

Effects of TRIPS on Growth, Welfare and Income Inequality in an R&D-Growth Model

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Abstract

What are the effects of the WTO's TRIPS Agreement on growth, welfare and income inequality? To analyze this question, we develop an open-economy R&D-growth model with wealth heterogeneity. Under TRIPS, the North experiences higher growth and higher welfare at the expense of higher income inequality while the South experiences higher growth at the expense of lower welfare and higher income inequality. As for global welfare, there exists a critical degree for the domestic importance of foreign goods below (above) which global welfare decreases (increases) under TRIPS. In light of these findings, we discuss policy implications on China's accession to the WTO in 2001. Furthermore, we analyze the effects of China's rising innovative capability on domestic and foreign income inequality.

Keywords: endogenous growth, heterogeneity, income inequality, patent policy, TRIPS

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1. Introduction

The WTO's Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property rights (IPR) protection that must be provided by each member country. Given that developed countries (i.e. the North) generally had a much higher level of IPR protection than developing countries (i.e. the South) prior to TRIPS, this agreement is likely to have asymmetric effects on the North and the South. As an example of the North (South), we consider the US (China). Table 1 presents Park's (2008) index of patent rights in the US and China,¹ and it shows that as a result of TRIPS, the level of patent protection in China is converging towards the level in the US.² Given the importance of TRIPS, what are its effects on economic growth, social welfare and income inequality?

Year	1985	1990	1995	2000	2005
China	1.33	1.33	2.12	3.09	4.08
United States	4.68	4.68	4.88	4.88	4.88

To analyze this question, we develop an open-economy quality-ladder model with heterogeneity in households' wealth. In the model, both the North and the South invest in R&D, but the North is assumed to have a higher degree of innovative capability than the South. Within this framework, we derive the following results. Firstly, an increase in the level of patent protection in either the North or the South increases both countries' (a) economic growth by increasing R&D and (b) income inequality by raising the return on assets. Then, following Lai and Qiu (2003) and Grossman and Lai (2004), we derive the pre-TRIPS Nash equilibrium level of patent protection that is sub-optimally low due to the positive externality of patent policy. We also find that the North would set a higher level of patent protection than

¹ The index is a scale of 0 to 5, and a larger number indicates stronger patent protection. See Park (2008) for details.

² As a result of TRIPS, the statutory term of patent in the US was extended from 17 years (counting from the issue date of the patent) to 20 years (counting from the earliest filing date). However, due to the difference in the starting date, the effective extension was minimal. As for China, it extended the patent length from 15 years to 20 years in 1992. Prior to joining the WTO, China reformed its patent system in 2000 in compliance with the TRIPS agreement.

the South and imposing the North's higher level of patent protection on the South as required by TRIPS increases (decreases) social welfare of the North (South).³

This welfare implication is consistent with Lai and Qiu (2003) and Grossman and Lai (2004). It is perhaps not surprising that the South would be worse off by deviating from its best response. Therefore, the intriguing question is whether TRIPS would improve or reduce global welfare. We find that there exists a critical degree for the importance of foreign goods in domestic consumption below (above) which global welfare is lower (higher) under TRIPS while Lai and Qiu (2003) find that global welfare is always higher under TRIPS. This difference arises because we introduce a parameter to allow for varying degree of domestic importance of foreign goods. In our model, the degree of the positive externality in the Nash equilibrium is determined by this parameter. When foreign goods are not very important for domestic consumption, the two countries are almost in autarky. In this case, imposing the North's level of patent protection on the South makes the South worse off without making the North much better off. This finding has important policy implications. Firstly, it implies that the North is not always able to compensate the South even in the presence of costless transfers. Secondly, the condition under which global welfare would increase under TRIPS is that foreign goods are sufficiently important for domestic consumption. In other words, a sufficient degree of globalization is a necessary condition for the harmonization of IPR protection to improve global welfare.

Furthermore, our model with heterogeneous households enables us to analyze the effects of TRIPS on income inequality within each country in addition to growth and welfare. Under TRIPS, the North experiences higher growth and higher welfare at the expense of higher income inequality. As for the South, it experiences higher growth at the expense of lower welfare and higher income inequality. Intuitively, a higher growth rate increases the rate of return on assets through the Euler equation, and this higher return on assets increases the income of asset-wealthy households relative to asset-poor households in each country. This result suggests that although the representative-agent welfare analysis of TRIPS in

³ See, for example, Reichman (1995) for a more detailed discussion on the requirements of TRIPS.

previous studies can be robust to an extension with heterogeneous households, it is useful to also analyze the distributional consequences within a country given that income inequality can be a social concern.

For example, China amended its patent law in 2000 in anticipation of its accession to the WTO in 2001.⁴ Since this amendment, the annual growth rate of the number of applications for invention patents in China has increased to 23% (compared to less than 10% before 2000). Hu and Jefferson (2009) provide empirical evidence to show that the patent-law amendment in 2000 is a major factor for China's recent surge in patenting activities. Also, R&D as a share of GDP in China increases from an average of about 0.7% in the 90's to 1.49% in 2007.⁵ At the same time, the rising income inequality in China poses the country a serious challenge on domestic stability. In 2007, China's Gini coefficient rises to 0.47 that is above the threshold of 0.45 considered by many to indicate potential social unrest. Our analysis suggests that increasing the level of patent protection in China as a result of TRIPS not only lowers China's social welfare as implied by previous studies but also exacerbates its rising income inequality. Given the current situation in China, the second consequence seems to be more alarming. In a panel regression, Adams (2008) finds that strengthening IPR protection indeed has a positive and statistically significant effect on income inequality in developing countries. His estimates imply that increasing Park's (2008) index by one (on a scale of five) is associated with an increase in the Gini coefficient of 0.01 to 0.02 (on a scale of one) in developing countries.

We should emphasize that there are also other factors contributing to the rising income inequality in China, and patent policy is only one of them. In addition, China's accession to the WTO carries other benefits, such as lower trade barriers, which are not considered in this partial analysis of patent policy. In the model, we introduce a parameter to capture trade barriers and find that lower trade barriers improve social welfare. Therefore, a complete welfare analysis on China's accession to the WTO should trade off the welfare gain from lower trade barriers against the welfare loss from TRIPS.

⁴ The changes include (a) providing patent holders with the right to obtain a preliminary injunction against the infringing party before filing a lawsuit, (b) stipulating standards to compute statutory damages, (c) affirming that state and non-state enterprises enjoy equal patent rights, and (d) simplifying the patent application process, examination and transfer procedures and unifying the appeal system.

⁵ This data can be obtained from China Statistical Yearbook.

Finally, it has often been suggested that China's innovative capability has risen recently. For example, Hua and Mathews (2008) provide an empirical analysis on the increase in innovative capability in China and consider different patent-citation indicators as well as R&D cost per patent as an efficiency indicator. Given this empirical pattern, we also consider the comparative statics of increasing China's innovative capability on income inequality in China and the US for given levels of patent protection in the two countries. We find that it has an unambiguously positive effect on income inequality in the US by increasing its economic growth and rate of return on assets. However, the effect on China's own income inequality is ambiguous. If the degree of innovative capability in the US is below (above) a critical value, then a higher degree of innovative capability in China increases (decreases) its income inequality. In the case of China, there is an additional negative effect on income inequality through the wage income, and this negative wage-income effect on income inequality is dominated by the positive interest-rate effect if and only if the degree of innovative capability in the US is below a threshold that is decreasing in the level of patent protection in each country.

1.1. Literature Review

There is an established literature on the asymmetric effects of increasing IPR protection on the North and the South. Early studies focus on the effects of IPR in reducing illegitimate imitation from the South and encouraging technology transfer from the North through licensing or foreign direct investment. In these studies, innovative activities are usually assumed to take place only in the North.⁶ However, two other important reasons for strengthening IPR in the South are (a) to provide incentives for the North to develop technologies that are also used by the South,⁷ and (b) to provide incentives for the South to invest in innovative activities. For example, in a panel regression, Chen and Puttitanun (2005) find that

⁶ See, for example, Helpman (1993), Lai (1998), Yang and Maskus (2001) and Glass and Saggi (2002b). Glass and Saggi (2002a) consider two symmetric innovating countries.

