跨時最適化的小型開放經濟模型之介紹—以通貨替代議題為例

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講題簡介

以通貨替代相關議題爲例，介紹跨時最適化的小型開放經濟模型之應用。

研習內容

單元一　雙元貨幣：要素需求理論 vs.資產選擇理論

單元二　通貨替代與匯率調整：小型開放經濟模型

單元三　跨時最適化的小型開放經濟模型的應用：實質匯率（貿易條件）動態

跨時最適化的小型開放經濟模型之介紹：通貨替代的應用

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緒論

通貨替代的概念(Miles 1978)

通貨替代在國際金融理論的應用

- 貨幣需求與貨幣自主性
  (Chen 1973, Miles 1978, Chen et al. 1981, Chen and Tsaur 1992)
- 匯率動態
  (Calvo and Rodriguez 1977, Liviatan 1981, Park 1987, Chen et al. 1989)

晚近的發展

- 最適通貨膨脹稅

  Vegh, 1989a, 1989b, 1989c, 1995

- 匯率內生性波動

  Karaken and Wallace, 1981; Barnett, 1992; King et al., 1992; Benhabib and Farmer, 1996

雙元通貨需求函數

Chen and Tsaur

[經濟論文 20:2 (Sept. 1992) pp.313-335]

x_1: *real balances of currency 1*

x_2: *real balances of currency 2*

jointly producing liquidity services, q

(1) $q = f(x_1, x_2); f_1, f_2 > 0; f_{11}, f_{22} < 0$

Optimization approach (utility function is separable)

(2) $\int_0^\infty [u(C) + f(x_1, x_2)] e^{-\delta t} dt$

Budget Constraint

(3) $\dot{x}_1 + \dot{x}_2 = Y + S - C - (g_1 x_1 + g_2 x_2)$

Y: *real income*; S: *transfer payments*

g_1, g_2: *inflation rates of* x_1 *and* x_2

Hamiltonian

(4) $\mathcal{H} = [u(C) + f(x_1, x_2)] + \lambda(Y + S - C - g_1 x_1 - g_2 x_2)$

λ: *shadow value of real money balances*

F.O.C

(5) $u_C = \lambda$

(6) $f_1 = \lambda(\delta + g_1) - \dot{\lambda}$

(7) $f_2 = \lambda(\delta + g_2) - \dot{\lambda}$

(8) $\dot{x}_1 + \dot{x}_2 = Y + S - C - (g_1 x_1 + g_2 x_2)$

In Steady State: $\dot{\lambda}=0,\ \dot{x}_1=\dot{x}_2=0$

$$S = g_1 x_1 + g_2 x_2$$

(9) $u_C = \lambda$

(10) $f_1 = \lambda(\delta + g_1)$

(11) $f_2 = \lambda(\delta + g_2)$

(12) $C = Y$

given Y, $\Rightarrow$ C is fixed $\Rightarrow$ U_C and λ become given constants

$$p=\frac{1}{u_C}=\frac{1}{\lambda},\ p_1=\delta+g_1,\ p_2=\delta+g_2$$

(13) $f_1(X_1,X_2)=\lambda p_1$

(14) $f_2(X_1,X_2)=\lambda p_2$

(15) $\frac{1}{\lambda}=p$

$$(17) \quad \frac{dx_1}{dp_1} = -\frac{x_1}{p_1}\left[(1-\alpha)\sigma + \alpha\frac{\varepsilon_1^2}{\varepsilon}\right] = -\frac{x_1}{p_1}\theta_{11}$$

$$(18) \quad \frac{dx_1}{dp_2} = \frac{x_1}{p_2}\left[(1-\alpha)\left(\sigma - \frac{\varepsilon_1\varepsilon_2}{\varepsilon}\right)\right] = \frac{x_1}{p_2}\theta_{12}$$

$$(19) \quad \frac{dx_2}{dp_1} = \frac{x_2}{p_1}\alpha\left(\sigma - \frac{\varepsilon_1\varepsilon_2}{\varepsilon}\right) = \frac{x_2}{p_1}\theta_{21}$$

$$(20) \quad \frac{dx_2}{dp_2} = -\frac{x_2}{p_2}\left[\alpha\sigma + (1-\alpha)\frac{\varepsilon_2^2}{\varepsilon}\right] = -\frac{x_2}{p_2}\theta_{22}$$

$$\textit{where} \quad \alpha = \frac{f_1 x_1}{f_1 x_1 + f_2 x_2} = p_1 x_1 / e, \quad \sigma = \frac{\partial ln(x_2/x_1)}{\partial ln(p_1/p_2)}, \quad e = p_1 x_1 + p_2 x_2$$

$$\varepsilon = \frac{p}{e}\frac{\partial e}{\partial p} > 0, \quad \varepsilon_i = \frac{p}{x_i}\frac{\partial x_i}{\partial p} = (\frac{e}{x_i}\frac{\partial x_i}{\partial e})(\frac{p}{e}\frac{\partial e}{\partial p}) = \eta_{ie}\varepsilon \quad i = 1,2$$

通貨替代相關議題的分析

§ Hayek的「自由通貨制度(Free Currency System)」

$$(21) \qquad \theta_{11} \gtrless \varepsilon_1 \quad as \quad \theta_{12} \gtrless 0$$

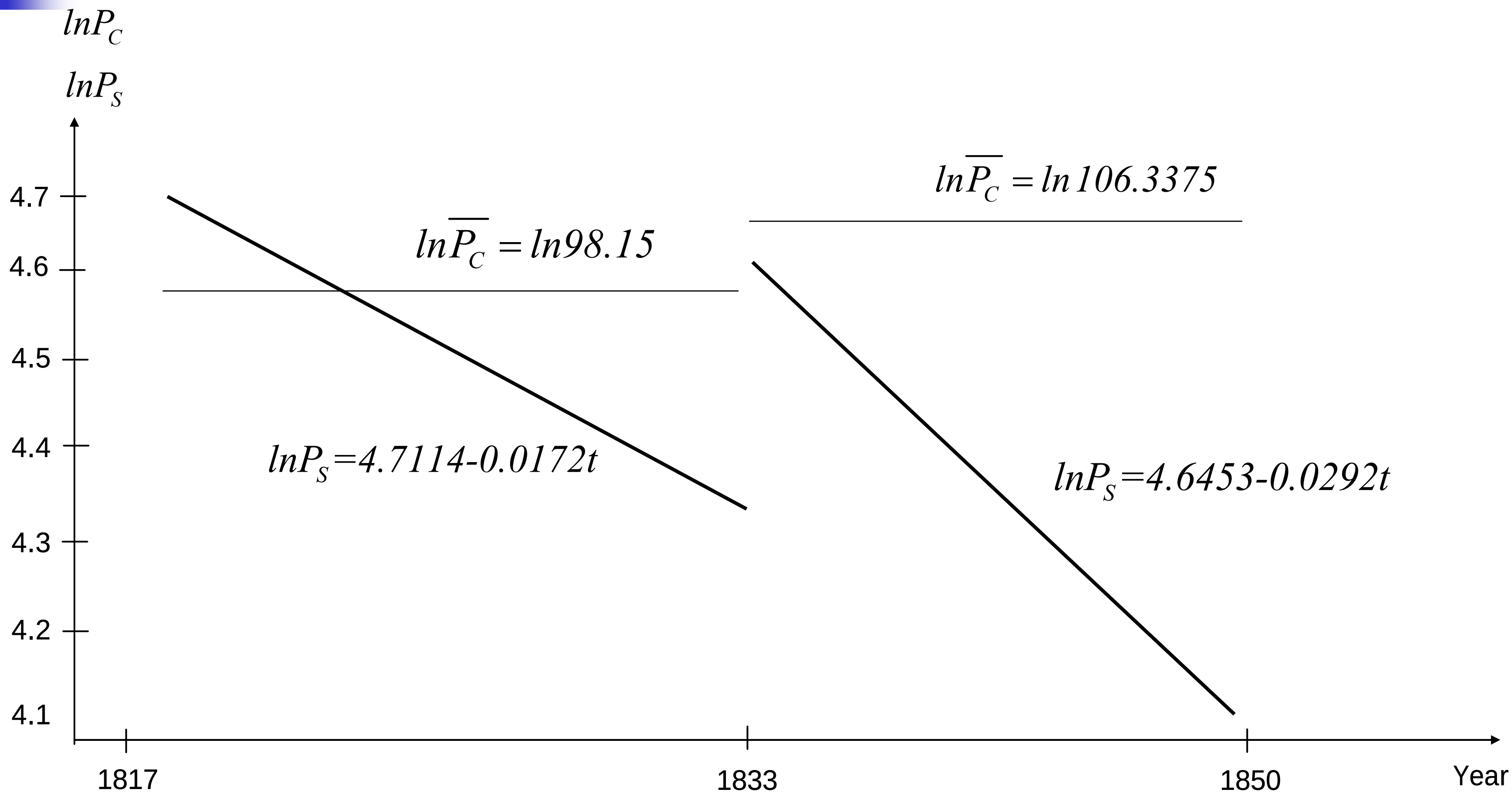
i.e., x_1 *and* x_2 *are substitutes or complements*

- Greshaw's Law

  Chen, Tsaur and Chou (1981)

- Ch'ing-dynasty monetary arrangement

  Chen, Chou and Tsaur (1977)



Base year (1820) : 100

通貨替代相關議題的分析(續)

§ Miles的「貨幣自主性(Monetary independence)」

- 固定匯率：供給面的通貨替代
- 浮動匯率：需求面的通貨替代

$$(17)\&(19) \Rightarrow$$

$$(22a) \qquad \frac{dx_2}{dp_1}\frac{dp_1}{dx_1} = -\frac{x_2}{x_1}\frac{\theta_{21}}{\theta_{11}} \overset{\leq}{\underset{>}{}} 0 \quad as \quad \theta_{21} \overset{\geq}{\underset{<}{}} 0$$

緊縮性的貨幣政策未必引起抵消性外幣內流

$$(22b)\qquad \frac{d(x_1+x_2)}{dx_1} = 1 - \frac{x_2}{x_1}\frac{\theta_{21}}{\theta_{11}} \begin{matrix}\le \\ \\ >\end{matrix} 1 \quad \underline{as} \quad \theta_{21} \begin{matrix}\ge \\ \\ <\end{matrix} 0$$

若設原先 $p_1 = p_2 \quad \Rightarrow \quad x_2 / x_1 = (1-\alpha)/\alpha$

$$(22c)\qquad \frac{d(x_1+x_2)}{dx_1} = \frac{\varepsilon_1}{\theta_{11}} = \frac{1}{\alpha\eta_{1e} + (1-\alpha)(\sigma/\varepsilon_1)}$$

即貨幣自主性的程度取決於$\alpha, \eta_{1e}, \sigma, \varepsilon_1$等參數，
而非僅看σ的絕對值而已。 *(Chen 1973; Miles 1978)*

通貨替代相關議題的分析(續)

§ Liviatan的補充性通貨「(“Cooperant Currencies)」

$$(23a) \qquad \theta_{21} > 0 \;\Rightarrow\; \frac{dx_2}{dp_1} > 0$$

$$(23b) \qquad 若\ \varepsilon = 0 \;\Rightarrow\; \sigma - \varepsilon > 0 \qquad (C - R\ Model)$$

通貨替代相關議題的分析(續)

§ 發行大鈔與通貨膨脹

若$p_1 = p_2$, 利用(17)&(19) $\Rightarrow$

$$(24) \quad \frac{dx_1}{dp_1} + \frac{dx_2}{dp_1} = -\frac{x_1}{p_1}\varepsilon_1 \quad \text{無論} \frac{dx_2}{dp_1} = \frac{x_2}{p_1}\theta_{21} \gtrless 0$$

通貨替代相關議題的分析(續)

§ Mckinnon的「兩階段貨幣需求(Two-Stage Demand of Money)」

Let r be the world interest rate, p_1 *the dollar rate, and* p_2 *the rowa rate then*

