Intertemporal Relation between Risk and Return: Panel Quantile Regression Approach

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[Very Preliminary Draft]:
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What this paper does...

- Explore the complete empirical linkage between risk and return after controlling for risk premia induced by other intertemporal hedging demand (the time-series relation)
- Key ingredients: ICAPM, ADCC, PQRFE, panel data
- Advantages of our methodology:
  - first-stage ADCC estimation provides characterization of time-series variation in large-scale conditional covariance
  - second-stage PQR with fixed effects explores the shape of conditional distribution of excess returns, and heterogeneity.

Roadmap Ahead

- Literature review, motivation, and contributions
- ICAPM and econometric methodology
  - Intertemporal Capital Asset Pricing Model
  - (Asymmetric) Dynamic Conditional Correlation Model
  - Panel Quantile Regression with Fixed Effects
- Data: Taiwan stock market, panel data
- Empirical results

Controversy in risk-return tradeoff

- Theory
  - positive relation between the conditional mean and variance of excess returns on the market portfolio is consistent with equilibrium: Merton's (1973, Econometrica) ICAPM
  - negative relation between the expected return and volatility is consistent with equilibrium: Abel (1988, JME), Gennotte and Marsh (1993, EER)
- Empirical studies
  - risk-return relation insignificant: French, Schwert, and Stambaugh (1987, JFE), Glosten, Jagannathan, and Runkle (1993, JF), Bollerslev and Zhou (2006, JoE)
  - negative intertemporal relation between risk and return: Campbell (1987, JFE), Nelson (1991, Econometrica), Harvey (2001, JEF)
  - positive relation between expected return and risk: Ghysels, Santa-Clara, and Valikanov (2005, JFE), Bali (2008, JFE), Bali and Engle (2010, JME)

Motivation

- The existing literature: estimating risk-return relation on average
  - asymmetric risk-return relation might result in the controversial results observed in the literature
- Thus, would like to recover the complete picture of risk-return relation
  - in addition, within the ICAPM framework, explore the complete relation between return and other state variables (macroeconomic variables, market uncertainty, liquidity, default risk, and etc.) which induce risk premia and intertemporal hedging demand

Contriutions

- This paper is closely related to Bali (2008) and Bali and Engle (2010): panel data
  - different assets
  - risk/volatility proxy
  - econometric methods
  - subsamples/finite-sample bias
- Explore the complete empirical linkage between risk and return after controlling for risk premia induced by other intertemporal hedging demand.
- PQRFE results are robust to inclusion of different state variables
- Minor: Taiwan stock market, large-scale panel data

Intertemporal Capital Asset Pricing Model

- Merton's (1973) ICAPM implies the following equilibrium relation between risk and return:

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = A \cdot Cov_t(r_{t+1}, r_{m,t+1}) + Cov_t(r_{t+1}, x_{t+1}) \cdot B, \qquad (\star)$$

where

- r_{t+1} is the stock returns,
- $r_{f,t}$ is the risk free rate,
- $r_{m,t+1}$ is the market return, and
- x_{t+1} is a vector of k state variables that shift the investment opportunity set.
- $Cov_t(r_{t+1}, r_{m,t+1})$ is the time-t expected conditional covariance between r_{t+1} and $r_{m,t+1}$.

Intertemporal Capital Asset Pricing Model (Contd.)

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = A \cdot Cov_t(r_{t+1}, r_{m,t+1}) + Cov_t(r_{t+1}, x_{t+1}) \cdot B \qquad (\star)$$

- Implications from the theory:
  - Intercepts in equation $(\star)$ are zero
  - the slope coefficient A is a scalar that is appropriate for all assets
  - B is a $k \times 1$ vector that prices all assets.
- Equation $(\star)$ is equivalent to non-exsitence of arbitrage when the pricing kernel is a linear function of the aggregate market return (additional factors could be added).

Intertemporal Capital Asset Pricing Model (Contd.)

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = A \cdot Cov_t(r_{t+1}, r_{m,t+1}) + Cov_t(r_{t+1}, x_{t+1}) \cdot B \qquad (\star)$$

- Common setting: A, B: time-invariant; Cov: non-stochastic market portfolio investigated only: $Cov = Var$
- Bali and Engle (2010): if the covariances are stochastic, they would represent additional sources of variation in the investment opportunity set and potential hedging demand terms.

Intertemporal Capital Asset Pricing Model (Contd.)

