A Dynamic Model of Auctions with Buy-Out: Theory and Evidences*

Jong-Rong Chen
Institute of Industrial Economics
National Central University
jrchen@cc.ncu.edu.tw

Kong-Ping Chen†
Institute of Economics
and
Research Center of Humanities and Social Sciences
Academia Sinica
kongpin@gate.sinica.edu.tw

Chien-Fu Chou
Department of Economics
National Taiwan University
cfchou@ntu.edu.tw

May 23, 2006 preliminary and incomplete

Abstract

The paper considers an ascending price independent value auction, in which the buyers are given the additional option to obtain the object immediately by paying a fixed price (the buy-out price). We completely characterize the optimal bidding strategy of the bidder and the optimal buy-out price of the seller. We show that the more risk-averse a buyer, the earlier is he willing to buy out the object. The seller's optimal buy-out price is decreasing in the his own degree of risk-aversion, and increasing in that of the buyer. The expected transaction price and the expected utility of the seller are higher with the buy-out option. Moreover, the higher the buy-out price, the higher is the expected transaction price. Finally, contrary to the usual ascending price auctions, the longer it takes for an item to be sold, the lower is its transaction price. All the theoretical predictions are confirmed in the data we collect.

* We thank Chia-Ming Yu and Ying-Tang Huang for excellent research assistance, and Chen-Ying Huang, Hui-Wen Koo and Mengyu Liang for useful comments. Financial support from the Center for Institution and Behavior Studies, Academia Sinica, is gratefully acknowledged.

† Corresponding author.

1 Introduction

An interesting feature of the recent on-line bidding auction format which is absent from the traditional English auction is the existence of the buy-out option.[1] There are two main explanations of why the seller will set up a buy-out price. The first is the seller's incentive to exploit bidders' time preference (see, e.g., Mathews, 2004). Under this explanation, the bidder is impatient, and is willing to pay a higher price to obtain an objective immediately, rather than through a time-consuming bidding process. The seller can then set up a buy-out price to satisfy this need and thereby make more profit. The second explanation is that if the buyers are risk-averse, then they will be willing to buy the object with a high, but fixed, price rather than to obtain the object through the bidding process, which has the risk of either losing to other bidders or, even if they win, paying an uncertain price.[2]

Both explanations imply that if we compare the auctions without buy-out prices and those of identical objects but with buy-out prices, the average transaction price will be higher for the latter. Although confirmed in the data we collect (see Section 3), both reasonings are, however, incomplete, because the same reasoning can also be applied to the seller. That is, not only buyers, but also the sellers can be impatient or risk-averse. In either case the sellers will be willing to set a lower buy-out price so that the objects can be sold earlier (if they are impatient) or at a fixed price (if they are risk averse). Thus the two mentioned explanations have implicitly assumed that it is the buyers, rather than

[1] See Lucking-Rieley (2000) and Bajari and Hortaçsu (2004) for genaral discussions on internet auctions.

[2] See, e.g., Budish and Takeyama (2001), Reynolds and Wooders (2004) and Mathews and Katzman (2006).

the sellers, who are impatient or risk averse. Therefore although the results derived from this assumption are consistent with the evidence, they still raise the question of why the buyers are risk-averse or impatient but the sellers are not.

In this paper we focus on how the buyer's and the seller's attitude toward risk affect the bidder's incentives to buy out and the seller's optimal buy-out price in auctions. We develop a dynamic model of English auction with two bidders who, at every prevailing price, need to decide whether to continue with bidding or to buy out. Either the buyer or the seller (or both) can be risk-averse. We solve for the optimal buy-out and bidding strategy of the bidders. This optimal strategy is in turn used to solve for the optimal buy-out price of the seller. Under the optimal strategy, the higher a bidder's valuation of the object, the earlier is he willing to buy out the object. Also, the optimal buy-out price is an increasing (decreasing) function of the bidders' (seller's) degree of risk-aversion. We also show that whether the auction will end with one bidder out-bidding his opponent or a buy-out depends on the configuration of the bidders' valuations of the object.

In our model, the buy-out price serves two purposes for the seller. First, if the buyers are risk-averse, it can be used as an instrument to exploit the buyer's aversion to risk resulting from the bidding process. Second, if the seller is risk-averse, it also serves to avoid risk for the seller himself. This implies that even if the buyers are risk-neutral, the seller still has incentives to evoke the buy-out option, not to make more profit, but to avoid the more risky outcome of the bidding process. Indeed, our result indicates that the only case in which the seller does not gain from a buy-out option is when both the buyers and sellers are risk-neutral. This is in contrast to Budish and Takeyama (2001), who show that the buy-out option is of value if and only if the buyers are risk-averse. Our paper thus makes two theoretical contributions to the literature. First, propose a dynamic model of auctions with buy-outs, and completely characterize the equilibrium, when both buyers and sellers are risk-averse. Second, we provide a rationale for buy-out option even when the buyers are risk-neutral.

Wang et. al. (2004) also propose a theory with buy-out price. In their model, a bidder decides whether to buy out the item in the beginning of the auction, or not to buy out and enter the bidding process. Thus it is impossible that a bidder buys out item in the middle of the bidding process. Their is thus a static model in which a bidder decides whether to buy out or to join the competitive bidding only in the beginning of auctions. In particular, it is impossible for a bidder to join the competitive bidding in the beginning, and to buy out the item half-way in the bidding process.

There are three main testable implications of our model: (i) The expected transaction price of an item will be higher in auctions with buy-out; (ii) the expected transaction price is in direct relationship with buy-out price; and (iii) the expected transaction price is in reverse relation with the time taken for an item to be sold. To test our theoretical model, we use date collected from Taiwan's Yahoo! on-line auctions to test the empirical implications above. All three predictions are confirmed by data.[3] In addition, our

[3] There is also other literature on buy-out price which is not directly related to our paper. For example, Dodonova and Khoroshilov (2004) empirically find that, other things being equal, bidders are willing to bid more for an item with a higher buy-out price than an identical item with lower buy-out price, consistent with the anchoring hypothesis. Reynolds and Wooders (2004) compare two formats of auctions with buy-out price, and find that when the bidders are risk-neutral, the eBay (temporary) and Yahoo! (permanent) types of auction with buy-out price have exactly the same expected revenue for the seller. However, when the bidders are risk-averse, the Yahoo! version raises more revenue.

empirical results also support some intuitive predictions. For example, good reputation of the seller increases both the probability an item is sold and its transaction price; and minimum bid increases pirce an item is sold at the cost of its probability.

2 The Model

A risk-averse seller conducts an English auction to sell an object. Two bidders ($i = 1, 2$) are participating in the auction. The value of the object to bidder i, v_i, is his own private information, but is known to be independently and uniformly drawn from $[0, \bar{v}]$.

A bidder can either buy the object by out-bidding the other bidder, or by buying the objective with the buy-out price, v_b, set by the seller in the beginning of the auction. The utility function of a bidder with valuation v is

$$u(v, p) = (v - p)^{\alpha}/\alpha \tag{1}$$

if he buys the objective with price p, and is 0 if he does not buy it; where $0 < \alpha \leq 1$. The value of α denotes the bidder's degree of relative risk-aversion. The smaller the value of α, the more risk-averse is the buyer. The utility function of the seller is assumed to be $u(x) = x^{\beta}/\beta$; where $\beta \in (0, 1]$ is the seller's degree of relative risk-aversion. Similarly, the smaller the value of β, the more risk-averse is the seller.

2.1 Equilibrium Buy-Out Strategy

In this subsection, we derive the optimal buy-out strategy of the bidder under a given buy-out price. Thus throughout this subsection, we assume that buy-out price is fixed at

v_b.

Even with a buy-out price, a basic result of the standard English auction remains true: It is a dominant strategy for a bidder to stay active in the auction as long as the prevailing price is lower than his valuation of the object. The complication comes from the fact that, at every prevailing price, now he has the additional option to pre-empt his opponent by buying the objective immediately at the buy-out price v_b.

Given v_b, let $p(v)$ be the buy-out strategy of the bidder whose valuation of the objective is v. That is, a bidder who values the object at v is willing to buy out the object (by paying v_b) when the prevailing price reaches $p(v)$. Since the greater the value of v, the more willing is the bidder to obtain the object immediately by paying v_b, we know that $p(v)$ is a decreasing function. It turns out to be easier to work with the inverse function of $p(v)$. Let $v(p)$ be the inverse function of $p(v)$. It relates the prevailing price p with bidder's valuation v, who at p is just willing to buy out the objective. That is, a bidder with valuation $v(p)$ is just willing to obtain the object by paying the buy-out price, when the prevailing price reaches p. Similarly, if a bidder with valuation v is willing to buy out the object, when the prevailing price is p, then a bidder with valuation $v' > v$ will be even more willing to do so at that moment. This implies that $v(p)$ is a decreasing function.