(25) $$dr = \alpha dp_1 + (1-\alpha) dp_2 = 0$$

$$\text{if } p_1 = p_2 = r$$

(26) $$dx_1 + dx_2 = x_1(\varepsilon_2 - \varepsilon_1)\frac{dp_1}{p_1} = 0 \quad \Rightarrow \quad \varepsilon_1 = \varepsilon_2$$

i.e., the monetary service production function is homogeneous

通貨替代與匯率動態

壹、模型

- 模型假設
- 實物面與金融面的運作
- 模型之長、短期特性

•模型假設

一. 兩財貨—貿易財(tradable goods)與非貿易財(non-tradable goods)。

二. 非貿易財價格P_N由國內市場結清條件決定。

三. 貿易財的國外價格P_T^*由國際經濟外生決定，假定固定為1，則貿易財在國內價格為E。E為名目匯率，為每單位外幣的國幣價格。

四. 資產選擇內容為兩種資產：無利息的國內貨幣或國外貨幣。

五. 財富(持有的金融資產)價值以貿易財來衡量，則

$W=M+F$, 其中$M=H/E$(H為國內貨幣供給，F代表本國大眾握有的國外貨幣)

六. 民眾具完全預期。

•實物面

$Q\equiv$相對價格=貿易財對非貿易財的價格比，即

$$Q \equiv \frac{E\,P_T^*}{P_N} = \frac{E}{P_N} \approx \text{實質匯率}$$

現代消費理論與貨幣理論：

- 商品與貨幣需求的決定並非取決於當期所得，而是以恆常所得來決定；而財富(W)是比較近似恆常所得概念衡量的變數。
- 因而兩財貨的消費，也是財富的函數：

$$(1.1)\quad C_T = C_T(Q,W),\quad C_{TQ} < 0,\quad C_{TW} > 0$$

$$(1.2)\quad C_N = C_N(Q,W),\quad C_{NQ} > 0,\quad C_{NW} > 0$$

在完全充分就業的假定下，兩財貨的生產：

$$(1.3)\quad X_T = X_T(Q),\quad X_{TQ} > 0$$

$$(1.4)\quad X_N = X_N(Q),\quad X_{NQ} < 0$$

非貿易財市場的均衡

$$C_N(Q,W) = X_N(Q)$$

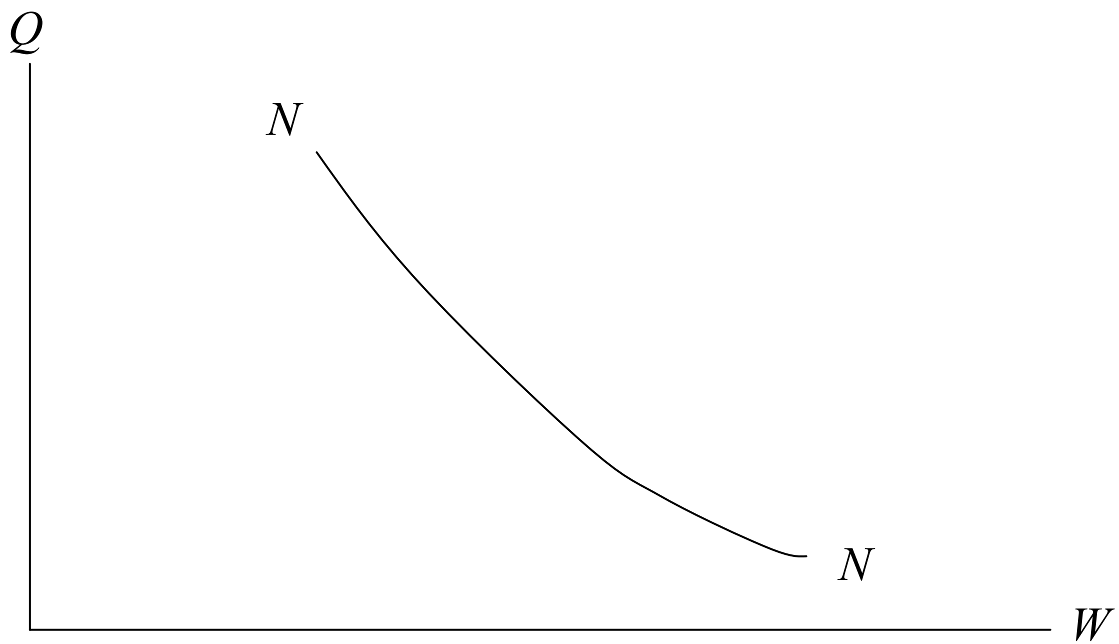
$$\quad (+)\ (+) \qquad\qquad (-)$$

$$Q\uparrow, \Rightarrow C_N\uparrow, X_N\downarrow(\text{非貿易財市場的超額需求})$$

$$\Rightarrow W\downarrow(for\quad equilibrium)$$

$$(1.5)\quad W = g(Q),\quad g' \equiv \frac{dW}{dQ} = \frac{X_{NQ} - C_{NQ}}{C_{NN}} < 0$$

$(1.5) \Rightarrow$ NN(非貿易財市場均衡條件)



圖一

貿易財市場的均衡

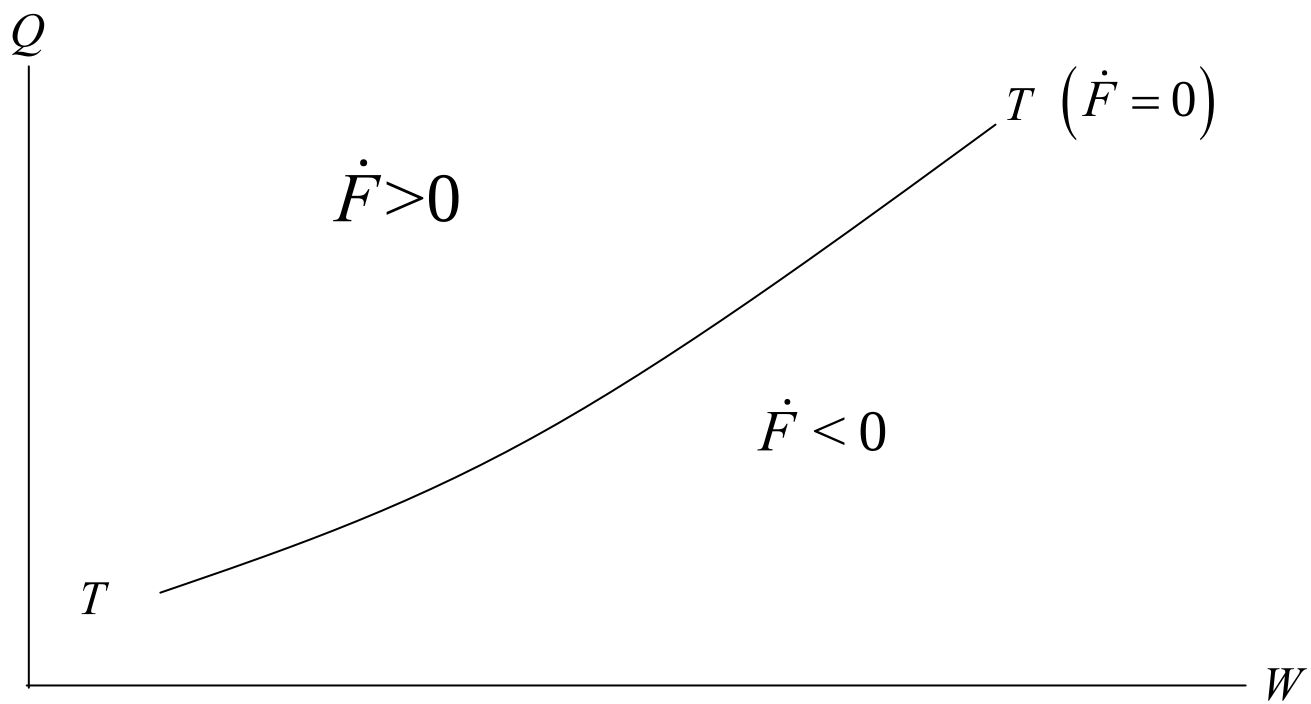
$$(1.6)\qquad \dot{F} = X_T(Q) - C_T(Q,W)$$

$$若\ \dot{F} = 0 \quad \Rightarrow \quad \underset{(+)}{X_T(Q)} = \underset{(-)\,(+)}{C_T(Q,W)}$$

$Q\uparrow \ \Rightarrow\ X_T\uparrow,\ C_T\downarrow$（貿易財市場的超額供給）

$$\Rightarrow W\uparrow\left(for\quad \dot{F} = 0\right)$$

$\dot{F}=0 \Rightarrow TT$(貿易財市場的均衡條件)

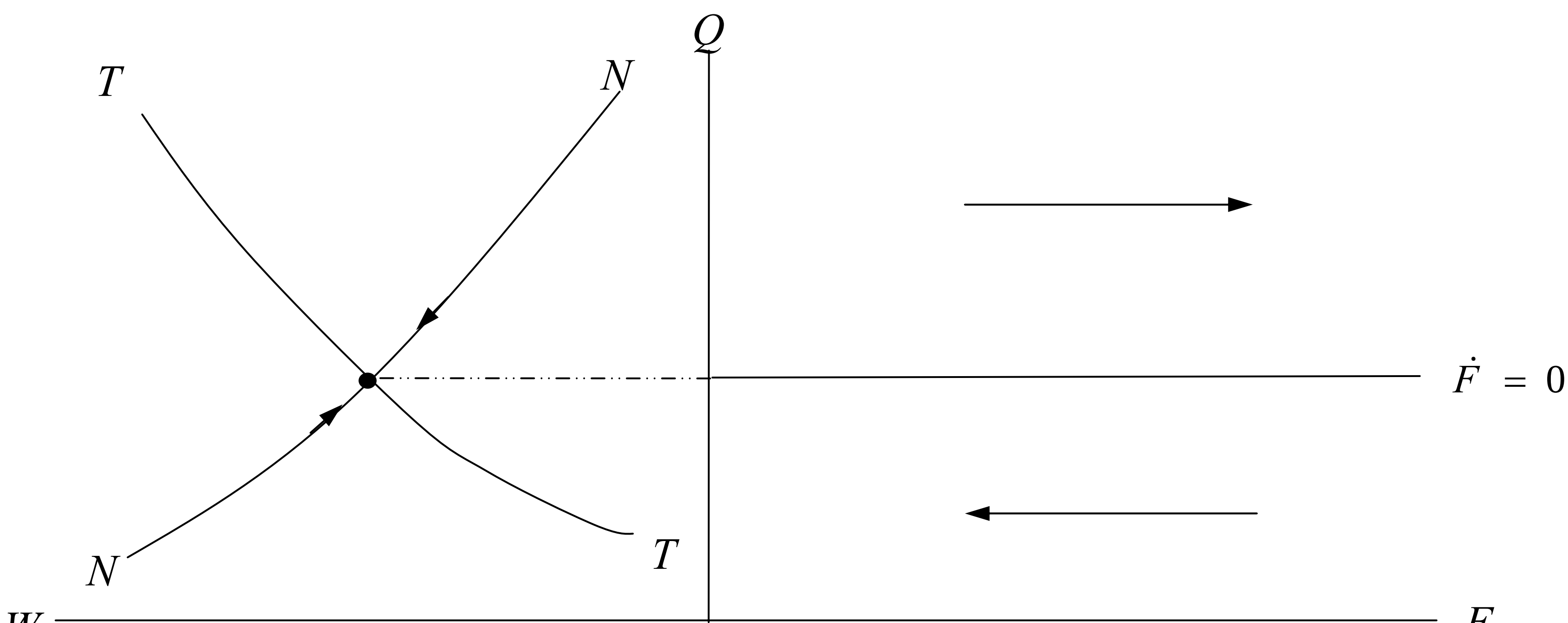


圖二

(1.5)代入(1.6)

$$(1.7)\quad \dot{F}=b(Q),\quad b' \equiv \frac{d\dot{F}}{dQ}=X_{TQ}-C_{TQ}-g'C_{TW}>0$$

$$(1.7)\quad \Rightarrow \dot{F}=0\quad Curve(Q \underset{<}{\overset{>}{}} \bar{Q} \Rightarrow \dot{F} \underset{<}{\overset{>}{}} 0)$$