- The expression in $(\star)$ can now be written as

$$Var_t \begin{pmatrix} r_{t+1} \\ r_{m,t+1} \\ x_{t+1} \end{pmatrix} = \begin{pmatrix} H_{r,r,t+1} & H_{r,m,t+1} & H_{r,x,t+1} \\ H_{m,r,t+1} & H_{m,m,t+1} & H_{m,x,t+1} \\ H_{x,r,t+1} & H_{x,m,t+1} & H_{x,x,t+1}, \end{pmatrix}$$

$$\mathbb{E}_t[r_{t+1}] = r_{f,t} + A \cdot H_{r,m,t+1} + H_{r,x,,t+1} \cdot B,$$

$$\mathbb{E}_t[r_{m,t+1}] = r_{f,t} + A \cdot H_{m,m,t+1} + H_{m,x,,t+1} \cdot B.$$

- If normality is assumed, then the first two moments are sufficient to define the distribution
- hence the equations above define the process regardless of how many assets are being priced.

Dynamic Conditional Correlation

- We'd like to estimate the time-varing conditional covariances.
- Engle's (2002) DCC parameterizes the volatilities and correlations separately.
- We follow Engle (2009, Anticipating Correlations) to briefly introduce the three general steps in the specification and estimation of a DCC model.
- Let

$$y_{t+1} = \begin{pmatrix} r_{t+1} \\ r_{m,t+1} \\ x_{t+1} \end{pmatrix}.$$

Dynamic Conditional Correlation (Contd.)

Step 1: DE-GARCHing

- Matrix notation:

$$\mathbb{E}_{t-1}[y_t y_t'] = H_t = D_t R_t D_t,$$

where $D_t^2 = \text{diag}\{H_t\}$.

- The conditional correlation matrix, R_t, is simply the covariance matrix of the standardized residuals, ε_t:

$$\begin{aligned} R_t &= Var_{t-1}(D_t^{-1} y_t) \\ &= Var_{t-1}(\varepsilon_t). \end{aligned}$$

Dynamic Conditional Correlation (Contd.)

Step 1: DE-GARCHing (Contd.)

- The diagonal elements of D_t are the square root of the expected variance of each asset; that is,

$$H_{i,i,t} = \mathbb{E}_{t-1}(y_{i,t}^2).$$

  - e.g., GARCH(1,1):

$$H_{i,i,t} = \omega_i + \alpha_i y_{t-1}^2 + \beta_i H_{i,i,t-1}.$$

  - The standardized residuals are defined by

$$\varepsilon_{i,t} = y_{i,t} / \sqrt{H_{i,i,t}}.$$

This process is called DE-GARCHing a series.

Dynamic Conditional Correlation (Contd.)

Step 1: DE-GARCHing (Contd.)

- Note: an asymmetric version of the GARCH model can be used.
- For example: GJR-GARCH, Threshold ARCH (TARCH)
- The volatility specification of TARCH is expressed as

$$H_{i,i,t} = \omega_i + \alpha_i y_{t-1}^2 + \gamma_i y_{t-1}^2 I_{\{y_{t-1}<0\}} + \beta_i H_{i,i,t-1}.$$

- In many equity series, negative returns are much more influential than positive returns.

Dynamic Conditional Correlation (Contd.)

Step 2: Estimating the Quasi-Correlations

- The quasi-correlation matrix: Q
- Mean-reverting model:

$$Q_t = \Omega + \alpha \varepsilon_{t-1} \varepsilon'_{t-1} + \beta Q_{t-1}.$$

  - This process has two unknown dynamic parameters and $\frac{1}{2}N(N-1)$ parameters in the intercept matrix.

Dynamic Conditional Correlation (Contd.)

Step 2: Estimating the Quasi-Correlations (Contd.)

- **Correlation Targeting:** estimating the intercept parameters using an estimate of the unconditional correlations among the volatility-adjusted random variables. That is,

$$\widehat{\Omega} = (1-\alpha-\beta)\overline{R},$$

$$\overline{R} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t\varepsilon_t',$$

  reduces the number of remaining unknown parameters to two regardless of the number of variables in the system.
  - This great economy in parameters arises from the use of the auxiliary estimator of the correlations.
  - Implementable in large-scale asset portfolio.

Dynamic Conditional Correlation (Contd.)

Step 2: Estimating the Quasi-Correlations (Contd.)

