Partial Privatization, Optimum-welfare and Maximum-revenue Tariffs

Leonard F.S. Wang*
Department of Applied Economics, National University of Kaohsiung

Jen-yao Lee
Department of International Business, National Kaohsiung University of Applied Science

Abstract

This paper examines the effect of releasing stocks on the priority of the maximum-revenue tariff and the optimum-welfare tariff under a mixed oligopoly with partial privatization. We find that in the mixed oligopoly with partial privatization and asymmetric marginal cost, the optimum-welfare tariff is higher than the maximum-revenue tariff if the marginal cost of domestic private firms is relatively lower; otherwise, the maximum-revenue tariff will be higher than the optimum-welfare tariff. In addition, when the marginal cost of the privatized firm is sufficiently high, and the relatively higher releasing stocks ratio drives the critical marginal cost further down, the maximum-revenue tariff will be higher than the optimum-welfare tariff, *ceteris paribus*.

Keywords: mixed oligopoly, partial privatization, tariff ranking

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*Correspondence: Leonard F. S. Wang, Department of Applied Economics, National University of Kaohsiung, No. 700, Kaohsiung University Road, Nan-Tzu District 811, Kaohsiung, Taiwan, R.O.C. Tel:886-7-5919322, Fax:886-7-5919320, E-mail address: lfswang@nuk.edu.tw

1. Introduction

The issue of maximum-revenue tariff versus optimum-welfare tariffs is interesting and should be noticed, because the tariffs revenue is an important income source of the government before building up an efficient tax system in a developing country. However, the government may adjust its goal from maximum-revenue to optimum-welfare along with the economic improvement and the need for fiscal reform.

In a traditional tariff analysis, Johnson (1951-1952) argued that the maximum-revenue tariff is higher than the optimum-welfare tariff because a 'large' country could change the terms of trade in order to raise its social welfare level. From the strategic trade aspect, Brander and Spencer (1984) have shown that government could improve its terms of trade through tariffs in an oligopoly market and take a leading position to transfer a foreign firm's revenue to a domestic firm by using tariff as a strategic instrument. Collie (1991) demonstrated that in a quantity competition oligopoly market with a linear demand function and an asymmetric marginal cost, the maximum-revenue tariff will be raised if domestic marginal cost is higher, and the maximum-revenue tariff will be higher than the optimum-welfare tariff if the domestic firm's marginal cost is relatively higher than that of the foreign firm. Larue and Gervais (2002) allowed asymmetric numbers of domestic and importing firms, and showed that if the numbers of producing and importing firms are the same, the maximum-revenue tariff is higher than the optimum-welfare tariff.

Recently, Clarke and Collie (2006) found that in a Bertrand price competition model, the optimum-welfare tariff is higher than the maximum-revenue tariff when the product is highly substitutable. Clarke and Collie (2008) have shown that in a game between two exporting countries, both countries may be better off if they both delegate to policymakers who maximize tax revenue rather than welfare. However, both countries delegating to policymakers who maximize revenue is not necessarily a Nash equilibrium. Wang *et al.* (2009) introduced market share delegation in a trade duopoly context, and demonstrated that the home government unambiguously imposes a higher optimum-welfare tariff than maximum-revenue regardless of the form of delegation. Wang *et al.* (2010) re-examined the tariff ranking issue under a linear mixed oligopoly model with foreign competitors and asymmetric costs. In particular, they demonstrated that under Cournot competition, when the sizes of domestic private and foreign private firms become more unequally distributed, the optimum-welfare tariff will exceed the maximum-revenue tariff.

Maw (2002) reviewed the empirical evidence and analyzed the justifications that have been put forward for adopting partial privatization in transitional economies. Chao and Yu (2006) used Mastsumura's partial privatization modeling (1998) to examine the effect of partial privatization or foreign competition on optimal tariffs and found that foreign competition lowers the optimal tariff rate but partial privatization raises it.

In this paper, we examine the effect of releasing stocks on the priority of the maximum-revenue tariff and the optimum-welfare tariff under a mixed oligopoly with partial privatization. A two stages game model with complete information is used to examine the government's setting of the stock ratios for maximum-revenue and optimum-welfare tariffs in a mixed oligopoly market. We find that in the mixed oligopoly with partial privatization and asymmetric marginal cost, the optimum-welfare tariff is higher than the maximum-revenue tariff if the marginal cost of domestic private firms is relatively lower; otherwise, the maximum-revenue tariff will be higher than the optimum-welfare tariff. In addition, when the marginal cost of the privatized firm is sufficiently high, and the relatively higher releasing stocks ratio forces the critical marginal cost further down, the maximum-revenue tariff will be higher than the optimum-welfare tariff, *ceteris paribus*.

This paper is organized as follows. Basic modeling is provided in Section 2. Sections 3 and 4 contain the tariff analysis without and with domestic private firms. Section 5 concludes the paper.

2. Basic Modeling

Assuming that domestic demand function is $P = a - Q$, there is one domestic public firm, n domestic private firms and m foreign private firms in the Cournot competition. The supply equation is given by $Q = q_s + \sum_i^n q_i + \sum_j^m q_{fj}$, where q_i and q_{fj} denote, respectively, domestic public firm's and foreign private firm's productions. We also assume that their cost functions are $C_s = c_s q_s$, $C_i = c_p q_i$ and $C_f = c_f q_f$, with $c_s > c_p > c_f > 0$, which means that the production efficiency of the public firm is lower than that of the private firm[1]. We assume that government could levy tariff on imports and the magnitude of tariff is given by t.