圖三

•金融面

- 民眾保有兩種資產的比例取決於這兩種資產相對的機會成本：

$$(1.8)\quad \frac{M}{F} = L(\hat{E}),\quad L' < 0$$

式中"∧"代表變動率

式(1.8)的反函數(Inverse function)

$$(1.9)\quad \hat{E} = l\left(\tfrac{M}{F}\right),\quad l' < 0$$

$\because \quad M = \frac{H}{E} \Rightarrow \hat{E} = \mu - \hat{M} \quad (\mu \equiv \hat{H}$為貨幣增長率$)$

$(1.9') \quad \hat{M} = \mu - l(\frac{M}{F})$

式$(1.9')$的左邊$= \dfrac{\dot{M}}{M_0} = \dfrac{\dot{W} - \dot{F}}{M_0}$

$$\left.\begin{aligned} (1.5) \quad &\Rightarrow \quad \dot{W} = g' \cdot \dot{Q} \\ (1.7) \quad &\Rightarrow \quad \dot{F} = b(Q) \end{aligned}\right\}$$代入上式

$$(1.10) \quad \hat{M} = \frac{g'\dot{Q} - b(Q)}{M_0}$$

式$(1.9')$的右邊$=\mu\text{-}l(\frac{W\text{-}F}{F})$

$$=\mu\text{-}l(\frac{W}{F}\text{-}1)$$

$$=\mu\text{-}l\left[\frac{g(Q)}{F}-1\right]$$

式(1.9')可改寫成

$$(1.11)\qquad \frac{g'\dot{Q}-b(Q)}{M_0}=\mu\text{-}l\left[\frac{g(Q)}{F}-1\right]$$

依據式(1.11)，設定下列方程式

$$(1.11')\quad \phi \equiv \frac{g'\dot{Q}-b(Q)}{M_0}-\mu+l\left[\frac{g(Q)}{F}-1\right]$$

$$=\phi(\dot{Q},Q,F,\mu)=0$$

式中

$$\phi_1=\frac{\partial\phi}{\partial\dot{Q}}=\frac{g'}{M_0}<0$$

$$\phi_2=\frac{\partial\phi}{\partial Q}=-\frac{b'}{M_0}+l'\cdot\frac{g'}{F}\gtrless 0$$

$$\phi_3=\frac{\partial\phi}{\partial F}=l'\left(\frac{-W_0}{F^2}\right)>0$$

$$\phi_4=\frac{\partial\phi}{\partial\mu}=-1$$

式(1.11')經線性化後，可推導出下列的動態方程式：

$$\dot{Q}=\varphi(Q,F;\mu)$$

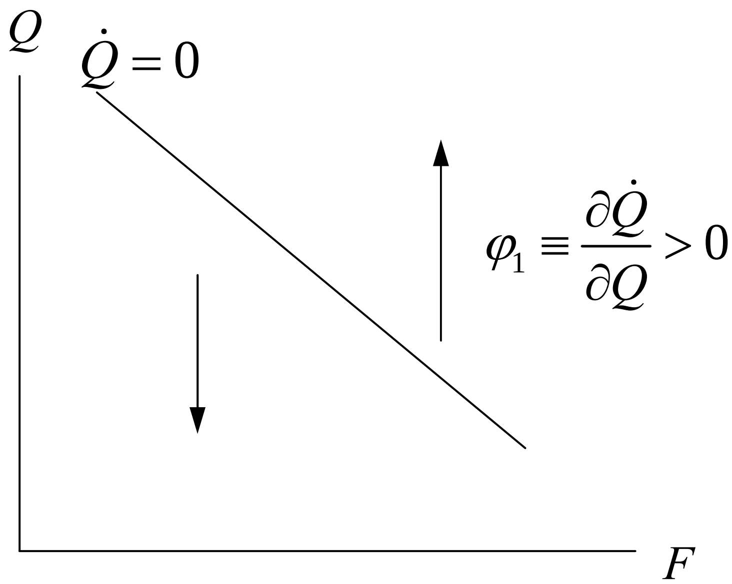
式中

$$\varphi_1 \equiv \frac{\partial \dot{Q}}{\partial Q}=-\frac{\phi_2}{\phi_1} \gtrless 0 \quad as \quad \phi_2 \gtrless 0$$

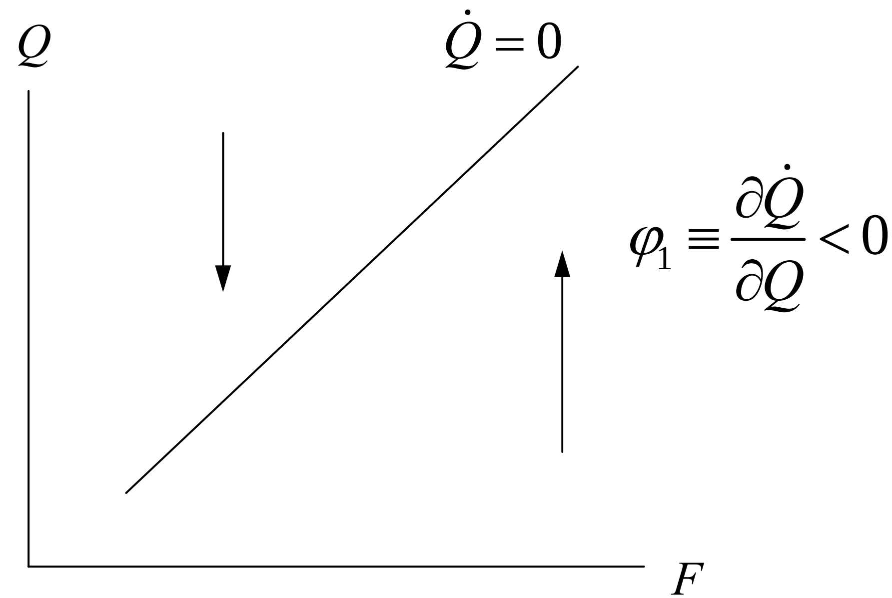
$$\varphi_2 \equiv \frac{\partial \dot{Q}}{\partial F}=-\frac{\phi_3}{\phi_1} > 0$$

$$\varphi_3 \equiv \frac{\partial \dot{Q}}{\partial \mu}=\frac{1}{\phi_1} < 0$$

$$\left.\frac{dQ}{dF}\right|_{\dot{Q}=0} = -\frac{\varphi_2}{\varphi_1} \underset{<}{\overset{>}{}} 0 \quad as \quad \varphi_1 \underset{>}{\overset{<}{}} 0$$

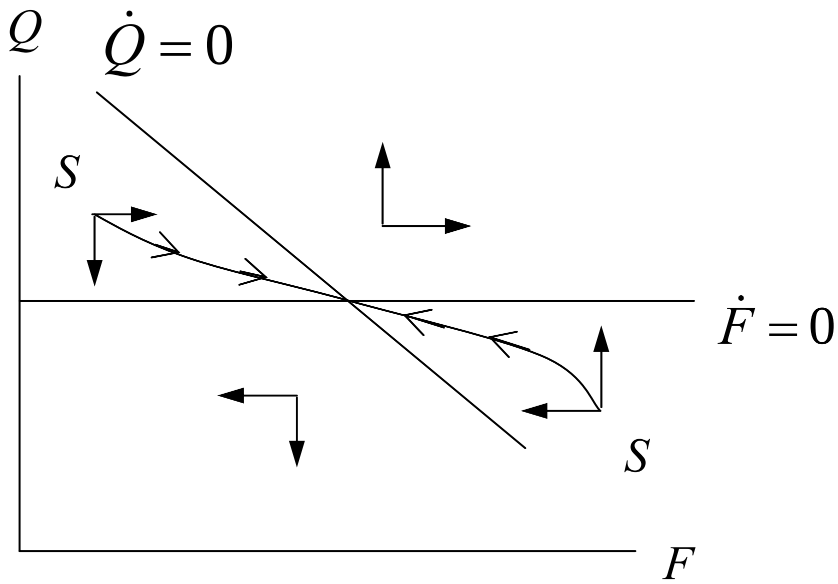


圖四 A

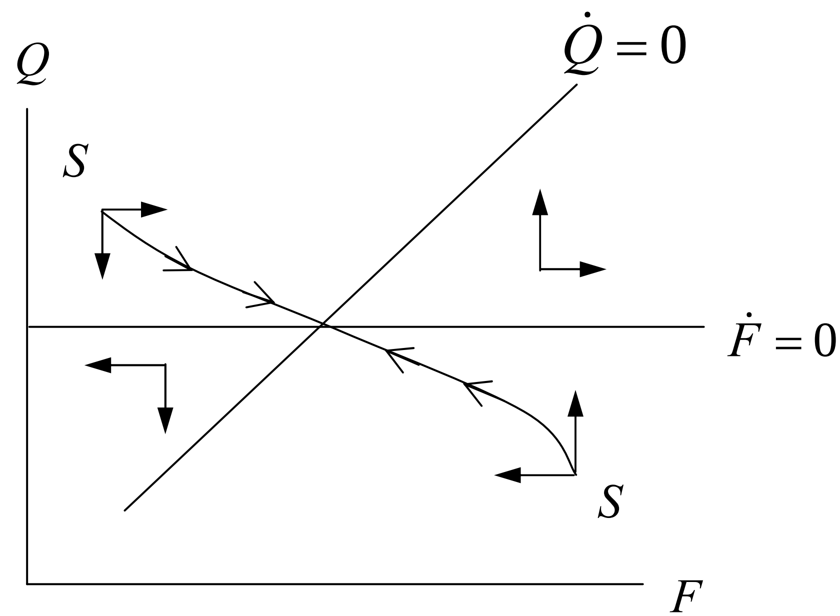


圖四 B

saddle-path equilibrium

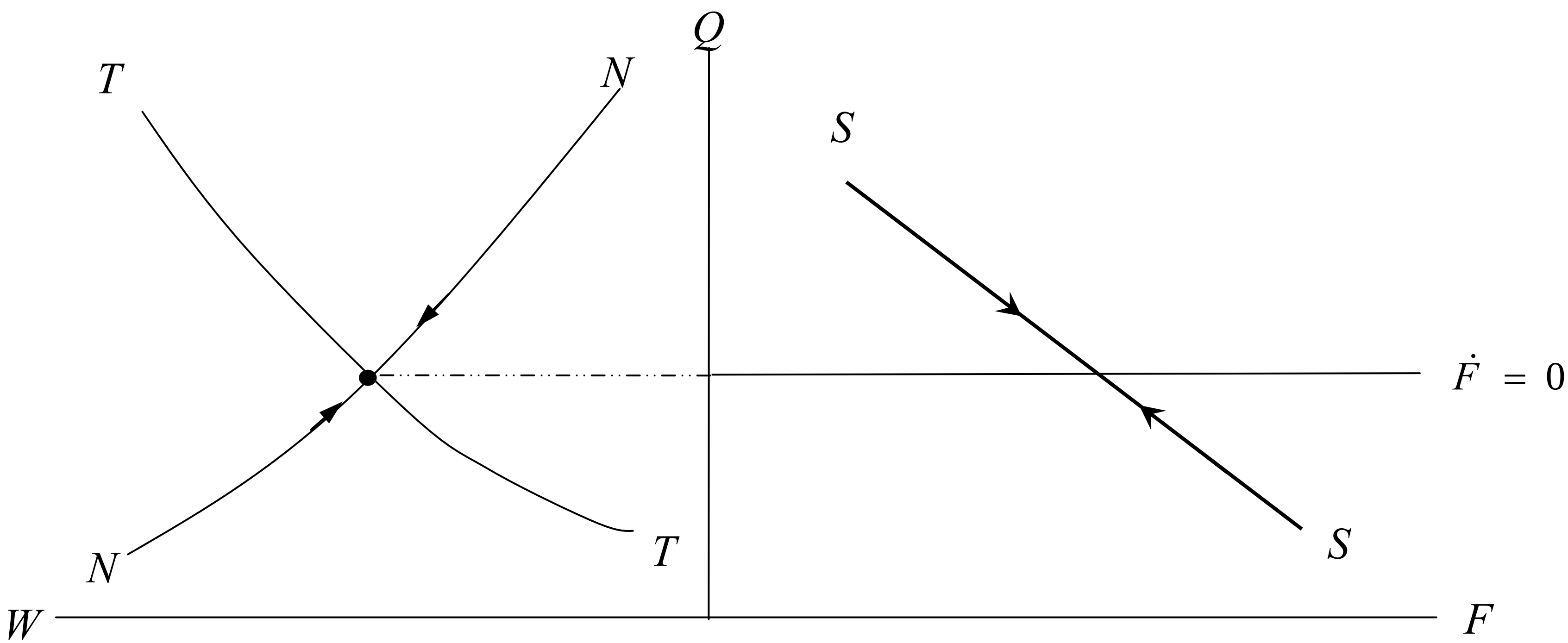


圖五　A



圖五　B

結合圖三



圖六

貨幣政策的效果

1. 貨幣量增加 $(H\uparrow)$

steady-state equilibrium：

Q, F, W, M 皆不變。

$$\because M = \frac{H}{E} \Rightarrow E\text{與}H\text{呈同幅度增加}(neutrality\ \ of\ \ money)$$