⁷ See, for example, Diwan and Rodrik (1991).

strengthening IPR in developing countries has a positive and significant effect on their innovations.⁸ To fill in this gap in the literature, recent theoretical studies, such as Lai and Qiu (2003) and Grossman and Lai (2004), have started to consider the important role of TRIPS in providing sufficient incentives for innovation in both the North and the South. Our paper follows this branch of studies to focus on this aspect of TRIPS.

Lai and Qiu (2003) and Grossman and Lai (2004) derive the Nash equilibrium level of patent protection in an open-economy variety-expanding model and analyze the welfare effects of harmonizing IPR protection. We complement these interesting studies by also considering the effects of TRIPS on income inequality and growth and by allowing for varying degree for the domestic importance of foreign goods. To our knowledge, our study is the first to analyze the effects of TRIPS on welfare, growth and income inequality simultaneously. The allowance of varying degree of domestic importance of foreign goods also has interesting implications on global welfare.⁹

Since the seminal study of Simon Kuznets (1955), the tradeoff between growth and inequality has been an important issue in economics. On one hand, early theoretical and empirical studies tend to find a negative growth-inequality relationship.¹⁰ On the other hand, the more recent studies tend to find a positive relationship.¹¹ Forbes (2000) finds a positive empirical growth-inequality relationship and argues that the different results in previous studies are due to omitted-variable bias and measurement error. García-Peñalosa and Turnovsky (2006, 2007) argue that the theoretical growth-inequality relationship should be ambiguous and depends on the underlying structural and policy changes. Incorporating wealth heterogeneity into an AK growth model, they show that a positive relationship is more likely to emerge.

⁸ Similarly, Falvey et al. (2006) find that strengthening IPR has a positive and significant effect on economic growth in low-income and high-income countries, but not in middle-income countries.

⁹ Lai (2005) extends Grossman and Lai (2004) to consider the effects of trade barriers. Lai (2005) is interested in deriving a condition under which IPR is under-protected before TRIPS and finds that this condition is likely to hold based on calibrated parameters. In contrast, we are interested in the change in the level of global welfare before and after TRIPS, which is a very particular policy regime that requires the harmonization of IPR protection.

¹⁰ See Galor and Zeira (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994) and Perotti (1996).

¹¹ See Saint-Paul and Verdier (1993), Benabou (1996), Galor and Tsiddon (1997), Li and Zou (1998) and Forbes (2000). Barro (2000) finds a positive (negative) growth-inequality relationship in developed (developing) countries.

Although the capital-accumulation-driven growth models are useful frameworks for analyzing many macroeconomic issues, they are not suitable for evaluating innovation policies. Therefore, this study incorporates wealth heterogeneity into an open-economy R&D-growth model to analyze the effects of TRIPS. In a related study, Chu (2010) analyzes the effects of patent policy on growth and inequality in the US using a closed-economy quality-ladder model with wealth heterogeneity. The current study differs from Chu (2010) by (a) developing an open-economy model, (b) modeling the level of patent protection as the outcome of a policy game between countries, and (c) considering the effects of patent policy on welfare in addition to growth and income inequality. There are other studies that also consider heterogeneity in R&D-growth models, such as Chou and Talmain (1996), Li (1998), Zweimuller (2000), Foellmi and Zweimuller (2006) and Bertola et al. (2006). However, these studies focus on the effects of inequality on growth but do not consider the effects of patent policy on income inequality.

This paper also relates to the literature on R&D underinvestment, patent policy and economic growth. Griliches (1992) provides a survey on empirical studies that find the social return to R&D to be much higher than the private return. Jones and Williams (1998, 2000) use these empirical estimates to show that in an R&D-growth model, the socially optimal level of R&D is at least two to four times higher than the market level. Given R&D underinvestment, Li (2001), O'Donoghue and Zweimuller (2004) and Chu (2009) analyze the effects of patent breadth on R&D and economic growth within the quality-ladder model that has a representative household.¹² Given that patent policy affects the distribution of income, the current study contributes to this literature by providing a growth-theoretic framework that highlights the distributional consequences of patent policy in an open-economy setting.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and analyzes its properties. Section 4 considers the effects of TRIPS on growth, welfare and income inequality simultaneously. Section 5 analyzes the effects of innovative capability on income inequality. Section 6 concludes with some suggestions for future research.

¹² Eicher and García-Peñalosa (2008) develop an R&D-growth model with endogenous IPR protection and derive multiple equilibria, in which one of the interior equilibria is characterized by strong IPR protection and a high level of R&D while the other equilibrium is characterized by weak IPR protection and a low level of R&D.

2. The Model

The underlying quality-ladder model is based on Grossman and Helpman (1991a). We modify the model by (a) extending it to a two-country setting with trade in intermediate inputs similar to Peng et al. (2006), (b) incorporating heterogeneity in households' wealth, and (c) considering incomplete lagging patent breadth (i.e. patent protection against imitation) as in Li (2001).¹³ There are two countries denoted by the North (n) and the South (s). As in Lai and Qiu (2003) and Grossman and Lai (2004), both countries invest in R&D, but the North is assumed to have a higher degree of innovative capability than the South. Also, trade is balanced as commonly assumed in the literature.

Given that quality-ladder models have been well-studied, the familiar components of the models are briefly described in Sections 2.1-2.4. To conserve space, we only present the equations for the North. However, the readers are advised to keep in mind that for each equation that we present, there is an analogous equation for the South.

2.1. Households

There is a continuum of identical households (except for the initial holding of wealth) on the unit interval $h \in [0,1]$ in each of the two countries indexed by a superscript $\in \{n, s\}$, and households are immobile across countries. In country n , household h 's utility function is given by

$$(1) \quad U^n(h) = \int_0^{\infty} e^{-\rho t} \ln C_t^n(h) dt. \quad ^{14}$$

$C_t^n(h)$ denotes household h 's consumption. $\rho > 0$ is the exogenous discount rate. Each household maximizes utility subject to a sequence of budget constraints given by

¹³ Lai and Qiu (2003) and Grossman and Lai (2004) consider patent protection in the form of patent length in their variety-expanding models. Given that we have a quality-ladder model, we consider patent protection in the form of patent breadth, which is an equally important patent-policy instrument commonly discussed in the patent-design literature. See O'Donoghue and Zweimuller (2004) for a discussion on different patent-policy instruments. Using China as an example, its statutory length of patent has been 20 years since 1993, and the patent-law amendments before its accession to the WTO in 2001 were related to other aspects of patent protection as mentioned in footnote 4.

¹⁴ In a similar (closed-economy) model, Chu (2010) considers a more general iso-elastic utility function and shows that the positive relationship between growth and income inequality is robust to this specification change. To simplify the analytical derivation, we focus on the log utility function in this study.

$$(2) \quad \dot{V}_t^n(h) = R_t^n V_t^n(h) + W_t^n - P_t^n C_t^n(h).$$

$V_t^n(h)$ is the value of financial assets owned by household h in country n at time t . Household h 's share of financial assets at time 0 is exogenously given by $s_{v,0}^n(h) \equiv V_0^n(h)/V_0^n$ that has a general distribution function with a mean of one and a standard deviation of σ_v^n (i.e. the coefficient of variation of wealth). R_t^n is the nominal rate of return on assets in country n . We assume home bias in asset holding such that the shares of monopolistic firms in each country are solely owned by domestic households.¹⁵ Household h inelastically supplies one unit of labor to earn a wage income W_t^n . P_t^n is the price of consumption in country n . From household h 's intertemporal optimization, the familiar Euler equation is given by

$$(3) \quad \frac{\dot{C}_t^n(h)}{C_t^n(h)} = \frac{\dot{C}_t^n}{C_t^n} = r_t^n - \rho,$$

where $r_t^n \equiv R_t^n - \dot{P}_t^n / P_t^n$ is the real interest rate. Also, consumption of households grows at the same rate.

2.2. Final Goods

Consumption in country n is an aggregate of domestic and foreign final goods given by

$$(4) \quad C_t^n = \frac{(C_t^{n,n})^{1-\alpha} (C_t^{n,s})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}.^{16}$$

$C_t^{n,s}$ refers to final goods consumed by country n and produced by inputs from country s . The parameter $\alpha \in [0,0.5]$ determines the importance of foreign goods in domestic consumption. A large number of perfectly competitive firms produce final goods using a standard Cobb-Douglas aggregator over a continuum of differentiated intermediates goods $i \in [0,1]$.