Suppose that both bidders are still active at the moment when the prevailing price is p. This implies that the valuations of both bidders are greater than p, which in turn implies that the possible valuations of any bidder must be distributed on $[p, \bar{v}]$. Moreover, by definition of $v(p)$, any bidder with valuation $v > v(p)$ would have bought out the object before the price has risen to p. The fact that this object has not been bought out at price

p implies that the bidder's possible valuations cannot lie in $(v(p), \bar{v}]$. As a result, both bidders' valuations must lie in $[p, v(p)]$. In other words, if both bidders are still active when the prevailing price is p, then (by Bayes rule) any bidder's possible valuations of the object must be distributed uniformly on $[p, v(p)]$.

Consider the decision of a bidder (whose valuation is v) at the moment when the prevailing price is $p < v$. If he buys the object immediately with buy-out price v_b, his utility will be $u(v, v_b) = (v - v_b)^\alpha/\alpha$. If instead he holds out and waits until price is $p + dp$ to buy out the object, then he will face three possible outcomes. First, his opponent buys out the object while he waits. Second, his opponent drops out between p and $p + dp$. Third, neither of the above happens so that he eventually buys out the object when the prevailing price is $p + dp$. Whether the bidder should buy the object immediately (by paying v_b), or waits until $p + dp$, depends on the difference of the utility between an immediate buy-out and the combined expected utility under the three possible outcomes of waiting until $p + dp$.

Figure 1 depicts the possible intervals at which outcomes 1 and 2 occur. When the valuation of the bidder's opponent lies in $[v(p) + dv(p), v(p)]$,[4] then his opponent will buy out the object while he waits. This is the first outcome we mentioned above, which occurs with probability $\frac{-dv(p)}{v-p}$, and his utility is 0. Similarly, if his opponent's valuation lies in $[p, p + dp]$, then his opponent will drop out while he waits, and he will win the bidding with price p. This is the second outcome mentioned, which occurs with probability $\frac{dp}{v-p}$, and his utility is $(v - p)^\alpha/\alpha$. Under the third outcome, which occurs with probability

[4] Since $dv(p) = v'(p)dp$ and $v'(p) < 0$, $dv(p) < 0$.

$1 - \left(-\frac{dv(p)}{v-p} + \frac{dp}{v-p}\right)$, his utility is $(v - v_b)^\alpha/\alpha$.

Figure 1: Possible outcomes of waiting.

The total expected utility of waiting until $p + dp$ to buy out is thus

$$\frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + \left[1 + \frac{dv(p)}{v-p} - \frac{d\,p}{v-p}\right]\frac{(v-v_b)^\alpha}{\alpha}. \tag{2}$$

The total change in utility of waiting until $p + d\,p$ to buy out, instead of buying out now, is

$$du = \frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + \frac{dv(p) - dp}{v-p}\frac{(v-v_b)^\alpha}{\alpha}. \tag{3}$$

For the function $v(p)$ to be the optimal buy-out strategy, it must be the case that $\frac{du}{dp} = 0$, i.e., the first-order condition must hold at any p. This implies that

$$(v-p)^\alpha - (v-v_b)^\alpha = -(v-v_b)^\alpha\,\frac{dv}{dp}. \tag{4}$$

Let $y = v - v_b$ and $x = v_b - p$, then $\frac{dv}{dp} = -\frac{dy}{dx}$, and equation (4) becomes

$$(x+y)^\alpha - y^\alpha = y^\alpha\frac{dy}{dx}. \tag{5}$$

It is difficult to directly solve for equation (5), but the initial condition and the fact that (5) is homogeneous of degree α on both sides supply a clue. Since $v(v_b) = v_b$,[5] the

[5] If the current price is v_b, and buy out price is v_b, then it must be optimal to buy out immediately.

solution of (5) must pass through $(x, y) = (0, 0)$. Let $x = \mu y$, then (5) becomes

$$(1+\mu)^{\alpha} = 1 + \frac{1}{\mu}. \tag{6}$$

Denote μ^* as the solution of (6). Figure 2 then depicts how μ^* is determined. It can be shown easily that $\mu^* \geq 1$ and that μ^* is decreasing in α. In particular, $\mu^* = 1$ when the buyers are risk neutral ($\alpha = 1$). We thus have $x = \mu^* \ y$. Substituting for $y = v - v_b$ and $x = v_b - p$ we eventually have

$$v(p) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}. \tag{7}$$

The function $v(p)$ is the explicit form of the inverse of a bidder's optimal buy-out strategy. Solving for the inverse of the function $v(p)$ we have

$$p(v) = (1 + \mu^*)v_b - \mu^* \ v. \tag{8}$$

The function $p(v)$ is exactly the optimal buy-out strategy of the bidder. It shows that a bidder, whose valuation of the object is v, will be willing to buy out the object (by paying v_b) when the prevailing price reaches $(1+\mu^*)v_b - \mu^* v$. Given the optimal buy-out policy, the optimal strategy of the bidder with valuation v is then easy to describe: Stay active as long as the prevailing price is lower than $p(v)$, and buy out the object when price reaches $p(v)$. Note that since a bidder will consider buying out only if $v > v_b$, we know $v - p(v) = (1 + \mu^*)(v - v_b) > 0$. That is, if a bidder will buy out the object, then he will do so before the price reaches his valuation. This also implies that the transaction

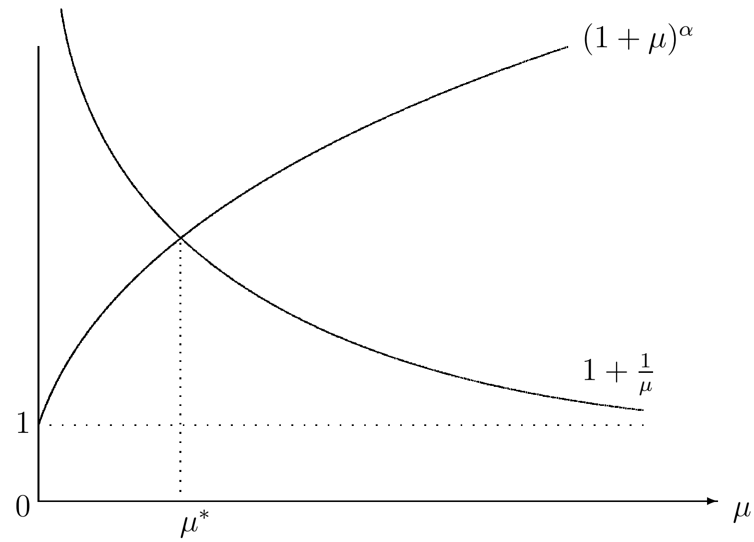

Figure 2: Determination of μ^*.

price cannot be higher than v_b. In other words, by setting v_b as the buy-out price, the seller essentially sets v_b as the upper-bound for the possible transaction prices.

The graph of equation $v(p)$ is depicted in Figure 3. It visualizes the relation between a bidder's valuation and the prevailing price at which he wants to buy out. The higher a bidder's valuation for the objective, the lower is the prevailing price at which he is willing to buy it out. In particular, if his valuation $v \geq (1 + \frac{1}{\mu^*})v_b$, then his valuation is so high that he is willing to buy out the objective right at the beginning of the auction (i.e., when $p = 0$). Moreover, since μ^* is decreasing in α, the more risk-averse a buyer, the earlier is he willing to buy out the object. This result is fairly intuitive. The more risk-averse a bidder, the less willing is he to face the uncertain outcome of the bidding process. Thus he