貨幣政策的效果(續)

2. 貨幣增長率提高 $(\mu\uparrow)$

steady-state equilibrium：*(super-neutrality of money)*

$$\mu\uparrow\Rightarrow\hat{E}\uparrow\Rightarrow\bar{M}\downarrow(\text{但因}\bar{Q},\ \bar{W}\text{皆固定不變}),\ \Rightarrow\bar{F}\uparrow$$

$$impact(F(0+)=\bar{F}_0):$$

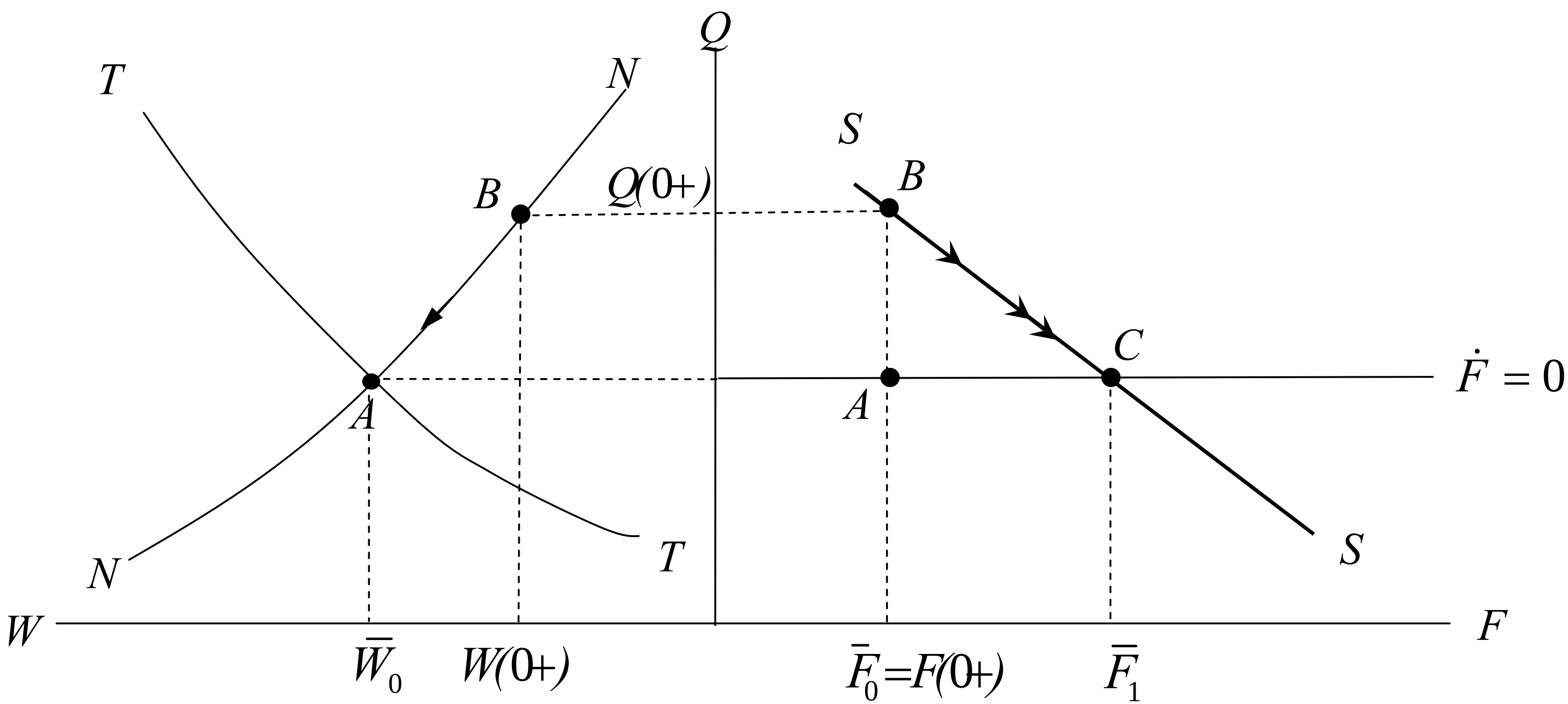
$$Q(0+)\uparrow(Overshoot)\Rightarrow W(0+)\downarrow\Rightarrow M(0+)\downarrow$$

$$dynamics:$$

$$F\uparrow,\ Q\downarrow\ \ (SS:B\rightarrow C)$$

$$Q\downarrow,\ W\uparrow\ \ (NN:B\rightarrow A)$$

短期效果及動態調整



圖七

貳、主要結果

1. *money neutrality:*

$$\bar{Q},\ \bar{W},\ \bar{C}_T,\ \bar{C}_N$$

2. *real-exchange-rate depreciation (overshooting):*

$$sign\frac{dQ(0+)}{d\mu}=sign\frac{d\dot{F}(0+)}{d\mu}=sign(\frac{d\bar{F}}{d\mu})$$

3. *currency substitution:*

$$given\ \ \bar{W}\Rightarrow\bar{M}\downarrow,\ \ \bar{F}\uparrow$$

- 參、檢討

1. *no foreign inflation*

2. *no microfoundation (ad hoc equations)*

Liviatan's Model：
The Utility Maximization Approach

壹、模型

特色

- 貨幣在效用函數
- 商品效用函數與貨幣效用函數是可分的

$$(2.1) \qquad \max \int_0^{\infty} e^{-\delta t}\left[U(C_N, C_T) + V(M, F)\right] dt$$

式中*U*和*V*為*strictly concave function*, 而 $\delta > 0$

實質所得(以貿易財衡量)：

$$(2.2) \qquad X = X_T + (1/Q)X_N, \quad Q \equiv \frac{E}{P_N}$$

預算限制式(*Budget Constraint*) ：

$$(2.3) \qquad \frac{\dot{H}}{E} + \dot{F} = X + S - \left[C_T + (1/Q)C_N\right]$$

式中*S*代表政府的移轉性支付(*transfer payment*) ，其與*X*之和為民眾之可支配所得。

依前述定義 $M \equiv \dfrac{H}{E} \Rightarrow \dfrac{\dot{H}}{E} = \dot{M} + M\hat{E}$

利用 $W = M + F$，式(2.3)可改寫成

$$(2.3') \qquad \dot{W} = X + S - \hat{E}M - \left[C_T + (1/Q)C_N\right]$$

The Hamiltonian for the optimization problem:

$$(2.4)\quad \mathcal{H}=U(C_N,C_T)+V(M,W\text{-}M)+\lambda\{X+S\text{-}\hat{E}M\text{-}[C_T+(1/Q)C_N]\}$$

FOC:

$$(2.5a)\quad \frac{\partial\mathcal{H}}{\partial C_N}=U_N\text{-}\lambda\left(\frac{1}{Q}\right)=0$$

$$(2.5b)\quad \frac{\partial\mathcal{H}}{\partial C_T}=U_T\text{-}\lambda=0$$

$$(2.5c)\quad \frac{\partial\mathcal{H}}{\partial M}=V_M\text{-}V_F-\lambda\hat{E}=0$$

$$(2.5d)\quad \frac{\partial\mathcal{H}}{\partial W}=V_F=\text{-}\dot{\lambda}+\delta\lambda$$

$$(2.5e)\quad \frac{\partial\mathcal{H}}{\partial\lambda}=X+S\text{-}\hat{E}M\text{-}[C_T+(1/Q)C_N]=\dot{W}$$

由(2.5a)和(2.5b)得

$$(2.6)\quad \frac{U_T}{U_N} = Q$$

由(2.5b)和(2.5c)得

$$(2.7)\quad \frac{V_M - V_F}{U_T} = \hat{E}$$

由(2.5b)和(2.5d)得

$$(2.8)\quad \frac{V_F}{U_T} - \delta = -\frac{\dot{U}_T}{U_T}$$

式(2.6)的左邊 $= \dfrac{U_T(C_N, C_T)}{U_N(C_N, C_T)}$

但非貿易財均衡條件：$C_N = X_N(Q)$，

式(2.6)可改寫成

$$(2.9) \quad \frac{U_T(X_N(Q), C_T)}{U_N(X_N(Q), C_T)} = Q$$

此式可解出 $C_T = C_T(Q)$，因此可得

$$(2.10) \quad U_T(C_N, C_T) = U_T(Q)$$

將(2.10)代入(2.8)，式(2.8)即為

$$(2.11) \quad G(\dot{Q}, Q, M, F) = 0$$

此式經線性化後可改寫成

$$(2.12) \quad \dot{Q} = \theta(Q, M, F)$$

依定義 $M \equiv \frac{H}{E} \Rightarrow \hat{E}=\mu-\hat{M}$ 代入(2.7)

$$(2.7') \quad \frac{V_M(M,F)-V_F(M,F)}{U_T(Q)}=\mu-\frac{\dot{M}}{M}$$

上式即為

$$J(\dot{M},M,F,Q,\mu)=0$$

上 式 線 型 化 後 可 以 改 寫 成

$$(2.13) \quad \dot{M}=\gamma(Q,M,F;\mu)$$

依$W=M+F$ ，式(2.5e)可改寫成$\frac{H}{E}$

$$(2.14)\quad \dot{M}+\dot{F}=[X_T+(1/Q)X_N]+S\text{-}\hat{E}M\text{-}[C_T+(1/Q)C_N]$$

利用非貿易財均衡條件：$X_N=C_N$，和

政府財政收支平衡條件：$S=\mu M$

(即政府移轉性支出等於鑄幣稅)

式(2.14)可簡化成下列的形式：

$$(2.15)\quad \dot{F}=X_T(Q)\text{-}C_T(Q)=B(Q),\ B'>0$$

綜結(2,12), (2.13), 與(2.15)三式,

可得動態體系如下：

$$\begin{cases} (2.12) & \dot{Q}=\theta(Q,M,F) \\ (2.13) & \dot{M}=\gamma(Q,M,F;\mu) \\ (2.15) & \dot{F}=B(Q) \end{cases}$$

線型化後而得

$$(2.16) \quad \begin{bmatrix} \dot{Q} \\ \dot{M} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} \theta_Q & \theta_M & \theta_F \\ \gamma_Q & \gamma_M & \gamma_F \\ B' & 0 & 0 \end{bmatrix} \begin{bmatrix} Q\text{-}\bar{Q} \\ M\text{-}\bar{M} \\ F\text{-}\bar{F} \end{bmatrix}$$

- 動態體系(2.16)有兩個跳躍變數(Q和M)，和一個緩慢調整變數(F)，而該體系的特性方程式有一負根(λ_1)和兩正根(λ_2, λ_3)，此即意味在此LRPF (long-run perfect-foresight)模型中，"Saddle path"是其確定的唯一解。該體系內生變數(例如F)解的形式如下：

$$F(t)=\bar{F}+b_1e^{\lambda_1 t}+b_2e^{\lambda_2 t}+b_3e^{\lambda_3 t} \tag{2.17}$$

- Saddle path的收斂條件要求正根的係數須為零：$b_2 = b_3 = 0$，意即在Saddle path上：

$$F(t)=\bar{F}+b_1e^{\lambda_1 t} \tag{2.18}$$

$\because \lambda_1<0$　　$F(t)$呈單調收斂至均衡值($\bar{F}$)