¹⁵ Note that home bias does not eliminate the positive externality of patent protection in generating profits to be earned by foreign households. When a country raises its level of patent protection, foreign firms owned by foreign households still earn a larger amount of profits. What home bias does is to naturally link the degree of this positive externality to the share of goods traded, which is determined by the domestic importance of foreign goods.

¹⁶ This type of Armington aggregator is commonly used in open-economy macroeconomic models for aggregating tradable goods across countries. See, for example, Obstfeld and Rogoff (2000).

$$(5) \quad C_t^{n,n} = \exp\left(\int_0^1 \ln C_t^{n,n}(i) di\right),$$

$$(6) \quad C_t^{n,s} = \exp\left(\int_0^1 \ln C_t^{n,s}(i) di\right).$$

$C_t^{n,s}(i)$ refers to intermediate goods i produced by inputs from country s .

2.3. Intermediate Goods

In country n , there is a continuum of industries indexed by $i \in [0,1]$. Each industry is dominated by a temporary monopolistic leader, who produces $X_t^{n,n}(i)$ and $X_t^{s,n}(i)$ that are the necessary inputs for $C_t^{n,n}(i)$ and $C_t^{s,n}(i)$ respectively. The leader holds a patent in each country on the industry's latest technology. Using the leader's input $X_t^{n,n}(i)$, the level of output for $C_t^{n,n}(i)$ is

$$(7) \quad C_t^{n,n}(i) = z^{N_t^n(i)} X_t^{n,n}(i).$$

$z > 1$ is the exogenous quality improvement from each invention, and $N_t^n(i)$ is the number of inventions that have occurred in industry i of country n as of time t . In other words, $z^{N_t^n(i)}$ represents the quality of each unit of input produced by the leader while $X_t^{n,n}(i)$ is the quantity of input produced. Similarly, using the leader's input $X_t^{s,n}(i)$, the level of output for $C_t^{s,n}(i)$ is

$$(8) \quad C_t^{s,n}(i) = (1 - \tau) z^{N_t^n(i)} X_t^{s,n}(i),$$

where $\tau \in [0,1)$ denotes an iceberg transportation cost that captures trade barriers.

To produce one unit of $X_t^{n,n}(i)$ or $X_t^{s,n}(i)$, the industry leader needs to employ one unit of workers. Therefore, the production function is

$$(9) \quad X_t^{n,n}(i) + X_t^{s,n}(i) = L_{x,t}^{n,n}(i) + L_{x,t}^{s,n}(i) = L_{x,t}^n(i).$$

$L_{x,t}^n(i)$ is the number of workers in industry i of country n . The marginal cost of producing one unit of $X_t^{n,n}(i)$ or $X_t^{s,n}(i)$ is

$$(10) \quad MC_t^n(i) = W_t^n.$$

Implicitly, we have assumed that the industry leader must employ domestic workers to produce for both domestic and foreign markets and abstracted from the issues of foreign direct investment, licensing and overseas imitation in order to keep the model tractable.¹⁷

As commonly assumed in quality-ladder models, the current and former industry leaders engage in Bertrand competition, and the familiar profit-maximizing price for the current industry leader is a constant markup over the marginal cost. The prices for $X_t^{n,n}(i)$ and $X_t^{s,n}(i)$ are respectively

$$(11) \quad P_t^{n,n}(i) = \mu(z, b^n) MC_t^n(i),$$

$$(12) \quad P_t^{s,n}(i) = \mu(z, b^s) MC_t^n(i),$$

where $\mu(z, b) = z^b$ for $b \in (0, 1]$. b^n (b^s) captures the level of patent breadth in country n (s). In Grossman and Helpman (1991a), there is complete patent protection against imitation (i.e. $b = 1$). Li (2001) generalizes the policy environment to capture incomplete patent protection against imitation (i.e. $b \in (0, 1]$).¹⁸ Because of incomplete patent protection, the former leader can imitate the current leader such that the quality of her product to be sold in country n (s) increases by a factor of z^{1-b^n} (z^{1-b^s}). In other words, the quality of the former leader's product to be sold in country n (s) can increase to $z^{N_t^n(i)-b^n}$ ($z^{N_t^n(i)-b^s}$) without infringing the current leader's patents.¹⁹ As a result, the limit-pricing markups for the current leader are z^{b^n} in country n and z^{b^s} in country s respectively. An increase in b in either country

¹⁷ These interesting issues have been studied in another strand of literature. See Grossman and Helpman (1991b), Helpman (1993), Lai (1998), Yang and Maskus (2001), and Glass and Saggi (2002a, b).

¹⁸ This is known as lagging patent breadth in the literature. See, for example, O'Donoghue and Zweimuller (2004) and Chor and Lai (2009) for an analysis of leading patent breadth in dynamic general-equilibrium models. For the purpose of the current study, the consideration of lagging patent breadth is more relevant for developing countries.

¹⁹ $z^{N-b} = (z^{N-1})(z^{1-b})$, in which the first term on the right is the quality of the former leader's product while the second term is the increase in the quality of her product by legally imitating the current leader's product.

enables the current leader to charge a higher markup in that country. The resulting increases in profits and the value of an invention improve the incentives for R&D. From now on, we denote patent protection as $\mu^n \equiv \mu(z, b^n)$ for convenience and consider changes in μ^n coming from changes in b^n only.

2.4. R&D

Denote the expected value of an invention for industry i in country n as $\tilde{V}_t^n(i)$. Due to the Cobb-Douglas specification in (5) and (6), the amount of monopolistic profits is the same across industries within a country (i.e. $\pi_t^{n,n}(i) = \pi_t^{n,n}$ and $\pi_t^{s,n}(i) = \pi_t^{s,n}$ for $i \in [0,1]$). As a result, $\tilde{V}_t^n(i) = \tilde{V}_t^n$ for $i \in [0,1]$. Also, denote the sum of profits generated by an invention from country n as $\pi_t^n \equiv \pi_t^{n,n} + \pi_t^{s,n}$. Because of complete home bias in asset holding, the market value of inventions in country n equals the total value of assets owned by domestic households (i.e. $\tilde{V}_t^n = V_t^n$). The familiar no-arbitrage condition for V_t^n is

$$(13) \quad R_t^n V_t^n = \pi_t^n + \dot{V}_t^n - \lambda_t^n V_t^n.$$

The left-hand side of (13) is the nominal return on this asset. The right-hand side of (13) consists of the sum of (a) the monopolistic profit π_t^n generated by this asset, (b) the potential capital gain \dot{V}_t^n , and (c) the expected capital loss $\lambda_t^n V_t^n$ due to creative destruction, in which λ_t^n is the Poisson arrival rate of inventions in country n .

There is a continuum of R&D entrepreneurs indexed by $j \in [0,1]$ in each country, and they hire workers to create inventions. The expected profit for entrepreneur j in country n is

$$(14) \quad \pi_{r,t}^n(j) = V_t^n \lambda_t^n(j) - W_t^n L_{r,t}^n(j).$$

The Poisson arrival rate of inventions for entrepreneur j in country n is $\lambda_t^n(j) = \varphi^n L_{r,t}^n(j)$, where φ^n captures the productivity of R&D workers (i.e. innovative capability) in country n . Without loss of generality, we assume $\varphi^n \geq \varphi^s$. Because of free entry, the zero-profit condition from the R&D sector is

$$(15) \quad V_t^n \phi^n = W_t^n.$$

This condition determines the allocation of labor between production and R&D within each country.

3. Decentralized Equilibrium

In this section, we define the equilibrium and show that the aggregate economy is always on a unique and stable balanced-growth path. Then, Section 3.1 shows that the distribution of assets is stationary on the balanced-growth path. Section 3.2 derives our measure of income inequality. Section 3.3 defines social welfare and characterizes the Nash equilibrium as well as the globally optimal level of patent protection.

The equilibrium in country n is a sequence of prices $\{R_t^n, W_t^n, P_t^n, P_t^{n,n}(i), P_t^{s,n}(i), V_t^n(h), V_t^n\}_{t=0}^\infty$ and a sequence of allocations $\{C_t^{n,n}(i), C_t^{s,n}(i), X_t^{n,n}(i), X_t^{s,n}(i), L_{x,t}^n(i), L_{r,t}^n(j), C_t^n(h), C_t^n, C_t^{n,n}, C_t^{n,s}\}_{t=0}^\infty$.