For private firms to maximize profit, the optimization problem is:

$$\underset{\{q_i\}}{Max.}\ \pi_i = Pq_i - C_i \qquad (1)$$

$$\underset{\{q_{fj}\}}{Max.}\ \pi_{fj} = Pq_{fj} - C_{fj} - tq_{fj} \qquad (2)$$

Following the specification of the literature[2] and considering the target function of the government is to maximize social welfare,

$$W = CS + \pi_s + \sum_{i=1}^{n} \pi_i + t\sum_{j=1}^{m} q_{fj} \qquad (3)$$

where $CS = \frac{Q^2}{2}$ is the consumer surplus; $\pi_s = Pq_s - C_s$, is the public firm's profit. When government privatizes the public firm partially, the optimization problem for the privatized firm is:

$$\underset{\{q_s\}}{Max.}\ S = \theta\pi_s + (1-\theta)W = \pi_s + (1-\theta)\left(CS + \sum_{i=1}^{n} \pi_i + t\sum_{j=1}^{m} q_{fj}\right) \qquad (4)$$

where θ is the ratio of government releasing stocks which is given exogenously and has values in the interval (0, 1). Government may choose an optimum tariff rate to maximize social welfare (W) or tariff revenue (R), which are denoted by:

$$\underset{\{t\}}{Max.}\ W = CS + \pi_s + \sum_{i=1}^{n} \pi_i + t\sum_{j=1}^{m} q_{fj}$$

$$\underset{\{t\}}{Max.}\ R = t\sum_{j=1}^{m} q_{fj}\ . \qquad (5)$$

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[1] When $c_s \le c_f$, no foreign private firm will be put into production. Also see Huang, Lee and Chen (2006) for the extended specification.

[2] Public firm may have other different targets, such as maximizing the profit, income, employee's income or with management of license, etc. In order to compare with the literature, we assume that public firm will maximize social welfare, see De Fraja and Delbono (1989), Katsoulacos (1994), Fjell and Pal (1996), Pal and White (1998).

The game structure is written as follows.

Figure 1 Game Structure

In this model setting, backward induction is used to solve the *sub-game perfect Nash equilibrium* (SPNE).

3. Mixed oligopoly without domestic private firms

As per the modeling in Chao and Yu (2006), we firstly discuss the case of mixed oligopoly without any domestic private firm, that is $n = 0$.

Firstly, we get the first-order derivatives from Eqs. (2) and (4):

$$\frac{\partial \pi_{fj}}{\partial q_{fj}} = a - c_f - t - (1+m)q_{fj} - q_s = 0 \qquad (6)$$

$$\frac{\partial S}{\partial q_s} = a - c_s - mq_{fj} - 2q_s + (1-\theta)(mq_{fj} + q_s) = 0 \qquad (7)$$

Combining Eqs. (6) and (7), the equilibrium quantities and price are:

$$q_s^* = \frac{(1+m-m\theta)a + \theta m(t + c_f) - (1+m)c_s}{H_1} \qquad (8a)$$

$$q_{fj}^* = \frac{a\theta - (1+\theta)(t + c_f) + c_s}{H_1} \qquad (8b)$$

$$Q^* = \frac{a - c_s + m(a - c_f - t)}{H_1} \qquad (8c)$$

$$P^* = \frac{a\theta + c_s + m(c_f + t)}{H_1} \qquad (8d)$$

where $H_1 = 1 + m + \theta$.

From Eq. (8b), if $t \geq \hat{t} \equiv \frac{a\theta - (1+\theta)c_f + c_s}{1+\theta}$, the foreign firm's production is zero, which means $\hat{t}$ is a prohibited tariff ratio.

Lemma 1:

(i) When $0<\theta<1$ ***and*** $0\le t<\hat{t}$***,*** q_s^* ***and*** P^* ***are increasing in*** t***, but*** q_{fj}^* ***and*** Q^* ***are decreasing in*** t***.***

(ii) When $0<\theta<1$ ***and*** $0\le t<\hat{t}$***,*** q_{fj}^* ***and*** P^* ***are increasing in*** θ***, but*** q_s^* ***and*** Q^* ***are decreasing in*** θ***.***

Proof:

We take the partial derivative of Eqs. (8a) and (8d) with respect to t and θ, respectively,

$$\frac{\partial q_s^*}{\partial t}=\frac{m\theta}{H_1}>0$$

$$\frac{\partial P^*}{\partial t}=\frac{m}{H_1}>0$$

$$\frac{\partial q_{fj}^*}{\partial t}=-\frac{1+\theta}{H_1}<0$$

$$\frac{\partial Q^*}{\partial t}=\frac{-m}{H_1}<0$$

$$\frac{\partial q_s^*}{\partial \theta}=\frac{-(1+m)[a-c_s+m(a-c_f-t)]}{H_1^2}<0$$

$$\frac{\partial P^*}{\partial \theta}=\frac{(a-c_s)+m(a-c_f-t)}{H_1^2}>0$$

$$\frac{\partial q_{fj}^*}{\partial \theta}=\frac{(a-c_s)+m(a-c_f-t)}{H_1^2}>0$$

$$\frac{\partial Q^*}{\partial \theta}=\frac{-[(a-c_s)+m(a-c_f-t)]}{H_1^2}<0$$

From Lemma 1, when the tariff rate is raising, the privatized firm increases output and foreign firms decrease output, the market price then increases in the domestic market; on the other hand, when the degree of privatization is increasing, when even the privatized firm decreases output and foreign firms increase output, market price increases in the domestic market as well.

