

# **On the Non-Equivalence of Specific and Ad Valorem Taxation in Competitive Markets**

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**[Abstract]** This note calls into question the canon of the equivalence between specific and ad valorem taxation in competitive markets. We conduct our analysis under the plausible assumption that a competitive firm knows the relevant prices it faces, but the tax authorities may not be well informed about all the prices of hundreds or even thousands of competitive firms. It is shown in competitive markets that, to collect a fixed amount of revenue for the government, ad valorem taxes entail a higher level of consumer surplus than specific taxes. The key to our finding lies in that evading specific taxes must be via underreporting quantities sold, whereas evading ad valorem taxes can be via underreporting selling prices as well as quantities sold.

Key words: Tax evasion; Ad valorem taxation; Specific taxation

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When comparing specific and ad valorem taxation on commodities, the focus of the literature has been on markets with imperfect competition at least since Wicksell (1896).<sup>1</sup> This focus may not be surprising. As Delipalla and Keen (1992, p. 351) put it: “Their equivalence under perfect competition has long been recognized.”<sup>2</sup>

In this note we call into question the canon of this equivalence. Once there is evasion, evading specific taxes must be via underreporting quantities sold, whereas evading ad valorem taxes can be via underreporting selling prices as well as quantities sold. We show that this difference causes a breakdown of the equivalence between these two tax types in the competitive market. In particular, we show that, to collect a fixed amount of revenue for the government, ad valorem taxes entail a higher level of consumer surplus than specific taxes.

Evading specific taxes must be via underreporting quantities sold, whereas evading ad valorem taxes can be via underreporting selling prices as well as quantities sold. As a result, a firm can implement evasion with a lower resource cost under ad valorem than specific taxation. The firm’s profit must be zero in equilibrium. The lower resource cost of evasion then leads to a lower consumer price and so a higher consumer surplus.

Firms in the competitive market are price takers, that is, they take prices as given, exogenous to their profit-maximizing behavior. We assume that a firm may evade taxes via reporting lower prices in competitive markets when ad valorem taxation is in operation. A key assumption underlying this possibility is that competitive firms know the prices they face, but the tax authorities may not know well enough about these prices. Put

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<sup>1</sup> Wicksell (1896) considers the monopoly case. Under the assumption of constant marginal cost, he shows that, given the same amount of revenue collected, ad valorem taxes will result in a lower consumer price and a higher level of quantity produced than specific taxes. Later work include Suits and Musgrave (1953), Kay and Keen (1983), Delipalla and Keen (1992), Skeath and Trandel (1994), and Anderson et al. (2001).

<sup>2</sup> This recognition can at least date back to Cournot (1838), in which there was a realization that specific and ad valorem taxes require separate consideration under monopoly.

differently, it is costly rather than costless for the tax authorities to obtain information regarding the true competitive prices. This may not be an unreasonable assumption. Prices in a competitive market may fluctuate a lot, commodities in question may be less familiar intermediate goods rather than more familiar final goods, and perhaps most importantly, while a single firm may only need to face prices of a single commodity, the tax authorities must deal with prices of hundreds or even thousands of commodities with a limited size of staff.

Evasion on commodity taxes is important. Slemrod (2007) cites a confidential study made in 2005 by the Forum on Tax Administration (a subsidiary body of the OECD's Committee on Fiscal Affairs). Estimated noncompliance rates for value-added tax range from 4.0 to 17.5 percent for some countries in the study. Silvani and Brondolo (1993) investigate noncompliance of value-added tax for 19 mostly developing countries. Comparing the actual value-added tax revenue with the potential tax base, they find a median evasion rate of 31.5 percent, with New Zealand the lowest at 5.1 percent while Peru the highest at 68.2 percent.

It is worth emphasizing at the outset that we do not claim that the equivalence between specific and ad valorem taxation in competitive markets will always break down in the presence of tax evasion. We only claim that, for some commodities or in some cases, this equivalence no longer holds when there is tax evasion.

## **I. Model**

Our model is based on Cremer and Gahvari (1993, hereafter CG), who analyze competitive firms' evasion of specific taxes. We amend the CG model straightforwardly to accommodate competitive firms' evasion of ad valorem taxes.