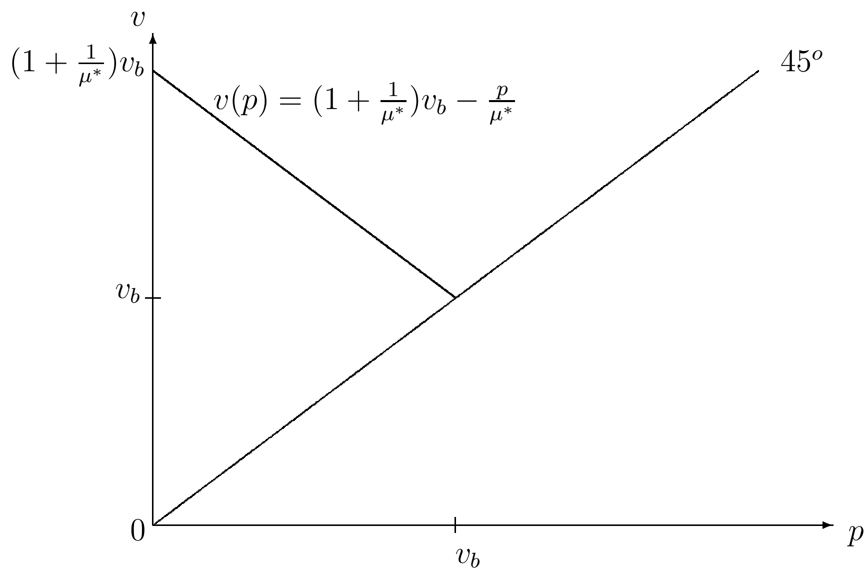

Figure 3: Buyer's optimal buy-out policy.

is more willing to buy it out early. (Note that to buy-out early is costly, as the expected gain of bidding is still high.) The solution $v(p)$ also makes it possible to characterize the outcomes of the auction as a function of v_1 and v_2. Note that by the symmetric nature of the equilibrium, a bidder will win if and only if his valuation of the object is greater than his opponent's. The question is only whether he will win by out-bidding his opponent or by direct buy-out. Since the line $v(p) = (1+\frac{1}{\mu^*})v_b - \frac{p}{\mu^*}$ characterizes the relation between a bidder's valuation and the prevailing price at which he is willing to buy out, bidder i will win by bidding if and only if $v_i > v_j$ and $v_i < (1+\frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. On the other hand, i will win by buy-out if and only if $v_i > v_j$ and $v_i > (1+\frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. We can thus characterize the outcomes of the auction as a function of the bidders' valuations of the item in Figure

4. In the figure, regions I and I' depict the case in which the winner wins by out-bidding his opponent. In regions II and II', the winner obtains the object by buy-out.

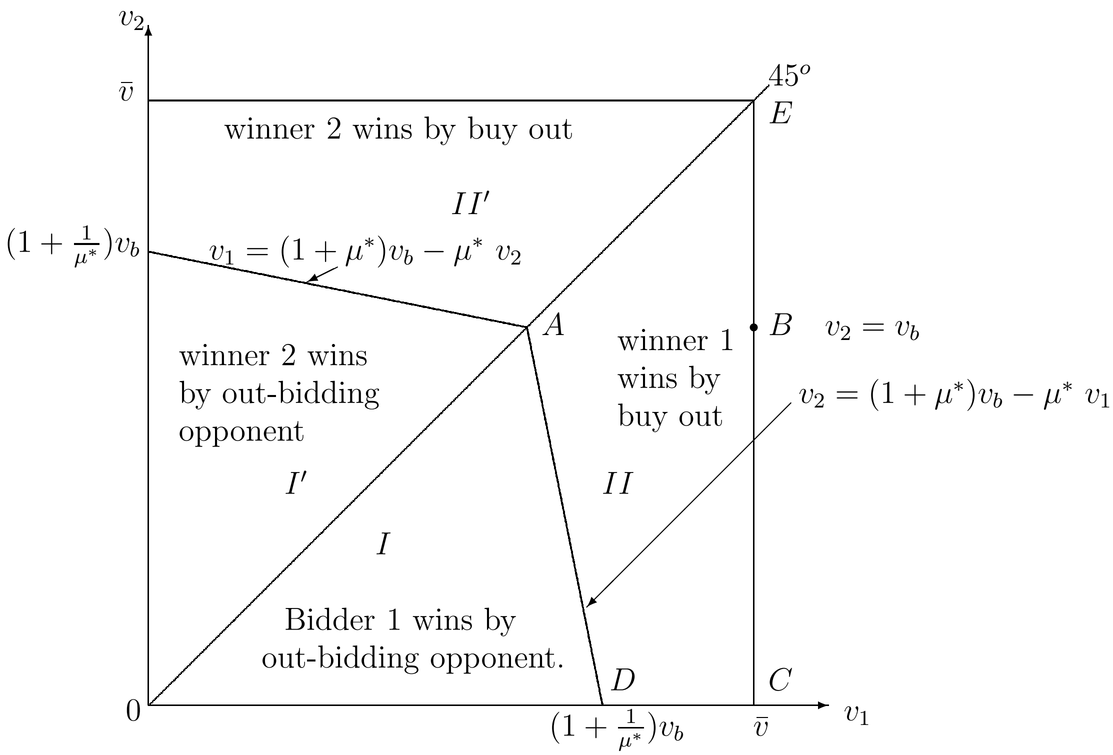

Figure 4: Outcomes of bidding as a function of bidders' valuations.

A technical problem occurs when the valuation of every bidder is greater than $(1+\frac{1}{\mu})v_b$. In that case both will want to buy out at beginning of the auction; i.e., when the prevailing price is 0. A reasonable assumption to make is to assume that every bidder prevails with probability 1/2. Therefore, there will be a mass of bidders (specifically, those whose valuations of the item are higher than $(1+\frac{1}{\mu^*})v_b$) who will buy out the object when the bidding price is still. This will create a discontinuity in the expected utility for bidders whose valuations are just below $(1+\frac{1}{\mu^*})v_b$. The reason is that according to the optimal strategy derived, a bidder whose valuation is slightly lower than $(1+\frac{1}{\mu^*})v_b$ will wait until the price is slightly higher than 0 to buy out. In that case he will lose for sure if his opponent's valuation is greater than his. If, however, rather than wait until price is slightly above 0, he buys out the item immediately (when prevailing price is 0), then his chance to win, when his opponent's valuation is greater than his, will surge from 0 to 1/2. Thus, the assumption that a bidder's winning chance is 1/2, when both propose to buy out when price is 0, creates a jump in expected utility for those bidders whose valuations are sufficiently close to $(1+\frac{1}{\mu})v_b$ if they buy immediately rather than follow the strategy $v(p)$. As a result, they will deviate instead of following the strategy $p(v)$. This prevents $p(v)$ from being the optimum for bidders whose valuations are close to $(1+\frac{1}{\mu})v_b$.

To overcome this technical problem, we will make the following strong assumption: Whenever two bidders propose to buy out at the beginning of the auction, the bidder with higher valuation will win. Although a strong assumption, it has certain justification. In an on-line auction an item is put up for bidding for much longer time span than the traditional English auction. Moreover, a bidder needs not be present during the whole auction process. A bidder can come in any time to bid as long as the item is still up for bidding. That means a bidder might miss the chance even if he is willing to buy out when the prevailing price is 0, as he might be absent. A bidder with higher valuation, being one having greater surplus from buying the item, is then more alert to stay on-line searching for the item. Therefore, he is more likely to be present when auction of the item in question starts, and thus has greater chance to buy out. Our assumption essentially says that this advantage for higher valuation bidder is absolute, in that he wins with probability 1. If this assumption is made, then the discontinuity in expected payoff mentioned above will cease to exist, as a bidder wins if and only if his valuation is greater, even if both bidders propose to buy out at price 0. Consequently, $v(p)$ is indeed the optimal buy-out strategy.[6]

The strategic effect of a buy-out option on the seller's revenue can be seen very clearly in Figure 4. Take the case when buyer 1 eventually wins (i.e., the region OEC). Without a buy-out option, bidder 1 will win by paying bidder 2's valuation, v_2. With a buy-out, there are three types of outcomes to consider. First, the outcomes in region OAD are the same as the case without buy-out: Bidder 1 wins by paying bidder 2's valuation v_2. Second, in region ABCD, bidder 1 wins by paying the buy-out price v_b. Note that in this region $v_2 < v_b$. What would have been sold with price v_2 is now sold with a higher price v_b. The seller thus gains by setting up a buy-out price in this region. Third, in region ABE, bidder 1 pays the buy-out price, v_b, to win the item, but now $v_b < v_2$. This is the region in which the seller actually loses with the buy-out option. The optimal buy-out price of the seller must thus balance the latter two types of outcomes; that is, to maximize the expected revenue from region ABCD net of the expected loss from region ABE.

