Does Demographic Change Matter for Growth?

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Abstract

How important is the demographic transition for economic growth? To answer this question, this paper constructs a general equilibrium overlapping generations model with endogenous fertility. Our results suggest that demographic transition contributed to more than one third of the output growth in Taiwan in the past four decades, while TFP growth explains another one third and the remaining is mainly due to skill-biased technological progress. As a complement to the literature, this paper shows that the role of demographic transition is important in the growth process.

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1 Introduction

The literature debates the role played by total factor productivity (TFP) growth and factor accumulation in explaining economic growth.[1] In additional to TFP growth and factor accumulation, it is one of the key regularities of economic development that economic growth tends to go together with demographic transition. Therefore, it raises the question of whether there are connections between demographic transition and growth.[2] If so, how important is demographic transition for economic growth?

This paper argues that economic growth is not only due to TFP growth and factor accumulation, but also to demographic transition and its relation with technological progress. The aim of this paper is to quantify the impacts of each of these sources on growth, especially sheds light on the role of demographic transition.

The framework is a general equilibrium overlapping generations model where an individual lives at most three periods. Children cannot work and depend on their parents for support. If they survive, children become young adults. Then, subject to a survival probability, young adults become old adults who consume their savings. Young adults supply labor and make decisions on consumption, savings, the number of children, and an education level for his children. A young adult derives utilities from both consumption and the utility of his children.[3] The production

[1]For example, Young (1994); Young (1995); Klenow and Rodrlguez-Clare (1997); Harberger (1998); Hsieh (1999); Easterly and Levine (2001); and Hsieh (2002).

[2]There is a strand of literature linking demographic change to growth. For example, Barro and Becker (1989); Dahan and Tsiddon (1998); Galor and Weil (2000); Fernández-Villaverde (2001); Galor and Moav (2002); Boucekkine, de la Croix, and Licandro (2003); Attanasio and Violante (2005); and Lagerlöf (2006).

[3]In the model, parents make decisions on whether sending their children to school or not. Thus, it is important to put the utility of children and future wages in the value function. It is a standard setup suggested by Barro and Becker (1989). Another strand of literature assumes that parents derive utilities from the number of children, such as Galor and Weil (1996); Tamura (1996); Dahan

side is perfectly competitive. There exists a representative firm using skilled labor, unskilled labor, and physical capital as inputs. The technology is capital-skill complementarity.[4]

The model encompasses three distinct channels through which demographic change (a decline in fertility) affect economic growth. First, a decline in fertility affects the age composition of the population and thereby the dependency ratio (i.e., the ratio of the number of children and retirees to the working-age population).[5] Second, decreases in mortality at different ages affect individual saving decisions.[6] Third, parents' reaction to a decline in fertility is to increase the education investment in each of these fewer children.[7] Therefore, a larger proportion of labor force, physical-capital accumulation, and human-capital accumulation all together push output up. This approach allows us to quantify the impact of demographic change on economic growth as well as the relative importance of each of the three channels.

Our model is a general model that can be applied to countries with demographic transition. The demographic changes in Europe and in the United State began around 1800, meaning that the precise data are unavailable. Alternatively, one may study the demographic changes of newly industrializing countries, such as Hong Kong, Korea, Singapore, and Taiwan. As shown in Figure 1, these rapidly-growing

and Tsiddon (1998); Lagerlöf (2003); and Conde-Ruiz, Giménez, and Pérez-Nievas (2004).

[4]Section 5.2 discusses the setting of capital-skill complementarity. A sensitivity test using a Cobb-Douglas production function is also provided. The main results are similar.

[5]Empirical studies find evidence to show that a decline in the dependency ratio contributes to growth. For example, Malmberg (1994); Bloom and Sachs (1998); Bloom, Canning, and Malaney (2000); and Williamson and Yousef (2002).

[6]Empirical studies find cross-country evidence to support the effects of demographic change on the accumulation of physical capital. For example, Mason (1981); Fry and Mason (1982); Mason (1987); and Kelley and Schmidt (1996).

[7]The quantity-quality tradeoff of children has been extensively discussed in the literature. For example, Cheng and Nwachukwu (1997); de la Croix and Licandro (1999); de la Croix and Doepke (2003); and de la Croix and Doepke (2004).

countries have also been characterized by especially rapid fertility decline in the past decades. Besides, the demographic changes went with technological progress in these countries. Studying these countries allows us to understand various sources of economic growth.

In our numerical analysis, the model is calibrated to the data from Taiwan due to the availability and completeness of the data. However, this paper is not investigating a special story about Taiwan. As shown in Figure 1 and Figure 2, the East Asia Tigers experienced similar patterns of demographic transition in the growth process: fast growth with a rapid decline in fertility; an increase in the proportion of working-age population; a growth in saving rates; and an improvement on the quality of labor. Therefore, our framework and the numerical results can be generalized to other developing countries, such as other East Asia Tigers.

There are various shocks to the environment (TFP growth, skill-biased technological progress, and demographic changes). In the experiments, we first solve steady states for Taiwan in 1970 and 2004. Starting with the steady state of 1970, various shocks take place and the corresponding transition paths are solved. If the shock of demographic changes occurs, the effects of demographic changes on growth are examined. In other words, this methodology allows us to distinguish between the causation from a decline in fertility to economic growth from the causation from growth to demographic changes.

The shock of demographic changes includes increases in the survival rates for children and for young adults and increases in time cost and education time cost associated with raising children.[8] On the one hand, parents are willing to have more

[8]Time costs may be increasing because children contribute less to home production in a modern economy, and because the government introduces restrictions on child labor. Education time costs may be increasing because of education reforms (so that a teacher takes care of fewer students), or because parents decide to spend more time educating children in response to technological changes. Bar and Leukhina (2010) summarize possible reasons for the increase in the time cost. They also

children when the survival probability of children goes up, so it is a positive shock to fertility. On the other hand, other shocks have negative impacts on fertility: longevity (an increase in the survival probability of young adults) and increases in the time costs of raising children and education all push fertility down.[9] In our experiments, the effects of time cost and education cost dominate, so demographic changes result in a decline in fertility.

A decline in fertility reduces the dependency ratio. Besides, parents' resources will be released from the expenditure of children. To response the decline in fertility, parents may provide more education to the fewer children and/or transfer to savings. Therefore, physical capital and human capital are both accumulated. A lower dependency ratio, a higher level of physical capital and a higher quality of population all together increase output.

The results suggest that demographic effects (i.e., the contribution of fertility decline) in Taiwan generated a per capita GDP growth of about 3.2 percent per year during the past 35 years, while the overall growth rate of per capita GDP is about 8.5 percent per year.[10] Thus, the contribution of demographic change to the overall growth rate is around 38 percent (3.2/8.5). Our results are close to the upper bound reported in Bloom and Williamson (1998).[11]

Furthermore, our experiments suggest that TFP growth explains about 28 per-

find that increasing the time cost helps explain the decline in fertility.

[9]A strand of literature argues that longevity is an important source of economic growth because of the Ben-Porath effect. This paper argues that longevity pushes fertility down, thereby contributes to growth. Also see Boucekkine, de la Croix, and Licandro (2002).

[10]Using a model with exogenous fertility, Attanasio and Violante (2005) suggested that changes in age structure accounted for a per capita income growth of about 0.55 percent per year in Latin America. In this paper, allowing parents to choose the number of children generates a larger demographic effect. The difference may also due to the difference across countries, such as culture difference and population policies that affect demographic transition.

[11]Bloom and Williamson (1998) estimate that the decline in the dependency ratio accounted for between 25 and 40 percent of East Asia's economic miracle during 1965-90.

cent of output growth and skill-biased technological progress explains another 29 percent. The remaining 5 percent has to be due to their interactions with demographic changes. Galor (2005) argues that technological progress increases the demand for human capital. Parents are motivated to substitute quality for quantity of children, leading to an increase in human capital and a higher level of output. In addition, longevity may reinforce the above effect because of the higher rate of return of human capital and skill-biased technological progress.

We conclude that the late demographic transition has contributed more than one-third of the growth rate in Taiwan in 1970-2004, while the TFP growth would explain another third, the remaining growth would be due to the skill-biased technological progress. As a complement to the literature, this paper shows that demographic transition plays an important role in the process of growth.[12]

One advantage of our framework is that we can decompose the contribution of demographic changes into the three channels in order to assess their relative importance. Tallman and Wang (1994) find the important roles of physical capital and human capital in explaining the growth experience of Taiwan. Young (1992) finds similar evidence in the growth experience of Singapore. When demographic and technological changes occur at the same period of time, our results are consistent with the literature. In contrast, in the experiment with demographic changes alone, our results suggest that changes in the dependency ratio is the most important channel.

This paper is organized as follows. Section 2 describes the model and defines a recursive competitive equilibrium. The equilibrium behavior of this model is also

[12] This paper discusses the contribution of a decline in fertility to output, rather than the efficiency or welfare changes during demographic transition. The recent literature studies the welfare analysis in models of endogenous fertility and argues that individual fertility choices may lead to a too low or too high population size and resource misallocations at the aggregate level. See Conde-Ruiz, Giménez, and Pérez-Nievas (2004) and Golosov, Jones, and Tertilt (2007).

discussed. Section 3 provides calibrated results for two steady states. In Section 4, quantitative experiments are provided. Section 5 is discussions and Section 6 concludes this paper.

2 Theoretical Analysis

2.1 The Model

We construct a three-period overlapping generations model with endogenous fertility in which demographic change affects economic growth through three channels: changes in the dependency ratio, physical-capital accumulation, and human-capital accumulation.[13] People in this economy can live for three periods: children, young adults, and old adults. Children do not work, but they do receive education, which is decided by their parents. The survival rate for children is π^c. If they stay alive until the next period, children become young adults. Young adults can be either skilled or unskilled, depending on the education they received in childhood. Young adults make decisions about consumption, labor supply, savings, fertility, and their children's education. They live until the next period with the probability π^y. Old adults neither make decisions nor work. They consume their own savings.

2.1.1 Evolution of Population

Assume an individual can have children at the beginning of his young-adult period. Define N^b as the population of b, where $b \in \{c, y, o\}$ which represents children, young adults, and old adults, respectively. Specifically, N^y_s is the population of

[13]Doepke (2004) provides a related two-period overlapping generations model with endogenous fertility and the accumulation of human-capital. However, Doepke (2004) abstracts from mortality, physical capital accumulation, and skill-biased technological change, which are central elements of our analysis.

skilled young adults and N_u^y is the population of unskilled young adults.

In this paper, we use i to represent the type of young adults and j to denote the type of children. $\{i, j\}$ can be either skilled (s) or unskilled (u). To describe the evolution of population, we define n_{ij} as the number of j-type children that each i-type young adult has. The population of children in this period is determined by the number of children that each young adult has:

$$N^c = (n_{ss} + n_{su})N_s^y + (n_{us} + n_{uu})N_u^y.$$

If children survive until the next period, they become young adults:

$$N^{y'} = \pi^c N^c.$$

In addition, the young adults who survive become old adults in the next period:

$$N^{o'} = \pi^y (N_u^y + N_s^y).$$

2.1.2 Preferences and Budget Constraint

In this economy, only young adults can make decisions. A young adult cares about his consumption today and tomorrow and the number of his children that survive. The lifetime utility of an i-type young adult is given by:

$$\frac{c_i^{1-\sigma}}{1-\sigma} + \beta\pi^y \left(\frac{c_i'^{1-\sigma}}{1-\sigma}\right) + \psi[\pi^c(n_{is} + n_{iu})]^{-\varepsilon}[\pi^c n_{is} V_s' + \pi^c n_{iu} V_u'], \tag{1}$$

where $0 < \beta < 1$, $0 < \varepsilon < 1$, and $0 < \sigma < 1$.[14] c_i is his consumption this period, and c_i' is his consumption at old age. V_s' is the utility that a child will receive with education, and V_u' is the utility of a child without education. Both utilities are foreseeable and are known when a young adult is making decisions. $\frac{1}{\sigma}$ denotes the elasticity of inter-temporal substitution. β is the subjective discount factor with respect to utility of consumption. ψ is the general level of altruism, representing how much a young adult loves his children. ε determines the elasticity of altruism with respect to the number of children.

Each young adult has one unit of time. We assume only skilled young adults can be teachers. Thus a skilled young adult can spend his time working in the production sector, teaching, and raising children. In contrast, an unskilled young adult only has two options: working in the production sector and raising children.[15] When an i-type young adult works in the production sector, he earns w_i for one

[14] In Section 2.2, we will show that in equilibrium a young adult only has one type of children. Thus, lifetime utility can be simplified as:

$$u(c_i) + \beta\pi^y u(c_i') + \psi(\pi^c n_j)^{1-\varepsilon} V_j'.$$

If $\varepsilon = 0$, we have $u(c_i) + \beta\pi^y u(c_i') + \psi\pi^c n_j V_j'$. The lifetime utility is linear in n_j. If $\varepsilon = 1$, the lifetime utility becomes $u(c_i) + \beta\pi^y u(c_i') + \psi V_j'$. It is independent of n_j. If $\varepsilon > 1$, the lifetime utility is reversely related to n_j. If $\varepsilon = \infty$, we have $u(c_i) + \beta\pi^y u(c_i')$. The lifetime utility is again independent of n_j. To avoid these problems, we assume that $0 < \varepsilon < 1$.

