.050	0.012	0.102**	0.113**
	(4.98)	(5.22)	(2.02)	(0.88)	(0.24)	(2.29)	(2.51)
Change in earnings	0.370***	0.367***	0.436***	0.423***	0.473***	0.433***	0.435***
	(5.66)	(5.63)	(6.21)	(5.27)	(5.74)	(6.19)	(6.19)
Change in R&D	0.307	0.292	−0.482	0.116	−0.229	−0.297	−0.318
	(0.30)	(0.29)	(−0.44)	(0.10)	(−0.15)	(−0.27)	(−0.29)
Change in interest expense	−2.613***	−2.629***	−2.004**	−1.870	0.622	−2.058**	−2.095**
	(−2.75)	(−2.79)	(−1.96)	(−1.49)	(0.51)	(−2.04)	(−2.06)
Change in dividends	1.321	1.331	0.271	−0.251	0.923	0.180	0.279
	(1.37)	(1.39)	(0.24)	(−0.17)	(0.55)	(0.16)	(0.24)
Net financing	−0.261***	−0.274***	−0.017	−0.019	−0.054	−0.041	−0.046
	(−2.79)	(−2.90)	(−0.17)	(−0.15)	(−0.51)	(−0.42)	(−0.48)
Leverage	−0.464***	−0.462***	−1.045***	−1.028***	−1.132***	−1.022***	−0.926***
	(−8.90)	(−8.87)	(−22.79)	(−16.28)	(−20.83)	(−22.26)	(−19.01)
Lagged cash	0.638***	0.636***	0.288***	0.346***	0.401***	0.308***	0.332***
	(8.69)	(8.79)	(3.86)	(3.79)	(4.03)	(4.14)	(4.42)
Lagged cash×Change in cash		−0.395	0.028	−0.195	−0.268	0.002	−0.000
		(−1.32)	(0.09)	(−0.56)	(−0.67)	(0.01)	(−0.00)
Overconfidence			−0.222***	−0.214***	−0.140***	−0.193***	−0.179***
			(−13.63)	(−9.92)	(−7.04)	(−11.66)	(−10.35)
Overconfidence×Change in cash			0.631**	0.496	0.917***	0.603**	0.607**
			(2.37)	(1.56)	(2.60)	(2.25)	(2.27)
Constrained dummy				−0.101***	−0.277***	−0.157***	−0.124***
				(−3.27)	(−5.33)	(−6.65)	(−6.70)
Constrained×Change in cash				0.552*	1.024**	0.093	0.032
				(1.73)	(2.15)	(0.26)	(0.12)
Number of observations	11,910	11,910	10,250	6,412	5,917	10,250	10,250
Adjusted R-squared	0.045	0.045	0.135	0.137	0.155	0.139	0.139

Value of Cash and CEO Overconfidence (Propensity Score Matching)