Figure 4 also shows clearly that a buy-out option reduces the risk that both the buyers and seller face in the bidding process. Again, consider the case in which bidder 1

[6] We also analytically solve for the case in which the winning chance is 1/2 for each bidder when both propose to buy out in the beginning. But since there exists no close-form solution, the comparative statics derivation and price comparison become extremely complicated and burdensome. We thus use simulation to check for the properties that we derived in Section 2 of the paper. All the results go through. These simulation results can be downloaded from the website www.sinica.edu.tw/ kongpin/auctionsimulation.pdf.

eventually wins, i.e., region OEC. In region OAD, the outcomes of bidding (and thus the uncertainty faced by both) are the same regardless of whether there is a buy-out option, since in both cases bidder 1 wins by paying bidder 2's valuation v_2. In region AECD, if there is no buy-out option, bidder 1 will win by paying bidder 2's valuation v_2, which is uncertain. However, with a buy-out option, bidder 1 will win by paying a fixed price v_b. Obviously, the price risk faced by both the buyers and the seller is reduced by the buy-out option.

2.2 Optimal Buy-Out Price

Given the outcomes of the bidding process depicted in Figure 4, it is straightforward to compute the expected utility of the seller under any buy-out price v_b:

$$\pi(v_b) = \frac{2}{\bar{v}^2}\left\{\int_0^{v_b}\int_0^{v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^*v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 \right.$$
$$\left. + \frac{v_b^\beta}{\beta}\left[\frac{\bar{v}^2}{2} - \frac{v_b}{2}\left(1+\frac{1}{\mu^*}\right)v_b\right]\right\}; \tag{9}$$

where the first two terms in the braces are profits from region I, and the third term is that from region II. $\pi(v_b)$ can be shown to be equal to

$$\frac{v_b^\beta}{\beta}\left[1 - \frac{\beta(3+\beta)(1+\mu^*)}{(\beta+1)(\beta+2)\mu^*}\cdot\left(\frac{v_b}{\bar{v}}\right)^2\right]. \tag{10}$$

The seller chooses the values of v_b to maximize $\pi(v_b)$. The first-order condition for v_b is

$$\frac{\partial\pi}{\partial v_b} = v_b^{\beta-1}\left[1 - \frac{(3+\beta)(1+\mu^*)}{(\beta+1)\mu^*}\ \left(\frac{v_b}{\bar{v}}\right)^2\right] = 0. \tag{11}$$

This implies that $v_b^* = \sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\bar{v}$. By plugging v_b^* into (10), we can compute the expected utility of the seller under the optimal buy-out price to be

$$\pi(\beta) \equiv \frac{2}{\beta(2+\beta)}\left(\sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\,\bar{v}\right)^{\beta}. \tag{12}$$

On the other hand, the expected utility of the seller without buy-out option is

$$\pi^0(\beta) \equiv \frac{2}{\bar{v}^2}\int_0^{\bar{v}}\int_0^{v_1}\frac{v_2^{\beta}}{\beta}dv_2dv_1 = \frac{2\bar{v}^{\beta}}{\beta(\beta+1)(\beta+2)}.$$

The difference in expected utility is thus

$$\pi(\beta) - \pi^0(\beta) = \frac{2\bar{v}^{\beta}}{\beta(\beta+1)(\beta+2)}\left[\left(\frac{\mu^*}{1+\mu^*}\right)^{\frac{\beta}{2}}(1+\beta)^{\frac{\beta}{2}+1}(3+\beta)^{-\frac{\beta}{2}} - 1\right]. \tag{13}$$

Let $\Phi(\mu^*,\beta)$ be the term in the bracket of (13). It is easy to see that Φ is increasing in μ^*. Moreover, $\Phi(1,\beta) = (\frac{1}{2})^{\beta}(1+\beta)^{\frac{\beta}{2}+1}(3+\beta)^{-\frac{\beta}{2}} - 1$, which is increasing in β initially, then decreasing in β. Note that $\Phi(1,0) = \Phi(1,1) = 0$, implying that $\Phi(1,\beta) \geq 0$ for all $\beta \in (0,1]$. By the fact that Φ is an increasing function of μ^*, we know that $\Phi(\mu^*,\beta) \geq 0$ for all $\beta \in (0,1]$ and $\mu^* \geq 1$. That is, the expected utility of the seller is always greater with the buy-out option. Moreover, $\Phi(\mu^*,\beta) = 0$ only if $\mu^* = 1$ (i.e., $\alpha = 1$) and $\beta = 1$, meaning that the expected utility of the seller is strictly higher with buy-out unless both agents are risk-neutral. We thus have the following proposition.

Proposition 1. *If both the buyers and the seller are risk-neutral, then the seller's expected revenues are the same in auctions with and without buy-out. If either the seller or the buyer is risk-averse, then the seller's expected utility is strictly higher with buy-out.*

Next we compare the expected utility of the bidder between cases with and without buy-outs. We will show that although seller gains from buy-out, unless the bidder has a very high valuation of an item and the seller is not very risk-averse, otherwise the bidder is worse off with the buy-out option. This is summarized in the next proposition.

Proposition 2. *Consider a bidder with valuation v. He has higher expected utility with buy-out if and only if* $v/\bar{v} > v_c(\alpha, \beta) \equiv \sqrt{(\frac{\mu}{1+\mu})(\frac{1+\beta}{3+\beta})}/(1-(\alpha+1)^{\frac{-1}{\alpha}}) > (1+\frac{1}{\mu^*})v_b$.

Proof: See Appendix B.

Proposition 2 shows that unless a bidder has very high valuation, otherwise he is worse off with buy-out option. Note that if α is close enough to 1 and β is close enough 0 (i.e., if the seller is sufficiently risk-averse and the bidder is close to risk-neutral), then $v_c(\alpha, \beta) > 1$, implying that all bidders, regardless of their valuations, are worse off under buy-out. Although the buy-out option might seem to help the bidders by offering them an option to buy the item with a more predictable transaction price, it actually serves as the seller's instrument to increase the competition between the bidders. With the buy-out option, the bidders not only have to compete in the bidding process, but have to compete in buy-out. The seller thus extracts more rent (Proposition 1) at the bidders' expense in the auction. Recall (see Figure 4) that buy-out price actually helps the bidders with high valuations, because it enables them to buy the items at the buy-out price rather than risk bidding into very high price. Consequently, a bidder who has very high valuation actually has higher expected utility in the case with buy-out (Proposition 2). Note that $v_c(\alpha, \beta)$ is increasing in β, implying that the more risk-averse the seller, the more likely the bidder will gain from buy-out.

There are other properties of the optimal buy-out price which deserve to be discussed. First, since v_b^* is an increasing function of μ^*, which is in turn decreasing in α, we know that optimal buy-out price is increasing in the degree of the buyer's degree of risk-aversion. This is an intuitive result, since one purpose of setting up a buy-out price is to make more profit by exploiting the aversion of bidders to the uncertainty of whether he will win or, if he wins, the uncertainty of price he needs to pay. What is surprising is that even if the bidders are risk-neutral, there is still incentives for the seller to set a non-trivial buy-out price (i.e., a buy-out price lower than $\bar{v}$). This can be seen clearly from the facts that when $\alpha = 1$, $\mu^* = 1$, implying $v_b^* = \sqrt{\frac{1+\beta}{2(3+\beta)}}\bar{v} < \bar{v}$, and that equation (13) is strictly positive.[7] This is in contrast to the conventional wisdom that the reason for the buy-out price is to satisfy the bidder's desire to avoid risks.[8] Again, the intuition for this is actually quite clear. In the case when both buyer and seller are risk-averse, the buy-out price serves two purposes for the seller. First, it can be used to exploit the bidder's aversion to risk and increase the seller's revenue. Second, it can also be used as a way to avoid risk for the seller. Therefore, even if the buyers are risk-neutral, the seller still has incentives to evoke the buy-out option, not to increase revenue, but to reduce his own risk.

Also note that v_b^* is increasing in β, meaning that the optimal buy-out price is decreasing in the seller's degree of risk-aversion. The reason behind this is transparent. The more risk-averse the seller, the more he abhors the uncertainty brought about by the result of the competitive bidding between the buyers. He is then more willing to set up a lower, but fixed and certain, buy-out price to avoid risk.

[7] If the seller sets $v_b > \bar{v}$, then the buy-out price is redundant. This is because the seller will then actually prefer the bidders to compete by bidding, rather than to sell it at a buy-out price lower than $\bar{v}$.

[8] For example, in Budish and Takeyama (2001), a buy-out price benefits the buyer only if the buyers are risk neutral. Since the seller can always set up an impossibly high buy-out price to make the auction equivalent to one without buy-out price, this implies that only if the buyers are risk-neutral will the buy-out price have any function.