[15] Assume a skilled young adult spends κ_s of his time on working, τ of his time on being a teacher, and $\phi(n_{ss} + n_{su})$ of his time on raising children at home. An unskilled young adult spends $\phi(n_{us} + n_{uu})$ on his children and κ_u on working. Then, the time constraint of each type is given by:

$$\kappa_s + \tau + \phi(n_{ss} + n_{su}) \leq 1;$$
$$\kappa_u + \phi(n_{us} + n_{uu}) \leq 1;$$

where $0 < \kappa_s < 1$, $0 < \kappa_u < 1$, and $0 < \tau < 1$. Following this notation, the supply of skilled labor is $\kappa_s N_s^y$ and the supply of unskilled labor is $\kappa_u N_u^y$. A recursive competitive equilibrium is defined unit of time. The compensation for being a teacher is w_s.[16]

Children cannot work. However, they are costly. Each child costs p units of consumption goods and a fraction ϕ of his parent's time. Parents can also educate children. The education time cost for a child is ϕ_s. Define p_{ij} as the total cost of raising a j-type child to an i-type young adult. Then the total cost of one child can be summarized as follows:

$$\begin{aligned} p_{is} &= \phi w_i + p + \phi_s w_s; \\ p_{iu} &= \phi w_i + p; \end{aligned}$$

Unskilled parents need to send children to school and pay $\phi_s w_s$ for a child. Skilled parents can teach their children either at home or at school. It also costs $\phi_s w_s$ to educate a child.

The budget constraint of an i-type young adult is given by:

$$c_i + \pi^y a_i' + p(n_{is} + n_{iu}) + \phi_s w_s n_{is} = [1 - \phi(n_{is} + n_{iu})]w_i, \tag{2}$$

where $0 < \phi_s < 1$. We assume every young adult signs a contract for life annuity. A young adult pays $\pi^y a_i'$ in this period. If he can survive to the next period, he will get the life annuity from the insurance company and consume $(1 + r')a_i'$ at old age; otherwise, he will get nothing. The insurance company is perfectly competitive.

in Section 2.1.4. In equilibrium, education demand is equal to education supply:

$$\phi_s(n_{ss}\lambda_{ss}N_s^y + n_{us}\lambda_{us}N_u^y) = \tau N_s^y$$

which implies $\tau = \frac{\phi_s(n_{ss}\lambda_{ss}N_s^y + n_{us}\lambda_{us}N_u^y)}{N_s^y}$ in equilibrium. λ_{ss} refers to the fraction of skilled young adults having skilled children; λ_{us} refers to the fraction of unskilled young adults having skilled children.

[16]If the marginal cost of being a teacher is larger (smaller) than the marginal benefit of being a teacher, a skilled young adult will reduce (increase) the time of being a teacher and increase (reduce) the time spent on the production sector. In equilibrium, a skilled young adult is indifferent between working and teaching.

The budget constraint for the next period is given by:

$$c_i' = (1 + r')a_i', \tag{3}$$

where r' is the interest rate in the next period.

By assumption, parents are not allowed to transfer savings to their children as a bequest. A strand of literature suggests that some parents are willing to leave bequests to their children, but the proportion is not large. First, the data in Gale and Scholz (1994) suggest that only 3.7 percent of U.S. households reported receiving bequests; and the average bequest received by those households was about 43000 dollars.[17] Second, Cox and Raines (1985) suggest that the majority of transfers occur *inter vivos*, not as a bequest.[18] Third, the data in Dynan, Skinner, and Zeldes (2002) suggest that only 8 percent of all households and 12 percent of retired households mentioned one's estate or children as a motivation for their savings.[19] Even in the top wealth class, Modigliani (1988) quotes previous findings and points out that only one-third of the households mentioned the bequest motivation. In this paper, allowing bequests is not essential but will introduce heterogeneity on family assets and complicate the model. Therefore, we assume old generation will consume all savings and leave nothing as a bequest to their children.

2.1.3 Production

There exists a representative competitive firm, using skilled labor (L_s), unskilled labor (L_u), and physical capital (K) as inputs.[20] The main purpose of the production setting is to generate the capital-skill complementarity. There are three ways to nest

[17]The Survey of Consumer Finances 1983-85. In the same survey, the average income of the whole sample was about 29500 dollars.

[18]Also see Kurz (1984) and Cox (1987).

[19]The Survey of Consumer Finances 1998.

[20]In our model, labor refers to the number of workers.

K, L_s, and L_u with the two-level CES function. Following Krusell, Ohanian, Rios-Rull, and Violante (2000), the production function in this economy is given by:

$$Y = A[\mu L_u^{\alpha} + (1-\mu)(\theta K^{\rho} + (1-\theta)L_s^{\rho})^{\alpha/\rho}]^{1/\alpha}, \tag{4}$$

where A denotes total factor productivity; μ and θ are parameters that govern income shares; α and ρ govern the elasticity of substitution between unskilled labor, physical capital, and skilled labor. In this setup, the elasticity of substitution between L_u and K is $\frac{1}{1-\alpha}$. The elasticity of substitution between L_s and K is $\frac{1}{1-\rho}$. In addition, the substitution elasticity of L_u and K equals the substitution elasticity of L_u and L_s. Finally, capital-skill complementarity requires that $\alpha > \rho$.

Capital-skill complementarity is essential for human-capital accumulation. In (4), physical capital and unskilled labor are more substitutes. During a demographic transition, unskilled labor is replaced by physical capital when physical capital is accumulated. Thus, the demand for unskilled labor declines. Since physical capital and skilled labor are more complements, the demand for skilled labor increases. Skill premium is given by:

$$\frac{w_s}{w_u} = \frac{1-\mu}{\mu}(\theta K^{\rho} + (1-\theta)L_s^{\rho})^{\frac{\alpha}{\rho}-1}(1-\theta)L_s^{\rho-1}L_u^{1-\alpha}.$$

We know that:

$$\frac{\partial}{\partial k}\left(\frac{w_s}{w_u}\right) = \frac{1-\mu}{\mu}(1-\theta)\left(\frac{\alpha}{\rho}-1\right)(\theta K^{\rho} + (1-\theta)L_s^{\rho})^{\frac{\alpha}{\rho}-2}\theta\rho K^{\rho-1}L_s^{\rho-1}L_u^{1-\alpha}.$$

The above equation is greater than zero as long as $\alpha > \rho$. In other words, skill premium increases when physical capital is accumulated. Skill premium attracts the parent to educate their existing children instead of having more children. The notion and the evidence that physical-capital accumulation benefits skilled labor (or increases skill premium) are discussed in Griliches (1969) and Fallon and Layard (1975).

This production function is constant return to scale. Thus, we can rewrite (4) to be:

$$y = A[\mu l_u^\alpha + (1-\mu)(\theta k^\rho + (1-\theta) l_s^\rho)^{\frac{\alpha}{\rho}}]^{\frac{1}{\alpha}},$$

where y denotes output per worker, k is the capital-labor ratio, l_u is the fraction of unskilled labor as a percentage of total labor, and l_s is the fraction of skilled labor.[21] Then output per capita is given by:

$$y_{pc} = \frac{Y}{N} = \frac{L}{N} y, \tag{5}$$

where y_{pc} denotes output per capita; L is the quantity of labor force working at the production sector; and N is total population. Thus, a demographic transition affects output per capita through three channels: the fraction of working-age population $\frac{L}{N}$, physical-capital accumulation k, and human-capital accumulation l_s.

2.1.4 Recursive Competitive Equilibrium

The aggregate state variables in this economy are the population of skilled young adults, the population of unskilled young adults, and physical capital. Thus, the state vector $x \equiv \{N_s^y, N_u^y, K\}$.[22]

The maximization problem of an i-type young adult is described by the Bellman equation:

$$V_i(x) = \max_{c_i, a_i', n_{is}, n_{iu}} \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + \beta \pi^y \frac{c_i'^{1-\sigma}}{1-\sigma} + \psi [\pi^c (n_{is} + n_{iu})]^{-\varepsilon} [\pi^c n_{is} V_s(x') + \pi^c n_{iu} V_u(x')] \right\}, \tag{6}$$

[21] We define $y = \frac{Y}{L}$, $k = \frac{K}{L}$, $l_s = \frac{L_s}{L}$, $l_u = \frac{L_u}{L}$, and $L = L_s + L_u$.

[22] The population of old generation (N^o) is also a state variable of this economy. It is not included in the state vector because K summarizes all relevant information about N^o.

subject to the budget constraint:

$$
\begin{aligned}
c_i + \pi^y a_i' + p(n_{is} + n_{iu}) + \phi_s w_s(x) n_{is} &= [1 - \phi(n_{is} + n_{iu})] w_i(x), && (7)\\
c_i' &= (1 + r'(x')) a_i', && (8)
\end{aligned}
$$

and a law of motion of the state vector $x' = G(x)$, where $i \in \{s, u\}$.

The firm's problem is given by:

$$
\max_{L_s^f, L_u^f, K^f} \quad Y - w_s(x) L_s^f - w_u(x) L_u^f - r(x) K^f, \tag{9}
$$

where Y is defined by (4).

We will show that a young adult has either skilled children or unskilled children. He does not want a mixture of skill types. Define λ_{ij} as the fraction of i-type young adults having j-type children. Then, the following conditions should be satisfied:

$$
\begin{aligned}
\lambda_{ss}(x) + \lambda_{su}(x) &= 1; && (10)\\
\lambda_{us}(x) + \lambda_{uu}(x) &= 1. && (11)
\end{aligned}
$$

By assumption, only skilled adults can be teachers at school. Therefore, the supply of skilled labor (L_s) and unskilled labor (L_u) in the production sector are given by:

$$
\begin{aligned}
L_s(x) &= [1 - (\phi + \phi_s) n_{ss}(x) \lambda_{ss}(x) - \phi n_{su}(x) \lambda_{su}(x)] N_s^y - \phi_s n_{us}(x) \lambda_{us}(x) N_u^y,\\
L_u(x) &= [1 - \phi n_{us}(x)] \lambda_{us}(x) N_u^y + [1 - \phi n_{uu}(x)] \lambda_{uu}(x) N_u^y.
\end{aligned}
$$

The supply of skilled labor is equal to total skilled labor supply minus skilled labor spent on raising children and on teaching. The supply of unskilled labor is total unskilled labor minus unskilled labor spent on raising children. We assume that skilled young adults can work as both skilled and unskilled workers, while unskilled young adults can only work as unskilled workers. Thus the market clearing conditions for the labor market are:

$$
\begin{aligned}
L_s^f(x) &\leq L_s(x), && (12)\\
L_u^f(x) &= L_u(x) + [L_s(x) - L_s^f(x)]. && (13)
\end{aligned}
$$

The equality of (12) holds if $w_s(x) > w_u(x)$.

Define A_s and A_u to be the aggregate asset holding per young skilled adult and per young unskilled adult, respectively. Aggregate supply of physical capital tomorrow is given by:

$$K'(x) = \pi^y (A'_s N^y_s + A'_u N^y_u). \tag{14}$$

where $A'_s = g_s(x)$ and $A'_u = g_u(x)$. In equilibrium, aggregate demand of physical capital ($K^f(x)$) has to equal aggregate supply of physical capital K. The market clearing condition for the physical-capital market is:

$$K^f(x) = K. \tag{15}$$

The final equilibrium condition is the law of motion of skilled and unskilled young adults. For each type, the population of young adults in the next period is determined by the fertility and the survival rates for children in this period. The evolution of young adults is given by:

$$N^{y'}_s = \pi^c[n_{ss}(x)\lambda_{ss}(x)N^y_s + n_{us}(x)\lambda_{us}(x)N^y_u]; \tag{16}$$

$$N^{y'}_u = \pi^c[n_{su}(x)\lambda_{su}(x)N^y_s + n_{uu}(x)\lambda_{uu}(x)N^y_u]. \tag{17}$$

A recursive competitive equilibrium consists of value functions $V_s(x)$ and $V_u(x)$, pricing functions $w_s(x)$, $w_u(x)$, and $r(x)$, mobility functions $\lambda_{ss}(x)$, $\lambda_{su}(x)$, $\lambda_{us}(x)$, and $\lambda_{uu}(x)$, policy functions $n_{ss}(x)$, $n_{su}(x)$, $n_{us}(x)$, $n_{uu}(x)$, $a'_s(x)$, and $a'_u(x)$, decision functions of the firm $K^f(x)$, $L^f_s(x)$, and $L^f_u(x)$, a law of motion of state variables $x' = G(x)$, and $A'_s = g_s(x)$ and $A'_u = g_u(x)$ such that:

1.Given $w_s(x)$, $w_u(x)$, and $r(x)$, the value functions and policy functions solve the household's dynamic programming problem (6).