National Taiwan University

Financial Constraint Criteria				Payout Ratio	Size	Bond Ratings	Paper Ratings
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Change in cash	1.148***	1.230***	0.387*	0.684**	1.706***	0.440*	0.473
	(6.53)	(5.55)	(1.71)	(2.31)	(4.43)	(1.87)	(1.48)
Change in non-cash assets	0.223***	0.223***	0.088	0.063	0.084	0.104*	0.123**
	(4.00)	(4.00)	(1.51)	(0.91)	(1.20)	(1.80)	(2.14)
Change in earnings	0.553***	0.559***	0.620***	0.639***	0.767***	0.628***	0.617***
	(6.27)	(6.10)	(6.70)	(6.53)	(6.94)	(6.76)	(6.53)
Change in R&D	0.446	0.458	−0.717	0.558	1.311	−0.438	−0.489
	(0.47)	(0.48)	(−0.74)	(0.53)	(1.02)	(−0.45)	(−0.51)
Change in interest expense	−3.474**	−3.528**	−3.469**	−3.784**	1.364	−3.623**	−3.541**
	(−2.47)	(−2.51)	(−2.24)	(−2.17)	(0.70)	(−2.36)	(−2.33)
Change in dividends	−0.408	−0.371	−0.290	−3.653**	−1.710	−0.378	−0.150
	(−0.31)	(−0.28)	(−0.21)	(−2.33)	(−0.95)	(−0.27)	(−0.11)
Net financing	−0.119	−0.124	0.132	0.265	−0.207	0.109	0.086
	(−0.84)	(−0.87)	(0.91)	(1.51)	(−1.15)	(0.75)	(0.60)
Leverage	−0.524***	−0.521***	−1.104***	−1.079***	−1.305***	−1.044***	−0.887***
	(−6.90)	(−6.84)	(−17.19)	(−12.69)	(−15.16)	(−16.15)	(−12.52)
Lagged cash	0.799***	0.799***	0.269***	0.404***	0.329***	0.306***	0.358***
	(8.51)	(8.54)	(3.07)	(4.01)	(2.67)	(3.50)	(4.07)
Lagged cash×Change in cash		−0.207	0.217	−0.106	−0.703	0.149	0.171
		(−0.58)	(0.63)	(−0.29)	(−1.01)	(0.43)	(0.51)
Overconfidence			−0.227***	−0.233***	−0.129***	−0.206***	−0.178***
			(−9.50)	(−8.05)	(−4.26)	(−8.58)	(−7.18)
Overconfidence×Change in cash			0.784**	0.711*	−0.048	0.847**	0.781**
			(2.30)	(1.85)	(−0.09)	(2.42)	(2.31)
Constrained dummy				−0.118***	−0.170***	−0.166***	−0.173***
				(−3.22)	(−3.49)	(−5.06)	(−6.81)
Constrained×Change in cash				0.173	−0.008	−0.137	−0.026
				(0.46)	(−0.01)	(−0.29)	(−0.08)
Number of observations	5,945	5,945	5,944	4,037	3,409	5,944	5,944
Adjusted R-squared	0.057	0.057	0.120	0.135	0.142	0.125	0.128

- We examine the source of cash for firms with and without overconfident CEOs using the model in McLean (2011).
- Our baseline model shows that firms save a greater portion of their proceeds from equity issuance on average, which is consistent with the findings in McLean (2011).
- The coefficients on the interaction terms of overconfidence and the source of cash are all positive, implying that overconfident CEOs save more cash from both external and internal sources. Such relation is especially significant for external source of cash.
- The coefficients on interaction terms of precautionary motive, equity/debt issuance, and overconfidence are positive, indicating that overconfident CEOs save larger portions of cash from equity and debt issuance if they have precautionary motive to do so. However, such effect is not found for internal source of cash.

Proxy for Precautionary Motive				R&D	Cash Flow Volatility	Dividend
Model	(1)	(2)	(3)	(4)	(5)	(6)
Equity	0.730***	0.730***	0.551***	0.567***	0.552***	0.562***
	(30.83)	(30.80)	(11.18)	(11.47)	(11.19)	(11.23)
Debt	0.059***	0.059***	0.031***	0.023**	0.031***	0.032***
	(3.86)	(3.86)	(3.31)	(2.50)	(3.27)	(3.34)
Cash flow	0.256***	0.256***	0.190***	0.191***	0.189***	0.193***
	(10.79)	(10.64)	(7.45)	(7.52)	(7.40)	(7.35)
Other	−0.029	−0.029	−0.026	−0.029*	−0.026	−0.023
	(−1.52)	(−1.52)	(−1.36)	(−1.70)	(−1.38)	(−1.22)
Assets	0.010***	0.010***	0.011***	0.008***	0.009***	0.011***
	(3.66)	(3.66)	(3.80)	(2.90)	(3.41)	(3.91)
Overconfidence		−0.001	−0.017***	−0.019***	−0.017***	−0.017***
		(−0.26)	(−3.54)	(−4.66)	(−3.36)	(−3.56)
Equity × Overconfidence			0.199***	0.049	0.465***	0.202***
			(3.83)	(0.88)	(3.11)	(3.82)
Debt × Overconfidence			0.037**	−0.024*	0.137**	0.048**
			(1.98)	(−1.80)	(2.10)	(2.28)
Cash flow × Overconfidence			0.093***	0.150***	0.140**	0.098***
			(2.69)	(4.86)	(2.11)	(2.68)
Precautionary Motive				−0.106	−0.007**	−0.431***
				(−1.36)	(−2.27)	(−5.03)
Precautionary × Equity × Overconfidence				0.371***	0.143**	−6.921***
				(3.38)	(1.97)	(−4.13)
Precautionary × Debt × Overconfidence				1.354***	0.045*	−0.964**
				(4.87)	(1.82)	(−2.06)
Precautionary × Cash flow × Overconfidence				0.022	0.018	0.361
				(0.14)	(0.68)	(0.63)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of observations	23,042	23,042	23,042	23,042	23,035	22,886
Adjusted R-squared	0.465	0.465	0.469	0.511	0.472	0.475