- Substituting $\widehat{\Omega}$ into the mean-reverting DCC model, we then have

$$Q_t = \overline{R} + \alpha(\varepsilon_{t-1}\varepsilon_{t-1}' - \overline{R}) + \beta(Q_{t-1} - \overline{R}).$$

- In practice, the dynamic adjustment of correlations may be different for negative random variables than it is for positive ones.
- It has been observed that correlations increase faster when markets are declining.
- Thus we could consider the asymmetric DCC (ADCC).

Dynamic Conditional Correlation (Contd.)

Step 2: Estimating the Quasi-Correlations (Contd.)

- Mean-reverting ADCC:

$$Q_t = \Omega + \alpha \varepsilon_{t-1} \varepsilon'_{t-1} + \gamma \eta_{t-1} \eta'_{t-1} + \beta Q_{t-1},$$

where $\eta_t = \min\{\varepsilon_t, 0\}$.

  - The product $\eta_{i,t} \eta_{j,t}$ will be nonzero only if both variables are negative.
  - A positive value of γ will give the desired result: that correlations increase more in response to market declines than they do in response to market increase.

Dynamic Conditional Correlation (Contd.)

Step 2: Estimating the Quasi-Correlations (Contd.)

- **Correlation Targeting:** Mean-reverting ADCC

$$\widehat{\Omega} = (1-\alpha-\beta)\overline{R} - \gamma\overline{N},$$

$$\overline{R} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t\varepsilon_t',$$

$$\overline{N} = \frac{1}{T}\sum_{t=1}^{T}\eta_t\eta_t'.$$

- Consequently, we then have

$$\Omega = (1-\alpha-\beta-\gamma)\overline{N} + (1-\alpha-\beta)\overline{P},$$

where $\overline{P} = \frac{1}{T}\sum_{t=1}^{T}\pi_t\pi_t'$, $\pi_t = \max\{\varepsilon_t, 0\}$, $\varepsilon_t = \eta_t + \pi_t$, $\varepsilon_t\varepsilon_t' = \eta_t\eta_t' + \pi_t\pi_t'$, $\eta_t\pi_t' = 0$, and $\overline{R} = \overline{N} + \overline{P}$.

Dynamic Conditional Correlation (Contd.)

Step 3: Rescaling in DCC

- To convert Q processes into correlations, they must be rescaled.
- The following equation is called rescaling

$$\rho_{i,j,t} = \frac{Q_{i,j,t}}{\sqrt{Q_{i,i,t}Q_{j,j,t}}}$$

and its matrix form is given by

$$R_t = \text{diag}\{Q_t\}^{-1/2} Q_t \text{diag}\{Q_t\}^{-1/2}.$$

Finally, estimation is done by QMLE. The log-likelihood function has a standard form and can be maximized with respect to all the parameters in the model.

Penalized Quantile Regression with Fixed Effects

- Consider the conditional quantile functions of the response of the tth observation on the ith firm y_{it},

$$Q_{y_{it}}(\tau|x_{it}) = x_{it}'\beta(\tau) + \alpha_i.$$

  - The α's have a pure location shift effect on the conditional quantiles of the response.
  - The effects of the covariates x_{it} are permitted to depend upon the quantile, τ, of interest, but the α's do not.

Penalized Quantile Regression with Fixed Effects (Contd.)

- PQRFE estimator

$$\min_{\alpha,\beta} \sum_{k=1}^{K} \sum_{t=1}^{T} \sum_{i=1}^{N} v_k \rho_{\tau_k} \left(y_{it} - x_{it}' \beta(\tau_k) - \alpha_i \right) + \lambda \sum_{i=1}^{N} |\alpha_i|,$$

  where
  - $\rho_\tau(u) = u(\tau - I_{\{u<0\}})$,
  - v_k are weights,
  - λ is tuning parameter,
  - $\sum_{i=1}^{N} |\alpha_i|$ is ℓ_1 penalty/shrinkage (Lasso)
- Like all shrinkage (regularization) estimators, asymptotic inference is somewhat problematic.
- Thus, bootstrap is the natural first resort.