從(2.16)第三式可知：Steady-state時，$\dot{F}=0 \Rightarrow Q=\overline{Q}$ 。

因此長期實質匯率固定不變。此外，當$Q \underset{<}{>} \overline{Q}$時，$\dot{F} \underset{<}{>} 0$

由此可以推論：

$$sign\frac{dQ(0+)}{d\mu}=sign\frac{d\dot{F}(0+)}{d\mu}=sign(\frac{d\overline{F}}{d\mu})$$

準此，了解steady-state的F的變化，

即可推知實質匯率(Q)的動態行為。

在steady-state：$Q=\bar{Q}$，$\hat{E}=\mu$，$\dot{U}_T=0$ ($\because \dot{Q}=0$)

$$(2.7) \quad \Rightarrow \frac{V_M(M,F)-V_F(M,F)}{\bar{U}_T}=\mu$$

$$(2.8) \quad \Rightarrow \frac{V_F(M,F)}{\bar{U}_T}=\delta$$

上列兩式比較靜態分析：

$$(2.17) \quad \begin{bmatrix} \frac{V_{MM}}{\bar{U}_T} & \frac{V_{MF}}{\bar{U}_T} \\ \frac{V_{MF}}{\bar{U}_T} & \frac{V_{FF}}{\bar{U}_T} \end{bmatrix} \begin{bmatrix} dM \\ dF \end{bmatrix} = \begin{bmatrix} d\mu \\ 0 \end{bmatrix}$$

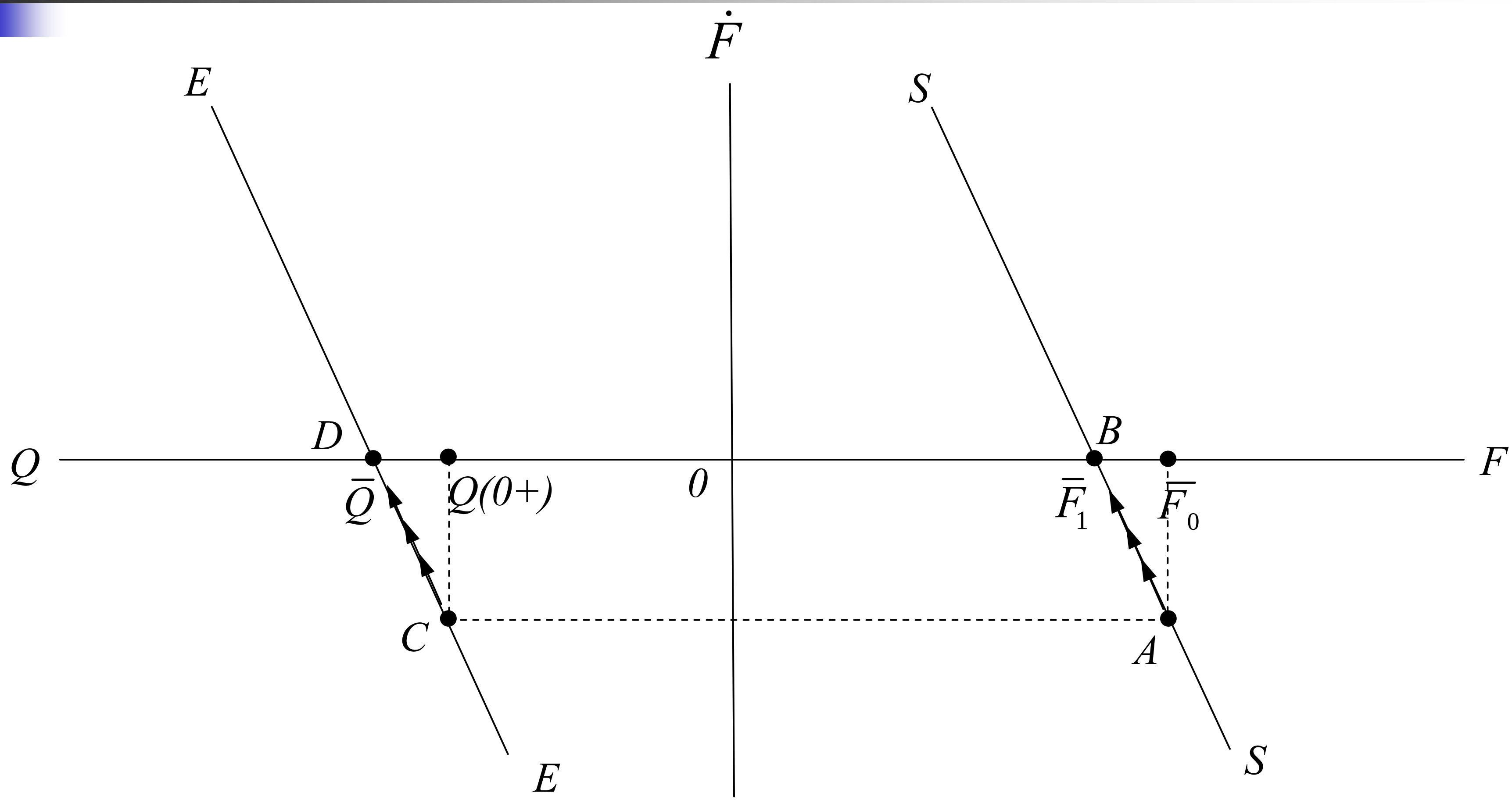
$$\frac{dM}{d\mu}=\frac{V_{FF}\bar{U}_T}{\Delta}<0 \qquad (\because V_{FF}<0)$$

$$\frac{dF}{d\mu}=\frac{-V_{MF}\bar{U}_T}{\Delta}\ \substack{<\\ >}\ 0 \qquad as\, V_{MF}\ \substack{>\\ <}\ 0)$$

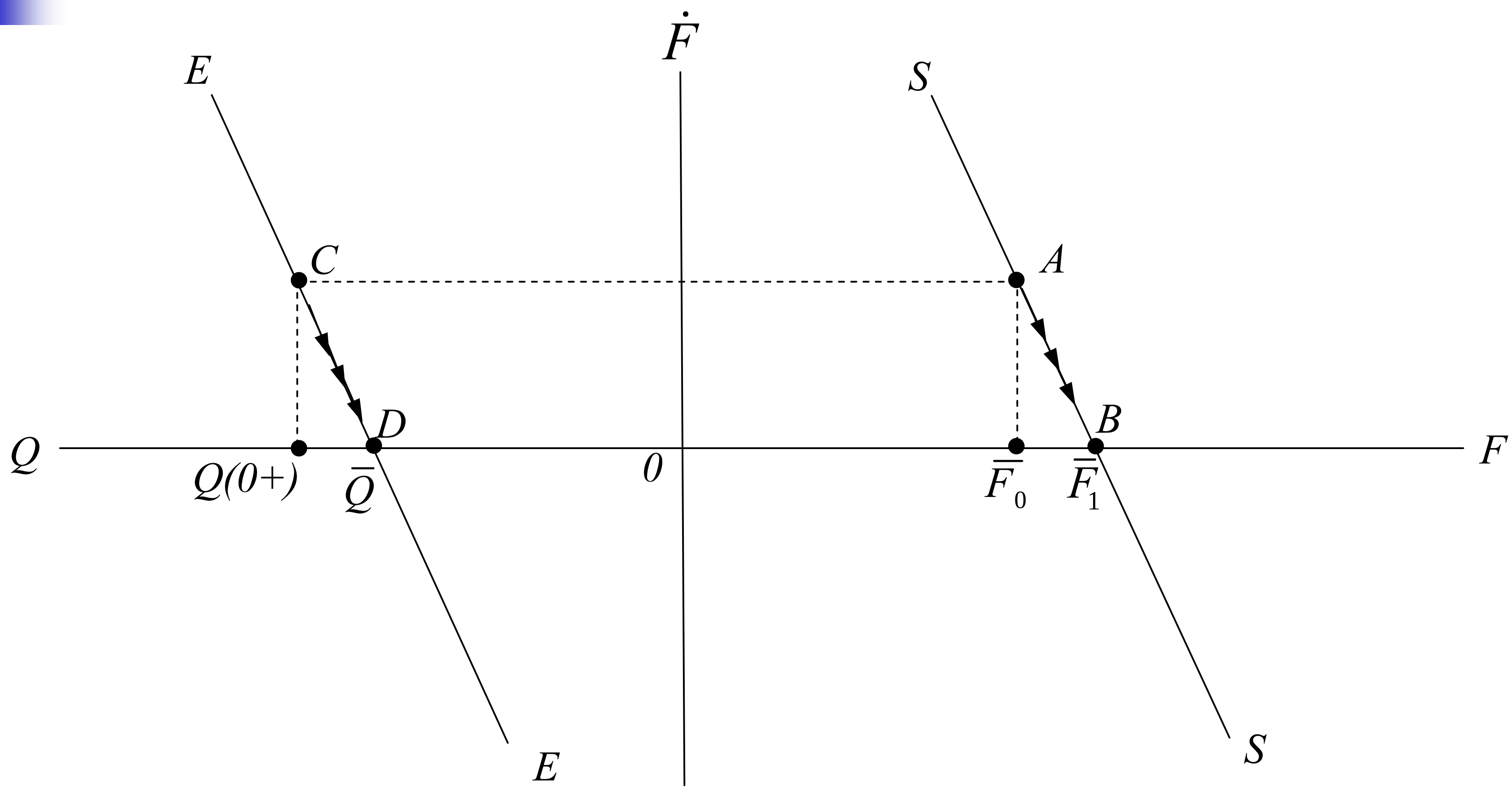
式中$\Delta=V_{MM}V_{FF}-V_{MF}^2>0$ [設$V(\bullet)$為concavity]

"cooperancy" for M&F，$V_{MF}>0$，$\frac{dF}{d\mu}<0 \Rightarrow Q$ real appreciation。

"substitution" for M&F，$V_{MF}<0$，$\frac{dF}{d\mu}>0 \Rightarrow Q$ real depreciation。



圖八 A $V_{MF}>0$，$\frac{d\bar{F}}{d\mu}<0$，*Q real appreciation*



圖八 B $V_{MF}<0$，$\dfrac{d\bar{F}}{d\mu}>0$，Q *real depreciation*

Calvo (*JIMF*, 1985)

模型特色：

商品效用函數與貨幣函數不可分割

$$(3.1) \quad \max \int_0^{\infty} e^{-\delta t} V\left[u\left(C_N, C_T\right) + l\left(M, F\right)\right] dt, \quad \delta > 0$$

$$(3.2) \quad V_u > 0, \qquad V_l > 0$$

$$u_N > 0, \qquad u_T > 0$$

$$l_m > 0, \qquad l_F > 0$$

Hamiltonian:

$$(3.3)\quad \mathcal{H}=V[u(C_N,C_T)+l(M,W\text{-}M)]+\lambda\{X+S\text{-}\hat{E}M\text{-}[C_T+(1/Q)C_N]\}$$

FOC:

$$(3.4a)\quad \frac{\partial\mathcal{H}}{\partial C_N}=V_u u_N-\lambda\left(\frac{1}{Q}\right)=0$$

$$(3.4b)\quad \frac{\partial\mathcal{H}}{\partial C_T}=V_u u_T-\lambda=0$$

$$(3.4c)\quad \frac{\partial\mathcal{H}}{\partial M}=V_l(l_M-l_F)-\lambda\hat{E}=0$$

$$(3.4d)\quad \frac{\partial\mathcal{H}}{\partial W}=V_l l_F=-\dot{\lambda}+\delta\lambda$$

$$(3.4e)\quad \frac{\partial\mathcal{H}}{\partial\lambda}=X+S\text{-}\hat{E}M\text{-}[C_T+(1/Q)C_N]=\dot{W}$$

$$X = X_T + (1/Q)X_N$$

$$X_T = X_T(Q), \quad X_T' > 0$$

$$X_N = X_N(Q), \quad X_N' < 0$$

由(3.4a)和(3.4b)得

$$(3.5) \qquad \frac{u_T}{u_N}=Q$$

由(3.4b)和(3.4c)得

$$(3.6) \qquad \frac{V_l(l_M-l_F)}{V_u u_T}=\hat{E}$$

由(3.4b)和(3.4d)得

$$(3.7) \qquad \frac{d(V_u u_T)}{dt}=-V_l l_F+\delta V_u u_T$$

非貿易財均衡條件：$C_N = X_N(Q)$

$$(3.5) \Rightarrow \quad \frac{u_T(X_N(Q), C_T)}{u_N(X_N(Q), C_T)} = Q$$

$$(3.8) \qquad C_T = C_T(Q)$$

$$(3.4e) \Rightarrow$$

$$(3.9) \qquad \dot{F} = X_T(Q) - C_T(Q),\ X'_T(Q) > C'_T(Q)$$

$$Steady\ state:\quad \dot{F}=0 \Rightarrow C_T(Q)=X_T(Q) \Rightarrow Q=\bar{Q}$$

$$(3.5)\ \&\ (3.9) \Rightarrow$$

$$(3.10)\qquad \frac{u_T(X_N(Q),X_T(Q))}{u_N(X_N(Q),X_T(Q))}=Q$$

$implies:\bar{Q}$獨立於貨幣變數

$$(3.11)\qquad \frac{d\bar{Q}}{d\mu}=0\qquad (money\ neutrality)$$

$$(3.12)\qquad \frac{d\bar{F}}{d\mu}\gtrless 0\qquad implies\qquad \frac{d\dot{F}(0+)}{d\mu}\gtrless 0\ and\ \frac{de(0+)}{d\mu}\gtrless 0$$