Source of Cash and CEO Overconfidence (Propensity Score Matching)

National Taiwan University

Proxy for Precautionary Motive				R&D	Cash Flow Volatility	Dividend
Model	(1)	(2)	(3)	(4)	(5)	(6)
Equity	0.793***	0.793***	0.648***	0.658***	0.652***	0.654***
	(21.76)	(21.71)	(7.61)	(7.81)	(7.65)	(7.75)
Debt	0.105***	0.106***	0.026	0.020	0.026	0.027
	(3.76)	(3.77)	(1.41)	(1.06)	(1.37)	(1.46)
Cash flow	0.345***	0.341***	0.288***	0.293***	0.287***	0.297***
	(8.95)	(8.80)	(4.97)	(5.06)	(4.96)	(4.99)
Other	−0.023	−0.023	−0.020	−0.035	−0.021	−0.019
	(−0.72)	(−0.72)	(−0.66)	(−1.20)	(−0.70)	(−0.61)
Assets	0.016***	0.016***	0.017***	0.016***	0.017***	0.017***
	(3.10)	(2.96)	(3.20)	(3.11)	(3.13)	(3.27)
Overconfidence		0.010**	−0.014	−0.017*	−0.016	−0.013
		(2.45)	(−1.48)	(−1.85)	(−1.64)	(−1.38)
Equity × Overconfidence			0.170**	0.117	0.502**	0.165**
			(2.01)	(1.33)	(2.45)	(1.96)
Debt × Overconfidence			0.137***	0.003	0.242**	0.171***
			(3.21)	(0.09)	(2.31)	(3.43)
Cash flow × Overconfidence			0.067	0.158**	0.005	0.066
			(0.99)	(2.10)	(0.04)	(0.92)
Precautionary Motive				−0.015	−0.001	−0.561
				(−0.13)	(−0.16)	(−1.58)
Precautionary × Equity × Overconfidence				0.042	0.183*	−2.273
				(0.31)	(1.75)	(−0.98)
Precautionary × Debt × Overconfidence				1.570***	0.054	−3.927***
				(4.16)	(1.25)	(−3.03)
Precautionary × Cash flow × Overconfidence				−0.254	−0.041	0.826
				(−0.87)	(−0.91)	(0.66)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of observations	12,507	12,507	12,507	12,507	12,507	12,442
Adjusted R-squared	0.546	0.546	0.555	0.598	0.557	0.563

- We conduct a comprehensive study on CEO overconfidence and corporate cash policy from three aspect, **level of cash holdings**, **value of cash**, and **source of cash**.
- We find that firms with overconfident CEOs tend to have higher cash holdings as well as higher marginal value of cash than those without. This finding verifies the costly external financing hypothesis in the CEO overconfidence literature and is consistent with the **trade-off theory** of cash holdings.
- The above effect is lower for firms with higher growth opportunity, because firms with higher growth opportunity have better accessibility and lower costs to external financing, reducing the **transaction motive** of cash holdings.

Conclusion

- We further show that firms with overconfident CEOs would save more cash than those without, especially from external sources. This finding indicates that overconfident CEOs would save as much cash as possible due to higher financing costs for investments.
- The above effect is especially significant when firms have large R&D expenditure or large volatility, being consistent with the **precautionary motive** of cash holdings.
- The empirical results remain robust when we deal with potential sample selection bias due to the endogeneity of firms' decision in hiring overconfident or non-overconfident CEOs.
- Overall, our empirical results indicate that in addition to the demand of capital and the economic condition, the **manager's belief** also demonstrates a strong impact on corporate cash policy.