A higher tariff rate will increase domestic production and the domestic firm's profit. This is the *profit shifting effect* as named by Brander and Spencer (1984). As mentioned above, government's goal is to choose an optimum t for maximizing social welfare or tariff revenue. Solving this maximizing problem, Eqs. (3) and (5) are differentiated with respect to t, respectively,

$$\frac{dW}{dt} = Q\left[\frac{dq_s}{dt} + m\frac{dq_j}{dt}\right] + \left[q_s\frac{dP}{dt} + \left(P - c_s\right)\frac{dq_s}{dt}\right] + m\left[q_{fj} + t\frac{dq_{fj}}{dt}\right] \tag{9}$$

$$\frac{dR}{dt} = m\left[q_f + t\frac{\partial q_f}{\partial t}\right] \tag{10}$$

Eq. (9) indicates the impact of tariff rate on domestic social welfare. It can be decomposed into three effects: firstly, *consumer-surplus effect* is negative, increasing the tariff will decrease consumer surplus; secondly, *profit-shifting effect* is positive, domestic firm's profit is raised when tariff is increased. Higher tariff increases the foreign firm's marginal cost which will make the domestic firm more competitive, profit is shifted from the foreign firm to the domestic privatized firm; thirdly, *tariff-revenue effect*, which could be either positive or negative. From Eq. (10), as is well known, the tariff revenue effect is zero when the government practices maximum-revenue tariff policy. Nevertheless, under optimum-welfare tariff policy, the sign of Eq. (9) depends on the relative magnitudes of *consumer-surplus effect* and *profit-shifting effect*. For example, if *consumer-surplus effect* plus *profit-shifting effect* is positive, and *tariff-revenue effect* is negative, then from Eq. (9), the optimum-welfare tariff rate is higher than that of the maximum-revenue tariff.

Substituting Eqs. (8a) and (8b) into (3) and (5), we then take the first-order derivatives with respect to t, giving the following optimum tariffs[3]:

$$t_W^* = \frac{a\theta(1+2\theta) - [1+\theta(2-m+\theta)]c_f + [1-\theta(m-1+\theta)]c_s}{m+2(1+\theta)^2} < \hat{t}$$

$$t_R^* = \frac{a\theta - (1+\theta)c_f + c_s}{2(1+\theta)}$$

Comparing these two tariff rates, we have:

$$t_W^* - t_R^* = \frac{a\theta[2\theta(1+\theta) - m] + m(1+\theta)(1+2\theta)c_f - [m + 2m\theta + 2(1+m)\theta^2 + 2\theta^3]c_s}{2(1+\theta)[m+2(1+\theta)^2]}$$

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[3] Second-order condition is always satisfied.

(11)

From Eq. (11), and let $\bar{c}_s^0 \equiv \dfrac{a\theta[2\theta(1+\theta)-m]+m(1+\theta)(1+2\theta)c_f}{2\theta^2(1+\theta)+m[1+2\theta(1+\theta)]}$, if $c_s < \bar{c}_s^0$ $t_W^* > t_R^*$; otherwise, if $c_s \geq \bar{c}_s^0$ $t_W^* \leq t_R^*$.

Proposition 1:

In a mixed oligopoly market without a domestic private firm, when the marginal cost of privatized firm exceeds $\bar{c}_s^0$, the optimum-welfare tariff can be lower than the maximum-revenue tariff.

In the following analysis, we review the findings of the relevant literature. Assuming $m=1$, if $\theta=1$ (i.e. privatized totally) and $c_s = c_p > c_f$, we have $t_W^* - t_R^* = \dfrac{a+2c_f-3c_p}{12}$. When $c_p < \bar{c}_p^1 \equiv \dfrac{a+2c_f}{3}$, $t_W^* > t_R^*$; otherwise, $t_W^* \leq t_R^*$. This result is obtained in Collie (1991). Under pure duopoly, the optimum welfare tariff may exceed the maximum revenue tariff due to the profit-shifting effect. If domestic marginal cost is such that $c_p > \bar{c}_p^1$, the government raises maximum-revenue tariff which will be higher than optimum-welfare tariff. Under the same marginal costs of domestic and foreign firms, optimum-welfare tariff must be higher than maximum-revenue tariff. Differentiating $\bar{c}_p^1$ with respect to a, we have $\dfrac{\partial \bar{c}_p^1}{\partial a} = \dfrac{1}{12} > 0$. The impact of market size on critical marginal cost is positive under pure duopoly; the optimum-welfare tariff will be higher than maximum-revenue tariff.

If $\theta = 0$ (i.e. public firm without privatized), $t_W^* - t_R^* = \dfrac{c_f - c_s}{6} < 0$. Under mixed duopoly, when production efficiency of the public firm is lower than that of the foreign private firm, government must lower tariff such that maximum-revenue tariff will be higher than optimum-welfare tariff.