2.For $(i, j) \in \{s, u\}$, if $\lambda_{ij}(x) > 0$, $n_{ij}(x)$ maximizes (6).

3.Given $w_s(x)$, $w_u(x)$, and $r(x)$, the decision functions maximize the firm's profit.

4.The market-clearing conditions (12), (13), and (15) are satisfied.

5.The mobility functions satisfy (10) and (11).

6.The law of motion G for the state variable x is given by (14), (16), and (17).

7. Perceptions are correct: $A'_i(x) = a'_i(x)$, where $i \in \{s, u\}$.

2.2 The Equilibrium Behavior

This section discusses the equilibrium behavior of the model. First, the maximization problem has corner solutions. A young adult will either educate all of his children or none of them. Second, since a skilled child is relatively cheaper than an unskilled child for a skilled young adult, an unskilled young adult is indifferent between educating all of his children or none of them. For a balanced growth path, skilled young adults always have skilled children, a fraction (λ_{us}) of unskilled young adults educate their children, and other unskilled young adults always have unskilled children.

2.2.1 Corner Solutions

The intuitive reason of the corner solutions is that children living in the same family are identical. Therefore, parent will either send all of his children to school or none of them. Skilled children and unskilled children will not live in the same family.

In the model, the decision problem can be broken down into two stages. In the first stage, a young adult allocates his income between consumption today, asset holdings, and total expenditures on children E_i.[23] In the second stage, he allocates the total expenditure on children between his skilled and unskilled children. Assume that a fraction f_i is spent on skilled children. Then the number of skilled children for an i-type young adult equals $n_{is} = \frac{f_i E_i}{p_{is}}$. The number of unskilled children is $n_{iu} = \frac{(1-f_i)E_i}{p_{iu}}$.

[23] $E_i = p_{is}n_{is} + p_{iu}n_{iu}$, where p_{ij} is the total cost for an i-type adult to raise a j-type child.

The adult chooses the fraction f_i to maximize his lifetime utility. The maximization problem can be rewritten as:

$$\max_{f_i} \quad \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + \beta\pi^y \frac{c_i'^{1-\sigma}}{1-\sigma} + \psi\pi^{c^{1-\varepsilon}} E_i^{1-\varepsilon} \left(\frac{f_i}{p_{is}} + \frac{1-f_i}{p_{iu}} \right)^{-\varepsilon} \left(\frac{f_i V_s}{p_{is}} + \frac{(1-f_i)V_u}{p_{iu}} \right) \right\}.$$

where $c_i = w_i - \pi^y a_i' - E_i$. By assumption, $0 < \varepsilon < 1$.[24] Thus, the objective function is convex in f_i and the adult will choose a corner solution. The proof is provided in Appendix A.

2.2.2 Indifference Condition

An i-type young adult only has one type of children. Given the type of children j and the total expenditure E_i, the number of children a young adult has is $\frac{E_i}{p_{ij}}$. The maximization problem can be written as:

$$\max \quad \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + \beta\pi^y \frac{c_i'^{1-\sigma}}{1-\sigma} + \psi\pi^{c^{1-\varepsilon}} \left(\frac{E_i}{p_{ij}} \right)^{1-\varepsilon} V_j \right\},$$

where $c_i = w_i - E_i - \pi^y a'_i$ and $c'_i = (1+r')a'_i$. The first two terms are independent of the type of children (independent of j). The last term contains the cost and the utility of a child. A young adult is indifferent between having skilled or unskilled children if and only if the following condition holds:

$$\frac{V_s}{p_{is}^{1-\varepsilon}} = \frac{V_u}{p_{iu}^{1-\varepsilon}}. \tag{18}$$

[24] As discussed in Footnote 14, the case $\varepsilon \geq 1$ is ruled out. In the case $\varepsilon = 0$, the objective function becomes:

$$\max_{f_i} \quad \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + \beta\pi^y \frac{c_i'^{1-\sigma}}{1-\sigma} + \psi\pi^c E_i \left(\frac{f_i V_s}{p_{is}} + \frac{(1-f_i)V_u}{p_{iu}} \right) \right\}.$$

If we have $\frac{V_s}{p_{is}} = \frac{V_u}{p_{iu}}$ (the indifference condition when $\varepsilon = 0$), the parent is indifferent between skilled and unskilled children and any combination of the two types. The solution is not necessary to be corner. Therefore, the case of $\varepsilon = 0$ is ruled out.

If this condition is satisfied, every i-type young adult faces the same maximization problem at the first stage, that is, he allocates resources between consumption, asset holdings, and total expenditures on children regardless of the type of children. At the second stage, given the optimal E_i, there is a trade-off between quantity and quality of children. The higher cost of having skilled children reduces the number of children that a young adult has.

Condition (18) can be rewritten as:

$$\frac{V_s}{V_u} = \left(\frac{p_{is}}{p_{iu}}\right)^{1-\varepsilon}.$$

The right side of this equation is the price of a skilled child relative to the price of an unskilled child. The relative price for a skilled young adult is given by:

$$\frac{p_{ss}}{p_{su}} = \frac{\phi w_s + p + \phi_s w_s}{\phi w_s + p};$$

the relative price for an unskilled young adult is given by:

$$\frac{p_{us}}{p_{uu}} = \frac{\phi w_u + p + \phi_s w_s}{\phi w_u + p}.$$

Furthermore, we know that $w_s > w_u$, so the relative price for a skilled young adult is always smaller than the relative price for an unskilled young adult. Since a skilled child is relatively cheaper for skilled parents, only one type of young adult can be indifferent between having skilled and unskilled children.

In summary, there are three possibilities. A typical case is that skilled young adults have skilled children and unskilled young adults are indifferent. That is, skilled adults always have skilled children; a fraction of unskilled parents have skilled children; and the others have unskilled children. This implies $\lambda_{ss} = 1$ and $0 < \lambda_{us} < 1$. In this case, there is an upward mobility: human-capital accumulation. Because we look for a balanced growth path in this economy, we focus on this case in the rest of this paper.[25]

[25]Other two possibilities are: (1) Skilled parents always have skilled children, while unskilled

2.2.3 Fertility

To simplify the notation, the subscript for the type of a young adult is ignored in this subsection. Given the type of a young adult, the maximization problem can be simplified as:[26]

$$\max_{n_j,a'} \left\{ \frac{[w - \pi^y a' - p_j n_j]^{1-\sigma}}{1-\sigma} + \beta\pi^y \frac{[(1+r')a']^{1-\sigma}}{1-\sigma} + \psi(\pi^c n_j)^{1-\varepsilon} V_j' \right\}$$

where $j = \{s, u\}$, representing the type of children the young adult chooses. The first order conditions are:

$$n_j^\varepsilon p_j = \psi(1-\varepsilon)\pi^{c^{1-\varepsilon}} V_j' [w - \pi^y a' - p_j n_j]^\sigma; \quad (19)$$

$$\frac{c'}{c} = [\beta(1+r')]^{1/\sigma}. \quad (20)$$

Equation (20) is the Euler equation. The rate of substitution between consumption today and tomorrow depends on the time preference discount factor and the interest rate tomorrow. Since there is an life annuity, it does not depend on π^y.

Equation (19) implies that given the type of his children j, a young adult will choose his fertility until the marginal utility of a child equals the marginal cost.[27] Fertility n_j is increasing in children's utility V_j', in the survival rate for children π^c,

parents always have unskilled children; (2) Skilled parents are indifferent, while unskilled parents always have unskilled children. Both cases cannot occur in a balanced growth path. In the first case, unskilled children is cheaper than skilled children so the fertility rate of unskilled young adults is higher. There are more and more unskilled workers in this economy. Finally, w_u becomes zero, which will not happen in the equilibrium. In the second case, there is a downward mobility. It will lead to $w_u = 0$.

[26] To deal with the life annuity, we assume that different types of young adults have different contract. Thus the type $s \to s$, $u \to s$, and $u \to u$ can get a'_{ss}, a'_{us}, and a'_{uu} at his old age respectively.

[27] The marginal cost of one child is the marginal disutility of consumption.

$$[w - \pi^y a' - p_j n_j]^{-\sigma} p_j = \psi(1-\varepsilon)\pi^{c^{1-\varepsilon}} V_j' n_j^{-\varepsilon}$$

and in the altruism coefficient ψ; but it is decreasing in longevity (π^y). There is an income effect: fertility increases as the wage rate increases. Thus children are normal goods in this model. On the other hand, since the total cost of a child is an increasing function of the wage rate, the fertility rate is decreasing in the total cost of a child. Working is then substituted for raising children. When the substitution effect is larger than the income effect, fertility declines.

In the case of $\lambda_{ss} = 1$ and $0 < \lambda_{us} < 1$, the evolution of population is rewritten as:

$$\begin{aligned} N^c &= n_{ss}N_s^y + [n_{us}\lambda_{us} + n_{uu}(1-\lambda_{us})]N_u^y; &(21)\\ N_s^{y'} &= \pi^c[n_{ss}N_s^y + n_{us}\lambda_{us}N_u^y]; &(22)\\ N_u^{y'} &= \pi^c n_{uu}(1-\lambda_{us})N_u^y; &(23)\\ N^{o'} &= \pi^y N^y. &(24)\end{aligned}$$

The supply of labor and physical capital are given by:

$$\begin{aligned} L_s &= [1-(\phi+\phi_s)n_{ss}]N_s^y - \phi_s n_{us}\lambda_{us}N_u^y; &(25)\\ L_u &= (1-\phi n_{us})\lambda_{us}N_u^y + (1-\phi n_{uu})(1-\lambda_{us})N_u^y; &(26)\\ K' &= \pi^y({a'}_{ss}N_s^y + {a'}_{us}\lambda_{us}N_u^y + {a'}_{uu}(1-\lambda_{us})N_u^y). &(27)\end{aligned}$$

3 Calibration

Our model is a general model that can be applied to every country. As discussed in the introduction, Europe and the US may not be suitable because their demographic transition began in the early 1800. In contrast, the newly industrializing countries experienced not only a rapid fertility decline but also a fast growth in the past few decades. Therefore, countries, such as East Asia Tigers, would serve a good example. Due to the data availability and completeness, we calibrate our model to the data from Taiwan. However, this paper is not studying a special experience of

Taiwan. Instead, the story can be generalized to other developing countries which have similar demographic changes currently or in the past decades.

The first part of this section provides the demographic experience of Taiwan. The second part discusses the parameters and the algorithm for the calibration. Finally the calibrated results are discussed.

3.1 Taiwan's Experience

There exist three distinct channels through which a demographic transition can affect economic growth. Consider Taiwan in the years of 1951-2006. Figure 3 displays the growth of GDP per capita and the three channels. The upper-left figure is the growth of GDP per capita. Taiwan experienced rapid growth during this period. The upper-right figure shows the fraction of the working-age population as a percentage of the total population. As demographic transition occurs, the fraction of the working-age population increased from 0.55 to 0.72 in 56 years. The lower-left figure shows the ratio of gross savings to GDP. The saving ratio started at 9 percent, increased to its peak (about 30 percent) in the late 1980s, and then declined to 25 percent. The lower-right figure is the fraction of skilled employees to total employees. Data was only available for 1978-2006. The fraction of skilled employees increased dramatically from 8.5 percent in 1978 to 36.7 percent in 2006.

Figure 3 shows a strong positive relationship between GDP growth and the three channels, especially before 1990. This clear relationship motivates us to choose Taiwan as an example to explore the impact of demographic transitions on economic growth.

3.2 Parameters

Table 1 summarizes the parameters in each steady state. Eight parameters differ across the steady states, while the others remain unchanged. The first two parameters are survival rates for children and for young adults. Survival rates are higher in 2004 because of improvements in sanitation, safer water, and the development of antibiotics and medical science. The increase in TFP reflects neutral technological progress. Labor income share (to total income) can be decomposed into two components: the skilled labor income share of total labor income and the unskilled labor income share of total labor income. We assume the labor income share of total income is unchanged. However, the shares of skilled labor income and unskilled labor income are different in the two steady states because they are determined by the skill premium and the ratio of skilled labor to unskilled labor. In addition, factor weights, μ and θ, of the production function are different between the two steady state since they are determined by the unskilled labor income share and the physical capital income share. Finally, we also believe that time costs and education costs had risen in 2004. Bar and Leukhina (2010) summarizes possible reasons discussed in the literature for the rise in time costs. For example, children contribute less to home production when the economy industrializes, and the government introduces child labor laws and education reforms. Mauldin, Mimura, and Lino (2001) empirically show that parents' after-tax income is positively related to expenditures on children's education. Furthermore, the skill premium grows over time, implying that skilled workers' time become more valuable relative to that of unskilled workers. Since only skilled young adults can provide education in our model, education cost rises in the second steady state. In Greenwood and Seshadri (2002), parents decide to spend more time on education in response to technological change. Thus increases in time costs and education costs are required to target fertility and the fraction of skilled labor of 2004. The details are discussed below.