2.3 Empirical Implications

In this subsection, we derive three empirical implications from our theoretical model. First, we compare the expected transaction price of an item between auctions with and without buy-out. The expected transaction price in the case without buy-out can be easily computed to be $\bar{v}/3$. The average transaction price, when buy-out price is v_b, is

$$\frac{2}{\bar{v}^2}[\int_0^{v_b}\int_0^{v_1} v_2 dv_2 dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^* v_1} v_2 dv_2 dv_1 + \frac{v_b}{2\mu^*}(\mu^*\bar{v}^2 - (1+\mu^*)v_b^2)]$$
$$= 2[\frac{v_b}{2} - \frac{(1+\mu^*)v_b^3}{3\mu^*\bar{v}^2}]. \tag{14}$$

The difference in expected transaction prices under optimal buy-out price, v_b^*, is thus

$$\frac{1}{3}[\bar{v} + \left(\frac{2(1+\beta)}{3+\beta} - 3\right)\ v_b^*]. \tag{15}$$

As a result, the expected transaction price is greater with buy-out option if and only if

$$\mu^* > \frac{(3+\beta)^3}{6\beta(6+\beta)+22}. \tag{16}$$

Note that $\mu^* \in [1,\infty)$ and $(3+\beta)^3/[6\beta(6+\beta)+22] \in (1, 27/22]$. That means the expected transaction price without buy-out can be greater only if μ^* falls in the narrow interval $[1, 27/22]$. (Even in this case the average transaction price with buy-out still has good chance to be greater). Consequently, for reasonable assumptions on the values of α and β, we will expect the transaction price to be greater for auctions having buy-out options than for ones without.[9] That is, unless in the extreme case when α is very close to 1 and

[9] For example, assuming that both α and β are uniformly distributed on $(0,1]$, then we can show that the probability that $\mu^* \geq 27/22$ (i.e., $\mu^* > (3+\beta)^3/[6\beta(6+\beta)+22]$ for *all* possible values of β) is 0.744.

β very close to 0 (i.e., the buyer is almost risk-neutral and the seller is very risk-averse), otherwise the transaction price is greater when there is buy-out option.[10] Since in the on-line auctions whether to set up a buy-out price is the option of the seller, if we look at auctions with identical objects, there will be ones that go with buy-out prices and those go without. The empirical implication for this fact and our discussions above is that, the average transaction price for items sold under buy-out (but not necessarily sold with buy-out price) will be greater than those without.

Second, by plugging the optimal value of v_b into (14), we can easily show that the transaction price is an increasing function of β. That means as the seller becomes more risk-averse, both the transaction price and the optimal buy-out price will be lower. This result has a strong empirical implication. If we assume that different sellers have different degree of risk-aversion, but they face the same pool of potential buyers, then different sellers will have different buy-out prices only if they differ in degree of risk-aversion. However, since both optimal buy-out price and average transaction price are decreasing in seller's degree of risk-averse, if we sample only those items which are sold without a reservation price (so that all items are sold), then the average transaction price will be in direct relationship with the buy-out price.

Third, for those items that come with buy-out options, we can also compare the average transaction price for items which are eventually sold under buy-out (regions II

[10] Note that when $\beta = 1$, the average transaction price is exactly the expected utility of the seller, $\pi(1)$. Although Proposition 1 shows that $\pi(1) \geq \pi^0(1)$, this does not prove that the transaction price is always higher with buy-out. It only shows that average transaction price is higher with buy-out *when the seller is risk-neutral*. In order to make the appropriate comparison, we have to do it for the general case when β is not necessarily 1.

and II') and those that are sold under competitive bidding (regions I and I'). It is obvious from the figure that the average price is higher in the former case. This fact has a strong empirical implication. Note that in an on-line auction, the length of time an item is put up for sale is fixed in advanced by the seller. The only possibility that the object is sold by a time shorter than this period is that someone buys it out. But since we already knows that the average transaction pirce is higher for items that are sold under buy-out, there should be an *inverse* relationship between the transaction price of an item and the time it takes to be sold. This is a fact that is in contrast to a usual ascending price auction.

There are thus three predictions from our model. First, for identical items the expected transaction price is higher under auctions with buy-out. Second, the expected transaction price is in positive relation with buy-out price. Third, the expected transaction price of an item is in inverse relation with the time it takes to be sold. In section 3, we will empirically test the three predictions.

3 Empirical Study

In this section, we perform two empirical studies to examine the three implications of our theoretical model. Our first test investigates whether the average transaction price in auctions with buy-out options is higher than that without the option. Our second test investigates whether in auctions with buy-out options, the average transaction price is increasing in buy-out price. They need to be tested separately mainly because they have different sample sizes. In both we also test our third implication, namely whether the transaction price of an item is in inverse relationship to the time it takes to be sold. Moreover, for every test we check for the robustness of our empirical results by eliminating the explanatory variables one at a time and going through the same estimating procedure. We also drop all the dummy variables and do the same estimation. None of these alternative specifications changes our results qualitatively.

3.1 Data Description and Variable Specification

We collect data from Taiwan's Yahoo! auction site during the period from April 1, 2005 to July 1, 2005. Our data contains 2182 observations (items) for the auction of digital cameras. There are totally 13 brands. Table 1 lists the sample distribution of these brands. Among these observations, we find that there are 11 whose buy-out prices (averaged at NT$ 111,253.9)[11] are well above the average of the rest of the observations (NT$ 10,070.3) to be credible buy-out prices.[12] We thus delete these outliners and have a total of 2171 observations.

Table 2 displays the bidding outcomes of our sample. Among the 2171 items, 1166 auctions resulted in a sale, while 1005 items remain unsold. Moreover, among the 1166 (1005) auctions that are eventually sold (remain unsold), 936 (805) have buy-out options, and 230 (200) have not. Thus, more than 80% of the items in our data are put up for sale with the buy-out option. Note that for the 936 items (with buy-out option) that resulted in a sale, there are 744 that are sold with buy-out prices, and the average transaction

[11] 1 U.S. dollar roughly equals 30 NT$.

[12] These outliers include Canon (6), Pentax (2), Casio (1), Fujifilm (1) and Nikon (1) with the number of cases shown in the parenthesis.

price equals to NT$ 9,674.874 (which is also the average buy-out price); the other 192 items are sold through the bidding process. The average transaction price in those cases is NT$ 6,293.33. (The average buy-out price is NT$ 7,859.) For the 230 items that are put up for sale without buy-out options, the average transaction price is NT$ 6,594.9. Finally, the average buy-out price for the 805 items that remain unsold is NT$ 10,963.02.

In our regressions, the dependent variables are whether the auction results in a sale or not (TRADE), and the transaction price (PRICE), if the item is sold. The former takes a value of one if the auction ends up with a deal, and is zero otherwise. The latter is equal to the buy-out price if it is sold with buy-out; otherwise it is the amount of winning bid.[13] For the explanatory variables, we include a number of controls such as BUYOUT (buy-out price), BUYOUTD (a dummy variable which equals to one if the auction has buy-out option; and is zero otherwise), LENGTH (the length of auction in terms of the number of days),[14] REP (seller's reputation, which is the accumulated ratings given by past buyers), NEW (a dummy variable with the value one if the auction subject is new; and is zero otherwise), and MINIBID(minimum price to start bidding, set by the seller).

The key variables in our test are buy-out price (BUYOUT) and the buy-out dummy (BUYOUTD). The buy-out dummy is used to test whether the average transaction price with buy-out option is higher than that without buy-out if an auction results in sale. If our theory is correct, it should have a positive coefficient. On the other hand, the buy-out price is used to verify whether the average transaction price is increasing in buy-out price

[13] The transaction price excludes shipping charge.

[14] Note that Length is zero, if the auction is ended within one day.

if we only look at the auctions with buy-out option.

In a usual ascending price auction, we expect that the longer the auction lasts, the higher will be the transaction price. However, this may not be the case for on-line auctions with buy-outs. This is because in an on-line auction the time period that an item is put up for sale is fixed in advanced. If no bidder buys out the object, then the seller will wait until that time period expires to determine who the winner is, and at what price the object is transacted. This implies that if it takes shorter time for the object to be sold, then it must be because some bidder buys it out. Given that our theoretical model has shown that the average transaction price is higher for the auction resulted in buy-out, we would expect there to be an inverse relationship between the transaction price and the time it takes until it is sold. That means the coefficient for LENGTH should be negative.