Corollary 1:

In a pure duopoly market, if $c_p < \bar{c}_p^1$, then $t_W^* > t_R^*$. In a mixed duopoly market,

when production efficiency of the privatized firm is lower than that of the foreign private firm, government must lower tariff, hence maximum-revenue tariff will be higher than optimum-welfare tariff.

When $n = 0$, $m = 1$, which is the case of mixed duopoly with partial privatization. The optimum-welfare tariff and maximum-revenue tariff are $t_W^* = \dfrac{(a - c_f + c_s)(1+\theta) - (a + c_f\theta) - (c_s - c_f)\theta + (2a - c_f - c_s)\theta^2}{1 + 2(1+\theta)^2}$ and

$t_R^* = \dfrac{(a - c_f)(1+\theta) - (a - c_s)}{2(1+\theta)}$ respectively.

Let $\bar{c}_s^1 \equiv \dfrac{a\theta[2\theta(1+\theta) - 1] + (1+\theta)(1+2\theta)c_f}{1 + 2\theta(1+\theta)^2}$, if $c_s < \bar{c}_s^1$ $t_W^* > t_R^*$; otherwise, if $c_s \geq \bar{c}_s^1$ $t_W^* \leq t_R^*$.

Differentiating $\bar{c}_s^1$ with respect to θ, we have

$$\frac{\partial \bar{c}_s^1}{\partial \theta} = \frac{(2\theta(2 + \theta(7 + 2\theta(3+\theta))) - 1)(a - c_f)}{\left(1 + 2\theta(1+\theta)^2\right)^2} > 0, \text{ when } \theta > 0.1546.$$

Notice that the impact on the critical marginal cost of government releasing ratio is non-linear, being negative first and then positive. When the government releases stock further, the optimum-welfare tariff will be higher than the maximum-revenue tariff only if the marginal cost of the privatized firm is greater than that of the foreign private firm.

Corollary 2:

In a mixed duopoly with partial privatization, when the marginal cost of the privatized firm exceeds a critical value, maximum-revenue tariff is higher than optimum-welfare tariff. Otherwise, optimum-welfare tariff is higher than maximum-revenue tariff.

Differentiating $\bar{c}_s^0$ with respect to θ, we have

$$\frac{\partial \bar{c}_s^0}{\partial \theta} = \frac{m[4\theta(1+\theta)^3 - m(1-2\theta^2)](a - c_f)}{\{2\theta^2(1+\theta) + m[1 + 2\theta(1+\theta)]\}^2} > 0, \text{ when } m < \bar{m} \equiv \frac{4\theta(1+\theta)^3}{1 - 2\theta^2}.$$

Again, the impact of the government-released ratio on the critical marginal cost is non-linear; depending on the number of foreign private firms and the government-released stock ratio. When the number of foreign firms is relatively small and the higher releasing stocks ratio makes the critical marginal cost increase, optimum-welfare tariff is higher than maximum-revenue tariff, *ceteris paribus*. However, when the number of foreign firms is relatively large and the degree of releasing stocks ratio is also significant, the optimum-welfare tariff will be higher than the maximum-revenue tariff only if the marginal cost of the privatized firm is greater than that of the foreign private firm.

Corollary 3:

In a mixed oligopoly with partial privatization, when the number of foreign firms is relatively large and ***the degree of releasing stocks ratio is also significant, the optimum-welfare tariff will be higher than the maximum-revenue tariff only if the marginal cost of the privatized firm is greater than that of the foreign private firm.***

4. Mixed oligopoly with domestic private firm

In this section, we generalize Chao and Yu (2006), and Wang *et al*. (2010) for "$1+n+m$" mixed model setting. As mentioned in section 3, backward induction is used to solve the SPNE. The first-order conditions are as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a - c_p - t - (1+n)q_i - mq_{fj} - q_s = 0 \tag{12}$$

$$\frac{\partial \pi_{fj}}{\partial q_{fj}} = a - c_f - t - (1+m)q_{fj} - nq_i - q_s = 0 \tag{13}$$

$$\frac{\partial S}{\partial q_s} = a - c_s - mq_{fj} - nq_i - 2q_s + (1-\theta)(mq_{fj} + q_s) = 0 \tag{14}$$

Combining Eqs. (12), (13) and (14), the equilibrium quantities and price are:

$$q_s^* = \frac{(1+m-m\theta)a - (n-\theta-n\theta)m(t+c_f) + [1+m-m\theta]nc_p - [1+n+m]c_s}{H_2} \tag{15a}$$

$$q_i^* = \frac{a\theta + m(t+c_f) - (1+m+\theta)c_p + c_s}{H_2} \tag{15b}$$

$$q_{fj}^{*} = \frac{a\theta - (1+\theta+n\theta)(t+c_f) + n\theta c_p + c_s}{H_2} \quad (15c)$$

$$Q^{*} = \frac{a - c_s + n\theta(a - c_p) + m(a - c_f - t)}{H_2} \quad (15d)$$

$$P^{*} = \frac{a\theta + c_s + n\theta c_p + m(c_f + t)}{H_2} \quad (15e)$$

where $H_2 = 1 + m + \theta + n\theta$.

From Eq. (15c), if $t \geq \hat{t} \equiv \dfrac{a\theta - (1+\theta+n\theta)c_f + n\theta c_p + c_s}{1+\theta+n\theta}$, the foreign firm's production is zero which means $\hat{t}$ is a prohibited tariff ratio.