We assume the time period in this economy is twenty-five years. Taiwanese annual data from 1956 to 2004 are thus used to construct a twenty-five year survival rate. The mortality rate is calculated by the number of deaths in one generation divided by the population in that generation. However, the number of deaths is reported in three age groups: 0-14 years old, 15-64 years old, and 65 years old or above. We assume that different ages in the same age group are subject to the same survival rate. For example, suppose the survival rate for 0-14 years old in the data is π_a, and the survival rate for 15-64 years old is π_b. Then we construct the survival rate for children as $\pi^c = {\pi_a}^{15}{\pi_b}^{10}$, and the young adult survival rate is $\pi^y = {\pi_b}^{25}$. Figure 4 shows the constructed sequences for both survival rates. Both survival rates increased before 1970 and then become much flatter. In 1956-1970, the survival rate for children rapidly increased from 84 percent to 93 percent while the survival rate for young adults only increased by about 5 percent. After 2000, the survival rate for children stayed around 96 percent and the survival rate for young adults was about 93 percent. In the calibration, we choose the 15th point ($\pi^c = 0.9305$ and $\pi^y = 0.9090$) to be the survival rates in the first steady state and the last point ($\pi^c = 0.9666$ and $\pi^y = 0.9351$) as the survival rates in the second steady state.

Preference parameters for the first steady state are chosen as follows. In 1982-2007, the annual stock market return was about 10 percent in Taiwan.[28] Thus, we choose the annual discount factor to be 0.93. Deaton and Paxson (1997) estimate consumption by age in Taiwan. Relative to consumption at the age of 25, consumption increases as age increases. At its peak, relative consumption is about 1.5 (in logarithm scale).[29] Lee, Mason, and Miller (2000) confirm this result. However, there are only two points in our model: young adults and old adults. We choose σ to match consumption at the peak relative to middle age.[30] Thus, σ is equal to 0.5. We choose the elasticity of altruism with respect to the number of children ε

[28] Annual TAIEX is used to calculate the stock market return. The 25-year accumulated return is computed. The annual stock market return is around 10 percent.

[29] Deaton and Paxson (1997) estimate that consumption at the peak relative to the age of 25, $log(\frac{C_{peak}}{C_{age25}})$, is about 1.5. The peak is around the age of 55-60. They also discuss the high consumption at the oldest ages in Taiwan.

[30] The middle age is about the age of 40. We choose σ to match relative consumption which is roughly equal to one.

to be 0.5. This number is chosen so that about 5 percent of children have parents educated at or above the college level in 1975.[31] The final preference parameter is the altruism coefficient ψ. We choose it to equal 0.238 so that we can match the fertility rate 3.365 in 1972. In the second steady state, we assume the preference does not change. Thus we apply the same preference parameters for the second steady state.

Following the suggestion by Hulten and Wykoff (1981), the annual depreciation rate δ_a of 9.05 percent is employed in Thangavelu and Heng (2004). Thus, we use this annual depreciation rate in our calibration.[32] Then the depreciation rate for 25 years is computed by $\delta = 1 - (1 - \delta_a)^{25}$. TFP in the first steady state is equal to one. TFP in the second steady state is chosen to be 1.628 in order to match the annual growth rate of per capita GDP in 1970-2004, 8.5 percent.[33]

In addition to the depreciation rate and TFP, we need to pin down four param-

[31]In 1975, about 3 percent of infants were born from mothers who were college or above; 7 percent of infants were from fathers who were college or above. Here, we use the average since we do not have sex distinction in our model. Data source: Macroeconomics Database, DGBAS, Executive Yuan, Taiwan.

[32]Following Hulten and Wykoff (1981), Sun (2004) employs depreciation rate of 9.25 percent for Taiwan's manufacturing sector. However, this is an average depreciation rate based on fewer industries. Thus we use the depreciation rate in Thangavelu and Heng (2004).

[33]We use 35 years to discount the growth rate of GDP per capita from the first steady state to the second steady state. The annual GDP per capita growth 8.5% is computed as follows. First, GDP per capita of 2004 relative to GDP per capita of 1970, 1971, 1972,..., 1978 is computed, respectively. GDP per capita is measured by local currency. Second, we average the above relative GDP per capita. This average is used to represent the growth of GDP per capita from 1970 to 2004. Third, 35 years is used to discount the growth rate. If only GDP per capita of 1970 is applied, the annual growth rate of GDP per capita in 1970-2004 is 10.32%. In comparison, the average growth rate of GDP per capita from 1970 to 2004 (compute the growth rate for every year and then take the average) is 7.65%. In our experiment, TFP grows from 1 to 1.628, which implies the annual TFP growth is about 1.4 percent.

eters (α, ρ, μ, and θ) in the production sector. Krusell et. al. (2000) estimates that the elasticity of substitution between unskilled labor and physical capital is about 1.67.[34] They also estimate the elasticity of substitution between skilled labor and physical capital to be about 0.67. Therefore, we choose α to be 0.401 and ρ to be -0.493.

We use physical capital income share and unskilled labor income share (the ratio of unskilled labor income to total labor income) to pin down the factor weights μ and θ in the production function. Young (1995) and Young (2003) estimate the total labor income share in Taiwan to be 0.67. Therefore, we set the physical capital income share to be 0.33. The unskilled labor income share is calculated by the following formula:

$$\frac{w_u L_u}{w_s L_s + w_u L_u} = \frac{1}{\frac{w_s}{w_u}\frac{l_s}{l_u} + 1},$$

where l_s is the ratio of skilled workers to total workers and l_u is the ratio of unskilled workers to total workers. A skilled worker in this paper is defined as a worker who has a college degree or above. In 1978, the fraction of skilled workers was about 8.51 percent, and the skill premium was 1.761. Thus, the ratio of unskilled labor income to total labor income for the first calibration is 0.8594. This implies the unskilled labor income share of output is 0.5758 and skilled labor income share of output is 0.0942. In 2004, the skill premium was about 2.199, and the fraction of skilled workers was 32.9 percent. Therefore, the ratio of unskilled labor income to total labor income for the second calibration is 0.4914. In this case, the unskilled labor income share of output is 0.3292 and skilled labor income share of output is 0.3408. Finally, μ and θ are determined by the physical capital income share and

[34] Johnson (1997) reports the elasticity of substitution between L_u and L_s is about 1.5, which is close to Krusell et. al. (2000).

the unskilled labor income share:

$$\frac{K(r+\delta)}{Y} = \frac{(1-\mu)(\theta k^{\rho}+(1-\theta)l_s^{\rho})^{\frac{\alpha}{\rho}-1}\theta k^{\rho}}{\mu l_u^{\alpha}+(1-\mu)(\theta k^{\rho}+(1-\theta)l_s^{\rho})^{\frac{\alpha}{\rho}}};$$

$$\frac{w_u L_u}{w_s L_s + w_u L_u} = \frac{1}{\frac{1-\mu}{\mu}(\theta k^{\rho}+(1-\theta)l_s^{\rho})^{\frac{\alpha}{\rho}-1}(1-\theta)l_s^{\rho}l_u^{-\alpha}+1}.$$

Solving the above equations in steady state, we obtain μ is 0.1755 and θ is 0.3281 for the first steady state; μ is 0.2025 and θ is 0.1963 for the second steady state. The details of this are discussed in the algorithm subsection.

There are three costs associated with children: the good cost p, the time cost ϕ, and the education time cost ϕ_s. We set p to be 0. In 1989-1998, Liu and Hsu (2004) estimate that parents need an extra 28.2 percent of their original income after the first child is born in order to maintain the same quality of life. The second child costs an extra 24.8 percent of their income. The marginal cost of the third child is 22.4 percent. The fourth child is an extra 20.8 percent. Thus, the more children parents have, the lower the marginal cost of a child is. To pin down the time cost in the first steady state, we average the marginal cost of the first four children. Thus, it costs an extra 24.05 percent of parents' income to raise a child. This implies that the time cost is about 0.11. We apply 0.1102 in the first steady state.[35] The fertility rate is 1.18 in 2004. Considering the economic scale of raising children, we use the marginal cost of the first child to pin down the time cost. Thus a family needs an extra 28.2 percent of its income to raise a child. The time cost is equal to 0.175. We employ 0.1775 for the second steady state.

[35] The marginal cost of raising a child is an extra 24.05 percent of parents' income. For example, in a family with the income of 50000 dollars, the extra cost is equal to 12025 dollars. In other words, this family need extra 12025 dollars to maintain the same quality of life. Thus, 12025 dollars is 19.39 percent of 62025 dollars. Then we obtain the time cost by solving the following equation:

$$\phi w_i = 0.1939(1-\phi n)w_i.$$

Applying $n = 3.365$ to the above equation, we have $\phi = 0.11$.

The number of teachers per student in public schools is used to determine the education time cost. In 1972, one student has 0.03 teachers. Thus, an education time cost of 0.0297 is used. A student in public schools has about 0.07 teachers in 2004. We employ the education time cost of 0.073 in the second steady state.

3.3 Algorithm for Solving Steady States

The two steady states are solved independently. The first steady state is solved using the parameters reported in Table 1. Then we change the parameters and repeat the procedure to solve the second steady state.

The population of young adults in 1972 is the initial population. Originally, the population was classified to three age groups: younger than 15, between the ages of 15-64, and older than 65. To match the time periods in the model, we adjust these categories to be younger than 24, between the ages of 25-49, and older than 50. The initial ratios of skilled workers for the first and the second calibrations are 8.5 percent and 32.9 percent, respectively. The population of skilled workers and unskilled workers are then normalized by the population of young adults. The initial physical capital per young adult is 0.0011, such that the annual capital-output ratio is equal to 0.83 in 1970. It is equal to 0.0091, so that the annual capital-output ratio is 1.55 in 2004.

We initially guess λ_{us}, w_s, and w_u. Then we solve the partial equilibrium. The first-order condition (19) implies that the fertility rate is a function of lifetime utility and wage rates. Furthermore, lifetime utility can be rewritten as a function of the fertility rate and wage rates.[36] Using the initial guess for wage rates, we can solve the fertility rate for each type: n_{ss}, n_{us}, and n_{uu}.

[36]For example, the lifetime utility for skilled parents having skilled children is given by:

$$V_{ss} = \frac{1}{1-\psi(\pi^c n_{ss})^{1-\varepsilon}} \frac{1}{1-\sigma}(c_{ss}^{1-\sigma} + \beta \pi^y {c'}_{ss}^{1-\sigma}),$$

where consumption is determined by the budget constraint.

Once we have fertility, we solve the fraction of skilled workers and the fraction of unskilled workers using labor supply (25) and (26). Physical capital per labor is calculated. Then the physical capital income share and the unskilled labor income share are solved together to pin down the production weights μ and θ.

Now we can solve for new wage rates and the interest rate by marginal product. Furthermore, we can determine asset holdings for each type using the budget constraint and the Euler equation. The lifetime utility for each type is calculated.

Finally, we update w_s and w_u using a linear combination of the initial wage rates and the new wage rates. λ_{us} is updated by the following equation:

$$\lambda_{us} = \lambda_{us} \left(\frac{V_{us}}{V_{uu}} \right)^{\upsilon}. \tag{28}$$

where υ is the update speed. Since λ_{us} is the fraction of unskilled parents having skilled children, in equilibrium the lifetime utility of having unskilled children should be equal to the lifetime utility of having skilled children. If the lifetime utility of having skilled children is higher, $V_{us} > V_{uu}$, unskilled parents prefer to educate their children, so λ_{us} has to increase. On the other hand, unskilled parents want to have unskilled children if $V_{us} < V_{uu}$. In this case, λ_{us} has to decrease.

For the next iteration, the population of skilled young adults and the population of unskilled young adults are updated using the population evolution equations. Aggregate physical capital is updated by (27).

The convergence procedure stops if two criteria are satisfied at the same time: (i) The initial wage rates and new wage rates are very close. (ii) V_{us} and V_{uu} are close to each other. After the convergence, we compute output per capita for this steady state. Then, we repeat the above process to solve for the second steady state.

