Introduction to Search Theory of Money

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The Art of Monetary Theory

- *This, as I see it, is really the central issue in the pure theory of money. Either we have to give an explanation of the fact that people do hold money when rates of interest are positive, or we have to evade the difficulty somehow... The great evaders ... would have put it down to "frictions," and since there was no adequate place for frictions in the rest of their economic theory, a theory of money based on frictions did not seem to them a promising field for economic analysis. This is where I disagree. I think we have to look frictions in the face, and see if they are really so refractory after all.* Hicks (1935)

- *Progress can be made in monetary theory and policy analysis only by modeling monetary arrangements explicitly.* Kareken and Wallace (1980)

Issues

- Existence and essentiality of fiat money
  Why would intrinsically worthless money have value, or more generally, how can asset prices differ from "fundamental" values? How can fiat money improve the efficiency of resource allocations?
- Return dominance
  Why is money dominated in the rate of return by other assets and, in particular, by government issued nominal bonds?
- Monetary policy
  How does monetary policy affect transactions, prices and allocations? Do we get new insight if using a model where the means-of-payment decisions and liquidity of assets are modeled explicitly?

Issues (con't)

- Money, credit, banking, and payments systems
  How does credit work absent commitment? How can credit and money coexist? What is the role of assets in credit arrangements? What are the roles of intermediation, and of inside and outside money?
- International monetary arrangements What determines the currency regime? What determines the nominal exchange rate between currencies? What are policy implications in an international currency regime?

Microfoundation of money

- Search monetary theory is about trying to understand the process of exchange in the presence of frictions, and how this process might be facilitated by institutions, including money, but also credit, intermediation, and the use of assets as payment instruments or as collateral.

Essentiality of money and liquidity

- Money is essential if it improves the efficiency of resource allocations relative to an economy without money.
- Money as a medium of exchange
  Whether an object circulates as a medium of exchange depends on its intrinsic properties and extrinsic beliefs.
- To derive endogenously assets with different rates of return and liquidity, one needs to model 'frictions' by making explicit assumptions about spatial, temporal, and information, and emphasize the fundamental characteristics of assets.

Multiple Assets

- money vs credit
- different commodity monies - gold and silver
- outside money vs inside money (private money)
- money vs deposits
- money vs government bonds (or, private debt)
- money vs equity
- genuine assets vs counterfeit assets
- money vs capital
- national money vs international money
- legal money vs illegal money

A Search-theoretic Approach to Monetary Economics

Kiyotaki and Wright (1993 AER)

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Introduction

- Using a double-coincidence-of-wants problem to motivate the medium of exchange role of money.
- Same coconut story as in Diamond (1982), only that they come in different colors (or, many goods).

Model: agents, goods, and production

- A continuum of infinitely lived agents $\in [\,0, 1\,]$.
- One indivisible good, zero storage cost, one unit at a time.
- Production technology:
  - potential projects arrive according to a Poisson process with constant arrival rate α. Each project yields one unit of good at cost c (in term of disutility).
  - For now, we assume production is free and instantaneous: $c = 0$ with prob. 1, and $\alpha = \infty$.

Model: consumption

- Utility function:

$$u(z) = \begin{cases} u & z \leq x \\ 0 & z > x \end{cases}$$

  where $u > 0$ and $x \in (0, 1)$.

- The prob. that a random trader accepts any real commodity is $prob(z \leq x) = x$.

- Consumption is a necessary input into production.

Model: Meeting technology

- Agents meet trading partner bilaterally according to a Poisson process with arrival rate β.
- Assume $\beta = \beta(N)$, N is number of traders.
- Underlying meeting technology:
  $m(N)$ the number of meeting per unit of time. $m'(N) > 0$
  - $\beta(0) = m(0) = 0$.
  - When $N > 0$, $\beta(N) = \frac{m(N)}{N}$
    If $m(N)$ displays IRS, then $\beta'(N) > 0$
    If $m(N)$ displays DRS, then $\beta'(N) < 0$
    If $m(N)$ displays CRS, then $\beta'(N) = 0$.

Model: money and transaction

- Indivisible fiat money $\rightarrow$ every trade is a one-for-one swap.
- $M =$ measure of agents endowed with one unit of money. $1 - M =$ measure of agents endowed with one unit of real commodity.
- Transaction cost $\varepsilon > 0$, when accepting any goods
  $\Rightarrow$ In equilibrium, an agent will never accept a commodity that is not his consumption good.
- The only non-trivial decision: whether to accept fiat money.