Also, in each period,

- a. household h chooses $\{C_t^n(h)\}$ to maximize (1) subject to (2) taking $\{R_t^n, W_t^n, P_t^n\}$ as given;
- b. perfectly competitive final-goods firms maximize profit taking prices as given;
- c. the leader in industry i produces $\{X_t^{n,n}(i), X_t^{s,n}(i)\}$ and chooses $\{P_t^{n,n}(i), P_t^{s,n}(i), L_{x,t}^n(i)\}$ to maximize profit according to the Bertrand competition and taking $\{W_t^n\}$ as given;
- d. R&D entrepreneur j chooses $\{L_{r,t}^n(j)\}$ to maximize profit taking $\{W_t^n, V_t^n\}$ as given;
- e. the market for consumption clears such that $\int_0^1 C_t^n(h) dh = C_t^n = \frac{(C_t^{n,n})^{1-\alpha} (C_t^{n,s})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}$;
- f. the market for domestic final goods clears such that $C_t^{n,n} = \exp\left(\int_0^1 \ln C_t^{n,n}(i) di\right)$;
- g. the market for foreign final goods clears such that $C_t^{n,s} = \exp\left(\int_0^1 \ln C_t^{n,s}(i) di\right)$;
- h. the domestic market for intermediate goods i clears, i.e. $C_t^{n,n}(i) = z^{N_t^n(i)} X_t^{n,n}(i)$;

- i. the overseas market for intermediate goods i clears, i.e. $C_t^{s,n}(i) = (1 - \tau)z^{N_t^n(i)}X_t^{s,n}(i)$;
- j. the labor market clears such that $\int_0^1 L_{x,t}^n(i)di + \int_0^1 L_{r,t}^n(j)dj = 1$; and
- k. the value of trade in intermediate goods is balanced such that $P_t^{n,s}C_t^{n,s} = P_t^{s,n}C_t^{s,n}$.²⁰

Lemma 1 shows that the aggregate economy always jumps immediately to a unique and stable balanced-growth path,²¹ in which all aggregate variables grow at some constant (possibly zero) rates.

Lemma 1: *The aggregate economy is always on a unique and stable balanced-growth path, in which the equilibrium allocation of labor in country n is given by*

$$(16) \quad L_x^{n,n}(\mu^n, \varphi^n) = \left(\frac{1-\alpha}{\mu^n} \right) \left(1 + \frac{\rho}{\varphi^n} \right),$$

$$(17) \quad L_x^{s,n}(\mu^s, \varphi^n) = \left(\frac{\alpha}{\mu^s} \right) \left(1 + \frac{\rho}{\varphi^n} \right),$$

$$(18) \quad L_r^{n,+}(\mu^n, \mu^s, \varphi^n) = 1 - \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) \left(1 + \frac{\rho}{\varphi^n} \right).$$

Proof: See Appendix A. ■

The properties of the equilibrium labor allocation are quite intuitive. An increase in μ^n , μ^s or φ^n improves the incentives for R&D. As a result, labor is reallocated away from the production sector to the R&D sector. To ensure that $L_r^n > 0$, we impose a lower bound on R&D productivity.

Condition R (R&D productivity): $\varphi^n > \rho / (\Gamma^n - 1)$,

²⁰ These price indices will be defined in the proof of Lemma 1.

²¹ As in Grossman and Helpman (1991a), the implicit assumptions behind this result are (a) at any point in time, each industry has an existing leader with a competitor one step down the quality ladder and (b) R&D entrepreneurs always implement their inventions immediately (i.e. ruling out endogenous implementation cycles).

where $\Gamma^n \equiv \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right)^{-1}$.

Given the equilibrium allocation of labor, the next lemma characterizes the equilibrium outcomes for the other aggregate variables.

Lemma 2: *On the balanced-growth path, the other aggregate variables are given by*

$$(19) \quad \lambda^n(\mu^n, \mu^s, \varphi^n) = \varphi^n L_r^n,$$

$$(20) \quad \frac{\dot{C}_t^n}{C_t^n} \equiv g^n(\mu^n, \mu^s, \varphi^n, \varphi^s) = [(1-\alpha)\lambda^n + \alpha\lambda^s] \ln z,$$

$$(21) \quad C_t^n = \left(1 + \frac{\rho}{\varphi^n} \right) \frac{W_t^n}{P_t^n}.$$

Proof: See Appendix A. ■

The arrival rate of inventions is increasing in domestic R&D. The growth rate of consumption in country n is an increasing function in the arrival rate of inventions in either country. Thus, an increase in μ^n , μ^s , φ^n or φ^s increases domestic and/or foreign R&D as well as the consumption growth rate.

3.1. Distribution of Assets

I adopt a similar approach as in García-Peñalosa and Turnovsky (2006, 2007) to show that the distribution of assets is stationary on the balanced growth path. The value of assets in country n evolves according to

$$(22) \quad \dot{V}_t^n = R_t^n V_t^n + W_t^n - P_t^n C_t^n.$$

Combining (2) and (22), the law of motion for $s_{v,t}(h) \equiv V_t(h)/V_t$ is given by

$$(23) \quad \frac{\dot{s}_{v,t}^n(h)}{s_{v,t}^n(h)} = \frac{W_t^n - P_t^n C_t^n(h)}{V_t^n(h)} - \frac{W_t^n - P_t^n C_t^n}{V_t^n}.$$

From (15) and (21), $s_{v,t}^n(h)$ evolves according to a simple linear differential equation given by

$$(24) \quad \dot{s}_{v,t}^n(h) = \rho s_{v,t}^n(h) + \left(1 - s_c^n(h) \left(1 + \frac{\rho}{\varphi^n}\right)\right) \varphi^n.$$

(24) describes the potential evolution of $s_{v,t}^n(h)$ given an initial value of $s_{v,0}^n(h)$. $s_c^n(h) \equiv C_t^n(h) / C_t^n$ is a stationary variable from (3), so that the last term in (24) is constant. The coefficient on $s_{v,t}^n(h)$ given by ρ is constant and positive. Therefore, the only solution consistent with long-run stability is $\dot{s}_{v,t}^n(h) = 0$ for all t . From (24), $\dot{s}_{v,t}^n(h) = 0$ for all t implies that $s_{v,t}^n(h) = s_{v,0}^n(h)$ and

$$(25) \quad C_t^n(h) = \left(1 + \frac{\rho s_{v,0}^n(h)}{\varphi^n}\right) \frac{W_t^n}{P_t^n}$$

for all t . Lemma 3 summarizes the stationarity of the wealth distribution in country n .

Lemma 3: For every household h in country n , $s_{v,t}^n(h) = s_{v,0}^n(h)$ for all t .

Proof: Proven in the text. ■

3.2. Income Inequality

This section derives our measure of income inequality. We consider inequality in real income, which is the appropriate measure because it is invariant to the unit of denomination. Household h 's real income $Y_t^n(h)$ is the sum of the real return on financial assets and the real wage rate given by

$$(26) \quad Y_t^n(h) = r_t^n V_t^n(h) / P_t^n + W_t^n / P_t^n.$$

From (3), (15) and Lemma 3, the share of real income earned by household h simplifies to

$$(27) \quad s_{y,t}^n(h) \equiv \frac{Y_t^n(h)}{Y_t^n} = \frac{(\rho + g^n) s_{v,0}^n(h) + \varphi^n}{\rho + g^n + \varphi^n}$$

for all t . The standard deviation of income share (i.e. the coefficient of variation of income) is

$$(28) \quad \sigma_y^n \equiv \sqrt{\int_0^1 [s_{y,t}^n(h) - 1]^2 dh} = \left(\frac{\rho + g^n}{\rho + g^n + \varphi^n} \right) \sigma_v^n,$$

where the coefficient of variation of wealth σ_v^n is exogenously given at time 0.²² We follow García-Peñalosa and Turnovsky (2006, 2007) to use the coefficient of variation of income as a measure of income inequality. Proposition 1 summarizes the effect of growth on income inequality.

Proposition 1: *Holding ρ and φ^n constant, income inequality σ_y^n is increasing in the growth rate g^n .*

Proof: See (28). ■

Intuitively, a higher growth rate drives up the real interest rate through the Euler equation, and the resulting higher return on assets increases the income share $s_y(h)$ of asset-wealthy households (i.e. $s_y(h) > 1$) while it decreases that of asset-poor households (i.e. $s_y(h) < 1$). This positive relationship between growth and inequality is consistent with recent empirical studies, such as Li and Zou (1998) and Forbes (2000). We now consider the effects of an exogenous increase in patent protection on growth and income inequality. Corollary 1 shows that a higher level of patent protection in either country increases R&D and hence economic growth as well as income inequality in both countries.