Steady-state equilibrium: $\mu = \bar{E},\ \dot{F} = 0,\ \frac{d(V_u u_T)}{dt} = 0$

$(3.6) \Rightarrow$

$$(3.13) \qquad \frac{l_M}{l_F} = 1 + \frac{\mu}{\delta}$$

$(3.7) \Rightarrow$

$$(3.14) \qquad \frac{V_l}{V_u} = \frac{\delta u_T}{l_F}$$

Linear Homogeneous (LH) Case：

假設$V(\bullet)$和$l(\bullet)$為LH

$$l_{MF} > 0 \quad (M\text{和}F\text{為}"cooperant")$$

定義：

$$\sigma_{ul} \equiv \frac{d\ln(u/l)}{d\ln(V_l/V_u)} = \text{消費和流動性勞動的替代彈性}$$

$$\sigma_{MF} \equiv \frac{d\ln(F/M)}{d\ln(l_M/l_F)} = \text{本國通貨和外國通貨的替代彈性}$$

Lemma 1

$V(\bullet)$和$l(\bullet)$皆為LH，則

$$\frac{d\overline{F}}{d\mu} \mathop{}_{<}^{>} 0 \quad \Leftrightarrow \quad \sigma_{ul} \mathop{}_{>}^{<} \sigma_{MF}$$

命題 1

若 $\sigma_{MF} > \sigma_{ul}$，則

$$\frac{d\bar{F}}{d\mu} > 0 \Rightarrow \frac{d\dot{F}(0+)}{d\mu} > 0 \ and \ \frac{dQ(0+)}{d\mu} > 0$$

(real depreciation)

引申 1

若$\sigma_{ul}=0$(即u與l呈固定比例)

在steady-state，u獨立於μ

一旦$\sigma_{ul}=0$，l亦獨立於μ

因此 $\frac{d\overline{M}}{d\mu}<0$ and $\frac{d\overline{F}}{d\mu}>0 \quad \Rightarrow \overline{l}(\overline{M},\overline{F})(\because l_M>0, l_F>0)$

從而 $\frac{dQ(0+)}{d\mu}>0$ (real depreciation)

引申 2

若$\sigma_{MF}=0$(即M與F呈固定比例)

$$\frac{d\bar{M}}{d\mu}<0 \Rightarrow \frac{d\bar{F}}{d\mu}<0(\because \sigma_{MF}=0)$$

$$\frac{d\bar{F}}{d\mu}<0 \Rightarrow \frac{d\dot{F}(0+)}{d\mu}<0 \text{ and } \frac{dQ(0+)}{d\mu}<0 \text{ (real appreciation)}$$

引申 3

強調Currency Substitution：$\sigma_{MF} > \sigma_{ul}$

$$\frac{d\bar{F}}{d\mu} > 0 \Rightarrow \frac{d\dot{F}(0+)}{d\mu} > 0 \text{ and } \frac{dQ(0+)}{d\mu} > 0 \text{ (real depreciation)}$$

考慮國外通貨膨脹(Engel, *JIMF* 1989)

$$\hat{P}_T^* \equiv \frac{\dot{P}_T^*}{P_T^*} = \mu^*$$，國外通貨膨脹率

$$q \equiv \frac{EP_T^*}{P_N}$$

$$w \equiv \frac{W}{P_T^*} = m + f$$

$$m \equiv \frac{H}{EP_T^*} = \frac{M}{P_T^*}$$

$$f \equiv \frac{F}{P_T^*}$$

(4.1) 跨時效用函數 $$\max \int_0^\infty e^{-\delta t} [U(C_N, C_T) + V(m, f)] dt$$

(4.2) 預算限制式 $$\frac{\dot{H}}{EP_T^*} + \frac{\dot{F}}{P_T^*} = [X_T + (1/q) X_N] + S - [C_T + (1/q) C_N]$$

依定義 $m \equiv \frac{H}{EP_T^*} \Rightarrow \frac{\dot{H}}{EP_T^*} = \dot{m} + m(\hat{E} + \mu^*)$

$$f \equiv \frac{F}{P_T^*} \Rightarrow \frac{\dot{F}}{P_T^*} = \dot{f} + f\mu^*$$

利用$w=m+f$，式(3.2)可改寫成

$$(4.3)\quad \dot{w} = [X_T + (1/q)X_N] + S - m\hat{E} - w\mu^* - [C_T + (1/q)C_N]$$

（假設X_T和X_N固定；即$X_T = \bar{X}_T$，$X_N = \bar{X}_N$）

Hamiltonian

$$(4.4)\quad \mathcal{H}=U(C_N,C_T)+V(m,w\text{-}m)+\lambda\{X+S\text{-}\hat{E}m\text{-}w\mu^{*}-[C_T+(1/q)C_N]\}$$

FOC:

$$(4.5a)\quad \frac{\partial\mathcal{H}}{\partial C_N}=U_N\text{-}\lambda\left(\frac{1}{q}\right)=0$$

$$(4.5b)\quad \frac{\partial\mathcal{H}}{\partial C_T}=U_T\text{-}\lambda=0$$

$$(4.5c)\quad \frac{\partial\mathcal{H}}{\partial m}=V_m\text{-}V_f-\lambda\hat{E}=0$$

$$(4.5d)\quad \frac{\partial\mathcal{H}}{\partial w}=V_f\text{-}\lambda\mu^{*}=\text{-}\dot{\lambda}+\delta\lambda$$

$$(4.5e)\quad \frac{\partial\mathcal{H}}{\partial\lambda}=[X_T+(1/q)X_N]+S\text{-}\hat{E}m\text{-}w\mu^{*}-[C_T+(1/q)C_N]=\dot{w}$$

基於商品產出固定及非貿易財市場均衡條件(即 $C_N = \overline{X}_N$)，可由(4.5a)和(4.5b)得

$$(4.6)\quad \frac{U_T(\overline{C}_N, C_T)}{U_N(\overline{C}_N, C_T)} = q$$

上式可求解 C_T 與 q 的關係如下

$$(4.6')\quad \frac{dq}{dC_T} = \frac{U_N U_{TT} - U_T U_{NT}}{U_N^2} < 0$$

由(4.5b)和(4.5c)得

$$(4.7)\quad \frac{V_m - V_f}{U_T(\overline{C}_N, C_T)} = \hat{E}$$

由(4.5b)和(4.5d)得

$$(4.8)\quad \frac{V_f}{U_T(\overline{C}_N, C_T)} - (\delta + \mu^*) = -\frac{\dot{U}_T(\overline{C}_N, C_T)}{U_T(\overline{C}_N, C_T)}$$

$$(4.6') \Rightarrow \quad C_T = C_T(q) \quad C_T' < 0$$

$$(4.7) \Rightarrow \quad \frac{V_m(m,f) - V_f(m,f)}{U_T(\bar{C}_N, C_T(q))} = \mu - \mu^* - \frac{\dot{m}}{m}$$

$$J(\dot{m}, m, f, q, \mu, \mu^*) = 0$$

$$\dot{m} = \gamma(m, f, q; \mu, \mu^*)$$

$$(4.8) \Rightarrow \quad \frac{V_f(m,f)}{U_T(\bar{C}_N, C_T(q))} - (\delta + \mu^*) = -\frac{\dot{U}_T(\bar{C}_N, C_T(q))}{U_T(\bar{C}_N, C_T(q))}$$

$$G(\dot{q}, q, m, f, \mu^*) = 0$$

$$\dot{q} = \theta(m, f, q; \mu^*)$$

利用非貿易財市場均衡條件：$\bar{X}_N = C_N$，

政府財政收支平衡條件：$S = \mu m$，以及$\dot{w} = \dot{m} + \dot{f}$

(4.3)簡化成

$$(4.9) \quad \dot{f} = \bar{X}_T - C(q) - f\mu^*$$

在*steady-state*：$\dot{m}=\dot{q}=\dot{f}=\dot{U}_T=0$，$\hat{E}=\mu\text{-}\mu^*$，$q=\bar{q}$，$m=\bar{m}$，$f=\bar{f}$

$$(4.7)\quad \Rightarrow \frac{V_m(\bar{m},\bar{f})-V_f(\bar{m},\bar{f})}{U_T(C_T(\bar{q}))}=\mu-\mu^*$$

$$(4.8)\quad \Rightarrow \frac{V_f(\bar{m},\bar{f})}{U_T(C_T(\bar{q}))}=\delta+\mu^*$$

$$(4.9)\quad \Rightarrow \bar{X}_T\text{-}C_T(\bar{q})=f\mu^*$$

steady state:比較靜態分析

$$(4.10)\qquad \begin{bmatrix} -V_m C'_T\left(\frac{U_{TT}}{U_T}\right) & V_{mm} & V_{mf} \\ -V_f C'_T\left(\frac{U_{TT}}{U_T}\right) & V_{mf} & V_{ff} \\ -C'_T & 0 & -\mu^* \end{bmatrix} \begin{bmatrix} d\bar{q} \\ d\bar{m} \\ d\bar{f} \end{bmatrix} = \begin{bmatrix} U_T d\mu \\ U_T d\mu^* \\ f d\mu^* \end{bmatrix}$$

解聯立方程式(4.10)，得

$$(4.11)\qquad \frac{d\bar{q}}{d\mu}=\frac{-\mu^{*}U_{T}V_{mf}}{\Delta}\gtrless 0 \quad as \quad V_{mf}\lessgtr 0$$

$$(4.12)\qquad \frac{d\bar{m}}{d\mu}=\frac{-C_{T}'U_{T}}{\Delta}\left\{\mu^{*}V_{f}\left(\frac{U_{TT}}{U_{T}}\right)+V_{ff}\right\}<0$$

$$(4.13)\qquad \frac{d\bar{f}}{d\mu}=\frac{C_{T}'U_{T}V_{mf}}{\Delta}\gtrless 0 \quad as \quad V_{mf}\lessgtr 0$$