Bellman equations

- $\Pi =$ prob. that a random commodity trader accepts fiat money.
- $\pi =$ best response of a representative agent.
- In steady state, Bellman equations are

$$\begin{aligned} rV_g &= \beta(1-M)x^2(u-\varepsilon) \\ &\quad +\beta M x \max_{\pi}(V_m - V_g, 0) \qquad (1) \\ rV_m &= \beta(1-M)x\Pi(u-\varepsilon+V_g-V_m) \qquad (2) \end{aligned}$$

Best response conditions

- Strategy: accept money with prob π, where

$$\pi = \begin{cases} 1 & \text{if } V_m > V_g \\ [0,1] & \text{if } V_m = V_g \\ 0 & \text{if } V_m < V_g \end{cases}$$

- V_m and V_g depend on Π:
  individual's best response correspondence is $\pi = \pi(\Pi)$.
- An equilibrium for the model is a fixed point $\Pi = \pi(\Pi)$.

Multiple equilibria

Claim. There are three equilibria:

- $\Pi > x \Rightarrow V_m > V_g \Rightarrow \pi = 1$: pure monetary eqm.
- $\Pi = x \Rightarrow V_m = V_g \Rightarrow \pi \in [0, 1]$: mixed eqm.
- $\Pi < x \Rightarrow V_m < V_g \Rightarrow \pi = 0$: nonmonetary eqm.
- Multiplicity results from a coordination problem in the social choice of media of exchange. $\rightarrow$ government's policy help influence this choice.

Search, Bargaining, Money and Prices

Trejos and Wright (1995 *JPE*)

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Motivation

- Introduce bargaining into Kiyotaki and Wright (1991,1993) in order to determine the price level endogenously.

Model: agents and meeting technology

- $M \in (0, 1)$: buyers; indivisible money
  $1 - M$: sellers; divisible good
- Agents do not consume their own output.
- Traders meet according to Poisson arrival rates that are proportional to the number of agents on the other side of market.
- $x =$ prob. (the trade partner likes my output), but both can produce each others' consumption goods with prob. 0.
  $\Rightarrow$ No double coincidence of wants, so all trades involve a buyer paying with cash.

Model: production and preferences

- $u(q)$: utility of consuming q units of good
  $c(q)$: disutility of producing q units of good
- Assume

  $u(0) = c(0)$,
  $u'(0) > c'(0) = 0$,
  $u'(q) > 0, c'(q) > 0$
  $u'' \leq 0, c'' \geq 0$ for $q > 0$
  with at least one of the weak inequalities strict.

- $\exists\ \hat{q} > 0 \quad$ s.t. $\quad u(\hat{q}) = c(\hat{q})$

Monetary theory

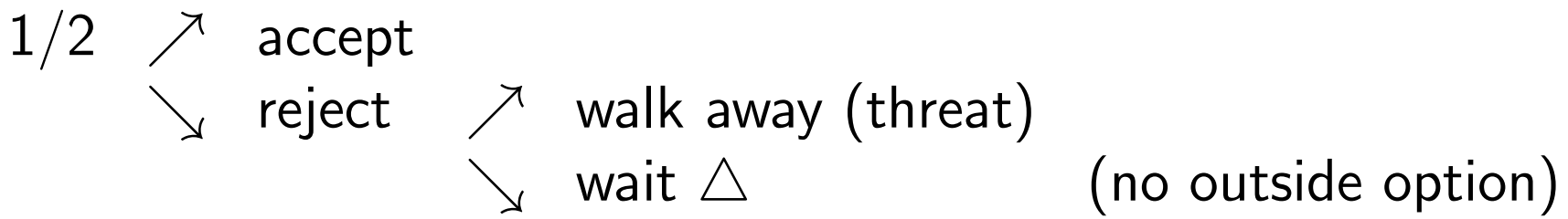
- If $q = Q$ is taken as given, then

$$rV_s = \alpha Mx[V_b - V_s - c(Q)] \quad (1)$$
$$rV_b = \alpha(1-M)x[u(Q) + V_s - V_b] \quad (2)$$

  can be solved for $V_b = V_b(Q)$ and $V_s = V_s(Q)$
- In the following, normalize $\alpha x = 1$.
- Sellers accept money implies
  $V_b - c(Q) > V_s > 0 \Leftrightarrow u(Q) > \frac{1+r-M}{1-M} c(Q)$.
  And $V_b > 0$ implies money has value.