Previous work on the impact of a seller's reputation on the transaction price of an auction indicates that the returns to reputation is either insignificant or small if it exists.[15] One recent exception is Livingston (2005), who argued that previous studies underestimate the returns to reputation because they assumed that the relationship between the transaction price and the seller's reputation is linear or log-linear, and they might fail to control for sample selection bias. Using the Probit model and sample selection model, he showed that the probabilities of the auction receives a bid, the probability that the auction results in a sale, and the amount of the winning bid all increase substantially with the first few positive reports of the seller. However, marginal returns to additional positive reports beyond the first few are severely decreasing. Given this result, in order to

[15] See Bajari and Hortaçsu (2004).

control for the effect of reputation on transaction price, we also put a reputation variable in our regression.

Minimum bid is the lowest possible bid set by the seller for the buyer to start the bidding process. Higher minimum bid may lower the chance that an auction results in a sale by discouraging the buyer's incentive of placing a bid. As is explained in Livingston (2005), minimum bid should only affect the buyer's participation decision, but not the decision of how much to bid. In term of our theory in the Section 2, there may be some observed and unobserved factors that affect the participation decision, but not the fact that a buyer is willing to buy the object as long as price is lower than his valuation. Thus, minimum bid is used to serve as an exclusion restriction in the sample selection model in our second test.

3.2 Transaction Price and Buy-out

As is explained, our theoretical model predicts that the average transaction price of auctions with buy-out option (regardless whether items are sold through buy-out or bidding) is higher than those without. Accordingly, in our first test, the sample contains all 1166 items that resulted in a sale no matter whether the objects are sold with buy-out prices or the highest biddings.

A simple look at the data indicates that the average transaction price with buy-out option (NT$8,981.2)[16] is greater than that without buy-out (NT$ 6,594.9). We first carry out a simple t-test to test the difference between their mean values. The null hypothesis is

[16] NT$ $9,674.9 \times \frac{744}{936}$+NT$ $6,293.3 \times \frac{192}{936}$ =NT$ 8,981.2.

that average transaction price with buy-out option is greater than that without buy-out. The value of t-statistics is 5.364, and the null hypothesis cannot be rejected at the 1% significance level. Given this, we then set up an empirical model that controls for the relevant variables.

Because the dependent variable (the log of the transaction price) is censored at 5.347 (left-censored observation) and 11.002 (right-censored observation), the appropriate model to use is a Tobit model. The data set we use contains the 1166 items that resulted in a sale in our sample. Also, we use the following explanatory variables to control for the influence of other factors on the transaction price: BUROUTD (a dummy variable which is equal to one if the seller of the auction set up a buy-out option; and zero otherwise), LENGTH (the length of an auction in terms of the number of days), REP (accumulated ratings of the seller given by past buyers), NEW (a dummy variable with the value one if the auction subject is new; and zero otherwise). We do not include minimum bid in the equation. This is because minimum bid affects the probability that the auction results in a sale or not, but not the transaction price.[17] Besides, we add twelve dummy variables (one for each brand) to control for the effects of brand names on the winning bids. Note that Kyocera digital camera is chosen as the base brand because it has the smallest number of items. Summary statistics of the related variables are displayed in Table 3.

The estimation results of the Tobit model by a censored-normal maximum likelihood estimation procedure are presented in Table 4.[18] The coefficient on the buyout dummy is

[17] More on this in the next sub-section.

[18] We use STATA Release 8 to estimate the empirical models in this paper.

positive and significant at the one percent level, confirming our prediction that the average transaction price for auctions with buy-out options are greater than those without.

The sign of the coefficient of other explanatory variables are also consistent with our theory. For example, the coefficient for the variable LENGTH is negative and significant, indicating that the transaction price of an object is in inverse relationship with the time it takes to be sold. This is a result that is in stark contrast with the usual ascending price auctions. The force that drives this result is, naturally, that an item is sold earlier only when some bidder buys it out, and our theoretical calculation shows that average transaction price for items that are eventually sold with buy-out is greater than those that are eventually sold under the winning bidding.

Consistent with Livingston (2005), our result also shows that good reputation of the seller has statistically and economically significant effects on the level of the transaction price.

3.3 Transaction Price and Buy-out Price

The second empirical implication of our theoretical model is that the value of the winning bid, given that a deal occurs, will be higher, the higher the buy-out price. Our sample consists of the 1741 cases with buy-out option no matter the auction ends up a deal or not. That is, we include the 936 items with buy-out option that result in sale, plus the 805 items with buy-out option but did not result in a sale. Our theoretical model assumes zero reservation price, thus the auction will always result in a sale. But in reality there exists a non-zero probability that the auction fails to reach a deal if the amount of the highest bid is less than the reservation price. In such cases, we do not observe a transaction price. Therefore, a sample selection model is adopted, and is specified as follows.[19]

$$P_j^* = x_j\beta + \mu_j, \tag{17}$$

$$W_j^* = z_j\alpha + v_j, \tag{18}$$

$$P_j = \begin{cases} P_j^* \ \text{ if } \ W_j^* > M_j, \ j = 1, 2, ..., n. \\ M_j \ \text{ if } \ W_j^* \leq M_j, \end{cases} \tag{19}$$

We call (17) the regression equation, and (18) the selection equation. In the specifications, P_j^* is the value of the transaction price in auction j, W_j^* is the bidding amount. Since some items are not sold, we will use P_j to denote the transaction price (P_j^*) if it is sold, and the minimum bid M_j if it is not. That is, since reservation price is not observable, minimum bid is used as a proxy, even though in some cases, a winner's bid is greater than the minimum bid does not necessarily result in a sale. (μ_j, v_j) are assumed to be iid normal with zero mean and variances σ^2 and 1, and correlation ρ. Also, P_j^* and W_j^* are assumed to be a linear function of observed variable x and z respectively.

The independent variable x includes BUYOUT, LENGTH, REP, and NEW. The observed variable in selection equation, z, contains LENGTH, REP, NEW and MINIBID. It is worthing noting that the explanation variable BUYOUT appears only in the regression equation but not the selection equation. The reason is that the buyout price specified by

[19] See Livingston (2005) for the reasons of using sample selection model rather than the censoring model such as Tobit model. In his paper, however, the purpose is to study the effects of a seller's reputation on bidders' participation decisions and on the decision of how much to bid.

a seller should affect the level of transaction price but not the probability that the auction results a sale.[20]

The dependent variable in the regression equation is the transaction price if the auction results in a sale. Otherwise, it is the minimum bid.[21] The dependent variable in the selection equation is a dummy variable which takes the value of one if the auction results in a sale, and zero otherwise. It is worth noting that, same in the case of buy-out option, the variable MINIBID affects the probability that the auction results in a sale but not the winning bid. Higher minimum bid lowers the probability that an auction ends up with a sale by decreasing the incentive of placing a bid, but not the transaction price. Thus, minimum bid is used to serve as an exclusion restriction in the sample selection equation, but not the regression equation. As in Section 3.2, we also control for the impact of different brands on the probability that the auction results in a sale and the level of transaction price by adding twelve dummy variables to the selection and regression equations. The base brand is Kyocera since it has the smallest number of auction subjects. Table 5 presents the summary statistics of the related variables.

The empirical model of sample selection contains a correlation term ρ. As long as ρ is not zero, OLS is biased. We thus use full information maximum likelihood to estimate the sample selection model. As a result, the estimates are consistent and asymptotically

[20] An item will be sold if and only if one of the bidders' valuation is greater than the reservation price. Since the price at which a bidder is willing to buy out an item must always be smaller than his valuation, buy-out option only affects the possible prices the item is eventually sold, but not its probability of being sold.

[21] As pointed out by Livingston (2005), setting the transaction price equal to the minimum bid when the auction did not result in a sale has no effect on the likelihood function, and merely represents the event that the bidding amount is less than the minimum bid. See also Amemiya (1985).

efficient. Table 6 shows the results of our estimation.

The standard errors appear in parentheses of Table 6 has been adjusted for clustering at the brand level of digital camera. The estimated value of ρ is small (0.10), but the Wald test rejects the null hypothesis that ρ is equal to zero at the one percent level of significance. Note that the coefficient on the buy-out price is positive and significant at the one percent level, indicating that if we consider the auctions with buy-out option, the average transaction price is increasing in buy-out price no matter the auction results in a sale or not. This result confirms the prediction of our theoretical model. Furthermore, the coefficient on the minimum bid is negative and significant at the one percent level. Therefore, a higher minimum bid, which is used as an exclusion restriction in the sample selection model, reduces the probability that the auction ends up with a deal. Finally, the signs of all other explanatory variables are consistent with that of the Tobit model in Section 3.2.