式中 $\Delta=-C_{T}'[A-\mu^{*}B(\frac{U_{TT}}{U_{T}})]>0$，

$A\equiv V_{mm}V_{ff}-V_{mf}^{2}>0$，

$B\equiv V_{m}V_{ff}-V_{f}V_{mm}>0$

(4.11)和(4.13)的意涵

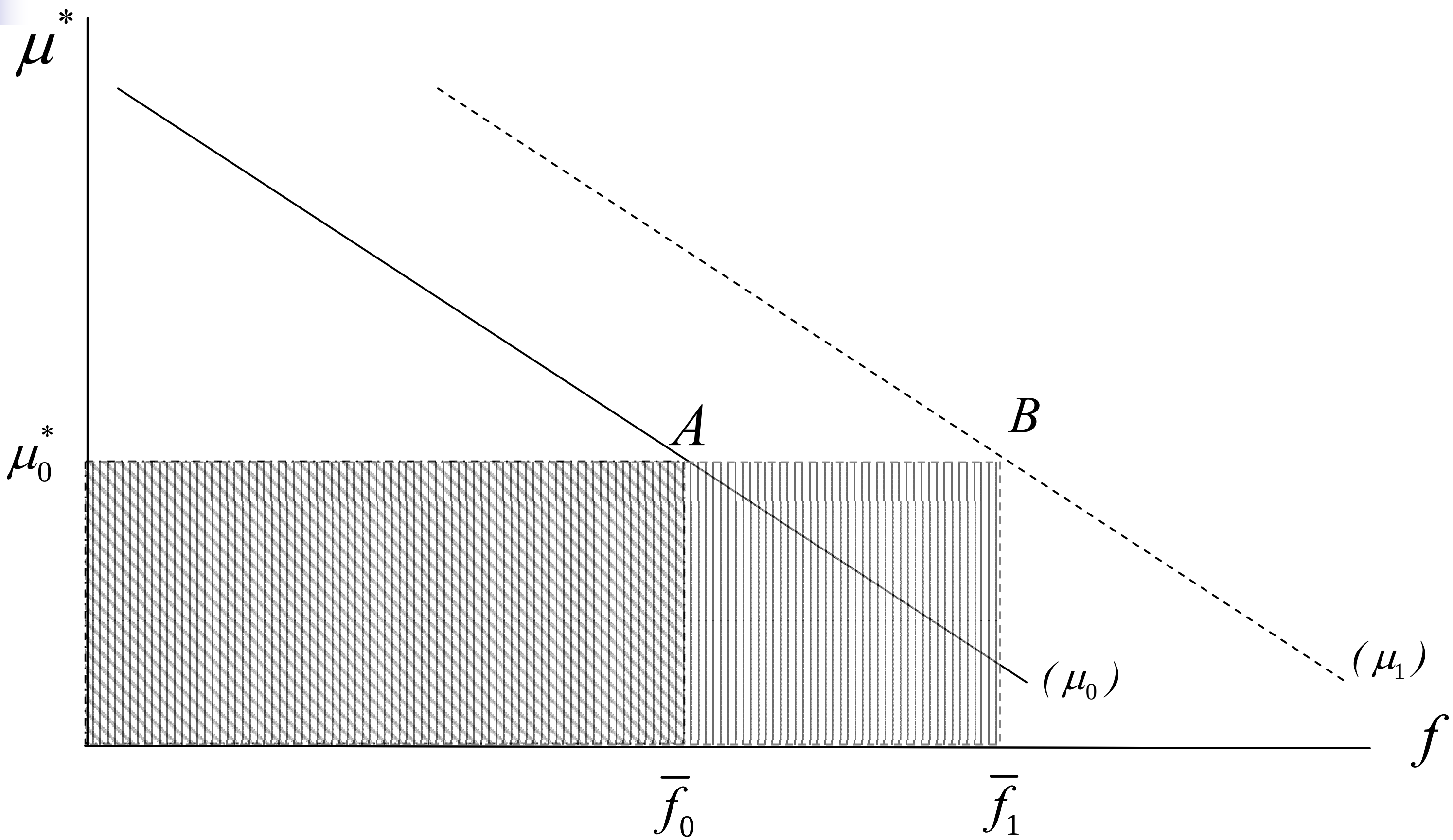
在 *steady-state*：

由(4.9) $\overline{X}_T - C_T(\overline{q}) = f\mu^*$

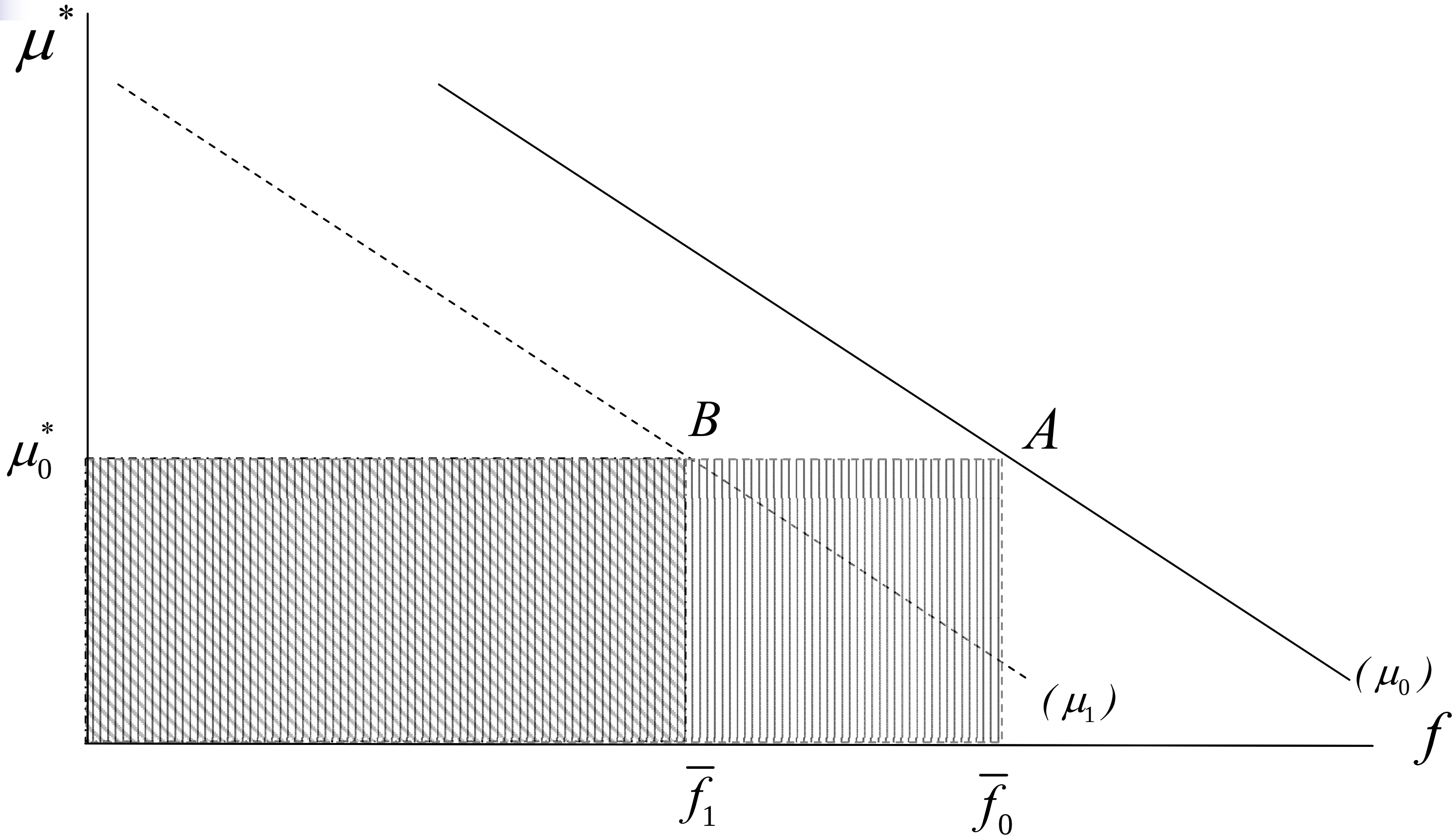
可得 $-(\frac{dC_T}{d\mu}) = \mu^*(\frac{d\overline{f}}{d\mu})$

上式，等號左邊$= -C'_T(\frac{d\overline{q}}{d\mu}) = \mu^*\left(\frac{C'_T U_T V_{mf}}{\Delta}\right) = \mu^*(\frac{d\overline{f}}{d\mu})$=等號右邊

(4.11) (4.13)



$$substitutes \quad \left(V_{mf} < 0\right), \quad \mu \uparrow \Rightarrow m \downarrow \Rightarrow f \uparrow$$



$$complements \quad \left(V_{mf} > 0\right), \quad \mu \uparrow \Rightarrow m \downarrow \Rightarrow f \downarrow$$

$$(4.14)\qquad \frac{d\overline{q}}{d\mu^*} = \frac{1}{\Delta}\left\{ fA + \mu^* U_T V_{mm} \right\} \begin{matrix} > \\ < \end{matrix} 0$$

$$(4.15)\qquad \frac{d\overline{m}}{d\mu^*} = \frac{C'_T}{\Delta}\left\{ V_m(\mu^* U_T + fV_{ff})(\frac{U_{TT}}{U_T}) + V_{mf}[U_T - fV_f(\frac{U_{TT}}{U_T})] \right\} \begin{matrix} > \\ < \end{matrix} 0$$

$$(4.16)\qquad \frac{d\overline{f}}{d\mu^*} = \frac{-C'_T}{\Delta}\left\{ fB(\frac{U_{TT}}{U_T}) + V_{mm} U_T \right\} < 0$$

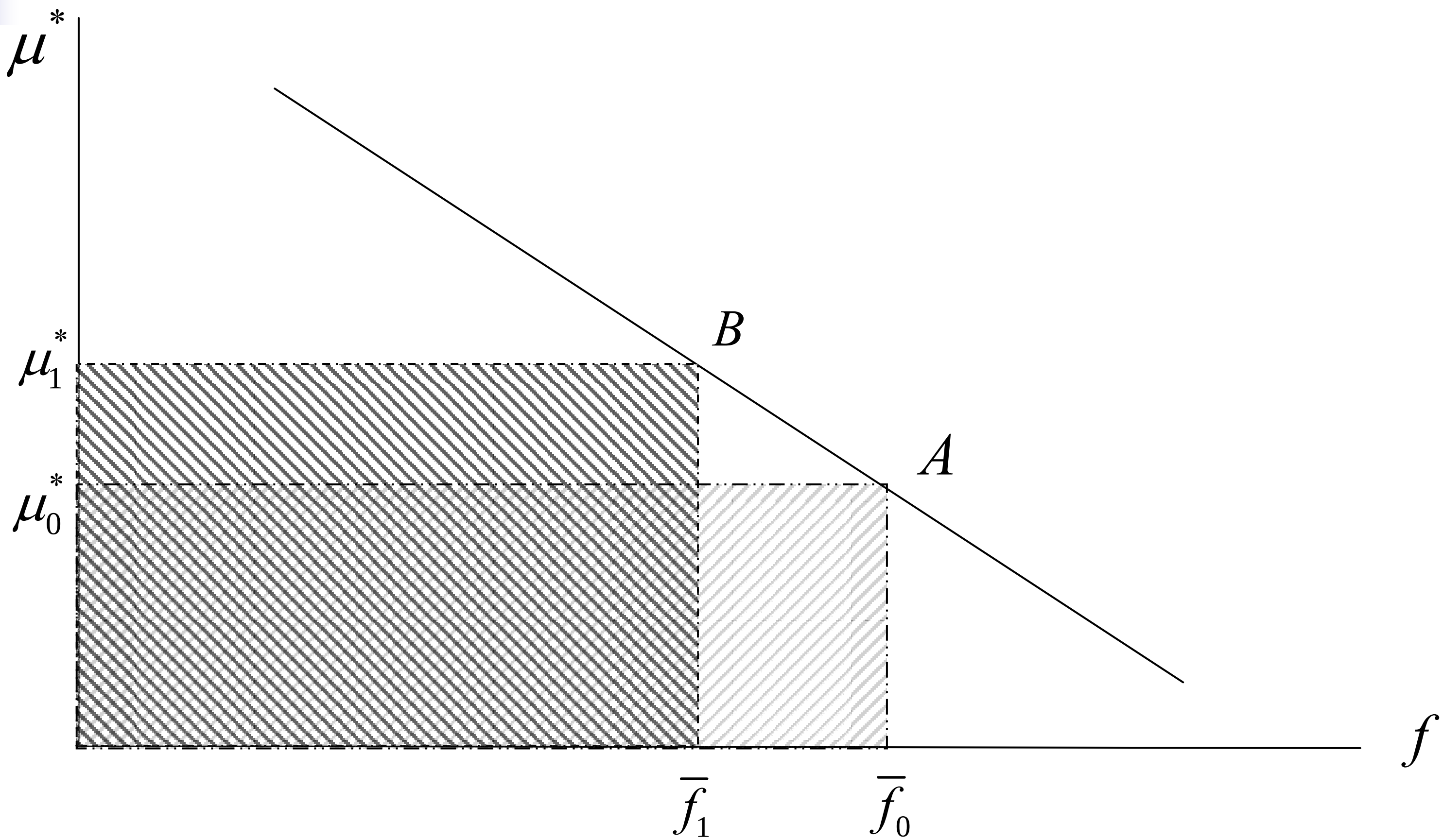
(4.14)和(4.16)的意涵

就(4.9)對μ^*微分

$$(4.17)\qquad -C_T' \frac{d\bar{q}}{d\mu^*} = \bar{f} + \mu^* \frac{d\bar{f}}{d\mu^*} = \bar{f}(1-\varepsilon)$$

$$\varepsilon \equiv -\frac{\mu^*}{\bar{f}} \frac{df}{d\mu^*} > 0$$

$$\frac{d\bar{q}}{d\mu^*} = \frac{\bar{f}(\varepsilon - 1)}{C_T'} \overset{\geq}{\underset{<}{}} 0 \quad as \quad \varepsilon \overset{\leq}{\underset{>}{}} 1$$