Data

- $r_t, r_{m,t}$: Stocks listed on Taiwan 50 Index (TW50)
- x_t: state variables shifting the investment opportunity set
  - macroeconomic variables (business cycle):
    - the term spread $TERM_{t+1}$
  - future market volatility:
    - following Campbell (1993, AER; 1996, JPE), investors are assumed to hedge against unexpected changes in future market volatility.
    - the future volatility is defined as the first-difference of the options implied volatility of TWSE index return. $\Delta VIX_{m,t+1}$.
  - funding liquidity risk: constructed specifically for Taiwan market. (could think of a proxy of default spread)
    - $Liquidity_{t+1}$
- Sample period: Jan. 4th, 2010 - Dec. 30th, 2011

Model

- Combinations:
  - GJR-GARCH, TARCH
  - DCC, ADCC
  - Controlling for the risk premium induced by conditional covariation with market volatility risk, macroeconomic variables and other state variables (ICAPM)
  - tuning parameter λ
- Our models: (A)DCC-PQRFE
- Benchmark: (A)DCC-panel least squares with fixed effects

Model (Contd.)

4 Models: different state variables

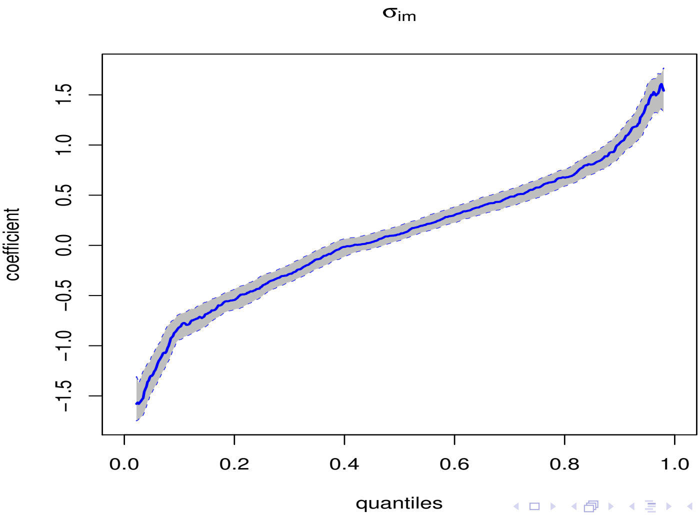
- e.g., system of equations:

$$R_{i,t+1}(\tau) = A(\tau)\cdot\sigma_{im,t+1} + B_V(\tau)\cdot\sigma_{i,\Delta VIX_{m,t+1}} + B_T(\tau)\cdot\sigma_{i,TERM,t+1} + B_L(\tau)\cdot\sigma_{i,Liquidity,t+1} + \alpha_i + e_{i,t+1}(\tau)$$

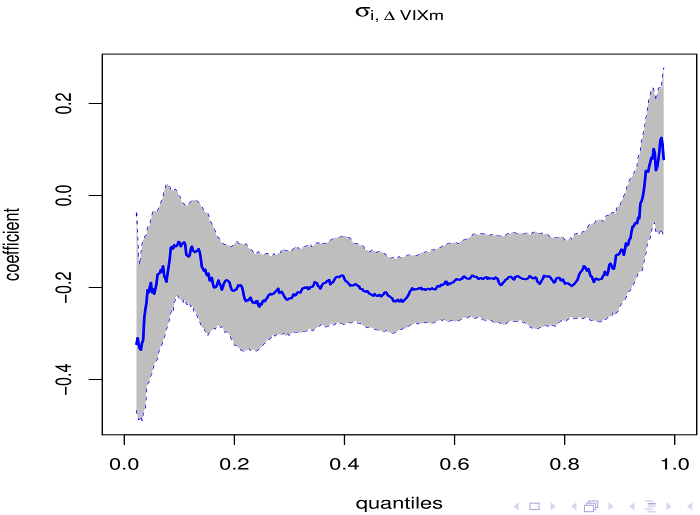
where

- $R_{i,t+1}$: excess returns
- $\sigma_{im,t+1}$: time-t expected conditional covariance between the excess returns on stock i and the market portfolio
- $\sigma_{i,\Delta VIX_{m,t+1}}$: time-$t$ expected conditional covariance between excess returns and the change in the option implied volatility of TWSE index
- $\sigma_{i,TERM,t+1}$: time-t expected conditional covariance between excess returns and the term spread
- $\sigma_{i,Liquidity,t+1}$: time-t expected conditional covariance between excess returns and the funding liquidity risk