Consider a competitive economy in which there are many industries. We focus on a representative industry in which there are many identical firms using the linear production technology to produce a single product, and  $c$  is the average and marginal cost

of production. Let  $x$  denote a firm's output in the industry and  $p$  its consumer price. The market demand of the industry is given by  $p = p(X)$  with  $p'(X) < 0$ .

The firm's taxes depend on its sales, which are subject to either specific or ad valorem taxes. Under specific taxation, a fixed amount tax of  $t > 0$  is imposed on per unit output so that the firm's net-of-tax revenue equals  $(p - t)x$  if there is no evasion. Under ad valorem taxation, a fixed percentage tax of  $0 < \tau < 1$  is levied on the output price so that the firm's net-of-tax revenue equals  $(p - p\tau)x$  if there is no evasion.

The firm may attempt to evade taxes. As in the CG model, we assume that it is costly for the firm to conceal its sale information from the tax authorities (say, costs involved in falsifying invoices or fabricating accounts). Evading taxes must be via underreporting quantities sold in the CG model since commodity taxes only take the form of specific taxation. By contrast, evading taxes may be via underreporting selling prices as well as quantities sold in our model since commodity taxes can take either the form of ad valorem taxation or that of specific taxation. This is the main difference between the two models.

Let  $\alpha$  and  $\beta$  denote, respectively, the fraction of the output and of the selling price that the firm chooses to report to the tax authorities. Since concealing activities are costly, it is clear that the firm will not underreport its selling price under specific taxation; that is,  $\beta = 1$  will always hold with specific taxes. This is not necessarily true under ad valorem taxation. Besides underreporting its output, the firm may underreport its selling price for the reported output,  $\alpha x$ , when ad valorem taxes are in operation. Since concealing activities are costly, it is clear that the firm will not carry out "price" concealment for the unreported output  $(1 - \alpha)x$ .

We follow CG to model the firm's cost of "quantity" concealment. Namely, for the fraction of the output not reported to the tax authorities, each unit of output *not reported* entails a quantity concealing cost  $G(1 - \alpha)$ , which is an increasing and convex function of  $1 - \alpha$  with  $G(0) = 0$  and  $G'(1) = \infty$ . Thus, the total concealment cost associated

with specific taxes is  $G(1-\alpha) \cdot (1-\alpha)x$  when the firm underreports a  $1-\alpha$  fraction of its output  $x$ .

We model the firm's cost of "price" concealment analogously. More specifically, for the fraction of the selling price not reported to the tax authorities, each unit of output *reported* entails a price concealing cost  $H(1-\beta)$ , which is an increasing and convex function of  $1-\beta$  with  $H(0)=0$  and  $H'(1)=\infty$ . Thus, the total concealment cost associated with ad valorem taxes is  $G(1-\alpha) \cdot (1-\alpha)x + H(1-\beta) \cdot \alpha x$  when the firm underreports a  $1-\alpha$  fraction of its output  $x$  and a  $1-\beta$  fraction of its selling price for the reported output  $\alpha x$ . Of course,  $\alpha$  and  $x$  under these two forms of taxation may not be the same.

Suppose that the audit probability faced by the firm is  $A > 0$ , and that a fine  $F > 1$  is levied on the amount of evaded tax if the firm's evasion is caught. As in CG, we assume that the firm's true sales will be revealed accurately once the tax authorities carry out audits. For convenience, let  $g(1-\alpha) \equiv (1-\alpha)G(1-\alpha)$ . The firm's expected profits,  $E(\pi)$ , under the two forms of taxation equal respectively

$$(1s) \quad E(\pi^s) = [p^s - c - g(1-\alpha^s) - \alpha^s t - AFt(1-\alpha^s)]x^s,$$

$$(1a) \quad E(\pi^a) = [p^a - c - g(1-\alpha^a) - \alpha^a H(1-\beta) - \alpha^a \beta p^a \tau - AFp^a \tau(1-\alpha^a \beta)]x^a,$$

where the superscript  $s$  denotes specific taxes and  $a$  ad valorem taxes. The firm's objective function under specific taxation, (1s), is identical to that in CG.  $\alpha^a \beta$  in (1a) denotes the fraction of sales (including both price and quantity) that the firm reports to the tax authorities. Thus, the firm's evasion rate equals  $1 - \alpha^a \beta$ .