$\bar{f}\mu^*$的變化，取決於ε的大小

通貨替代與貨幣對流

兩國模型

符號說明

H=第一種通貨的總存量(由本國發行)

F=第二種通貨的總存量(由外國發行)

H_1=本國居民保有的第一種通貨量

H_2=外國居民保有的第一種通貨量

F_1=本國居民保有的第二種通貨量

F_2=外國居民保有的第二種通貨量

符號說明(續)

X=本國產品的產出量

Y=外國產品的產出量

P_{X1}=以第一種通貨表示的X的價格

P_{X2}=以第二種通貨表示的X的價格

P_{Y1}=以第一種通貨表示的Y的價格

P_{Y2}=以第二種通貨表示的Y的價格

符號說明(續)

π=匯率$(=P_{X1}/P_{X2}=P_{Y1}/P_{Y2})$

q=貿易條件$(=P_{Y1}/P_{X1}=P_{Y2}/P_{X2})$

B=以本國產品計算的本國貿易收支順差

兩國雙元通貨需求

$$(5.1) \qquad M_1 \equiv \frac{H_1}{P_{X1}} = M_1(\underset{(-)}{\hat{P}_{X1}}, \underset{(?)}{\hat{P}_{X2}})$$

$$(5.2) \qquad M_2 \equiv \frac{F_1}{P_{X2}} = M_2(\underset{(?)}{\hat{P}_{X1}}, \underset{(-)}{\hat{P}_{X2}})$$

$$(5.3) \qquad M_1' \equiv \frac{H_2}{P_{Y1}} = M_1'(\underset{(-)}{\hat{P}_{Y1}}, \underset{(?)}{\hat{P}_{Y2}})$$

$$(5.4) \qquad M_2' \equiv \frac{F_2}{P_{Y2}} = M_2'(\underset{(?)}{\hat{P}_{Y1}}, \underset{(-)}{\hat{P}_{Y2}})$$

貨幣供給

$$(5.5) \qquad H = H_0 e^{g_1 t} = H_1 + H_2$$

$$(5.6) \qquad F = F_0 e^{g_2 t} = F_1 + F_2$$

式中g_1和g_2分別代表H和F的增長率

$$(5.5)\&(5.6) \Rightarrow$$

$$(5.7) \qquad g_1 = w_1 \hat{H}_1 + (1-w_1)\hat{H}_2$$

$$(5.8) \qquad g_2 = (1-w_2)\hat{F}_1 + w_2 \hat{F}_2$$

$$w_1 \equiv \frac{H_1}{H_0}, \quad w_2 \equiv \frac{F_2}{F_0}$$

$$steady-state:$$

$$(5.9) \qquad \frac{dg_1}{dt}=\frac{d\hat{H}_1}{dt}=\frac{d\hat{H}_2}{dt}=\frac{d\hat{P}_{X1}}{dt}=\frac{d\hat{P}_{Y1}}{dt}=0$$

$$(5.10) \qquad \frac{dg_2}{dt}=\frac{d\hat{F}_1}{dt}=\frac{d\hat{F}_2}{dt}=\frac{d\hat{P}_{X2}}{dt}=\frac{d\hat{P}_{Y2}}{dt}=0$$

$$(5.7) \Rightarrow \qquad g_1 = w_1(\hat{M}_1 + \hat{P}_{X1}) + (1-w_1)(\hat{M}'_1 + \hat{P}_{Y1})$$

$$= w_1\hat{M}_1 + (1-w_1)\hat{M}' + w_1\hat{P}_{X1} + (1-w_1)\hat{P}_{Y1}$$

$$(5.8) \Rightarrow \qquad g_2 = (1-w_2)(\hat{M}_2 + \hat{P}_{X2}) + w_2(\hat{M}'_2 + \hat{P}_{Y2})$$

$$= (1-w_2)\hat{M}_2 + w_2\hat{M}'_2 + (1-w_2)\hat{P}_{X2} + w_2\hat{P}_{Y2}$$

$$steady-state: \qquad \hat{M}_1 = \hat{M}'_1 = \hat{M}_2 = \hat{M}'_2 = 0$$

$$(5.11) \qquad g_1 = w_1 \hat{P}_{X1} + (1-w_1)\hat{P}_{Y1}$$

$$(5.12) \qquad g_2 = (1-w_2)\hat{P}_{X2} + w_2 \hat{P}_{Y2}$$

$$(5.7)\&(5.8)\ and\ (5.11)\&(5.12) \quad \Rightarrow$$

$$steady-state:$$

$$(5.13) \qquad \hat{P}_{X1}=\hat{P}_{Y1}=\hat{H}_1=\hat{H}_2=g_1$$

$$(5.14) \qquad \hat{P}_{X2}=\hat{P}_{Y2}=\hat{F}_1=\hat{F}_2=g_2$$

Demand for Money

(5.15) $$M_1 = M_1(g_1, g_2)$$

(5.16) $$M_2 = M_2(g_1, g_2)$$

(5.17) $$M_1' = M_1'(g_1, g_2)$$

(5.18) $$M_2' = M_2'(g_1, g_2)$$

Money interflow:

$$f\,(\text{Money net inflow}) = B\ (\text{Trade Surplus})$$

$$(5.19) \qquad B\left(X-f, Y+\frac{f}{q}, q\right) = f \equiv \left(\pi \frac{dF_1}{dt} - \frac{dH_2}{dt}\right) \Big/ P_{X1}$$

$$(5.20) \qquad f = M_2 g_2 - q M_1' g_1$$

$$(5.21) \qquad \frac{dq}{dg_1}=\frac{(1-\lambda-\lambda')\left[(\frac{M_2 g_2}{g_1})\varepsilon_{21}+M_1'(\varepsilon_{11}'-1)\right]}{B_q-\lambda' M_2 g_2+(1-\lambda)M_1' g_1}$$

式中 $\varepsilon_{21}\equiv\frac{g_1}{M_2}\frac{\partial M_2}{\partial g_1}\gtrless 0$ ， $\varepsilon_{11}'\equiv-\frac{g_1}{M_1'}\frac{\partial M'}{\partial g_1}>0$

小國家： $M_1'=0$ ， $\lambda_1'=0$

$$(5.21a) \qquad \frac{dq}{dg_1}=\frac{(1-\lambda)M_2 g_2 \varepsilon_{21}}{B_q g_1} \gtrless 0$$

端視 $\varepsilon_{21} \gtrless 0$ 而定

沒有國外通貨膨脹(C-R Model)： $g_2=0$

$$(5.21b) \qquad \frac{dq}{dg_1}=0$$

$$(5.21') \qquad \frac{dq}{dg_1}=\frac{(1-\lambda-\lambda')M_1'\left[(\frac{M_2g_2}{M_1'g_1})\varepsilon_{21}+\varepsilon_{11}'-1\right]}{B_q-M_1'g_1\left[1-\lambda-\frac{M_2g_2}{M_1'g_1}\lambda'\right]}$$

若原先國際貿易收支平衡 $B=f=0$，

即$M_2g_2=M_1'g_1$，則

$$(5.22) \qquad \frac{dq}{dg_1}=\frac{(1-\lambda-\lambda')M_1'(\varepsilon_{21}+\varepsilon_{11}'-1)}{B_q-M_1'g_1(1-\lambda-\lambda')}$$

結論

Q & A

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參考文獻

Barnett, R. C. (1992), “Speculation, Incomplete Currency Market Participation, and Nonfundamental Movements in Nominal and Real Exchange Rates,” *Journal of International Economics*, 33, 167–186.

Benhabib, J. and R. Farmer (1996), “Indeterminacy and Sector-Specific Externalities,” *Journal of Monetary Economics*, 37, 421–443.

Calvo, G. (1985), “Currency Substitution and the Real Exchange Rate: the Utility Maximization Approach,” *Journal of International Money and Finance*, 4, 175-188.

Calvo, G. and C. Rodriguez (1977), “A Model of Exchange Rate Determination under Currency Substitution and Rational Expectation,” *Journal of Political Economy*, 85, 617-625.

Chen, C. N. (1973), “Diversified Currency Holdings and Flexible Exchange Rates,” *Quarterly Journal of Economics*, 87, 96-111.

Chen, C. N. (1976), “Currency Denominations and the Price Level,” *Journal of Political Economy*, 84, 179-184.

Chen, C. N. and T. W. Tsaur (1983a), “Currency Substitution and Foreign Inflation,” *Quarterly Journal of Economics*, 98, 177-184.

Chen, C. N. and T. W. Tsaur (1983b), “Currency Denominations, Currency Substitution and the Price Level,” *Journal of Macroeconomics*, 5, 511-513.

Chen, C. N. and T. W. Tsaur (1992), “Currency Substitution, Factor Demand Theory, and the Dynamics of Exchange Rates,” *Academia Economic Papers*, 20:2, 313-335.

Chen, C. N., C. F. Chou and T. W. Tsaur (1977), “The Flexible Bimetallic Exchange Rate System Revisited,” in Chi-ming Hou and Tzong-shian Yu eds. Conference on Modern Chinese Economic History, The Institute of Economics, Academia Sinica, Taipei, PP. 457-66.

Chen, C. N., T. W. Tsaur and C. F. Chou (1981), “Gross Substitution, Greasham's Law, and Hayek's Free Currency System,” *Journal of Macroeconomics*, 3, 547-557.

Chen, C. N., T. W. Tsaur, and S. C. Liu (1989), “Foreign Inflation, Currency

Substitution, and Terms of Trade Dynamics," *Journal of Political Economy*, 97, 955-964.

Hayek, F. A. (1976), "Choice in Currency," *Occasional Paper*, 48, (Institute of Economic Affairs).

Karaken, J. and N. Wallace (1981), "On the Indeterminacy of Equilibrium Exchange Rates," *Quarterly Journal of Economics*, 96, 207–222.

King, R., N. Wallace, and W. Weber (1992), "Nonfundamental Uncertainty and Exchange Rates," *Journal of International Economics*, 32, 883–908.

Liviatan, N. (1981), "Monetary Expansion and Real Exchange Rate Dynamics," *Journal of Political Economy*, 89, 1218-1227.

McKinnon, R. I. (1982), "Currency Substitution and Instability in The World Dollar Standard," *American Economic Review*, 68, 320-333.

Miles, M. A. (1978), "Currency Substitution, Flexible Exchange Rates, and Monetary Independence," *American Economic Review*, 68, 428-436.

Park, W. (1987), "Crawling Peg, Inflation Hedges and Exchange Rate Dynamics," *Journal of International Economics*, 23, 131–150.

Vegh, C. A. (1989a), "The Optimal Inflation Tax in the Presence of Currency Substitution," *Journal of Monetary Economics*, 24, 139-146.

Vegh, C. A. (1989b), "Currency Substitution and the Optimal Inflation Tax under Labor Income Taxation," International Monetary Fund, Unpublished manuscript.

Vegh, C. A. (1989c), "Government Spending and Inflationary Finance: A Public Finance Approach," *International Monetary Fund Staff Papers*, 14, 679-694.

Vegh, C. A. (1995), "Inflationary Finance and Currency Substitution in a Public Finance Framwork," *Journal of International Money and Finance*, 14, 679-693.