3.4 Steady State Properties of the Model

The calibrated results are reported in Table 2. Young adults would like to have more children if the survival rate for children increases, regardless of the type of young adults. However, people become longevity and the cost of children in the second steady state goes up. Thus, the fertility rate declines. The fertility rate of skilled parents decreases by around 68 percent, the fertility rate of unskilled parents with skilled children declines by 68 percent, and the fertility rate of unskilled parents with unskilled children declines by 61 percent.

The ratio of skilled workers to total workers increases to 32.9 percent in the second steady state. Three channels drive this result. The first is human-capital accumulation. The skill premium provides an incentive for parents to send their children to school. Thus, in the second steady state, more parents educate their children. Second, although the time cost goes up, the total time that a young adult spends on his children declines. For example, a skilled young adult spends 30 percent of his time on his children at the first steady state, while a skilled young adult only spends 15 percent of his time raising his children at the second steady state.[37] Thus, skilled young adults spend more time working and teaching. Third, the ratio of teachers to skilled labor declines from 18 percent to 12 percent.[38] Due to lower fertility rates, the demand for teachers decreases. More skilled young adults work in the production sector. Thus, there is more skilled labor in the second steady state and the fraction of skilled workers goes up.

4 Experiments

This section provides several counterfactual experiments. In each case, we solve a transition path from one steady state to another steady state. Specifically, each

[37]By assumption, each young adult has one unit of time. A young skilled adult spends ϕn_{ss} of his time on raising children. ϕn_{ss} is equal to 0.3 at the first steady state and is about 0.15 at the second steady state.

[38]The fraction of teachers as a percentage of available skilled labor is computed by $\frac{\phi_s(n_{ss}N_s^y+n_{us}\lambda_{us}N_u^y)}{(1-\phi n_{ss})N_s^y}$.

transition path starts with the first steady state, which is reported in the last section. Permanent shocks take place at period four.[39] After shocks, a transition path converges to a new steady state.

The algorithm for a transition path is slightly different from Section 3.3. Given the initial steady state, we first compute the state variables $\{K, N_s^y, N_u^y\}$ for the second period. All state variables are normalized by the population of young adults. To solve a transition path, we initially guess a sequence of $\{w_s, w_u, r, \lambda_{us}, V_{ss}, V_{uu}\}$ for each period. In our model, the interest rate tomorrow and children's utility are foreseeable to a young adult. Thus, we also need to guess r, V_{ss}, V_{uu} for period $t+1$ in order to solve the last period. Then, for each period, we solve the partial equilibrium, compute the ratio of skilled workers to total workers and physical capital per worker, calculate new wage rates and the new interest rate, and compute asset holdings and consumption to obtain new lifetime utilities. At this point, we already have a new sequence of these variables $\{w_s, w_u, r, V_{ss}, V_{us}, V_{uu}\}$. We then construct tests to check the distance between our initial guesses and new values. The difference between V_{us} and V_{uu} is also tested for λ_{us}. Finally, we update the sequence of each variable using a linear combination of the original value and the new value. λ_{us} is updated by (28). This procedure stops if the following criteria are satisfied at the same time: (i) the original values of $\{w_s, w_u, r, V_{ss}, V_{uu}\}$ are close to the new values; and (ii) V_{us} and V_{uu} are close to each other.

4.1 Contribution to Growth

In 1970-2004, Taiwan experienced not only demographic change but also technological progress. This sub-section discusses their contributions and interactions to

[39] In the model, parents have full information. Therefore, parents will adjust his decisions a few periods earlier. To observe the early changes, the permanent shocks are imposed at period four rather than period one. Period four is arbitrary. It is not interpreted as the shocks take place after 100 years.

economic growth.

In the experiments, the following cases are considered: (1) TFP growth; (2) A change in the factor weight μ; (3) A change in the factor weight θ;[40] (4) Changes in the both factor weights (or skill-biased technological progress);[41] (5) Demographic changes; (6) TFP growth and skill-biased technological progress; (7) Demographic changes and TFP growth; (8) Demographic changes and skill-biased technological progress; (9) Total changes (i.e., demographic changes, TFP growth, and changes in factor weights).

In every case, the corresponding parameter permanently changes from the first steady state value to the second steady state value, as reported in Table 1. Other parameters remain unchanged. For example, TFP growth refers to the TFP increases from 1 to 1.628 and other parameters stay at the first steady state.

Demographic changes include changes in four parameters: the survival rate for children, the survival rate for young adults, the time cost, and the education time cost. It is important to note that we include rising time costs and education costs in demographic changes. Section 3.2 has discussed possible reasons for rising time costs and education costs. Furthermore, Bar and Leukhina (2008) find that increasing the cost of raising children helps to explain the decline of the general fertility rate. Therefore, we group the rise of the two survival rates, the time cost, and the education cost together in order to generate a decline in fertility.

Table 3 summarizes the results. The growth rates of output per capita (annual growth rate) are reported in column 3.[42] The overall annual growth rate of per

[40] As discussed in Section 3.2, factor weights are jointly determined by the income share of physical capital and the income share of unskilled labor. In case (2) and (3), we change one factor weight at one time. Thus, the income share of physical capital will change and may not be equal to 0.33. The income share of unskilled labor may also change.

[41] The case of changes in the both factor weights is named "Tech" in all tables and figures.

[42] In the experiment, it takes 7 periods (175 years) to converge to the second steady state. If we use 175 years to convert the growth rate of per capita output into annual basis, the annual growth

capita output is 8.5 percent (the last row). Thus, the contribution of each case is reported as the percentage of the overall growth rate (column 4). For example, demographic changes result in 3.2 percent of annual growth rate of per capita output. Therefore, the demographic changes contribute about 38 percent (3.25/8.50) of the overall growth rate in Taiwan during the past four decades. Figure 5 provides the transition path of per capita output that is affected by demographic changes.

Table 3 shows that TFP growth explains about 28 percent of simulated output growth. Skill-biased technological progress explains another 29 percent. The contribution of demographic changes is about 38 percent. The remaining 5 percent has to be due to their interactions.

Summarizing from the above experiments, this paper concludes that the late demographic change has contributed more than one-third of the growth rate observed in Taiwan in 1970-2004, while the standard catching-up argument (TFP growth) would explain another third, the remaining growth would be due to the skill-biased technological progress.

The contribution of demographic changes to growth is significant but we may still under-estimate the impact of demographic changes. First, the demographic dividends of Taiwan started in the 1950s but our transition path starts with 1970 because of data availability. We ignore the first twenty years, which had rapid decline in fertility. Second, instead of efficiency units of labor, we use the number of workers as inputs. It is believed that in 1970-2004, workers' health improved remarkably due to health policies and medical advances.

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rate due to demographic change is 0.64 percent. Since the overall growth rate of per capita GDP (8.5% per year) is computed from 1970 to 2004, to be comparable, one period is equal to five years when we convert the growth rate of per capita output into annual basis. The inconsistency is the common problem in OLG models.

4.2 Interactions between Technological and Demographic Changes

Table 3 shows that the interaction between technological and demographic changes explain about 5 percent of the overall growth: 3 percent can be explained by the interaction between TFP growth and demographic changes; more than 1 percent is resulted from TFP growth and skill-biased technological progress; and less than 1 percent is due to the joint effects of skill-biased technological progress and demographic changes.

To further explore the interactions, we calculate the percentage changes of labor-population ratio ($\frac{L}{N}$), physical capital per labor (k), and the ratio of skilled labor (l_s) that are affected by the interactions. For example, the percentage change of labor-population ratio due to the interaction between skill-biased technological progress and demographic changes are computed as follows. First, transition paths for changes in factor weights and for demographic changes are separately calculated. Second, the two individual changes occur together to obtain a joint transition path. Third, the percentage change ($\Delta\frac{L}{N}$) is calculated by:

$$\Delta\frac{L}{N} = \left(\frac{\frac{L}{N}_{joint,ss2}}{\frac{L}{N}_{ss1}} - 1\right) - \left(\frac{\frac{L}{N}_{Tech,ss2}}{\frac{L}{N}_{ss1}} - 1\right) - \left(\frac{\frac{L}{N}_{Dem,ss2}}{\frac{L}{N}_{ss1}} - 1\right),$$

where $\frac{L}{N}_{joint,ss2}$ is the labor-population ratio caused by the joint effect in the period in which the transition converges to; $\frac{L}{N}_{ss1}$ is the labor-population ratio at the beginning, which refers to the first steady state reported in the last section. "Tech" refers to the skill-biased technological progress. "Dem" is the demographic changes.

Figures 6-8 are the transition paths of interactions. All transition paths are plotted on a logarithmic scale. Table 4 summarizes their magnitudes. Percentage changes due to interactions are reported in Part A. For example, the interaction between skill-biased technological progress and demographic changes results in an increase in the labor-population ratio by about 5 percent, physical capital per labor by 177 percent, and the ratio of skilled labor by 20 percent in the past four decades.

Greenwood and Seshadri (2002) show that technological change increases the marginal costs of raising children, reduces fertility rates, and gives rise to human-capital accumulation. Similar results are observed in Column 3 of Table 4. Besides, our results also support the argument in Galor (2005a) that technological progress rises the demand for skill labor and therefore motivates parents to substitute quality for quantity of children. Ultimately, the fertility declines and the quality of population is improved.[43]

In Part B, the percentage changes are converted into the percentage contribution of total changes. We find that the three interactions (not the synergy effects) explain almost 50 percent of the physical-capital accumulation; more than one-fourth of the human-capital accumulation; and about 7 percent of the increase in the labor-population ratio.

Among the three interactions, the interaction between skill-biased technological progress and demographic changes gives a higher rise to labor-population ratio. One possibility is the decline in the gender wage gap suggested by Galor and Weil (1996) and Lagerlöf (2003): Capital-skill complementarity reduces the gender wage gap (because capital is more complement to women's labor than to men's), and thereby leads to a lower fertility rate and higher women's labor force participation rate.

In 1980 of Taiwan, women's wages were about 64 percent of men's. The gender wage gap was stable in the early 1980s and then started to decline. In 2008, women's wages rose to about 80 percent of men's.[44] At the same period of time, women's

[43] Also see Galor and Weil (1999); Galor and Weil (2000); and Galor (2005b) for the connection between technological change, demographic transition, and human capital accumulation.

[44] The wages refers to the average monthly earnings (on payrolls) of employees who work in industry and services. The average monthly earnings include regular wages (salary and fixed monthly subsides and bonuses) and other non-regular wages (for example, monthly bonuses, holiday and performance bonuses, employee bonuses, travel expenses, meal allowances, and wage increment reimbursements).

labor force participation rate increased from 39 percent of 1980 to 50 percent of 2008. Thus, the decline in gender wage gap would lower fertility, give a rise in women's labor force participation, and thereby increases the fraction of labor force as a percentage of total population.

There is no gender distinction in our model, therefore it is not easy to directly measure the effects of the gender wage gap on fertility. However, higher women's wages provide economic incentives for women to work, meaning an increase in the cost of raising children. In our numerical analysis, the effects of gender wage gap would be reflected in the increase in the time cost of raising children (from 0.1102 to 0.1775 in Table 1). A more complicated model is required if we want to separate the effects of gender wage gap from the overall effects.

4.3 Decomposition into Three Channels

Equation (5) shows that demographic changes affect output per capita through three channels: the labor-population ratio ($\frac{L}{N}$), physical capital (k), and human capital (l_s). Define output per worker as $y = F(k, l_s, 1 - l_s)$. The impact of demographic changes through each channel is measured as follows:

Channel of the fraction of working-age population (and the corresponding dependency ratio):

$$\frac{y_{pc,2}}{y_{pc,1}} = \frac{(\frac{L}{N})_2}{(\frac{L}{N})_1} \frac{F_1(k_1, l_{s,1}, 1 - l_{s,1})}{F_1(k_1, l_{s,1}, 1 - l_{s,1})};$$

Channel of physical-capital accumulation:

$$\frac{y_{pc,2}}{y_{pc,1}} = \frac{(\frac{L}{N})_1}{(\frac{L}{N})_1} \frac{F_1(k_2, l_{s,1}, 1 - l_{s,1})}{F_1(k_1, l_{s,1}, 1 - l_{s,1})};$$

Channel of human-capital accumulation:

$$\frac{y_{pc,2}}{y_{pc,1}} = \frac{(\frac{L}{N})_1}{(\frac{L}{N})_1} \frac{F_1(k_1, l_{s,2}, 1 - l_{s,2})}{F_1(k_1, l_{s,1}, 1 - l_{s,1})};$$

where x_2 refers to variable x in the steady state that the transition path converges to. x_1 denotes variable x in the steady state that the transition path starts from. $F_1(.)$ is the production technology at the first steady state. To explore the importance of each channel, for example, the first channel, GDP per capita is calculated using the labor-population ratio in the second steady state, while other variables and the production technology are taken from the first steady state.