Corollary 1: *An increase in μ^n or μ^s increases growth and income inequality in both countries.*

Proof: See (20) and (28). ■

3.3. Social Welfare

Due to the balanced-growth behavior of the model, the utility of household h in country n simplifies to

²² (28) shows an interesting difference between the AK model and the quality-ladder model. The AK model in García-Peñalosa and Turnovsky (2006, 2007) requires elastic labor supply to generate an endogenous income distribution while the quality-ladder model generates an endogenous income distribution even with inelastic labor supply. See Chu (2010) for a quality-ladder model with heterogeneous households and elastic labor supply.

$$(29) \quad U^n(h) = \frac{\ln C_0^n(h)}{\rho} + \frac{g^n}{\rho^2}.$$

Substituting (25) into (29) yields

$$(30) \quad U^n(h) = \frac{1}{\rho} \left(\ln \left(1 + \frac{\rho S_{v,0}^n(h)}{\varphi^n} \right) + \ln \left(\frac{W_0^n}{P_0^n} \right) + \frac{g^n}{\rho} \right).$$

The lifetime utility of a household depends on the growth rate and the initial level of consumption, which in turn depends on the initial real wage rate and the share of assets owned by the household. Although the ownership of assets varies across households, (30) shows that this household-specific term is independent of patent protection. This property is a result of the log utility function, and this convenient feature allows us to abstract from choosing a social welfare function for the government.

Lemma 4: *After dropping the exogenous terms, the initial real wage in country n can be decomposed into*

$$(31) \quad \ln(W_0^n / P_0^n) = -\ln \mu^n + \alpha \ln(1 - \tau) + \alpha \ln(W_0^n / W_0^s).$$

Proof: See Appendix A. ■

Lemma 4 shows that the initial real wage in country n has three components (a) the negative effect of markup pricing, (b) the negative effect of trade barriers,²³ and (c) the relative wage rate across the two countries. An expression for the relative wage can be derived from the balanced-trade condition

$P_t^{s,n} C_t^{s,n} = P_t^{n,s} C_t^{n,s}$, which simplifies to

$$(32) \quad \frac{W_t^n}{W_t^s} \equiv \omega^n(\varphi^+, \varphi^-) = \frac{\mu^n}{\mu^s} \left(\frac{L_x^{n,s}}{L_x^{s,n}} \right) = \left(1 + \frac{\rho}{\varphi^s} \right) / \left(1 + \frac{\rho}{\varphi^n} \right) \geq 1$$

for all t . (32) shows that the relative wage is independent of patent protection and depends on the relative R&D productivity between the North and the South. Substituting (31) and (32) into (30) and dropping the

²³ For example, the benefit of China's accession to the WTO may be captured by a reduction in trade barriers that increases social welfare in China.

terms that are independent of patent protection yield the welfare of any household h in country n as a function of μ^n and μ^s given by

$$(33) \quad \Omega^n(\mu^n, \mu^s) \equiv -\ln \mu^n + \frac{g^n(\mu^n, \mu^s)}{\rho}.$$

(33) has three interesting properties. Firstly, the welfare component that depends on patent protection is the same across households. Secondly, (33) captures the tradeoff between the static cost $-\ln \mu^n$ and the dynamic benefit g^n / ρ from raising patent protection that was first studied by Nordhaus (1969). Thirdly, (33) and the analogous condition for the South show that while the cost of increasing μ^n falls entirely on the North, the benefit is shared by the South because g^s is increasing in μ^n . As a result of this positive externality, the Nash equilibrium level of patent protection is suboptimal. $g^s = [(1 - \alpha)\lambda^s + \alpha\lambda^n] \ln z$ is increasing in μ^n via two channels (i) λ^n as an increasing function in μ^n and (ii) λ^s as an increasing function in μ^n . Channel (i) captures technology spillovers from one country to the other country while channel (ii) captures the positive effect of raising patent protection in one country on the profitability of R&D in the other country. Both of these channels are present if and only if $\alpha > 0$.

Upon deriving the welfare function, we firstly characterize the Nash equilibrium level of patent protection in the two countries denoted by (μ_{NE}^n, μ_{NE}^s) . As in Grossman and Lai (2004), the policymaker in each country chooses the domestic level of patent protection once and for all at time 0 to maximize domestic households' welfare (33) taking the foreign level of patent protection as given. In other words, the policymakers in the two countries play a one-shot game at time 0. Also, we assume an interior solution for the equilibrium level of patent protection such that $\mu < z$ (i.e. $b < 1$) in each country.

Proposition 2: *The Nash equilibrium level of patent protection is given by*

$$(34) \quad \mu_{NE}^n(\varphi^n, \varphi^s) = \left((1 - \alpha)^2 \left(\frac{\varphi^n}{\rho} + 1 \right) + \alpha^2 \left(\frac{\varphi^s}{\rho} + 1 \right) \right) \ln z,$$

$$(35) \quad \mu_{NE}^s(\varphi_+^n, \varphi_+^s) = \left((1-\alpha)^2 \left(\frac{\varphi_+^s}{\rho} + 1 \right) + \alpha^2 \left(\frac{\varphi_+^n}{\rho} + 1 \right) \right) \ln z .$$

Proof: See Appendix A. ■

As in Lai and Qiu (2003) and Grossman and Lai (2004), we find that the Nash equilibrium level of patent protection is stronger in the North than in the South unless either (a) $\alpha = 0.5$ or (b) $\varphi^n = \varphi^s$. For the rest of the analysis, we assume that neither (a) nor (b) hold such that $\mu_{NE}^n > \mu_{NE}^s$. Next, we derive the globally optimal level of patent protection denoted by $(\mu_{GO}^n, \mu_{GO}^s) \equiv \arg \max(\Omega^n + \Omega^s)$.²⁴ If $\alpha = 0$, then $\mu_{NE}^n = \mu_{GO}^n$ and $\mu_{NE}^s = \mu_{GO}^s$. Otherwise, $\mu_{NE}^n < \mu_{GO}^n$ and $\mu_{NE}^s < \mu_{GO}^s$ (i.e. a suboptimally low level of patent protection in the Nash equilibrium). For the rest of the analysis, we assume that $\alpha > 0$.

Proposition 3: *The globally optimal level of patent protection is given by*

$$(36) \quad \mu_{GO}^n(\varphi_+^n, \varphi_+^s) = \left((1-\alpha) \left(\frac{\varphi_+^n}{\rho} + 1 \right) + \alpha \left(\frac{\varphi_+^s}{\rho} + 1 \right) \right) \ln z > \mu_{NE}^n ,$$

$$(37) \quad \mu_{GO}^s(\varphi_+^n, \varphi_+^s) = \left((1-\alpha) \left(\frac{\varphi_+^s}{\rho} + 1 \right) + \alpha \left(\frac{\varphi_+^n}{\rho} + 1 \right) \right) \ln z > \mu_{NE}^s .$$

Proof: See Appendix A. ■

Corollary 2 shows that because the positive externality in the Nash equilibrium is increasing in α , the Nash equilibrium deviates further from the globally optimal level of patent protection as α increases.

Corollary 2: *An increase in α increases $\mu_{GO}^n - \mu_{NE}^n$ and $\mu_{GO}^s - \mu_{NE}^s$.*

Proof: See Appendix A. ■

²⁴ To be consistent with previous studies, we use this utilitarian approach to define global welfare.

4. Effects of TRIPS

In this section, we analyze the effects of TRIPS on growth, welfare and income inequality. We follow Lai and Qiu (2003) to define the policy regime under TRIPS as $\mu_{TRIPS}^s = \mu_{TRIPS}^n = \mu_{NE}^n$. In summary, the North experiences higher growth and higher welfare at the expense of higher income inequality while the South experiences higher growth at the expense of lower welfare and higher income inequality.