Lemma 2:

(i) When $0 < \theta < 1$ ***and*** $0 \leq t < \hat{t}$***,*** q_i^{*}***,*** $(q_s^{*} + nq_i^{*})$ ***and*** P^{*} ***are increasing in*** t ***while*** q_{fj}^{*} ***and*** Q^{*} ***are decreasing in*** t***.***

(ii) When $0 < \theta < 1$ ***and*** $0 \leq t < \hat{t}$***,*** q_i^{*}***,*** q_{fj}^{*} ***and*** P^{*} ***are increasing in*** θ ***while*** q_s^{*}***,*** $(q_s^{*} + nq_i^{*})$ ***and*** Q^{*} ***are decreasing in*** θ***.***

Proof:

We take the partial derivative of Eqs. (15a) and (15e) with respect to t and θ, respectively,

$$\frac{\partial q_i^{*}}{\partial t} = \frac{m}{H_2} > 0$$

$$\frac{\partial q_s^{*}}{\partial t} + n\frac{\partial q_i^{*}}{\partial t} = \frac{(1+n)m\theta}{H_2} > 0$$

$$\frac{\partial P^{*}}{\partial t} = \frac{m}{H_2} > 0$$

$$\frac{\partial q_{fj}^{*}}{\partial t} = -\frac{1+\theta+n\theta}{H_2} < 0$$

$$\frac{\partial Q^{*}}{\partial t} = \frac{-m}{H_2} < 0$$

$$\frac{\partial q_s^{*}}{\partial \theta} = \frac{-(1+n+m)A_1}{H_2^2} < 0$$

$$\frac{\partial q_i^*}{\partial \theta} = \frac{A_1}{H_2^2} > 0$$

$$\frac{\partial q_s^*}{\partial \theta} + n\frac{\partial q_i^*}{\partial \theta} = \frac{-(1+m)A_1}{H_2^2} < 0$$

$$\frac{\partial P^*}{\partial \theta} = \frac{A_1}{H_2^2} > 0$$

$$\frac{\partial q_{fj}^*}{\partial \theta} = \frac{A_1}{H_2^2} > 0$$

$$\frac{\partial Q^*}{\partial \theta} = \frac{-A_1}{H_2^2} < 0$$

where $A_1 \equiv [a + nc_p - (1+n)c_s] + m(a - c_f - t) + mn(c_p - c_f - t) > 0$.

From Lemma 2, when the tariff rate is raising, total output produced by the domestic firms $(q_s^* + nq_i^*)$ increases output while foreign firms decrease output, and market price then increases in the domestic market; on the other hand, when the degree of privatization is raising, domestic firms decrease output while foreign firms increase output, so the market price increases in the domestic market.

Lemma 3:

When $0 < \theta < 1$ ***and*** $0 \le t < \hat{t}$ ***,*** π_i^* ***and*** R^* ***are increasing in*** θ***.***

Proof:

We take the partial derivative of Eqs. (1) and (5) with respect to θ, respectively,

$$\frac{\partial \pi_i^*}{\partial \theta} = \frac{A_2\{[a + nc_p - (1+n)c_s] + m(a - c_f - t) + mn(c_p - c_f - t)\}}{H_2^3} > 0$$

where $A_2 \equiv [\theta(a - c_p) + c_s - c_p - m(c_p - c_f - t)] > 0$

$$\frac{\partial R^*}{\partial \theta} = \frac{mt\{[a + nc_p - (1+n)c_s] + m(a - c_f - t) + mn(c_p - c_f - t)\}}{H_2^2} > 0$$

Lemma 3 indicates that raising the degree of privatization will be beneficial for the domestic firms' profit and tariff revenue.

As presented in the previous section, we can rewrite (9) and (10), as follows:

$$\frac{dW}{dt} = Q\left[\frac{dq_s}{dt} + n\frac{dq_i}{dt} + m\frac{dq_j}{dt}\right] + \left[q_s\frac{dP}{dt} + \left(P - c_s\right)\frac{dq_s}{dt}\right]$$

$$+n\left[q_i\frac{dP}{dt}+\left(P-c_i\right)\frac{dq_i}{dt}\right]+m\left[q_{fj}+t\frac{dq_{fj}}{dt}\right] \qquad (9')$$

$$\frac{dR}{dt}=m\left[q_f+t\frac{\partial q_f}{\partial t}\right] \qquad (10')$$

Eq. $(9')$ indicates the impact of the number of domestic private firms on the *consumer-surplus effect* and *profit-shifting effect*. In the model with domestic private firms, even raising tariff rate will make the summed output of domestic firms increase, but the output of the privatized firm is ambiguous. Similarly, raising tariff rate will make the summed profit of domestic firms increase, but the profit of the privatized firm is ambiguous. Therefore, in the context of the mixed oligopoly with partial privatization, the presence of domestic private firms will result in additional channels of resource-allocation and profit-shifting effect which will affect the ranking of optimum-welfare tariff and maximum-revenue tariff.