Table 5 summarizes the effects of demographic change through each channel in terms of annual growth rate of output per capita. Overall, the dependency ratio is the most important channel through which demographic changes influence economic growth. The ratio of labor force to total population increases from 0.099 to 0.222.[45] Thus, the decline in the dependency ratio generates about 2.32 percent per year of per capita output growth. This contribution accounts for about 72 percent of total growth resulted by demographic change. Besides, the contribution of human-capital accumulation is small. The increase in human capital (from 0.085 to 0.115) generates only a 0.02 percent growth per year.

If we take joint effects into account, our conclusion will be different. Table 6 reports annual per capita output growth generated by demographic change and technological progress through the three channels. In general, the accumulation of physical capital becomes the most important channel. In addition, compared to the pure effects of demographic change, the channel of human-capital accumulation becomes more important when demographic change and technological progress occur at the same time. For example, the second column is the decomposition caused by the joint effect of demographic change and skill-biased technological progress. In

[45]The ratio of labor force to total population is computed by the following equation:

$$\frac{L}{N} = \frac{[1-(\phi+\phi_s)n_{ss}]\frac{N_s^y}{N^y} - \phi_s n_{us}\lambda_{us}\frac{N_u^y}{N^y} + (1-\phi n_{us})\lambda_{us}\frac{N_u^y}{N^y} + (1-\phi n_{uu})(1-\lambda_{us})\frac{N_u^y}{N^y}}{\frac{N}{N^y}}.$$

In the first steady state, $\frac{N_s^y}{N^y}$ is 0.079; $\frac{N_u^y}{N^y}$ is 0.921; λ_{us} is 0.043; and $\frac{N^y}{N}$ is 0.189. In the second steady state, they are equal to 0.112, 0.888, 0.094, and 0.314, respectively.

this case, the effects of a decline in fertility through human-capital accumulation generate 1.4 percent per year of per capita output growth, whereas the channel of human-capital accumulation in Table 5 only accounts for 0.02 percent per year. The intuition is easy. When an economy has a labor-intensive technology, such as an agricultural society, a lower dependency ratio (the relative quantity of workers) can generate a higher GDP per capita. However, when the production technology becomes more skill-intensive, as in an industrial society, the quality of workers becomes relatively more important.

There are three points we should keep in mind here. First, the annual GDP growth rates in Table 5 come from demographic changes only while the annual GDP growth rates reported in Table 6 are caused by joint effects (not interactions). Thus Table 6 is the synergy effects. Furthermore, the second row of Table 6, the impact of the dependency ratio on GDP growth, is calculated by changes in the ratio of $\frac{(\frac{L}{N})_2}{(\frac{L}{N})_1}$, instead of $\frac{(\frac{L}{N})_2}{(\frac{L}{N})_1}\frac{F_2(k_1,l_{s,1},1-l_{s,1})}{F_1(k_1,l_{s,1},1-l_{s,1})}$. This is because the second part is the growth due to technological progress, not directly due to a change in the dependency ratio. Third, adding up the decomposition of the annual growth rate in Table 5 (also Table 6) is not necessarily equal to the annual growth rate reported in Table 3. Because of the method of decomposition, they are not orthogonal.

5 Discussions

5.1 Land Reforms in Taiwan

Galor, Moav, and Vollrath (2009) argue that land reforms reduces the economic incentives of landlords to block education reforms, leading an increase in education. Therefore, the land reforms may play an important role in the growth process of Taiwan.

The land reforms in Taiwan were implemented in 1949-1953. It included farmland rent reduction, sale of public land to tenants, and the land-to-the-tiller program (to help tenants acquire landownership and to enforce landlords to convert landholding into industrial stocks).

For farmers who had been tenants, the reforms increased their income.[46] Therefore, children's education investments became affordable, leading an increase in the accumulation of human capital and an increase in the supply of educated labor. In addition, the land reforms in Taiwan were accompanied by educational reforms (the implementation of Nine-Year Universal Compulsory Education), leading further human-capital accumulation. Therefore, the land reforms and education reforms together brought an increase in the supply of skilled labor. Others being equal, the return to human capital would decline.

If land reforms are sufficient to bring about human-capital formation, a decline in skill premium would be observed. Unfortunately, the data of wage rate by education for the period of 1950-1970 are unavailable. In contrast, as shown in Table 2, the skill premium increased from 1.76 of 1970 to 2.20 of 2004. One possible explanation is the contents of the land-to-the-tiller program. For the land sold under the program, the landlords were compensated 70 percent in food bond (rice bonds and sweet potato bonds) and 30 percent in government enterprise stocks.[47] Thus, the financial resources of the landlords were transferred from land to the industrial enterprises. Besides, holding farmland was not profitable anymore after the rent reduction. On the other hand, the government of Taiwan introduced industrial policies in order to develop industrial sectors. For example, the policy of Import Substitution in the 1950s; the policy of Export Orientation and the establishment of Export Processing Zone in the 1960s. These policies provided the economic incentives of the landlords

[46]See Table 13 and 14 in Koo (1968).

[47]The government enterprise stocks included the stocks in the Taiwan Cement Corporation, the Taiwan Paper and Pulp Corporation, the Taiwan Agricultural and Forestry Development Corporation, and the Taiwan Industrial and Mining Corporation.

to transfer their financial resources to industrial enterprises. Therefore, physical capital was accumulated and the demand of skilled labor increased. Labor force moved from agriculture to industrial sectors.

The land reforms and industrial policies of Taiwan influenced both the demand and supply of educated labor in the 1950s and the 1960s. Thus, land reforms may help explaining a part of the subsequent human-capital accumulation and the growth process in Taiwan. Besides, the development of the high-tech industry in the early 1980s would further increase the demand of skilled labor, leading further human-capital formation.[48]

5.2 Capital-skill Complementarity

This paper employs a CES production function with capital-skill complementarity. Fernández-Villaverde (2001) suggest that more productive capital raises the skill premium when physical capital and skilled labor are more complement than physical capital and unskilled labor. The increase in the skill premium motives parents to substitute quality for quantity of children and leads to a decline in fertility.

The empirical research finds evidence to support the capital-skill complementarity hypotheses, such as Griliches (1969); Fallon and Layard (1975); Hamermesh (1993); Goldin and Katz (1998); Duffy, Papageorgiou, Perez-Sebastian (2004); and Yasar and Paul (2008). In particular, Papageorgiou and Chmelarova (2005) find strong evidence for the non-OECD countries, while the hypothesis of capital-skill complementarity is rejected by the OECD countries.

To test if our results are sensitive to capital-skill complementarity, we use a Cobb-Douglas production function and do the numerical experiments again. As reported in Section 3.2, the income shares are 0.33, 0.0942, and 0.5758 at the first steady state; and 0.33, 0.3408, and 0.3292 at the second steady state for physical

[48]Hsinchu Science Park was established in 1980.

capital, skilled labor, and unskilled labor, respectively. We apply the same income shares when we do the calibration with a Cobb-Douglas production function. TFP at the second steady state increases to 3.85 in order to match the annual growth rate of per capita output 8.5%. Other parameters remain unchanged.

The results are consistent. With a Cobb-Douglas production function, demographic transition alone generates about 2.96% per year of a growth rate of output. Therefore, the contribution of demographic transition is about 35% of the simulated output growth (2.96/8.50), meaning that more than one-third of the total growth is due to the demographic transition. In addition, when the effects of demographic transition are decomposed into the three channels, changes in the dependency ratio is the most important channel, which generates an annual growth rate of 2.27%. The accumulation of physical capital and human capital generate 0.70% and -0.04%, respectively. Finally, if the joint effects are considered, physical-capital accumulation is the most important channel and human-capital formation becomes more important than changes in dependency ratio.

5.3 Steady-state Assumption

The experiments in this paper begin at 1970 due to data availability. We assume that 1970 is a steady state. All transition paths start with this steady state. However, the demographic change in Taiwan actually started earlier than 1970. The fertility rate in 1951 was about 7 children per woman, decreased to 5.6 in 1961 and to 3.7 in 1971. Therefore, the 1970s were actually in the midst of the demographic change and were not a relatively stable period of time.

To test the influence of this assumption, the initial state variables ($\frac{K}{N^y}$ and $\frac{N_s^y}{N^y}$) are artificially modified as a percentage of those in 1970, such as 70 percent or 50 percent of 1970. The modified state variables capture that Taiwan in 1970 was not yet in steady state, but was on a transition path from an initial stage with relatively few skilled workers and little physical capital. Using the modified initial state variables as the initial condition, a new transition path is calculated.[49] The results are compared to the case with 100 percent of the state variables in 1970 (called the primary case below).

Table 7 reports the results. In the primary case, the demographic effect is 2.6 percent per year. If both state variables are 90 percent of those in 1970, the demographic effect slightly increases to 2.7 percent per year. When they are 70 percent of those in 1970, the demographic effect becomes 3 percent per year. In conclusion, the demographic effect increases as the state variables are getting smaller. The demographic effect is about 3.8 percent per year if the initial state variables are only 50 percent of those in 1970. In this case, the demographic change accounts for 40 percent (3.8/9.2) of the total growth rate of per capita output. We also estimate the demographic change by changing one state variable at one time. The results are similar.

The results show that the assumption of a steady state in 1970 does not have a significant influence on the main conclusions. In addition, the demographic effect is larger and is more important if the initial condition is smaller. It supports our explanation that the demographic effect could be larger if we can extend our estimate to the 1950s.

6 Conclusions

As a supplement to the literature, this paper attempts to quantify the contribution of various sources to growth. In particular, we shed light on the role of demographic transition.

The results suggest that more than one-third of the growth in Taiwan during 1970-2004 could be explained by the demographic change, while others are mainly

[49]The demographic change then occurs in the second period of the new transition path.

due to TFP growth and skill-biased technological progress. At the same period of time, the percentage decline in fertility was similar in the East Asia Tigers: the fertility declined 72 percent in Taiwan; 73 percent in Hong Kong; 76 percent in Korea; and 60 percent in Singapore. Therefore, our results are not a special story for Taiwan. It can be generalized to other developing countries which has both a rapid decline in fertility and a fast growth.

It is important to note that we do not conclude the effect of demographic change is about 3 percent per year in every country which is enjoying demographic dividends. The contribution of demographic transition heavily depends on a country's health policies, education policies, family planning policies, and economic policies. Different policy environments will influence the impacts of demographic changes. In further studies, it would be interesting to quantify the effect of demographic transitions in countries with very different population policies, such as China.

In the Solow model, economic growth is independently affected by population growth, the accumulation of physical capital, or the accumulation of human capital. This paper constructs a general equilibrium overlapping generations model in which population growth, physical capital, and human capital are connected to each other. The occurrence of demographic change directly affects the dependency ratio. Besides, due to a decline in fertility, the resources are re-allocated. Thus, physical capital and human capital are also accumulated in the process of demographic change.

Our model provides a convenient way to decompose the impact of demographic changes on GDP per capita. Our results suggest that changes in dependency ratios is relatively more important than the channel of human capital and the channel of physical capital in an agricultural-alike economy. However, if technological progress occurs together, physical-capital accumulation is the most important channel. The role of the human-capital channel becomes more important. Thus, in an industrial society, the quality of workers matters.

Appendix A: Proof for Corner Solutions

To simplify the notation, we ignore the subscript of the adult's type, i. At the first stage, the optimal total expenditure of children is determined. Given this E and the adult's type, the following maximization problem is considered:

$$\max_{0\leq f\leq 1}\left\{\frac{c^{1-\sigma}}{1-\sigma}+\beta\pi^{y}\frac{c'^{1-\sigma}}{1-\sigma}+\psi\pi^{c^{1-\varepsilon}}E^{1-\varepsilon}\left(\frac{f}{p_s}+\frac{1-f}{p_u}\right)^{-\varepsilon}\left(\frac{fV_s}{p_s}+\frac{(1-f)V_u}{p_u}\right)\right\},$$

where $c = w - \pi^y a' - E$. By assumption, β, π^c, π^y, and σ are between zero and one; $0 < \varepsilon < 1$; $\psi > 0$; $p_s > p_u > 0$; $V_s > 0$; and $V_u > 0$. We want to show that there is no interior solution, that is, the optimal f to satisfy the above maximization problem is either zero or one.