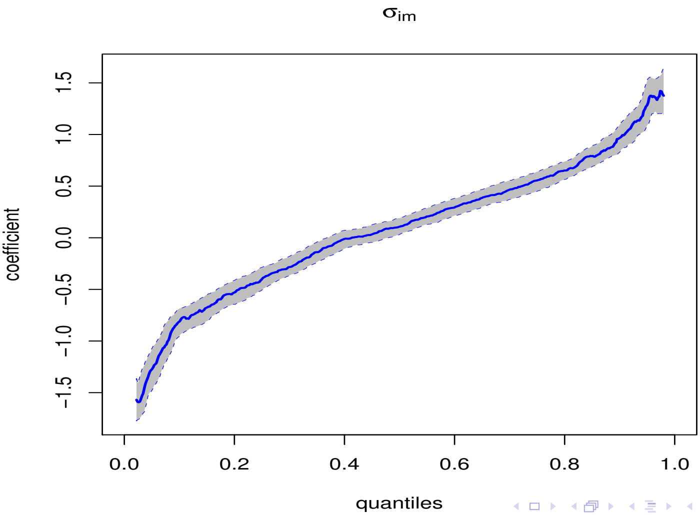
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$ $\lambda = 1$



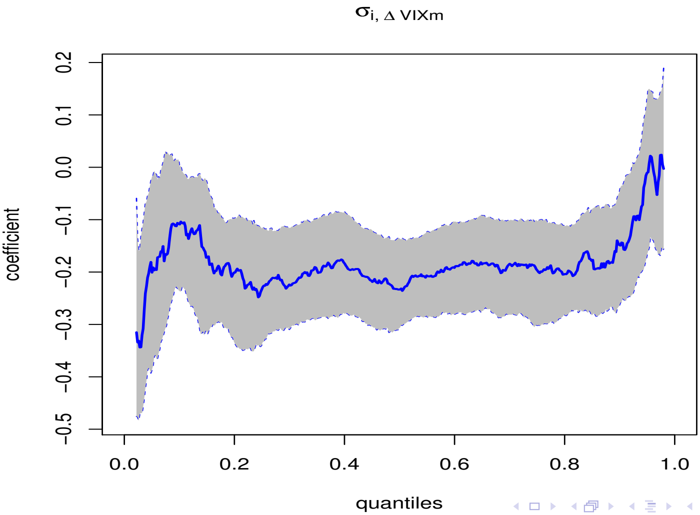
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$ $\lambda = 1$



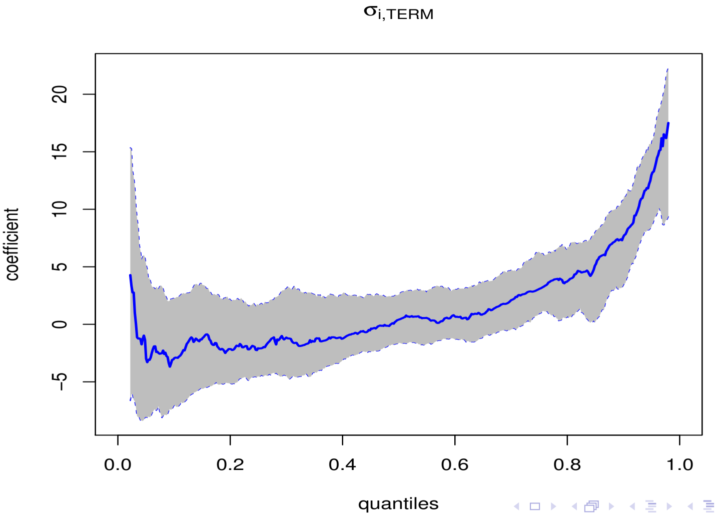
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM} $\lambda = 1$



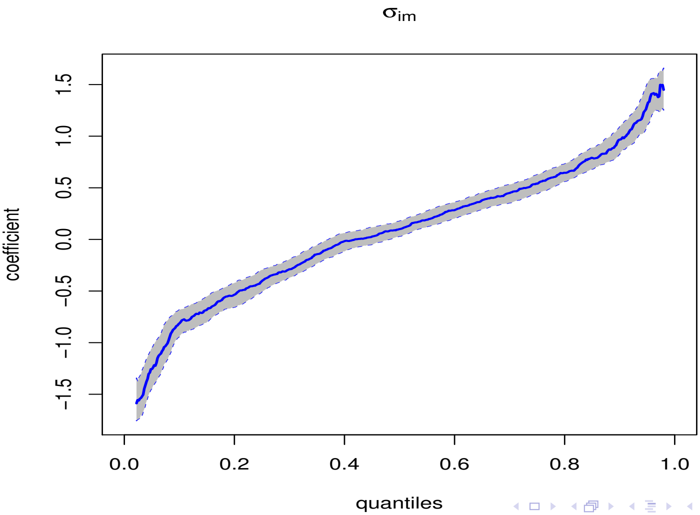
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM} $\lambda = 1$