This completes the description of our model.

## II. Comparing Specific with Ad Valorem Taxes in the Presence of Evasion

The production technology of the industry is linear, and hence  $p = c$  would hold in equilibrium if there were no taxes.

If specific taxes are in operation, we will have  $p^s = c + t$  in equilibrium. On the

other hand, if ad valorem taxes are in operation,  $p^a = c + p^a \tau$  in equilibrium. To compare specific with ad valorem taxes, the standard approach is to impose two tax types at the same magnitude at the competitive equilibrium, that is,  $t = p^a \tau$ . In the absence of evasion, specific and ad valorem taxation in competitive markets can be shown to be equivalent in that both yield the same equilibrium price, output and tax revenue. This result is one of the canons of undergraduate public finance textbooks; see, for example, Stiglitz (2000, pp. 488-490).

What would happen to this equivalence when there is tax evasion?

In the absence of tax evasion,  $t = p^a \tau$  must imply  $p^s = p^a$  and vice versa at the competitive equilibrium. This may no longer be true in the presence of tax evasion. Indeed, we show below that  $p^s > p^a$ , given  $t = p^a \tau$ .

#### *A. Specific Taxes*

As observed in CG, the firm's choice of  $\alpha^s$  is independent of its choice of  $x^s$  in the case of specific taxation. Given  $x^s > 0$ , the first- and second-order conditions for an interior  $\alpha^s$  from (1s) are respectively

$$(2s) \quad g'(1 - \alpha^s) = (1 - AF)t,$$

$$(3s) \quad g''(1 - \alpha^s) > 0,$$

where (3s) is satisfied because the assumption that  $G$  is an increasing and convex function of  $1 - \alpha^s$ . (2s) and (3s) are the same as (3)-(4) in the CG model

The firm's choice of  $\alpha^s$  must be positive because  $G'(1) = \infty$  by assumption. Note that

$$\left. \frac{dE(\pi^s)}{d\alpha^s} \right|_{\alpha^s=1} = g'(0) - (1 - AF)t = -(1 - AF)t,$$

where  $g'(0) = 0$  because  $G(0) = 0$  by assumption. Thus, a condition to support  $\alpha^s < 1$  is that  $AF < 1$ . We assume as in CG that this condition is satisfied.

## B. Ad Valorem Taxes

Similar to specific taxation, the firm's choice of  $\alpha^a$  and  $\beta$  is independent of its choice of  $x^a$  in the case of ad valorem taxation. Given  $x^a > 0$ , the first- and second-order conditions for interior  $\alpha^a$  and  $\beta$  from (1a) are respectively

$$(2a-1) \quad g'(1 - \alpha^a) = H(1 - \beta) + (1 - AF)\beta p^a \tau,$$

$$(2a-2) \quad H'(1 - \beta) = (1 - AF)p^a \tau,$$

$$(3a) \quad g''(1 - \alpha^s) > 0, \quad H''(1 - \beta) > 0,$$

where (3a) is satisfied because the convexity assumption about  $G$  and  $H$ .

The logic governing the firm's "price" evasion behind (2a-2) is not different from the logic governing the firm's "quantity" evasion behind (2s). Indeed, (2a-2) and (2s) resemble to each other in formula. A more interesting result is (2a-1), which shows that the firm's underreporting output has the additional benefit of saving cost of "price" concealment (the first right-hand-side term). The reason for this additional benefit is obvious: firms will not engage in costly price concealments for the quantities that they do not intend to report to the tax authorities.

The firm's choice for both  $\alpha^a$  and  $\beta$  must be positive because  $G'(1) = \infty$  and  $H'(1) = \infty$  by assumption. Note that

$$\left. \frac{dE(\pi^a)}{d\alpha^a} \right|_{\alpha^a=1} = g'(0) - H(1 - \beta) - (1 - AF)\beta p^a \tau = -H(1 - \beta) - (1 - AF)\beta p^a \tau.$$

This result indicates that the condition  $AF < 1$ , which supports the firm's choice of  $\alpha^s < 1$  in specific taxation, will support its choice of  $\alpha^a < 1$  in ad valorem taxation too.