Substituting Eqs. (15a), (15b) and (15c) into (3) and (5), we then take the first-order derivatives with respect to t, giving the following optimum tariffs[4]:

$$t_W^*=\frac{B_1(a-c_f)+\theta^2(a-c_s)+B_2(c_s-c_f)+B_3(c_s-c_p)+n^2\theta^2(c_p-c_f)}{H_3}<\hat{t}$$

$$t_R^*=\frac{a\theta-(1+\theta+n\theta)c_f+n\theta c_p+c_s}{2(1+\theta+n\theta)}$$

where $H_3=m+2(1+\theta+n\theta)^2$, $B_1\equiv\theta(1+\theta+2n\theta)$, $B_2\equiv 1+[1+2n-nm-m]\theta$, and $B_3\equiv n\{1+m+\theta[(1-\theta)n-2\theta]\}$.

Comparing these two tariff rates, we have:

$$t_W^*-t_R^*=f(c_s,c_p,c_f,n,m,\theta) \qquad (16)$$

From Eq. (16), and let $\bar{c}_p^0\equiv\dfrac{C_1 a+C_2(1+\theta)(1+2\theta)c_f+C_3c_s}{H_4}$, if $c_p<\bar{c}_p^0$ $t_W^*>t_R^*$; otherwise, if $c_p\geq\bar{c}_p^0$ $t_W^*\leq t_R^*$, where $C_1\equiv\theta[2(1+n)\theta(1+\theta+n\theta)-m]$, $C_2\equiv m[1+2(1+n)\theta](1+\theta+n\theta)$,
$C_3\equiv 2n(1+m)+2(1+n)[m(n-1)+2n]-m-2(1+m-n)(1+n)^2\theta^2-2(1+n)^3\theta^3$ and $H_4\equiv n\{m(2+3\theta+2n\theta)+2(1+\theta+n\theta)[1+(1+n)(1-\theta)]\}$.

4 Second-order condition is always satisfied.

Proposition 2:

Under mixed oligopoly with domestic private firms, when the marginal cost of domestic private firms exceeds $\bar{c}_p^0$, the optimum-welfare tariff can be lower than the maximum-revenue tariff.

In the following analysis, we review the findings of the relevant literature as special cases of our results. When $\theta = 1$ and $c_s = c_p$, and assuming the total number of domestic private firms is n, which is the equivalent expression of optimum-welfare tariff in Larue and Gervais (2002) considering a linear demand version, we have $t_R^* = \dfrac{a - c_f + n(c_p - c_f)}{2(1+n)}$ and $t_W^* = \dfrac{2n(a - c_f) + n(n-m)(c_p - c_f)}{m + 2(1+n)^2}$.

If $c_p < \bar{c}_p^2 \equiv \dfrac{2n(1+n)a - (a - c_f)m + (3+2n)nmc_f}{n[2+3m+2(1+m)n]}$, then $t_W^* > t_R^*$. Taking the derivative of $\bar{c}_p^2$ with respect to a, c_f, n and m respectively, we have

$$\frac{d\bar{c}_p^2}{da} = \frac{2n(1+n) - m}{2+3m+2(1+m)n} > 0 \text{ if } m < 2n(1+n)$$

$$\frac{d\bar{c}_p^2}{dc_f} = \frac{m(1+n)(1+2n)}{2+3m+2(1+m)n} > 0$$

$$\frac{d\bar{c}_p^2}{dn} = \frac{m[2(1+n)^2 + m(3+4n)](a - c_f)}{n^2[2+3m+2(1+m)n]^2} > 0,$$

$$\frac{d\bar{c}_p^2}{dm} = \frac{-2(1+n)^2(1+2n)(a - c_f)}{n[2+3m+2(1+m)n]^2} < 0.$$

Notice that the impact of the domestic market size on the critical marginal cost depends on the number of foreign private firms; the optimum-welfare tariff will be higher than the maximum-revenue tariff only if the number of foreign private firms is relatively smaller than that of domestic private firms, *ceteris paribus*. When n gets larger but m becomes smaller; that is, when the sizes of domestic private and foreign private firms become more unequally distributed, the optimum-welfare tariff in relative magnitude will be greater than the maximum-revenue tariff. The economic reasoning for such a result is that when the number of domestic firms is large and the domestic market is highly competitive, to protect domestic firms, the domestic government will raise its tariff rate, and the optimum-welfare tariff rate will be raised more than the maximum-revenue tariff.

Corollary 4:

In a pure oligopoly market with n ***domestic private firms and*** m ***foreign private firms, if*** $c_p < \bar{c}_p^2$***, then*** $t_W^* > t_R^*$***, the critical cost is non-linear in a, increasing in*** n ***and decreasing in*** m***.***

When $\theta = 0$ and $c_s > c_p > c_f$, we have $t_W = \frac{(c_s - c_f) + (1+m)n(c_s - c_p)}{2+m}$ and $t_R = \frac{(c_s - c_f)}{2}$. If $c_p < \bar{c}_p^3 \equiv \frac{2(1+m)nc_s - m(c_s - c_f)}{2n(1+m)}$, then $t_W^* > t_R^*$. This is the result obtained by Wang *et al.* (2010)[5]. They argued that with the domestic private and foreign firm's competition, when the domestic government imposes a tariff rate, the public firm will further reduce its output when the public firm's marginal cost exceeds both that of the domestic and foreign private firms, and then due to that, the profit shifting effect is less than the consumer surplus loss, so the domestic government imposes a higher optimum-welfare tariff versus maximum-revenue tariff. Taking the partial derivative of $\bar{c}_p^3$ with respect to c_s, c_f, n and m respectively, we have