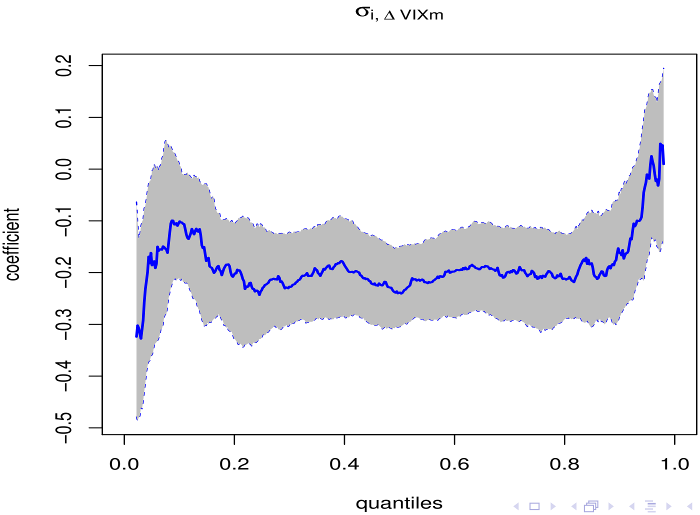
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM} $\lambda = 1$



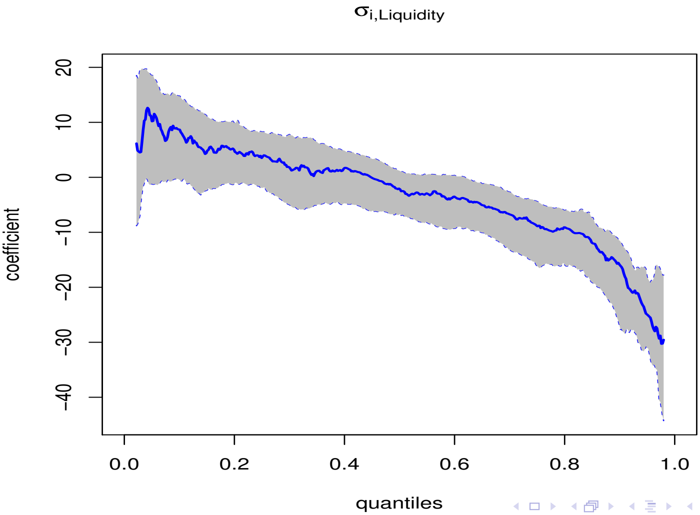
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, $\sigma_{iLiquidity}$ $\lambda = 1$



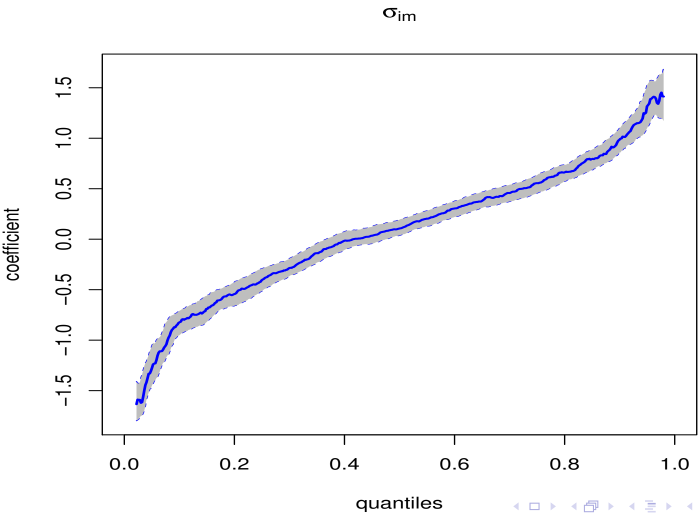
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, $\sigma_{iLiquidity}$ $\lambda = 1$