Under TRIPS, the South's level of patent protection increases from μ_{NE}^s to μ_{TRIPS}^s . This higher level of patent protection increases growth in both countries (i.e. $g_{TRIPS}^n > g_{NE}^n$ and $g_{TRIPS}^s > g_{NE}^s$). (28) shows that higher growth increases income inequality (i.e. $\sigma_{y,TRIPS}^n > \sigma_{y,NE}^n$ and $\sigma_{y,TRIPS}^s > \sigma_{y,NE}^s$). Also, (33) shows that the higher growth in the North unambiguously increases its welfare (i.e. $\Omega_{TRIPS}^n > \Omega_{NE}^n$). As for the South, the increase in μ^s leads to two opposing effects on its welfare. One is the positive growth effect, and the other is the negative effect of markup pricing. However, from the definition of the Nash equilibrium, a unilateral deviation from the best response must render the South worse off (i.e. $\Omega_{TRIPS}^s < \Omega_{NE}^s$). Proposition 4 summarizes these findings.

Proposition 4: *In the North, the effects of TRIPS on growth, welfare and income inequality are (a) $g_{TRIPS}^n > g_{NE}^n$, (b) $\Omega_{TRIPS}^n > \Omega_{NE}^n$, and (c) $\sigma_{y,TRIPS}^n > \sigma_{y,NE}^n$. In the South, the effects of TRIPS on growth, welfare and income inequality are (a) $g_{TRIPS}^s > g_{NE}^s$, (b) $\Omega_{TRIPS}^s < \Omega_{NE}^s$, and (c) $\sigma_{y,TRIPS}^s > \sigma_{y,NE}^s$.*

Proof: Proven in the text. ■

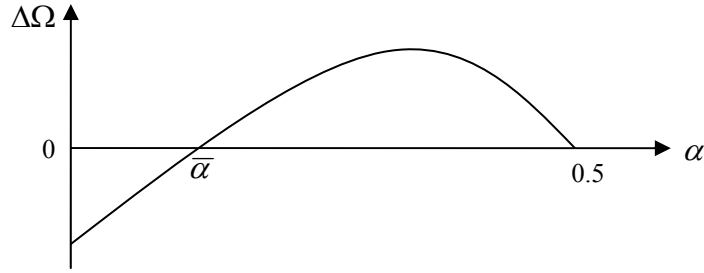
The above implication on each country's welfare is perhaps not surprising given the definition of Nash equilibrium. Therefore, the intriguing question is whether global welfare increases or decreases as a result of TRIPS, and we compare the level of global welfare between the Nash equilibrium and the policy regime under TRIPS. It turns out that there exists a critical degree for the importance of foreign goods in

domestic consumption below (above) which global welfare is lower (higher) under TRIPS. Proposition 5 summarizes this result, and Figure 1 plots $\Delta\Omega \equiv (\Omega_{TRIPS}^n + \Omega_{TRIPS}^s) - (\Omega_{NE}^n + \Omega_{NE}^s)$ against α .

Proposition 5: There exists a cutoff value $\bar{\alpha} \in (0, 0.5)$ such that (a) $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s$ if $\alpha \in (0, \bar{\alpha})$, and (b) $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s$ if $\alpha \in (\bar{\alpha}, 0.5)$.

Proof: See Appendix A. ■

Figure 1: Changes in Global Welfare under TRIPS



Firstly, as $\alpha \rightarrow 0$, $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s$ because the two economies are in autarky and the South's optimal level of patent protection is lower than that of the North. Forcing the South to adopt the North's level of patent protection causes the South to experience a welfare loss while the North's welfare is unchanged. When α is slightly above 0, $(\Omega_{TRIPS}^n + \Omega_{TRIPS}^s) - (\Omega_{NE}^n + \Omega_{NE}^s)$ is increasing in α because the positive externality in the Nash equilibrium reduces the welfare loss in the South and leads to a small welfare gain in the North under TRIPS. As $\alpha \rightarrow 0.5$, $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s = \Omega_{NE}^n + \Omega_{NE}^s$ because the Nash equilibrium is the same as the policy regime under TRIPS, such that $\mu_{NE}^s = \mu_{TRIPS}^s$. When α is slightly less than 0.5, $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s$ because $\mu_{NE}^s < \mu_{TRIPS}^s < \mu_{GO}^s$. In other words, the South's level of patent protection under TRIPS is moving towards the globally optimal level. For intermediate values of α , there exists a critical degree $\bar{\alpha}$ below (above) which global welfare under TRIPS is lower (higher) than in the Nash equilibrium.

5. Effects of China's Rising Innovative Capability on Income Inequality

To analyze the inequality effects of an increase in China's innovative capability, we consider the comparative statics of increasing the South's innovative capability while holding μ^s and μ^n constant. We find that increasing φ^s has a positive effect on σ_y^n by increasing g^n and r^n . However, the effect on σ_y^s is ambiguous and depends on the North's innovative capability. If φ^n is below (above) a threshold, then increasing φ^s increases (decreases) σ_y^s . Increasing φ^s has an additional negative effect on σ_y^s by increasing the South's wage income, and this negative wage-income effect on income inequality is dominated by the positive interest-rate effect if and only if φ^n is below a threshold that is decreasing in μ^s and μ^n . Differentiating the analogous condition of (28) for the South with respect to φ^s yields

$$(38) \quad \frac{\partial \sigma_y^s}{\partial \varphi^s} = \frac{1}{(\rho + g^s + \varphi^s)^2} \left(\varphi^s \frac{\partial g^s}{\partial \varphi^s} - (\rho + g^s) \frac{\partial \varphi^s}{\partial \varphi^s} \right) \sigma_y^s,$$

where $-(\rho + g^s)$ determines the negative effect of φ^s on income inequality through wage income. This negative wage-income effect is likely to be dominated by the positive interest-rate effect at a low growth rate g^s . Given that g^s is increasing in φ^n , a smaller φ^n is more likely for $\partial \sigma_y^s / \partial \varphi^s > 0$. When there is an increase in either μ^s or μ^n , a smaller φ^n is needed in order to offset the effects on g^s and σ_y^s .

Proposition 6: *Increasing innovative capability in the South has the following effects: (a) $\partial \sigma_y^n / \partial \varphi^s > 0$, and (b) $\partial \sigma_y^s / \partial \varphi^s > 0$ if $\varphi^n < \bar{\varphi}^n$ while $\partial \sigma_y^s / \partial \varphi^s < 0$ if $\varphi^n > \bar{\varphi}^n$, where*

$$(39) \quad \bar{\varphi}^n(\mu_-^n, \mu_-^s) \equiv \frac{\rho}{\alpha} \left((1-\alpha) \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) + \alpha \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) - \frac{1}{\ln z} \right) / \left(1 - \frac{1-\alpha}{\mu^n} - \frac{\alpha}{\mu^s} \right).^{25}$$

Proof: See Appendix A. ■

²⁵ Note that $\bar{\varphi}^n$ can be negative. In this case, it is always the case that $\partial \sigma_y^s / \partial \varphi^s < 0$.

6. Conclusion

This paper analyzes the effects of TRIPS on growth, welfare and income inequality simultaneously. In summary, strengthening patent protection in developing countries as a result of TRIPS increases global economic growth but also worsens global income inequality. Whether it increases global welfare depends on the domestic importance of foreign goods. To derive these results, this paper incorporates wealth heterogeneity into an open-economy quality-ladder model. Our model belongs to the class of first-generation R&D-growth models that may exhibit scale effects, in which a larger economy experiences faster growth. We eliminate scale effects by normalizing each country's population size to unity.²⁶ A possible direction for future research is to analyze the effects of TRIPS on growth and income inequality in later vintages of R&D-growth models.

In our model, we have abstracted from some interesting issues, such as licensing, foreign direct investment, and North-South product cycles. In reality, both of (a) technology transfer from the North to the South and (b) providing sufficient incentives for the South to innovate are important reasons for strengthening IPR in the South. For analytical tractability and the relative lack of attention to the latter issue in the literature, we follow Lai and Qiu (2003) and Grossman and Lai (2004) to focus on (b) instead of (a). Therefore, another possible direction for future research is to account for these issues in a model with heterogeneous households. Furthermore, given that the enforcement of IPR is as important as the statutory law in the real world, it is interesting for future studies to consider the issue of IPR enforcement in developing countries.

Although our model is designed to analyze the positive externality associated with IPR protection provided by developed and developing countries, the two countries in the model can easily be relabeled as two developed countries by assuming that they have similar levels of R&D productivity. In this case, the Nash equilibrium level of patent protection continues to be lower than the globally optimal level as long

²⁶ The literature has two other ways of dealing with scale effects (a) the semi-endogenous growth model and (b) the second-generation endogenous-growth model that combines quality improvement and variety expansion. Our model's implication that devoting a larger *share* of labor to R&D would increase growth is consistent with the second-generation models. See, for example, Jones (1999) for a discussion on scale effects in R&D-growth models.

as α is greater than zero. In other words, a coordination failure of patent policy can exist even among developed countries suggesting the importance of also evaluating whether the level of IPR protection chosen by developed countries is indeed optimal from the perspective of global welfare.