The first derivatives of U with respect to f is:

$$\begin{aligned}\frac{\partial U}{\partial f} &= \psi\pi^{c1-\varepsilon}E^{1-\varepsilon}\Bigg[(-\varepsilon)\left(\frac{f}{p_s}+\frac{1-f}{p_u}\right)^{-\varepsilon-1}\left(\frac{1}{p_s}-\frac{1}{p_u}\right)\left(\frac{fV_s}{p_s}+\frac{(1-f)V_u}{p_u}\right)\\ &\quad+\left(\frac{f}{p_s}+\frac{1-f}{p_u}\right)^{-\varepsilon}\left(\frac{V_s}{p_s}-\frac{V_u}{p_u}\right)\Bigg]. \qquad (29)\end{aligned}$$

The terms outside the bracket are positive. The first term in the bracket is also positive. If the last term is $\frac{V_s}{p_s} \geq \frac{V_u}{p_u}$, the first-order condition cannot be satisfied. Then there is no interior solution. For an interior solution to be possible, it has to be the case that $\frac{V_s}{p_s} < \frac{V_u}{p_u}$. Thus, in the following discussion, we focus on this case. First, we set the first derivative to be zero to solve the unique f. Then we plug this f into the second derivative to show that the second-order condition for a maximum can not be satisfied. Setting (29) to be zero, f is given by:

$$f=\frac{\varepsilon(\frac{V_u}{p_s}-\frac{V_u}{p_u})-(\frac{V_s}{p_s}-\frac{V_u}{p_u})}{(1-\varepsilon)p_u(\frac{1}{p_s}-\frac{1}{p_u})(\frac{V_s}{p_s}-\frac{V_u}{p_u})}. \qquad (30)$$

The second derivatives of U with respect to f is:

$$\begin{aligned}\frac{\partial^2 U}{\partial f^2} &= \psi\pi^{c1-\varepsilon}E^{1-\varepsilon}\varepsilon\left(\frac{f}{p_s}+\frac{1-f}{p_u}\right)^{-\varepsilon-1}\left(\frac{1}{p_s}-\frac{1}{p_u}\right)\Bigg[(\varepsilon+1)\\ &\quad\left(\frac{f}{p_s}+\frac{1-f}{p_u}\right)^{-1}\left(\frac{1}{p_s}-\frac{1}{p_u}\right)\left(\frac{fV_s}{p_s}+\frac{(1-f)V_u}{p_u}\right)-2\left(\frac{V_s}{p_s}-\frac{V_u}{p_u}\right)\Bigg].\end{aligned}$$

The whole term outside the bracket is negative. Therefore, the second derivative is positive if the whole term in the bracket is negative. Plugging in f, the inequality becomes:

$$(1+\varepsilon)\left(\frac{V_u}{p_s}-\frac{V_u}{p_u}\right)<\varepsilon\left(\frac{V_u}{p_s}-\frac{V_u}{p_u}\right)+\frac{V_s}{p_s}-\frac{V_u}{p_u}.$$

After some algebra, this inequality yields $V_u < V_s$. Thus, if $V_u < V_s$, the second-order condition for a maximum can not be satisfied. The interior solution does not exist. On the other hand, if $V_u > V_s$, parents always prefer unskilled children for sure because they are cheaper. We conclude that there exists only corner solutions. Parents have either skilled or unskilled children. They do not want a mixture of children types.

Appendix B: Data Sources

Hong Kong

Data are from WDI, except the fraction of skilled labor. The fraction of skilled labor is from Census and Statistics Department, the Government of the Hong Kong Special Administrative Region. "Skilled labor" is defined as sixth form and post secondary.

Korea

Data are from WDI, except the fraction of skilled labor. Before 2000, the fraction of skilled labor is from Lee (1997); after 2000 is from Korean Statistical Information Service (KOSIS). "Skilled labor" is defined as college, university, and above.

Taiwan

GDP per capita, gross savings (as a percentage of GNP), GNP, average earnings of employees on payrolls by gender, and women's labor force participation rate are from Macroeconomics Database, DGBAS, Executive Yuan. Fertility and population age structure are from Statistics, Department of Household Registration, Ministry of the Interior, Executive Yuan. The fraction of skilled labor is from Council of Labor Affairs, Executive Yuan. "Skilled labor" is defined as college, university, and above. The number of deaths is obtained from Health Statistics, Department of Health, Executive Yuan. Annual TAIEX is obtained from Taiwan Stock Exchange Corp. Wages by education in 1978 are from Report on the Survey of Personal Income Distribution in Taiwan Area, Republic of China 1978, DGBAS. Wages by education in 2004 are from The Survey of Family Income and Expenditure 2004, DGBAS. The number of students per teacher is obtained from Department of Statistics, Ministry of Education.

Singapore

Data are obtained from WDI, except the following items. The data source of gross savings as a percentage of GDP in 2005 is the Department of Statistics, Singapore. The fraction of skilled labor in 1970 and 1980 are both from Bercuson and Carling (1995), other years are obtained from the Department of Statistics, Singapore. in 1970, "skilled labor" is defined as university and above because of the lack of data; others are defined as post-secondary and above.

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Table 1: Parameters for Calibration

Parameter		1970 SS1	2004 SS2
Survival rate for children	π^c	0.9305	0.9666
Survival rate for young adults	π^y	0.9090	0.9351
Time period		25 years	25 years
Annual discount factor	β_a	0.93	0.93
Risk aversion	σ	0.5	0.5
Elasticity of altruism	ε	0.5	0.5
Altruism coefficient	ψ	0.238	0.238
Annual depreciation rate	δ_a	0.0905	0.0905
Elasticity of substitution (L_u and K)	$\frac{1}{1-\alpha}$	1.67	1.67
Elasticity of substitution (L_s and K)	$\frac{1}{1-\rho}$	0.67	0.67
Total factor productivity	A	1	1.628
Physical capital income share		0.33	0.33
Unskilled labor income share		0.8594	0.4914
Factor weight (unskilled labor)	μ	0.1755	0.2025
Factor weight (physical capital)	θ	0.3281	0.1963
Education time cost	ϕ_s	0.0297	0.073
Time cost	ϕ	0.1102	0.1775
Good cost	p	0	0

Table 2: Calibrated Results

	1970		2004	
	Data	SS1	Data	SS2
n_{ss}		2.695		0.873
n_{us}		2.947		0.930
n_{uu}		4.453		1.738
Average fertility	3.365	3.365	1.180	1.180
l_s	0.085	0.085	0.329	0.329
$\frac{w_s}{w_u}$	1.761	1.759	2.199	2.113
y_{pc} (annual growth)			8.50%	8.50%

Table 3: Contribution to Growth

Changes in parameters	$y_{pc,SS1}$	$y_{pc,SS2}$	y_{pc} (annual growth)	% of total
TFP	0.0047	0.0108	2.41%	28.35
Factor weight μ	0.0047	0.0060	0.70%	8.24
Factor weight θ	0.0047	0.0094	2.00%	23.53
Tech (μ & θ)	0.0047	0.0109	2.43%	28.59
Dem	0.0047	0.0144	3.25%	38.24
TFP & Tech	0.0047	0.0259	5.00%	58.82
Dem & TFP	0.0047	0.0352	5.92%	69.65
Dem & Tech	0.0047	0.0332	5.74%	67.53
Dem & TFP & Tech (total)	0.0047	0.0780	8.50%	100

Note: "Tech" refers to skill-biased technological progress; "Dem" denotes demographic changes.

Table 4: Magnitude of Interaction

	Tech & Dem	TFP & Dem	TFP & Tech	Total Effects
	A. Percentage changes due to interactions			
$\Delta(L/N)$	4.96%	3.52%	2.46%	152.32%
Δk	177.23%	217.01%	168.75%	1156.25%
Δl_s	19.99%	18.93%	41.13%	286.25%
	B. Percentage of total effects			
$\Delta(L/N)$	3.26%	2.31%	1.62%	100%
Δk	15.33%	18.77%	14.59%	100%
Δl_s	6.98%	6.61%	14.37%	100%

Note: "Tech" refers to skill-biased technological progress; "Dem" denotes demographic changes.

Table 5: Three Channels-Demographic Effects

	GDP per capita (annual growth)
Dependency ratio	2.32%
Physical capital	0.83%
Human capital	0.02%

Note: Because of the way we do the decomposition, adding up the decomposition of the annual growth rate is not necessarily equal to 3.25 %.

Table 6: Three Channels-Joint Effects

GDP per capita (annual growth)	Tech & Dem	TFP & Dem	Total Effects
Dependency ratio	2.47%	2.41%	2.68%
Physical capital	2.55%	3.13%	4.53%
Human capital	1.42%	1.41%	2.77%

Note: "Tech" refers to skill-biased technological progress; "Dem" is demographic changes. Because of the way we do the decomposition, adding up the decomposition of the annual growth rate is not necessarily equal to the growth rates reported in Table 3.

Table 7: Sensitivity Tests

State Variable $\frac{K}{N^y}$ and $\frac{N_s^y}{N^y}$	Total Growth (A)	Demographic Effects (B)	Contribution (B)/(A)
100% for both	8.42%	2.61%	31.0%
90% for both	8.55%	2.73%	31.9%
70% for both	8.86%	2.99%	33.8%
50% for both	9.24%	3.75%	40.6%
90% for K/N^y	8.53%	2.75%	32.2%
70% for K/N^y	8.77%	3.02%	34.5%
50% for K/N^y	9.08%	3.70%	40.7%
90% for N_s^y/N^y	8.45%	2.59%	30.7%
70% for N_s^y/N^y	8.54%	2.90%	34.0%
50% for N_s^y/N^y	8.66%	2.96%	34.2%

Note: Column (A) and (B) are the growth rate of per capita output per year.

Figure 1: Growth and Fertility in the East Asia Tigers

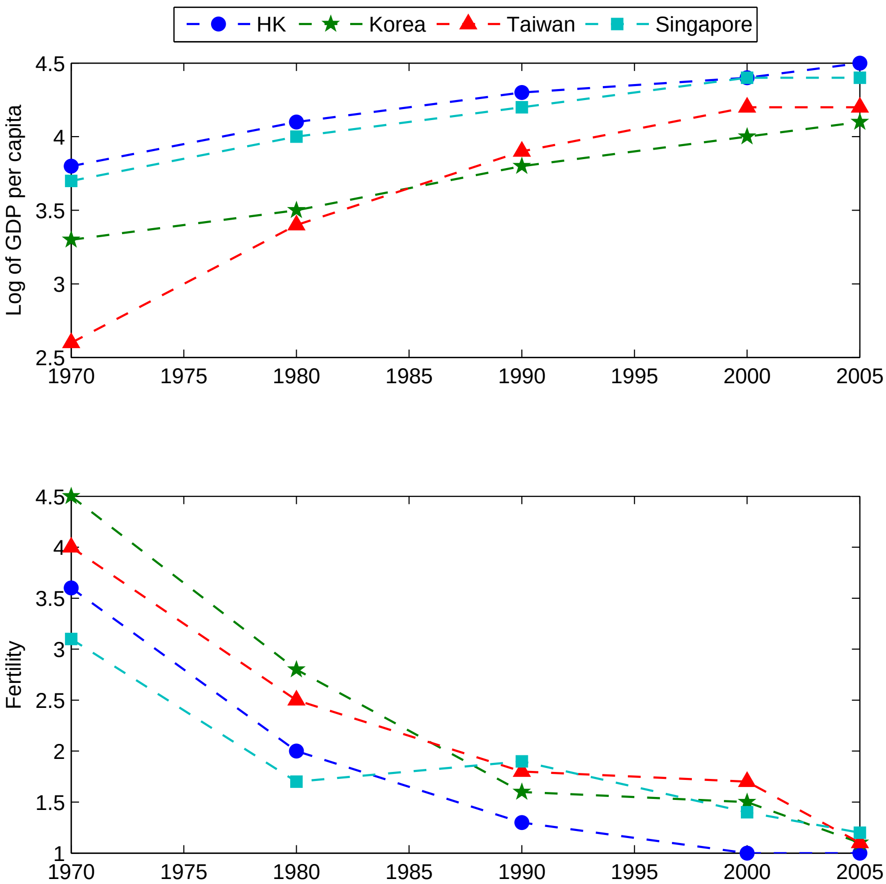
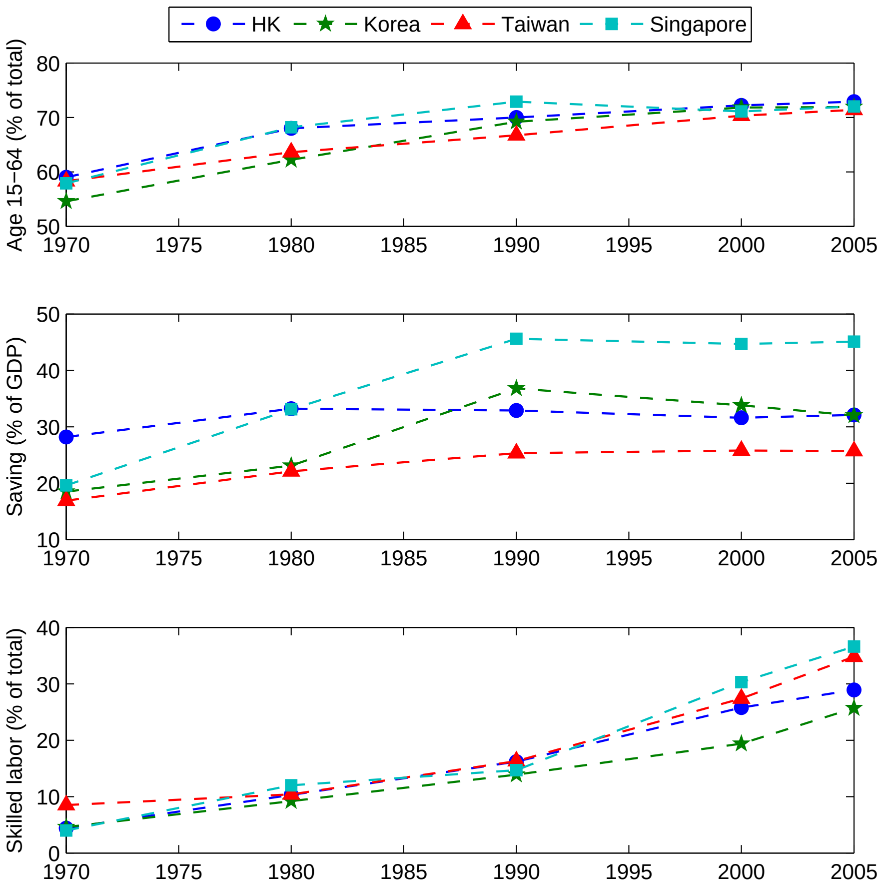
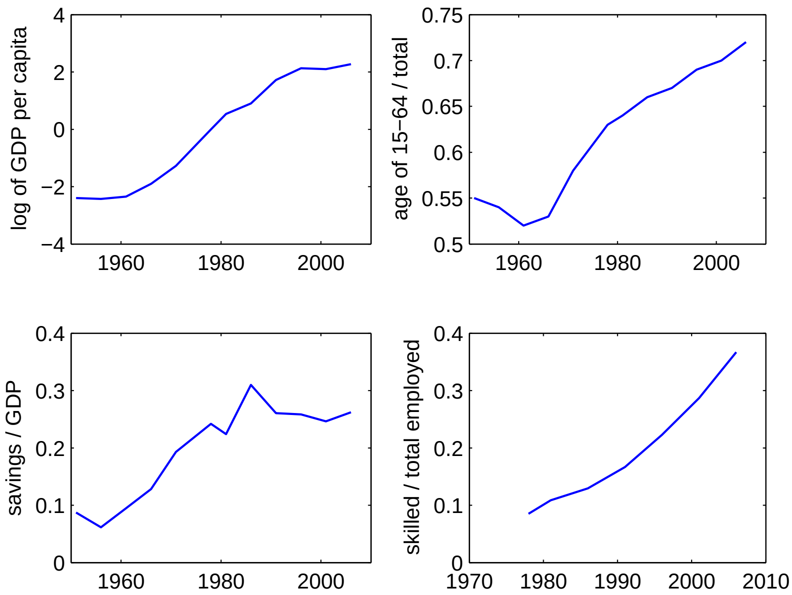