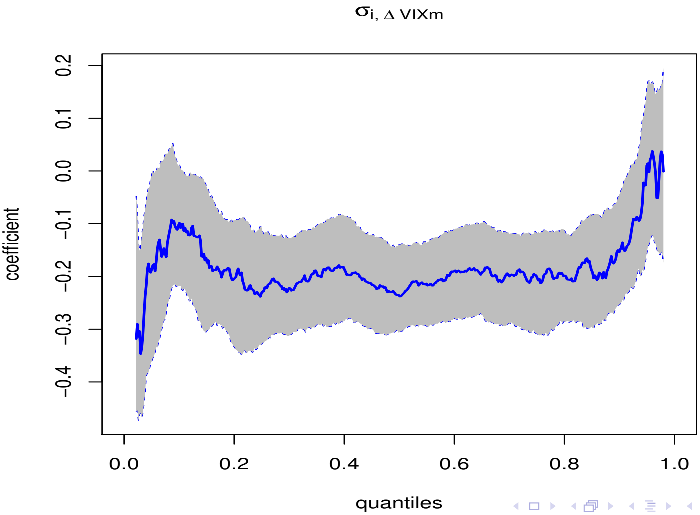
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, $\sigma_{iLiquidity}$ $\lambda = 1$



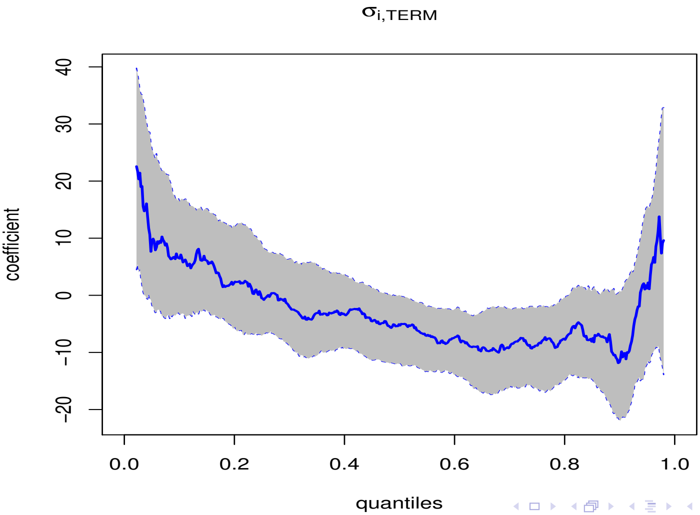
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM}, $\sigma_{iLiquidity}$ $\lambda = 1$



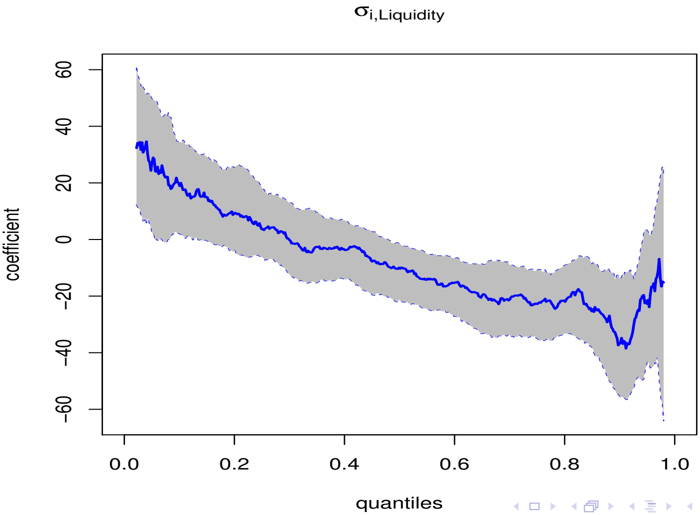
DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM}, $\sigma_{iLiquidity}$ $\lambda = 1$



DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM}, $\sigma_{iLiquidity}$ $\lambda = 1$



DCC-PQRFE: σ_{im}, $\sigma_{i\Delta VIX}$, σ_{iTERM}, $\sigma_{iLiquidity}$ $\lambda = 1$



Concluding Remarks

- Similar results: 4 large-scale panel data models with different state variables
- Similar results:
  - GJR-GARCH(1,1,1)-DCC, GJR-GARCH(1,0,1)-DCC
  - GJR-GARCH(1,1,1)-ACC, GJR-GARCH(1,0,1)-ACC
  - TARCH(1,1,1)-DCC, TARCH(1,0,1)-DCC
  - TARCH(1,1,1)-ACC, TARCH(1,0,1)-ACC
- Bali (2008): panel data methods mitigate finite-sample bias. Results are likely robust to the choice of subsamples.