Finally, in our model, income inequality is generated by an unequal distribution of (financial) capital income, and patent policy affects income inequality through the rate of return on assets. Therefore, even if inventions do not represent a significant share of assets in reality,²⁷ the effect of patent policy on income inequality can still be significant in the presence of other capital incomes that depend on the real interest rate. Although the prevailing wisdom is that income inequality is mainly caused by an increase in the skill premium (i.e. the relative wage between skilled and unskilled workers), some studies, such as Atkinson (2000, 2003), argue that inequality in capital income is also playing an increasingly important role. For example, Reed and Cancian (2001) show that capital income contributes to one quarter of the increase in income inequality in the US in the 90's while it accounts for less than one-tenth of the increase in the 70's. Therefore, the current study also serves the purpose of providing an open-economy R&D-growth model that highlights the increasing importance of capital income on income inequality. For analytical tractability, our model features only one dimension of heterogeneity in the form of an unequal distribution of wealth.²⁸ However, it is not difficult to see that strengthening IPR protection would also increase wage inequality when there are skilled and unskilled workers. Intuitively, strengthening IPR protection increases the return to R&D and hence the wage rate of R&D workers, who are mostly skilled labor. As a result, the relative wage between skilled and unskilled workers increases.

²⁷ Nakamura (2003) calculates that the market value of intangible assets in the US is at least \$5 trillion in 2000 (i.e. about 50% of US GDP). Although intangible assets include patents and copyrights that are innovation-related, they also include trademarks and goodwill that may be unrelated to innovation.

²⁸ See Bougheasa and Riezman (2007) for an interesting analysis on wage inequality and human-capital distribution in an open-economy model.

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Appendix A

Proof of Lemma 1: In this proof, we first show that aggregate expenditure on consumption $E_t^n \equiv P_t^n C_t^n$ in country n always jumps immediately to a unique and stable steady-state value. Then, we show that this steady-state value determines a unique and stationary equilibrium allocation of labor in country n . Choosing labor as the numeraire in country n (i.e. $W_t^n = 1$ for all t) implies that $V_t^n \varphi^n = 1$ for all t from (15). Given that φ^n is constant, $\dot{V}_t^n = 0$. Integrating (2) over $h \in [0,1]$ and then setting \dot{V}_t^n to zero yield

$$(A1) \quad E_t^n = W_t^n + R_t^n V_t^n = 1 + R_t^n / \varphi^n .$$

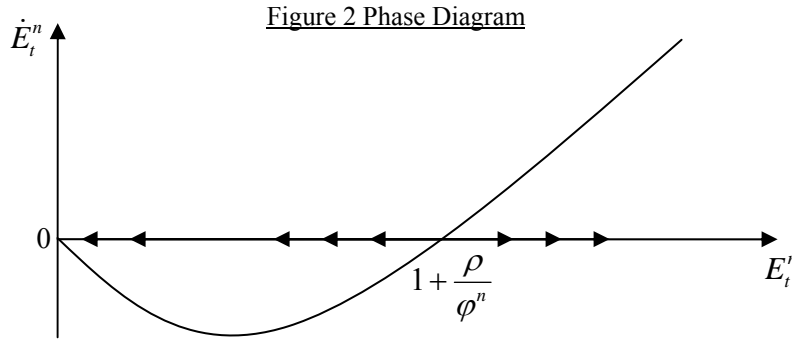
Using its definition, the law of motion for aggregate expenditure on consumption is given by

$$(A2) \quad \frac{\dot{E}_t^n}{E_t^n} = \frac{\dot{P}_t^n}{P_t^n} + \frac{\dot{C}_t^n}{C_t^n} = R_t^n - \rho$$

from (3) because $\dot{C}_t^n / C_t^n = \dot{C}_t^n(h) / C_t^n(h)$ for all $h \in [0,1]$. Substituting (A1) into (A2) yields

$$(A3) \quad \dot{E}_t^n / E_t^n = \varphi^n (E_t^n - 1) - \rho ,$$

which is plotted in Figure 2.



For any initial value of E_t^n below $1 + \rho / \varphi^n$, E_t^n eventually converges to zero violating the households' utility maximization. For any initial value of E_t^n above $1 + \rho / \varphi^n$, E_t^n eventually increases to a point in which all the workers are allocated to production. A zero allocation of R&D workers violates the R&D

entrepreneurs' profit maximization. Therefore, to be consistent with long-run stability, E_t^n must always jump to its unique non-zero steady state given by

$$(A4) \quad E^n = 1 + \rho / \varphi^n .$$

From (A2), $\dot{E}_t^n = 0$ implies that $R_t^n = \rho$ for all t .

Next, we derive the equilibrium allocation of labor. The price index for $C_t^n = \frac{(C_t^{n,n})^{1-\alpha} (C_t^{n,s})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}$

is $P_t^n \equiv (P_t^{n,n})^{1-\alpha} (P_t^{n,s})^\alpha$. The price index for $C_t^{n,n}$ is $P_t^{n,n} \equiv \exp\left(\int_0^1 \ln\left(\frac{P_t^{n,n}(i)}{z_t^{N_t^n(i)}}\right) di\right) = \frac{\mu^n W_t^n}{Z_t^n}$, where

$Z_t^n \equiv \exp\left(\int_0^1 N_t^n(i) di \ln z\right)$. Similarly, the price index for $C_t^{n,s}$ is $P_t^{n,s} = \frac{\mu^n W_t^s}{(1-\tau)Z_t^s}$. From (5), (7) and (9),

the aggregate production function for $C_t^{n,n} = Z_t^n L_{x,t}^{n,n}$. Similarly, from (6), (8) and (9), the aggregate production function for $C_t^{n,s} = (1-\tau)Z_t^s L_{x,t}^{n,s}$. For country n , the value of export is $P_t^{s,n} C_t^{s,n}$ while the value of import is $P_t^{n,s} C_t^{n,s}$. The balanced-trade condition is

$$(A5) \quad P_t^{s,n} C_t^{s,n} = P_t^{n,s} C_t^{n,s} \Leftrightarrow L_{x,t}^{s,n} = \left(\frac{\mu^n}{\mu^s \omega_t^n}\right) L_{x,t}^{n,s},$$

where $\omega_t^n \equiv W_t^n / W_t^s$ denotes the relative wage rate. The conditional demand functions in country n for domestic and foreign final goods are $P_t^{n,n} C_t^{n,n} = (1-\alpha)P_t^n C_t^n$ and $P_t^{n,s} C_t^{n,s} = \alpha P_t^n C_t^n$. Combining these two conditions yield

$$(A6) \quad \frac{P_t^{n,n} C_t^{n,n}}{(1-\alpha)} = \frac{P_t^{n,s} C_t^{n,s}}{\alpha} \Leftrightarrow L_{x,t}^{n,s} = \left(\frac{\alpha}{1-\alpha}\right) \omega_t^n L_{x,t}^{n,n}.$$

Substituting (A6) into (A5) yields

$$(A7) \quad L_{x,t}^{s,n} = \left(\frac{\mu^n}{\mu^s}\right) \left(\frac{\alpha}{1-\alpha}\right) L_{x,t}^{n,n}.$$

Substituting $E_t^n = P_t^{n,n} C_t^{n,n} / (1 - \alpha) = \mu^n L_{x,t}^{n,n} / (1 - \alpha)$ into (A4) yields (16). Then, substituting (16) into (A7) yields (17). Finally, substituting (16) and (17) into the labor-market clearing condition yields (18). A similar exercise yields the unique, stable and stationary equilibrium allocation of labor in country s . ■

Proof of Lemma 2: The arrival rate of inventions in country n is

$$(A8) \quad \lambda_t^n = \varphi^n L_{r,t}^n.$$

The growth rate of $Z_t^n = \exp\left(\int_0^1 N_t^n(i) di \ln z\right) = \exp\left(\int_0^t \lambda_\tau^n d\tau \ln z\right)$ is given by

$$(A9) \quad \frac{\dot{Z}_t^n}{Z_t^n} = \lambda_t^n \ln z.$$

The balanced-growth rate of consumption in country n is

$$(A10) \quad \frac{\dot{C}_t^n}{C_t^n} = (1 - \alpha) \frac{\dot{Z}_t^n}{Z_t^n} + \alpha \frac{\dot{Z}_t^s}{Z_t^s} = [(1 - \alpha)\lambda_t^n + \alpha \lambda_t^s] \ln z.$$