Note also that

$$\left. \frac{\partial E(\pi^a)}{\partial \beta} \right|_{\beta=1} = \alpha^a [H'(0) - (1 - AF)p^a \tau].$$

Thus, as long as one is willing to make the assumption that  $H'(0) = 0$ , the condition  $AF < 1$  will also support the firm's choice of  $\beta < 1$  in ad valorem taxes. Even if

$H'(0) > 0$  , the firm's choice of  $\beta < 1$  will still hold, provided that  $H'(0) < (1 - AF)p^a\tau$  . In what follows, we examine the equivalence between two tax types with the possibility of  $\beta < 1$ , that is, the firm underreports its selling price when ad valorem taxation is in operation. The purpose of our paper is to provide counter examples to the canon that specific and ad valorem taxes are equivalent in competitive market. In view of this purpose, it is legitimate to allow for  $\beta < 1$ .

### *C. Comparison*

Both price  $p$  and cost  $c$  are exogenous to the firm. Furthermore, the firm's production and evasion decisions are independent of each other. Putting them together implies that the firm's maximizing its expected profits (1s) and (1a) is equivalent to minimizing its expected unit costs

$$(4s) \quad E(C^s) = CC^s + ER^s ,$$

$$(4a) \quad E(C^a) = CC^a + ER^a ,$$

with

$$(4s-1) \quad CC^s \equiv g(1 - \alpha^s)$$

$$(4a-1) \quad CC^a \equiv g(1 - \alpha^a) + \alpha^a H(1 - \beta) ,$$

$$(4s-2) \quad ER^s \equiv [\alpha^s + AF(1 - \alpha^s)]t ,$$

$$(4a-2) \quad ER^a \equiv [\alpha^a \beta + AF(1 - \alpha^a \beta)]p^a \tau ,$$

where  $CC$  denotes the concealment cost incurred by the firm per unit output, and  $ER$  denotes the expected tax revenue collected by the government per unit output.

From (2s), we have

$$(5s) \quad \frac{d\alpha^s}{d[(1 - AF)t]} = -\frac{1}{g''} < 0 .$$

From (2a), we have

$$(5a) \quad \frac{d\alpha^a}{d[(1 - AF)p^a \tau]} = -\frac{\beta}{g''} < 0 .$$

Thus, given  $AF$ ,  $\alpha^s$  is a strictly decreasing function of  $t$  (denoted by  $\alpha^s(t)$ ), and  $\alpha^a$

is a strictly decreasing function of  $p^a \tau$  (denoted by  $\alpha^a(p^a \tau)$ ). We first derive a lemma, showing that specific taxation is a degenerate case of ad valorem taxation in a sense.

**Lemma 1.** *Let  $H(1 - \beta) = r \cdot h(1 - \beta)$ .*

(i)  $\frac{\partial \beta}{\partial r} > 0, \frac{\partial \alpha^a}{\partial r} < 0;$

(ii) *There exists an  $\bar{r}$  such that  $\beta \rightarrow 1$  and  $\alpha^a(p^a \tau) \rightarrow \alpha^s(t)$  as  $r \rightarrow \bar{r}$ .*

Proof: (i) is obtained by performing the comparative statics of (2a).

(ii) There are two cases to consider.

First, suppose that  $H'(0) = r \cdot h'(0) > 0$ . Then given the RHS of (2a-2), there must exist an  $\bar{r} < \infty$  such that  $\bar{r} \cdot h'(0) = (1 - AF)p^a \tau$ . This implies that  $\beta \rightarrow 1$  if  $r \rightarrow \bar{r}$ . Let  $\beta = 1$ , one can rewrite (2a-1) as

(2a-1')  $g'(1 - \alpha^a) = (1 - AF)t'$ , where  $t' = p^a \tau$ .

Observe that (2a-1') is identical to (2s) in form. This proves that  $\alpha^a(p^a \tau) \rightarrow \alpha^s(t)$  if  $r \rightarrow \bar{r}$ .

Next, suppose that  $H'(0) = r \cdot h'(0) = 0$ . The logic for our arguments remains the same, except that  $\bar{r} \rightarrow \infty$ . Q.E.D.