$$\frac{d\bar{c}_p^3}{dc_s} = 1 - \frac{m}{2n(1+m)} > 0,$$

$$\frac{d\bar{c}_p^3}{dc_f} = \frac{m}{2n(1+m)} > 0,$$

$$\frac{d\bar{c}_p^3}{dn} = \frac{m(c_s - c_f)}{2(1+m)n^2} > 0,$$

$$\frac{d\bar{c}_p^3}{dm} = \frac{-(c_s - c_f)}{2n(1+m)^2} < 0.$$

When the marginal cost of the public firm is higher, the critical marginal cost will be pushed up, and the optimum-welfare tariff in relative magnitude will be greater than the maximum-revenue tariff. The economic reasoning for such a result is

5 Wang *et al*. (2010) not only discuss the tariff ranking under Cournot competition, but also consider the impact on the tariff ranking of the order of the firms' move.

that when the marginal cost of the public firm is higher than the competitors, the capability for the public firm to adjust output for welfare improvement will be mitigated, and under such circumstance, the domestic government will raise its tariff rate to improve social welfare, which will make the optimum-welfare tariff higher than the maximum-revenue tariff. Once again, when n gets larger but m becomes smaller; that is, when the sizes of domestic private and foreign private firms become more unequally distributed, the optimum-welfare tariff in relative magnitude will be greater than the maximum-revenue tariff. We have the following corollary.

Corollary 5:

In a mixed oligopoly market with one public firm, n ***domestic private firms and*** m ***foreign private firms, if*** $c_p < \bar{c}_p^3$***, then*** $t_W^* > t_R^*$***. The critical cost is increasing in*** c_s ***and*** n***, and decreasing in*** m***.***

From the above discussions, we can see that the relative magnitudes of domestic firms' and foreign firms' marginal cost, the number of firms and the degree of privatization will all influence the determination of the optimum-welfare tariff and the maximum-revenue tariff. Taking the derivative of $\bar{c}_p^0$ with respect to a, c_s, c_f, n, m and θ, we obtain[6]:

$$\frac{d\bar{c}_p^0}{da} = \frac{\theta[2(1+n)\theta(1+\theta+n\theta)-m]}{H_3} > 0 \quad \text{if} \quad m < 2(1+n)\theta(1+\theta+n\theta)$$

$$\frac{d\bar{c}_p^0}{dc_s} = \frac{g_1(n,m,\theta)}{H_3} < 0 \quad \text{if} \quad m < \overline{m} \equiv \frac{2n+4n(1+n)\theta+2(n-1)(1+n)^2\theta^2-2(1+n)^3\theta^3}{1-2n-2(n^2-1)\theta+2(1+n)^2\theta^2}$$

$$\frac{d\bar{c}_p^0}{dc_f} = \frac{m(1+\theta+n\theta)[1+2(1+n)\theta]}{H_3} > 0$$

$$\frac{d\bar{c}_p^0}{dm} = \frac{g_2(a,c_s,c_f,n,\theta)}{H_3^2} < 0 \quad \text{if} \quad c_s > \bar{c}_s^2 \equiv \frac{-a\theta+[1+(1+n)(1-\theta)\theta]c_f}{(1-\theta)(1+\theta+n\theta)}$$

$$\frac{d\bar{c}_p^0}{dn} = \frac{g_3(a,c_s,c_f,n,m,\theta)}{H_3^2} > 0 \quad \text{if} \quad c_s > \bar{c}_s^3(a,c_f,n,m,\theta)$$

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[6] Some comparative analyses are provided in the Appendix.

$$\frac{d\bar{c}_p^0}{d\theta} = \frac{g_4(a, c_s, c_f, n, m, \theta)}{H_3^2} < 0 \quad \text{if} \qquad c_s > \bar{c}_s^4(a, c_f, n, m, \theta)$$

Here, the impact of the market size on the critical marginal cost hinges on the relative number of the domestic and foreign firms, and the degree of privatization. When the number of domestic firms is more than the foreign firms and the degree of privatization is large, the critical marginal cost will be pushed up by the market size, and, other things being equal, the optimum-welfare tariff will be higher than maximum-revenue tariff. However, when the marginal cost of the privatized firm is rising, and the number of domestic firms is more than the foreign firms accompanied with a sufficiently high degree of privatization, the critical marginal cost will be reduced which reflects the phenomenon of production inefficiency of the privatized firm; the domestic government will be inclined to lower the optimum-welfare tariff rate and allow the foreign firms to replace domestic firms on output produced.

It should be noticed that when the government implements partial privatization policy, the impact of the number of domestic and foreign firms on the critical marginal cost depends on the marginal cost of the privatized firm. When the marginal cost of the privatized firm is sufficiently high, the impact of the number of domestic and foreign firms on the critical marginal cost is in the same direction with nationalization and completes privatization situations. Otherwise, when the marginal cost of the privatized firm is sufficiently low, the influence of partial privatization will be in the opposite direction when it compares with the nationalized and completely privatized situations. So, when the number of the foreign firms is increasing, due to production inefficiency of the privatized firm, the domestic government will be inclined to lower the tariff rate and allow the foreign firms to replace domestic firms on output produced.