Price theory

Strategic bargaining game:



Strategic bargaining game

- Although offers are never rejected in eqm, it is the threat of rejecting and delaying settlement that drives the solution.
- A unique subgame perfect equilibrium to the bargaining game (V_b and V_s taken as given).
- This equilibrium has the property that a seller always proposes $q_s = q_s(\Delta)$, a buyer always proposes $q_b = q_b(\Delta)$ and these proposals are always accepted.
- As $\Delta \to 0$, $q_s \to q$ and $q_b \to q$ where q is a Nash bargaining solution.

Bargaining solution

- Offers q_s and q_b are constructed so that the proposer gets as much utility as possible, subject to the responder's acceptance of his offer.

$$q_s: \qquad V_s + u(q_s) = \frac{1}{1+r\Delta}[V_s + \frac{1}{2}u(q_s) + \frac{1}{2}u(q_b)]$$

$$q_b: \qquad V_b - c(q_b) = \frac{1}{1+r\Delta}[V_b - \frac{1}{2}c(q_s) - \frac{1}{2}c(q_b)]$$

Bargaining solution (con't)

- As $\Delta \to 0$, $q_s \to q$, and $q_b \to q$; i.e.

$$\frac{V_s + u(q)}{V_b - c(q)} = \frac{u'(q)}{c'(q)}$$

- The solution to the symmetric Nash bargaining game without threat points:

$$q = \arg\max[u(q) + V_s(q)][V_b(q) - c(q)]$$

- Bargaining solution with outside options

$$q = \arg\max[V_s + u(q) - V_b][V_b - c(q) - V_s].$$

Definition of equilibrium

- When bargaining over q, agents take $V_b(Q)$ and $V_s(Q)$ as given. In eqm, $q = Q$.
- A steady-state equilibrium is a list (Q, V_s, V_b) satisfying: (i) $q = Q$ solves the bargaining problem

$$q = \arg\max[u(q) + V_s(q)][V_b(q) - c(q)]$$

s.t. $\underbrace{u(q) + V_s(Q) \geq V_b(Q) \text{ and } V_b(Q) - c(q) \geq V_s(Q)}_{\text{participation conditions}}$

taking $V_b(Q)$ and $V_s(Q)$ as given; and (ii) V_s and V_b satisfy (1) and (2), taking Q as given.

Equilibrium

- Taking $V_b = V_b(Q)$ and $V_s = V_s(Q)$ as given, FOC:

$$\frac{c'(q)}{u'(q)} = \frac{V_b(Q) - c(q)}{u(q) + V_s(Q)} \tag{3}$$

- Eq. (3) defines a function $q = e(Q)$ [or, $T(q) = 0$] goes though the origin and intersect the 45° line at a unique point q^e
- Show $T(0) = 0,\ T'(0) > 0,\ T(q_1) < 0.$
  By continuity, there exists a $q \in (0, q_1)$ s.t. $T(q) = 0.$

Proposition 1

For any $r > 0$ and $M \in (0, 1)$, there exists a nonmonetary steady state equilibrium and a unique monetary steady state equilibrium. The monetary equilibrium is unconstrained, and it satisfies $u'(q) > c'(q)$.

搜尋模型在貨幣經濟的應用

本課程將介紹貨幣搜尋模型及其應用。自從 Kiyotaki and Wright (1989) 提出貨幣搜尋模型 (search monetary models) 來解釋貨幣如何出現，如何有價值等問題以來，該模型已被廣泛用來研究許多貨幣經濟的重要問題。傳統總體貨幣模型通常假設貨幣帶給人們效用而將貨幣直接放在效用函數中 (money-in-the-utility function)，或者假設交易一定須使用貨幣而採取「交易付現限制」(cash-in-advance constraint) 等等。雖然這些模型在研究許多總體經濟的問題上已有很好的成果，但是如果我們關心的問題是貨幣為什麼有價值、在什麼情況下人們會使用貨幣或信用進行交易、在多種資產存在的的情況下，人們會使用哪一種資產作為支付工具、而人們的決策和資產本身的特性如何影響資產之間的流動性差異、以及何種貨幣會成為國際通貨等問題，我們就不能先驗上對貨幣的使用作假設。貨幣搜尋模型的特色在於能夠明確描述交易型態與交易障礙，因此適於因此適於研究以上議題。

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