4 Conclusion

In this paper we propose a dynamic model of auction with buy-out option, in which both the seller and bidders are risk-averse. We completely characterize the optimal bidding strategy of the bidder and the optimal buy-out price of the seller. The seller is shown to benefit from the buy-out option from two sources. He can either use it to exploit the bidder's aversion to price risk in the bidding process, or to reduce risk in the bidding process for himself. In contrast to the literature, the buy-out option benefits the seller even if the bidders are risk-neutral. Since buy-out is also used as an additional instrument to intensify the competition between the bidders, unless a bidder has a high valuation of the item, otherwise he is worse off when there is buy-out.

Our model predicts three testable implications. First, the transaction price of an identical object will be higher when there is buy-out. Second, the transaction price of an item is in direct relation wiht its buy-out price. Third, the longer it takes for an item to be sold, the lower will be its transaction price, which is opposite to what is expected from a usual ascending price auction. All predictions are confirmed by the data we collected from Taiwan's Yahoo! on-line auction site.

An interesting option in auctions that is omitted from our model is the reserve price. In contrast to the buy-out price, which essentially sets an upper-bound on the possible transaction prices, the reserve price sets a lower bound. Moreover, like the buy-out price, the reserve price can also serve as a strategic instrument for the seller. That is, the seller can strategically set a reserve price to increase his revenue. When a reserve price consideration is incorporated, our model becomes substantially complicated, and requires major modification. But it also points to a promising direction for future research.

Appendix A: Uniqueness of Solution

In this appendix we show that the solution we find in Section 2.1 is the unique solution satisfying the initial condition $v(v_b) = v_b$. That is, $x = \mu^* y$ is the only solution of (4) that passes through $(0, 0)$. Rewrite (4) as $(\frac{x}{y} + 1)^\alpha - 1 = \frac{dy}{dx}$. Then by the definition of μ^* we know that for all points on line $x = \mu^* y$, $\frac{dy}{dx} = \frac{1}{\mu^*}$, i.e., direction of change points to the origin. Figure A1 depicts the phase diagram of solution of (4).

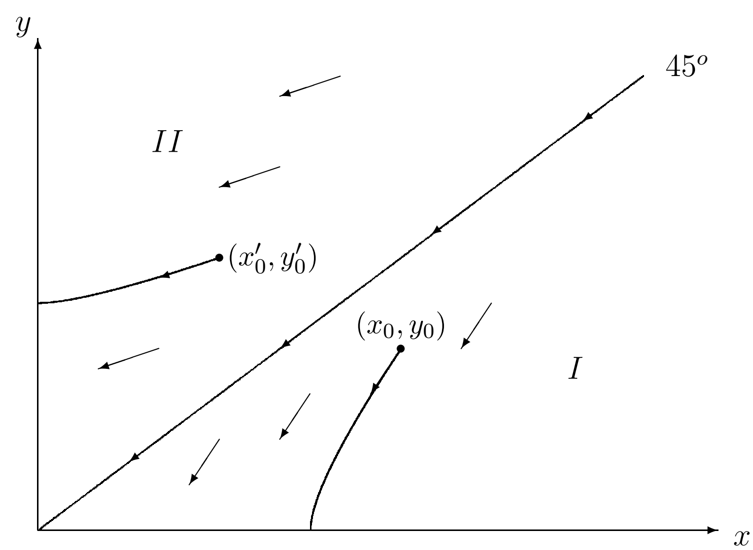

Figure A1

Consider a solution of (4) that passes through a point (x_0, y_0) in region I. Since $\frac{x_0}{y_0} > \mu^*$, $(\frac{x_0}{y_0} + 1)^\alpha - 1 > (\mu^* + 1)^\alpha - 1 = \frac{1}{\mu^*}$ by definition of μ^*. But a solution passing (x_0, y_0) must satisfy (4), namely, $(\frac{x_0}{y_0} + 1)^\alpha - 1 = \frac{dy}{dx}|_{(x_0,y_0)}$. That means $\frac{dy}{dx}|_{(x_0,y_0)} > \frac{1}{\mu^*}$. By the same argument we know that $\frac{dy}{dx}|_{(x_0,y_0)} < \frac{1}{\mu^*}$ for any solution passing (x_1, y_1) in region II. These are shown in Figure A1. The result implies that any solution of (4) not on the line $x = \mu^* y$ will not pass through $(0, 0)$. It is then obvious that, among solutions of (4), only the solution $x = \mu^* y$ can pass through $(0, 0)$, i.e., only $v(v_b) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}$

satisfies the boundary condition $v(v_b) = v_b$.

Appendix B: Proof of Proposition 2

The expected utility of a buyer with valuation v, when there is no buy-out option, is $\int_0^v \frac{1}{\bar{v}} \frac{(v-x)^\alpha}{\alpha}\, dx = v^{\alpha+1}/\alpha(\alpha+1)$. In the case with buy-out, the way to calculate buyer's expected utility depends on the value of v. For $v < v_b^*$, a buyer's expected utility is identical to the case without buy-out. If $v_b^* \le v < (1+\frac{1}{\mu^*})v_b^*$, the buyer's expected utility is

$$\int_0^{(1+\mu)v_b-\mu v} \frac{1}{\bar{v}} \frac{(v-x)^\alpha}{\alpha}\, dx + \int_{(1+\mu)v_b-\mu v}^{v} \frac{1}{\bar{v}} \frac{(v-v_b)^\alpha}{\alpha}\, dx$$
$$+ \frac{1}{\bar{v}} \frac{v^{\alpha+1}}{\alpha(\alpha+1)} - \frac{1}{\bar{v}} \frac{(1+\mu)^{\alpha+1}(v-v_b)^{\alpha+1}}{\alpha(\alpha+1)} + \frac{1}{\bar{v}} \frac{(v-v_b)^{\alpha+1}}{\alpha}(1+\mu). \tag{20}$$

The difference in utilities between the cases with and without buy-out is

$$\frac{1}{\bar{v}} \frac{(1+\mu)^{\alpha+1}(v-v_b)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{1}{\bar{v}} \frac{(v-v_b)^{\alpha+1}}{\alpha}(1+\mu)$$
$$= \frac{1}{\alpha\bar{v}}(v-v_b)^{\alpha+1}(1+\mu)[\frac{(1+\mu)^\alpha}{\alpha+1} - 1]$$
$$= \frac{1}{\alpha\bar{v}}(v-v_b)^{\alpha+1}(1+\mu[\frac{\frac{1}{\mu}-\alpha}{\alpha+1}]), \tag{21}$$

where the last equality comes from the fact that $(1+\mu)^\alpha = 1+\frac{1}{\mu}$. Since $v > v_b^*$, the sign of (21) depends on that of $\frac{1}{\mu} - \alpha$. From the fact that $(1+\mu)^\alpha = 1+\frac{1}{\mu}$, we have $\alpha = \log(1+\frac{1}{\mu})/\log(1+\mu)$. Thus

$$\frac{1}{\mu} - \alpha = \frac{1}{\mu} - \frac{\log(1+\frac{1}{\mu})}{\log(1+\mu)}$$
$$= \frac{1}{\mu\log(1+\mu)}\left[\log(1+\mu) - \mu\log(1+\frac{1}{\mu})\right]$$

$$= \frac{1}{\mu \log(1+\mu)} \left[(1-\mu)\log(1+\mu) + \mu \log(\mu) \right]. \tag{22}$$

First note that $g(\mu) \equiv (1-\mu)\log(1+\mu) + \mu\log(\mu) = 0$ when $\mu = 1$. Moreover,

$$\begin{aligned} g'(\mu) &= \frac{2}{1+\mu} + \log\ \mu - \log(1+\mu) \\ &> \frac{1}{\mu} + \log\ \mu - \log(1+\mu); \end{aligned} \tag{23}$$

which is positive by the concavity of the log function. This implies that, $\frac{1}{\mu} - \alpha > 0$ and, thus, the expected utility of the buyer is lower when there is buy-out.