Lemma 1 is intuitive. There are two instruments (price plus quantity) available for the firm to conceal its sales from the tax authorities in the case of ad valorem taxation, while there is only one instrument (quantity) available in the case of specific taxation. As the price concealment becomes more costly under ad valorem taxation, the firm will utilize less the price instrument and more the quantity instrument. When the price concealment is sufficiently costly, the firm will give up the price concealment completely and only engage in the quantity concealment. In such a situation, as far as evading taxes is concerned, there is no difference for the firm whether taxes imposed are specific or ad valorem.

We need another lemma.

**Lemma 2.**  $\frac{\partial p^s}{\partial t} > 0$  and  $\frac{\partial p^a}{\partial \tau} > 0$  at competitive equilibrium.

Proof: The firm minimizes  $E(C^s)$  under specific taxes and  $E(C^a)$  under ad valorem taxes. Applying the envelope theorem to (4s) and (4a) yields respectively

$$(6s) \quad \frac{\partial E(C^s)}{\partial t} = \alpha^s + AF(1 - \alpha^s),$$

$$(6a) \quad \frac{\partial E(C^a)}{\partial p^a \tau} = \alpha^a \beta + AF(1 - \alpha^a \beta).$$

Utilizing  $E(\pi^s) = E(\pi^a) = 0$  in equilibrium gives

$$(7s) \quad p^s - c = E(C^s),$$

$$(7a) \quad p^a - c = E(C^a).$$

Combining (6) with (7) leads to

$$(8s) \quad \frac{\partial p^s}{\partial t} = \frac{\partial E(C^s)}{\partial t} = \alpha^s + AF(1 - \alpha^s),$$

$$(8a) \quad \frac{\partial p^a}{\partial(p^a \tau)} = \frac{\partial E(C^a)}{\partial(p^a \tau)} = \alpha^a \beta + AF(1 - \alpha^a \beta).$$

Because  $\partial p^a / \partial \tau = [\partial p^a / \partial(p^a \tau)][\partial(p^a \tau) / \partial \tau] = [\partial p^a / \partial(p^a \tau)][p^a + \tau(\partial p^a / \partial \tau)]$ , we obtain from (8a)

$$(9a) \quad \frac{\partial p^a}{\partial \tau} = \frac{[\alpha^a \beta + (1 - \alpha^a \beta)AF]p^a}{1 - \tau[\alpha^a \beta + (1 - \alpha^a \beta)AF]}.$$

(8s) is obviously positive. Since  $AF < 1$  by assumption, (9a) is also positive. Q.E.D.

With Lemmas 1-2 at hand, consider the following problem for the government under ad valorem taxation:

$$\text{Min}_{\tau} \quad p^a(\tau) = E[C^a(\tau)] = CC^a(\tau) + ER^a(\tau)$$

$$\text{subject to } ER^a(\tau) \geq \bar{R}$$

where  $CC^a(\tau) = g(1 - \alpha^a) + \alpha^a r \cdot h(1 - \beta)$  and  $\bar{R} > 0$  is some revenue requirement per unit output. The equality  $p^a(\tau) = E[C^a(\tau)]$  is due to that  $E(\pi^a) = 0$  must hold in equilibrium. If we confine to the ascending part of the ‘‘Laffer curve’’ in which  $ER^a(\tau)$

is increasing in  $\tau$ , then by Lemma 2 the above problem can be simplified to

$$\text{Min}_{\tau} CC^a(\tau)$$

$$\text{subject to } ER^a(\tau) = \bar{R}.$$

Let  $\tau^*$  denote the solution to the above government problem. There may exist other  $\tau \neq \tau^*$  that meet  $ER^a(\tau) = \bar{R}$ . However, to minimize  $CC^a(\tau)$  and hence  $p^a(\tau)$ ,

Lemma 2 dictates that  $\tau^* < \tau$ .