Finally, aggregating (2) over $h \in [0,1]$ yields the level of consumption in country n given by

$$(A11) \quad C_t^n = \frac{W_t^n + R_t^n V_t^n}{P_t^n} = \left(1 + \frac{\rho}{\varphi^n}\right) \frac{W_t^n}{P_t^n}$$

because $\dot{V}_t^n = 0$, $R_t^n = \rho$ and $V_t^n \varphi^n = W_t^n$. ■

Proof of Lemma 4: The price index for consumption at time 0 is $P_0^n \equiv (P_0^{n,n})^{1-\alpha} (P_0^{n,s})^\alpha$, where

$P_0^{n,n} = \mu^n W_0^n / Z_0^n$ and $P_0^{n,s} = \mu^n W_0^s / [(1 - \tau) Z_0^s]$ from the proof of Lemma 1. The initial levels of

technology $Z_0^n \equiv \exp\left(\int_0^1 N_0^n(i) di \ln z\right)$ and $Z_0^s \equiv \exp\left(\int_0^1 N_0^s(i) di \ln z\right)$ are exogenous. After dropping

these exogenous terms, $\ln(W_0^n / P_0^n)$ simplifies to (31). ■

Proof of Proposition 2: After dropping the terms that are independent of patent protection, the welfare of any household h in country n is

$$(A12) \quad \Omega^n = -\ln \mu^n + \frac{g^n}{\rho}.$$

The arrival rates of inventions in the two countries are

$$(A13) \quad \lambda^n = \varphi^n - \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) (\varphi^n + \rho),$$

$$(A14) \quad \lambda^s = \varphi^s - \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) (\varphi^s + \rho).$$

Substituting (A13) and (A14) into (A10) yields

$$(A15) \quad g^n = \left((1-\alpha) \left(\varphi^n - \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) (\varphi^n + \rho) \right) + \alpha \left(\varphi^s - \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) (\varphi^s + \rho) \right) \right) \ln z.$$

Substituting (A15) into (A12) and then dropping the exogenous terms yield

$$(A16) \quad \Omega^n = -\ln \mu^n - \left((1-\alpha) \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) (\varphi^n + \rho) + \alpha \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) (\varphi^s + \rho) \right) \frac{\ln z}{\rho}.$$

Differentiating (A16) with respect to μ^n yields

$$(A17) \quad \frac{\partial \Omega^n}{\partial \mu^n} = -\frac{1}{\mu^n} + \left(\left(\frac{1-\alpha}{\mu^n} \right)^2 (\varphi^n + \rho) + \left(\frac{\alpha}{\mu^n} \right)^2 (\varphi^s + \rho) \right) \frac{\ln z}{\rho} = 0.$$

Solving (A17) yields (34), and (35) can be obtained by a similar derivation. ■

Proof of Proposition 3: Combining (A16) and the analogous condition for country s yields

$$(A18) \quad \Omega^n + \Omega^s = -\ln \mu^n - \ln \mu^s - \left(\left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) (\varphi^n + \rho) + \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) (\varphi^s + \rho) \right) \frac{\ln z}{\rho}.$$

Differentiating (A18) with respect to μ^n yields

$$(A19) \quad \frac{\partial(\Omega^n + \Omega^s)}{\partial \mu^n} = -\frac{1}{\mu^n} + \left(\left(\frac{1-\alpha}{(\mu^n)^2} \right) (\varphi^n + \rho) + \left(\frac{\alpha}{(\mu^n)^2} \right) (\varphi^s + \rho) \right) \frac{\ln z}{\rho} = 0.$$

Solving (A19) yields (36), and (37) can be obtained by a similar derivation. ■

Proof of Corollary 2: Subtracting (34) from (36) and differentiating $\mu_{GO}^n - \mu_{NE}^n$ with respect to α show that the sign of $\partial(\mu_{GO}^n - \mu_{NE}^n) / \partial \alpha$ is given by the sign of $(1 - 2\alpha) > 0$ for $\alpha < 0.5$. Similarly, from (35) and (37), differentiating $\mu_{GO}^s - \mu_{NE}^s$ with α shows that the sign of $\partial(\mu_{GO}^s - \mu_{NE}^s) / \partial \alpha$ is also given by $1 - 2\alpha$. ■

Proof of Proposition 5: As $\alpha \rightarrow 0$, $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s$ because the two countries are in autarky so that $\mu_{GO}^s < \mu_{TRIPS}^s$. As $\alpha \rightarrow 0.5$, $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s = \Omega_{NE}^n + \Omega_{NE}^s$ because the Nash equilibrium is the same as the policy regime under TRIPS such that $\mu_{NE}^s = \mu_{TRIPS}^s$. The rest of the proof shows that there must exist an intermediate range of α , in which $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s$. From (34) and (37), $\mu_{GO}^s - \mu_{TRIPS}^s$ is an increasing function in α . As $\alpha \rightarrow 0.5$, $\mu_{GO}^s > \mu_{TRIPS}^s$. Therefore, there must exist a threshold denoted by $\tilde{\alpha} \in (0, 0.5)$ above which $\mu_{NE}^s < \mu_{TRIPS}^s < \mu_{GO}^s$. When $\alpha \in [\tilde{\alpha}, 0.5)$, it is sufficient for $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s$ to hold, and there exists a lower critical value $\bar{\alpha} \in (0, \tilde{\alpha})$ above which $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s$ still holds. In this case, the South's level of patent protection moves from one suboptimal level to another suboptimal level (i.e. $\mu_{NE}^s < \mu_{GO}^s < \mu_{TRIPS}^s$). In summary, for low values of α , $\Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s$. As α increases above $\bar{\alpha}$, the reverse is true. ■

Proof of Proposition 6: For (a), (20) shows that g^n is increasing in φ^s while (29) shows that σ_y^n is increasing in g^n . For (b), we need to consider both the positive indirect effect of φ^s on σ_y^s through g^s and the negative direct effect of φ^s on σ_y^s through its denominator.

$$(A20) \quad \sigma_y^s = \left(\frac{\rho + g^s}{\rho + g^s + \varphi^s} \right) \sigma_v^s.$$

Differentiating (A20) with respect to φ^s yields

$$(A21) \quad \frac{\partial \sigma_y^s}{\partial \varphi^s} = \frac{1}{(\rho + g^s + \varphi^s)^2} \left(\varphi^s \frac{\partial g^s}{\partial \varphi^s} - (\rho + g^s) \right) \sigma_v^s.$$

Therefore, the sign of $\partial \sigma_y^s / \partial \varphi^s$ is determined by the sign of $\varphi^s \partial g^s / \partial \varphi^s - (\rho + g^s)$. Combining the analogous conditions of (18), (19) and (20) for the South yields

$$(A22) \quad g^s = \left(\begin{array}{l} (1-\alpha)\varphi^s \left(1 - \frac{1-\alpha}{\mu^s} - \frac{\alpha}{\mu^n} \right) + \alpha\varphi^n \left(1 - \frac{1-\alpha}{\mu^n} - \frac{\alpha}{\mu^s} \right) \\ - (1-\alpha) \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) \rho - \alpha \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) \rho \end{array} \right) \ln z.$$

Differentiating (A22) with respect to φ^s yields

$$(A23) \quad \frac{\partial g^s}{\partial \varphi^s} = (1-\alpha) \left(1 - \frac{1-\alpha}{\mu^s} - \frac{\alpha}{\mu^n} \right) \ln z.$$

From (A22) and (A23), $\varphi^s \partial g^s / \partial \varphi^s - (\rho + g^s) > 0$ is equivalent to

$$(A24) \quad \varphi^n < \bar{\varphi}^n \equiv \rho \left(\frac{\left((1-\alpha) \left(\frac{1-\alpha}{\mu^s} + \frac{\alpha}{\mu^n} \right) + \alpha \left(\frac{1-\alpha}{\mu^n} + \frac{\alpha}{\mu^s} \right) - \frac{1}{\ln z} \right)}{\alpha \left(1 - \frac{1-\alpha}{\mu^n} - \frac{\alpha}{\mu^s} \right)} \right). \blacksquare$$