In the case when $v \geq (1 + \frac{1}{\mu})v_b$, the expected utility with buy-out is

$$\int_0^v \frac{1}{\bar{v}} \frac{(v - v_b)^\alpha}{\alpha}\ dx = \frac{1}{\bar{v}} \frac{(v - v_b)^\alpha}{\alpha} v. \tag{24}$$

We thus have

$$\frac{1}{\bar{v}} \frac{v^{\alpha+1}}{\alpha(\alpha+1)} \Big/ \frac{1}{\bar{v}} \frac{(v - v_b)^\alpha}{\alpha} v = \frac{v^\alpha}{(\alpha+1)(v - v_b)^\alpha}. \tag{25}$$

Plugging $v_b = \sqrt{\frac{\mu(1+\beta)}{(1+\mu)(3+\beta)}}$ into (25), setting (25) to be 1, and solving for v we have

$$\frac{v}{\bar{v}} = \frac{\sqrt{(\frac{\mu}{1+\mu})(\frac{1+\beta}{3+\beta})}}{1 - (1+\alpha)^{\frac{-1}{\alpha}}} \equiv v_c(\alpha, \beta). \tag{26}$$

Since (25) is decreasing in v, we know that (25) is smaller than 1 (i.e., a bidder has higher expected utility with buy-out) if and only if $v \geq v_c(\alpha, \beta)$. QED

References

[1] Amemiya, T. (1985), *Advanced Econometrics*, Basil Blackwell, Oxford, UK.

[2] Bajari, P. and A. Hortaçsu (2004), "Economic Insights from Internet Auctions: A Survey," *Journal of Economic Literature*, 42, 457-86.

[3] Budish, Eric B. and Lisa N. Takeyama (2001), "Buy Price in Online Auctions: Irrationality on the Internet?" *Economics Letters*, 72, 325-33.

[4] Dodonova, A. and L. Khoroshilov (2004), "Anchoring and Transaction Utility: Evidence from on-line Auctions," *Applied Economics Letters*, 11, 307-10.

[5] Kirkegaard, R. and P.B. Overgaard (2004), "Buy-Out Prices in Auctions: Seller Competition and Multi-Unit Demand," Working Paper, University of Aarhus, Denmark.

[6] Livingson, J.A. (2005), "How Valuable is a Good Reputation? A Simple Selection Model of Internet Auctions," *Review of Economics and Statistics*, 87, 453-65.

[7] Lopomo, G. (1998), "The English Auction is optimal Among Simple Sequential Auctions," *Journal of Economic Theory*, 82, 144-66.

[8] Lucking-Reiley, D. (2000), "Auctions on the Internet: What's Being Auctioned and How," *Journal of Industrial Economics*, 48, 227-52.

[9] Mathews, T. (2004), "The Impact of Discounting on an Auction with a Buy-out Option: a Theoretical Analysis Motivated by eBay's But-It-Now Feature," *Journal of Economics*, forthcoming.

[10] Mathews, T. and B. Katzman (2006), "The Role of Varying Risk Attitudes in an Auction with a Buyout Option," *Economic Theory*, 27, 597-613.

[11] Reynolds, S. and J. Wooders (2004), "Auctions with a Buy Price" draft, University of Arizona, Tucson.

[12] Wang, X., A.L. Montgomery and K. Srinivasan (2004), "When Auction Meets Fixed Price: A Theoretical and Empirical Examination of Buy-It-Now Auctions," mimeo.

Table 1. Sample Distribution of Digital Camera.

Brand Name	Number of Observations
BenQ	124
Canon	336
Casio	215
Fujifilm	407
Kodak	79
Konica	137
Kyocera	21
Nikon	315
Olympus	59
Panasonic	232
Pentax	177
Ricoh	28
Sanyo	52
Total	2,182

Table 2. Bidding Outcome

<table>
<tr><td colspan="5">Total number of observations (2,171)</td></tr>
<tr><td colspan="3">Auction resulted in a sale (1,166)</td><td colspan="2">Auction did not result in a sale (1,005)</td></tr>
<tr><td colspan="2">Auctions with buyout option (936)</td><td>Auctions without buyout option (230)</td><td>Auctions with buyout option (805)</td><td>Auctions without buyout option (200)</td></tr>
<tr><td>Auctions with buyout price (744)</td><td>Auctions without buyout price (192)</td><td></td><td></td><td></td></tr>
<tr><td>Average transaction price: NT$9,674.874
Average buyout price: NT$9,674.874</td><td>Average transaction price: NT$6,293.33
Average buyout price: NT$7,859</td><td>Average buyout price: NT$6,594.9</td><td>Average buyout price: NT$10,963.02</td><td></td></tr>
</table>

Table 3. Summary Statistics of Related Variables in Tobit Model

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	608.76	1,161.17	-7	5,734
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.5214	.4998	0	1
BUYOUT	Buyout price	9,302.59	6,176.09	230	5,999
BUYOUTD	A dummy variable with the value one if the auction has buyout option; zero otherwise.	.8027	.3981	0	1
MINIBID	Minimum bid	7,894.52	6,233.43	1	59,999
LENGTH	Length of auction in terms of the number of days	6.59	3.23	0	11
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	1	0	1	1
PRICE	Transaction price	8,510.51	6,117.06	210	59,999
	Number of Observations		1,166		

Table 4. Regression Results of Tobit Model

Independent Variable	Transaction Price Equation
Constant	8.306065
	(.1844098)
Buyout Dummy	.1442162***
	(.0455197)
Reputation	.0000524***
	(.000016)
Length of Auction	-.0129374**
	(.0053768)
New Subject Dummy	.4079538***
	(.0395794)
Brand Dummy 1	-.3628905**
	(.1844767)
Brand Dummy 2	.4588292***
	(.1782668)
Brand Dummy 3	.3584773**
	(.1821942)
Brand Dummy 4	.1053558
	(.1773482)
Brand Dummy 5	-.1237965
	(.1947197)
Brand Dummy 6	.4301358**
	(.184843)
Brand Dummy 8	.3685628**
	(.1791298)
Brand Dummy 9	.1447664
	(.1946923)
Brand Dummy 10	.445412**
	(.1816543)
Brand Dummy 11	.2711898
	(.183737)
Brand Dummy 12	.0626323
	(.2234455)
Brand Dummy 13	.4463631**
	(.2041104)
Number of Observations	1,166

Notes: standard errors appear in parentheses.

*denote significance at the 10% level, ** at the 5% level,*** at the 1% level.

Table 5. Summary Statistics of Related Variables in Sample Selection Model

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	544.23	1,054.45	-25	5,806
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.6238	.4846	0	1
BUYOUT	Buyout price	10,070.34	6,472.57	100	65,250
BUYOUTD	A dummy variable with the value one if the auction has buyout option; zero otherwise.	1	0	0	1
MINIBID	Minimum bid	9,429.14	6,606.35	1	65,000
LENGTH	Length of auction in terms of the number of days	7.502	3.1326	0	11
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	.5376	.4987	0	1
PRICE	Transaction price	9,726.26	6,441.09	100	65,000
	Number of Observations		1,741		

Table 6. Regression Results of Sample Section Model

Independent Variable	Transaction Price Equation	Selection Equation
Constant	143.6738	1.791904
	(148.5132)	(.1528565)
Buyout Price	.9606718***	
	(.0123787)	
Reputation	.0553453**	.0002859***
	(.0227461)	(.000025)
Length of Auction	-81.8251***	-.1915817***
	(19.53664)	(.0162607)
New Subject Dummy	571.2208***	-.2685813***
	(84.27094)	(.0937472)
Minimum Bid		-.0000312***
		(7.81e-06)
Brand Dummy 1	364.5755***	.5368261***
	(35.81291)	(.0336813)
Brand Dummy 2	77.82769	.1482385***
	(82.10603)	(.0384809)
Brand Dummy 3	21.77229	-.1482891***
	(85.7244)	(.0338344)
Brand Dummy 4	-172.1399***	.0376227
	(49.23785)	(.0255614)
Brand Dummy 5	216.2498***	.0352377***
	(56.57014)	(.0183974)
Brand Dummy 6	165.2127**	.2327526***
	(82.28988)	(.0505312)
Brand Dummy 8	62.48953	.0235395
	(70.75916)	(.0346171)
Brand Dummy 9	-27.36123	.5640501***
	(41.92824)	(.0152641)
Brand Dummy 10	27.95332	-.025568
	(85.95286)	(.0446228)
Brand Dummy 11	54.42857	.0815529***
	(69.90363)	(.0316344)
Brand Dummy 12	-10.38036	.4683582***
	(72.79341)	(.0359326)
Brand Dummy 13	124.0816	.0661565*
	(107.8573)	(.036175)
Number of Observations	1,741	1,741

Notes: standard errors adjusted for clustering at the brand level of digital camera appear in parentheses. * denote significance at the 10% level, ** at the 5% level, *** at the 1% level.