Figure 2: Three Channels in the East Asia Tigers



Note: The fraction of skilled labor of Hong Kong is the data of 1971, 1981, 1991, 2001, and 2006. The fraction of skilled labor in Taiwan 8.5% is the data of 1978. For the fraction of skilled labor in Singapore, 1970 and 1980 are the percentage of total work force while others are the percentage of citizens. There is no education data for post-secondary in the 1970 of Singapore, so only "university and above" are included. The fraction of skilled labor may not be comparable across countries because of different education systems.

Figure 3: Facts in Taiwan



Note: GDP per capita is in logarithm scale and 1978 is normalized to zero. “Skill” refers to college and above. “Savings” refers to gross savings.

Figure 4: Constructed Survival Rates in Taiwan

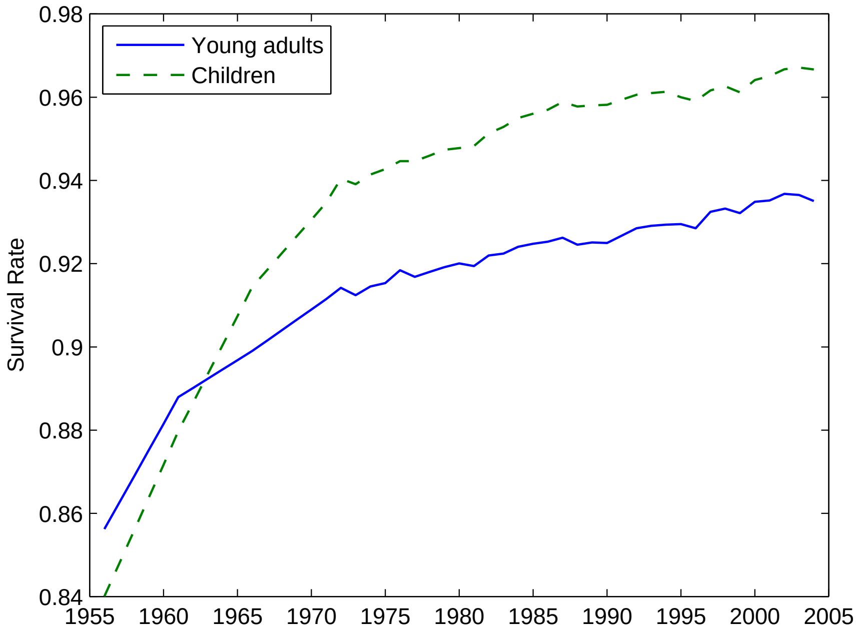


Figure 5: Transition of Demographic Effects

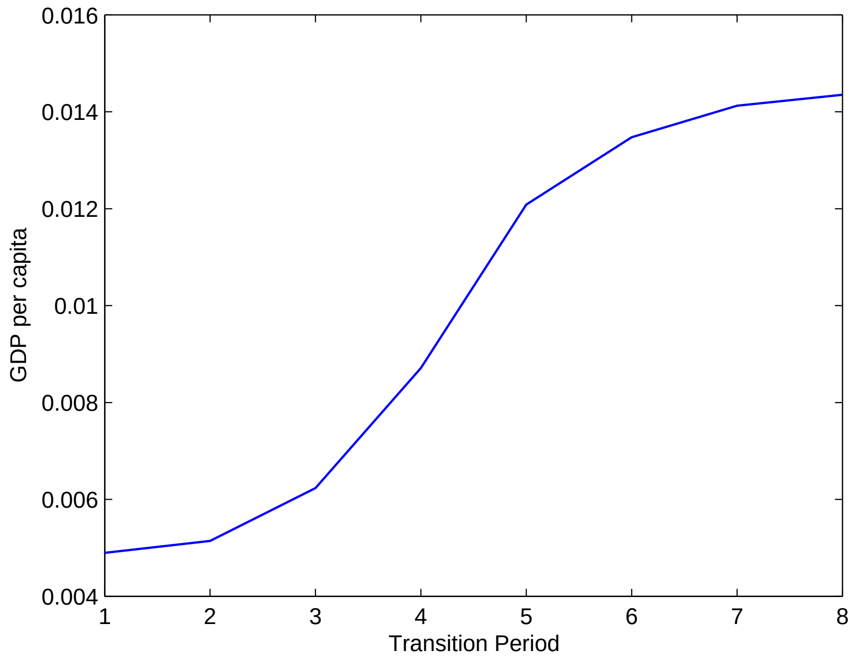
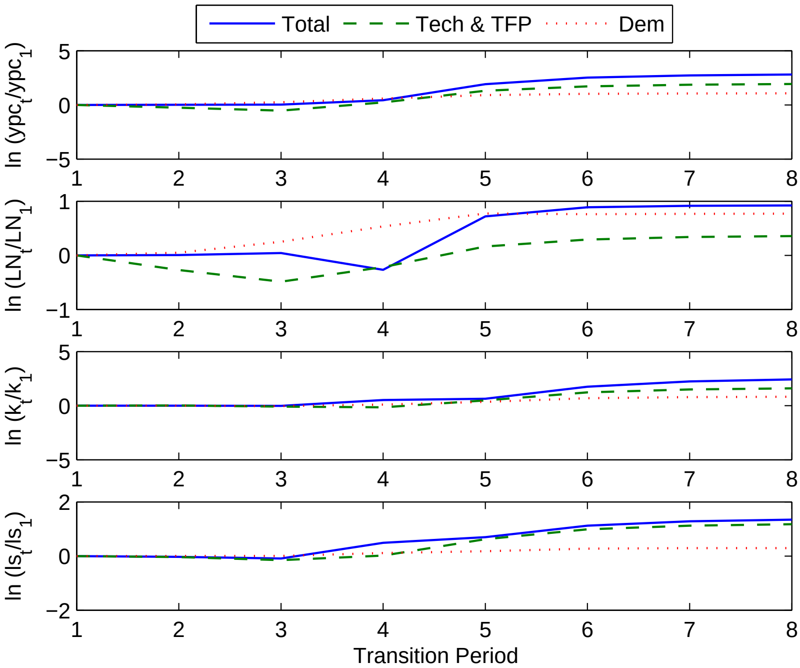
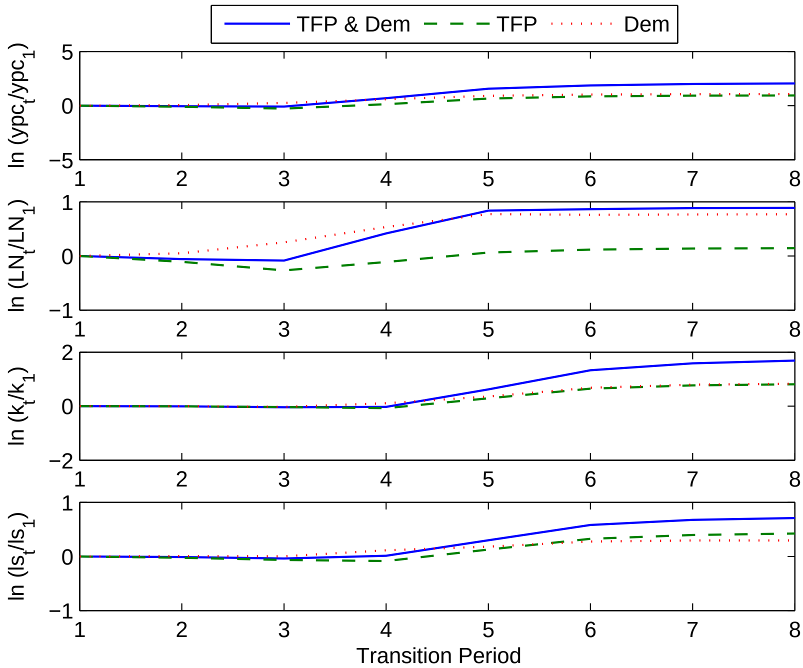


Figure 6: Total Changes



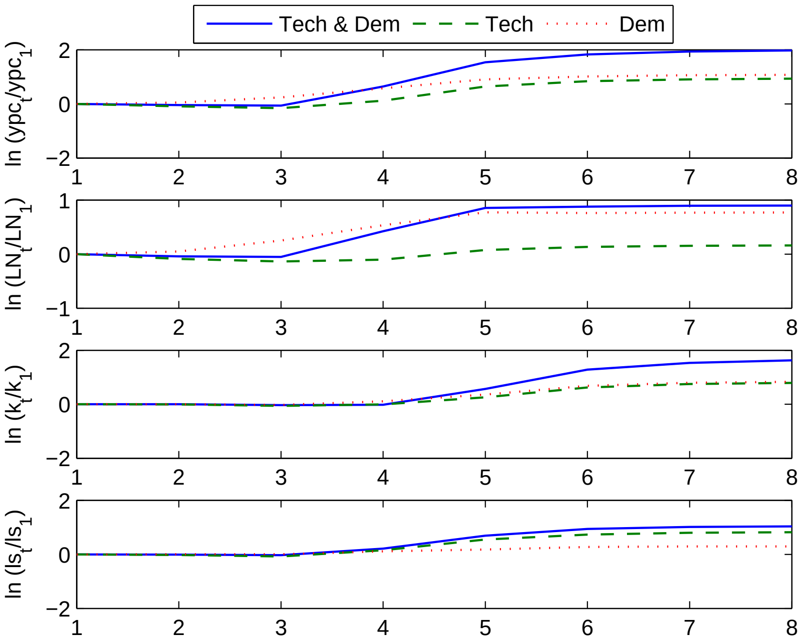
Note: "Tech" refers to skill-biased technological progress. "LN" denotes L/N.

Figure 7: TFP Growth and Demographic Changes



Note: "LN" denotes L/N.

Figure 8: Technological and Demographic Changes



Note: “Tech” refers to skill-biased technological progress. “LN” denotes L/N.