Forming the Lagrangean for the government problem:

$$L = CC^a(\tau) + \lambda[ER^a(\tau) - \bar{R}].$$

Applying the envelope theorem yields

$$(10a) \quad \frac{\partial L}{\partial r} = \alpha^a h(1 - \beta) > 0,$$

which implies that  $CC^a(\tau^*)$  is increasing in  $r$ , given that  $ER^a(\tau^*) = \bar{R}$ . This in turn implies that  $p^a(\tau^*)$  is increasing in  $r$ .

Now consider the corresponding problem for the government under specific taxation:

$$\text{Min}_{t} CC^s(t)$$

$$\text{subject to } ER^s(t) = \bar{R}.$$

Let  $t^*$  denote the solution to the above government problem. The imposition  $ER^a(\tau) = ER^s(t) = \bar{R}$  implies that  $p^a(\tau^*)\tau^* \rightarrow t^*$  as  $\beta \rightarrow 1$ . By Lemma 1, we then know that  $\beta \rightarrow 1$  and  $\alpha^a[p^a(\tau^*)\tau^*] \rightarrow \alpha^s(t^*)$  as  $r \rightarrow \bar{r}$ . Since both  $CC^a(\tau^*)$  and  $p^a(\tau^*)$  are increasing in  $r$ , we obtain  $CC^a(\tau^*) < CC^s(t^*)$  and  $p^a(\tau^*) < p^s(t^*)$  with  $ER^a(\tau^*) = ER^s(t^*) = \bar{R}$  if  $r < \bar{r}$ .

By assumption,  $p = p(X)$  with  $p'(X) < 0$ . Thus,  $p^a(\tau^*) < p^s(t^*)$  with  $ER^a(\tau^*) = ER^s(t^*)$  implies that  $ER^a(\tau^*)X(\tau^*) > ER^s(t^*)X(t^*)$ . In other words, with  $ER^a(\tau^*) = ER^s(t^*)$ , the government will be able to collect a larger total amount of expected revenue under ad valorem than specific taxation. Thus, to collect a given

amount of expected revenue under both tax types,  $\tau$  under ad valorem taxation can be lowered further and so  $p^a$  will become smaller further by Lemma 2. Since  $p'(X) < 0$ ,  $p^a < p^s$  indicates a higher level of consumer surplus under ad valorem than specific taxation..

To sum up, we state the main result of the paper.

**Proposition.** *Confining to efficient ways of collecting a fixed amount of expected revenue with  $0 < \beta < 1$  at competitive equilibrium, ad valorem taxation entails a higher level of consumer surplus than specific taxation.*

The efficient way of collecting taxes has two meanings in our context: (i) tax rates must be on the ascending part of the “Laffer curve,” and (ii) tax rates must minimize consumer price, given that the government collects a fixed amount of expected revenue.

Tax evasion by itself does no harm to an economy. As put it by Slemrod (2007, p. 41):

*“If all Americans were genetically predisposed to underpay their legal tax liability by 20 percent, at no cost to them, tax evasion wouldn’t matter at all. Government would simply readjust everyone’s “sticker price” tax liability upward so that the desired amount of tax would be collected, even after the 20 percent “discount” was taken.”*

The government must collect a fixed amount of revenue and, therefore, it can respond to evasion by adjusting tax rates appropriately in our model. However, Slemrod (2007, p. 42) points out:

*“Tax evasion also imposes efficiency costs. The most obvious are the resources taxpayers expend to implement and camouflage noncompliance, and the resources the tax authority expends to address it.”*

In this paper we focus on the resources taxpayers expend. There are price and quantity instruments for the firm to implement evasion under ad valorem taxation, whereas there is only quantity instrument for the firm to implement evasion under

specific taxation. As a result, other things being equal, the firm can evade taxes with a lower resource cost under ad valorem than specific taxation. The lower resource cost leads to a lower consumer price since the firm's profit must be zero in equilibrium. This is the key to our main result.

### **III. Conclusion**

Once there is evasion, evading specific taxes must be via underreporting quantities sold, whereas evading ad valorem taxes can be via underreporting selling prices as well as quantities sold. Built on this key observation, we amend the CG model to analyze competitive firms' evasion of ad valorem taxes, showing that, to collect a fixed amount of revenue for the government, ad valorem taxes entail a higher level of consumer surplus than specific taxes. Specific and ad valorem taxation is not equivalent even in the competitive market.

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