Lastly, the impact on the critical marginal cost of the degree of privatization is ambiguous depending on the marginal cost of the privatized firm. When the marginal cost of the privatized firm is high, the higher degree of privatization will push down the critical marginal cost which will make the maximum-revenue tariff higher than optimum-welfare tariff, *ceteris paribus*. The economic implication is that when the cost efficiency gap between the privatized firm and domestic private firms is mitigated, the domestic government will further reduce optimum-welfare tariff rate coupled with a higher degree of privatization that will decrease the output of the privatized firm and improve social welfare

4. Conclusion

Collie (1991) argued that when the marginal cost of the domestic firm is higher than that of the foreign firm under a pure Cournot duopoly market, the maximum-revenue tariff may be higher than the optimum-welfare tariff. This paper has re-examined this important tariff ranking issue in mixed oligopoly with partial privatization and extended the findings of Larue and Gervais (2002), Chao and Yu (2006) and Wang *et al*. (2010) in linear-demand mixed oligopoly.

Major findings of this paper are that: firstly, in a partial-privatization mixed oligopoly without domestic private firm, when the marginal cost of a privatized firm is higher than a critical value, the optimum-welfare tariff will be higher than the maximum-revenue tariff.

Secondly, the impact of government releasing ratio on the critical marginal cost is non-linear and depends on the number of foreign private firms, and the government releases stock ratio. When the government releases more stock, the optimum-welfare tariff will be higher than the maximum-revenue tariff only if the marginal cost of the privatized firm is greater than that of the foreign private firm. Otherwise, the maximum-revenue tariff is higher than the optimum-welfare tariff. In addition, when the number of foreign firms increases, it is highly possible that the optimum-welfare tariff will be lower than the maximum-revenue tariff, *ceterius paribus*.

Thirdly, in a partial-privatization mixed oligopoly with domestic private firms, the impacts of the number of domestic and foreign private firms, and government releasing ratio on the critical marginal cost are all ambiguous, depending on the marginal cost of the privatized firm. When the marginal cost of the privatized firm is relatively high, it is highly possible that the maximum-revenue tariff will be higher than optimum-welfare tariff, *ceteris paribus*.

In this paper, we have examined the tariffs ranking in a mixed oligopoly model with exogenous partial privatization. This research could be extended by considering the cases of optimal privatization degree, or/and increasing marginal cost with domestic free entry, so as to consider a variety of conditions.

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Appendix

(i) $$\frac{d\bar{c}_p^0}{dn} = \frac{g_3(a,c_s,c_f,n,m,\theta)}{H_3^2} > 0 \quad \text{if} \quad c_s > \bar{c}_s^3(a,c_f,n,m,\theta)$$

$$c_s^3(\cdot) \equiv \frac{S_1 + S_2}{H_4 + H_5}$$

where,

$$S_1 \equiv a\theta\{-m^2(2+3\theta+4n\theta)+4\theta(1+\theta+n\theta)^2[1+(1+n)^2(1-\theta)\theta] \\ +2m(1+\theta+n\theta)\{\theta[1-3n+4(1+n)\theta]-1\}\}$$

$$S_2 \equiv m\{m(1+\theta)\{2+\theta[7+4n+2(1+n)(3+n)\theta]\} \\ +2(1+\theta+n\theta)^2\{1+\theta\{3+2n+[1+2n(1+n)]\theta+2(1+n)^2\theta^2\}\}\}c_f$$

$$H_4 \equiv 4\theta^2(1+\theta+n\theta)^2[1+(1+n)^2(1-\theta)\theta]+m^2\{2+\theta\{7+4n+2[5+n(4+n)]\theta \\ +2(1+n)(3+n)\theta^2\}\}$$

$$H_5 \equiv 2m\{1+\theta[4+4n+8\theta+n(10+7n)\theta+2(1+n)[4+n(2+3n)]\theta^2 \\ +(1+2n)^2[1+2(n-1)n]\theta^3-2(1+n)^4\theta^4]\}$$

(ii) $$\frac{d\bar{c}_p^0}{d\theta} = \frac{g_4(a,c_s,c_f,n,m,\theta)}{H_3^2} < 0 \quad \text{if} \quad c_s > \bar{c}_s^4(a,c_f,n,m,\theta)$$

$$c_s^4(\cdot) \equiv \frac{S_3 + S_4}{H_6 + H_7}$$

where,

$$S_3 \equiv 2a\theta\{m^2-(1+n)\theta(1+\theta+n\theta)^2(2+\theta+n\theta) \\ +m\{1-(1+n)\theta\{4-(1+n)\theta[9+4(1+n)\theta]\}\}\}$$

$$S_4 \equiv -m\{m[3+4n+8(1+n)^2\theta+2(1+n)^2(3+2n)\theta^2]+2(1+n)(1+\theta+n\theta)^2 \\ +2(1+n)(1+\theta+n\theta)^2[1+2\theta(1+\theta+n\theta)]\}c_f$$

$$H_6 \equiv m(m+2n+4mn)+4(1+n)\{2+m[4+n+2m(1+n)]\}\theta \\ +2(1+n)^2\{10+m[16+n+m(3+2n)]\}\theta^2$$

$$H_7 \equiv 4(4+5m)(1+n)^3\theta^3+4(1+m)(1+n)^4\theta^4$$