

# Privatization and Efficiency Gain in an International Mixed Oligopoly with Asymmetric Costs

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## Abstract

This paper examines the two policy instruments, privatization of the domestic public firm and imposition of a tariff on the foreign firm's output, which are undertaken by the domestic government to enhance domestic welfare in an international mixed oligopoly model with asymmetric costs. It focuses on the impacts of the order of moves and the efficiency gain by privatization, and shows that different order of moves of firms will imply different government decisions on whether to privatize the public firm. In particular, the efficiency gain that highlights the importance of foreign competition is crucial in determining welfare-improving privatization policy.

**Keywords:** Mixed Oligopoly; Privatization; Efficiency Gain; Leadership; Asymmetric Costs.

**JEL Classification:** D43; L13; L33.

## 1. Introduction

Many empirical observations suggest that the public firms in many developing countries run less efficient than private firms, especially less than the foreign private firms.<sup>1</sup> No one will deny that privatization has a significant impact on social welfare and as a result, it is not surprising that the privatization policy has received a growing attention in economic research. An important objective of privatization must be allocative efficiency.<sup>2</sup> Sappington and Stiglitz (1987) describe a privatization procedure which implements the efficient allocation and demonstrate that a privatized firm should always be at least as efficient as a public firm. Though empirical observations and the mixed oligopoly literature do suggest that there are some gains in the efficiency of a privatized firm, but for the broad researchers in the fields of industrial organization, international trade, and/or development economics, especially who are interested in the privatization of public firms may want to know how the competition between more foreign private firms and the public firm affects the desirability of privatization of the public firm.

In early studies of the mixed oligopoly markets (Fershtman, 1990; De Fraja and Delbono, 1989 and 1990), the analysis is mostly based on the framework in which welfare-maximizing public firms compete against profit-maximizing private firms in a closed economy. It was noted by Matsumura (1998) that in fact partially privatized are quite common in many countries. Lee and Hwang (2003) allow managerial

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<sup>1</sup> See Vickers and Yarrow (1991) and Megginson and Netter (2001) for discussions and surveys on the concerned issues.

<sup>2</sup> Privatization also helps the government pursue other objectives, including revenue generation, employment promotion and foreign investment encouragement. Martimort (2006) provides an agency perspective on the costs and benefits of privatization.

inefficiency in Matsumura's (1998) model and argue that neither full privatization nor full nationalization is optimum. In the real world, along with the wave of trade liberalization, more foreign firms' entering the domestic market is prevalent. Industries such as oil, electricity, telecommunication and postal sector are usually occupied by public monopolies in developing countries. To promote free trade and foster economic growth, the multilateral agreements within the GATT/WTO framework have inspired the foreign firms' entry into the market of developing countries. The modeling of mixed oligopoly with foreign competitors began with Fjell and Pal (1996) who studied the effect of introducing foreign private firms on the equilibrium price and allocation of production.<sup>3</sup>

An interesting issue is to investigate privatization policy in an international mixed oligopoly with cost asymmetry. In a seminal paper, Pal and White (1998) explored the interactions between privatization and strategic trade policies with and without efficiency gain. Pal and White (1998) argued that "cost asymmetry between the public and the private firms does not give rise to significantly different results than those obtained under the assumption of symmetric costs" (p. 278). Recently, Chang (2005) adopt Matsumura (1998) and analyzes the optimal trade and privatization policies in an international mixed duopoly in which a public firm competes with a more efficient foreign private firm, and discusses how the degree of cost asymmetry between foreign and domestic firms affects the ownership structure in the context of strategic trade policy. Efficiency gap is also allowed in Tomaru (2007), which

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<sup>3</sup> See Chao and Yu (2006), Bárcena-Ruiz and Garzón (2006) and Matsushima and Matsumura (2006) for introducing the mixed oligopoly model into the field of international trade, environmental policy and location choice, respectively.

examines efficiency gain by cost-reducing R&D investment and how the decision on R&D investment is affected by privatization.

Most quantity-setting studies concerned with the case that firms move simultaneously. Nevertheless, economists often debate the relative merits of the order of moves. Sherali (1984) argues that a single firm is always better off as a Stackelberg leader with all the other firms having declared their strategy. Fjell and Heywood (2002) explore the equilibrium effects of Stackelberg leadership by a public firm, and discuss the effect of an open market policy allowing foreign and domestic firms to compete, while Matsumura (2003) studies the endogenous timing of a mixed duopoly and shows that the public firm evolves to become the leader in the presence of foreign competition. Lu (2006) examines endogenous timing in a mixed oligopoly with both domestic and foreign private firms, and proves that the domestic public firm does not move in advance of all foreign private firms. Lu (2007) also investigates endogenous timing in an international mixed oligopoly in the absence of domestic private firms and shows that foreign leadership is a SPNE. Hence, it is not unreasonable to allow firms to move sequentially or simultaneously.

This contribution investigates a mixed oligopoly model with  $m$  foreign private firms and one domestic public firm or one domestic private firm when the public firm is privatized, and within such context the import tariff is endogenized.<sup>4</sup> The public firm is assumed to be less efficient than the private firms. We want to emphasize that

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<sup>4</sup> The mixed oligopoly model that includes both domestic and foreign private firms is presented in Fjell and Pal (1996), Pal and White (1998), Lu (2006), etc. In order to simplify the model and emphasize the role of asymmetric cost structure, as in Matsumura (2003), Bárcena-Ruiz and Garzón (2003), Chang (2005), Chao and Yu (2006) and Lu (2007), we do not consider domestic private firms.

in our paper, privatization reduces production cost of the domestic firm, while Chang (2005) assumes that privatization does not affect cost structure. This paper considers three time structure: domestic leadership, foreign leadership, and simultaneous moves, and two situations (either the domestic firm is public or private). In particular, we discuss how the number of foreign firms affects the desirability of privatization of the public firm, and find that the relationship between the number of foreign private firms and the desirability of privatization crucially depends on the time structure.<sup>5</sup>

The remainder of this paper is structured as follows. Section 2 presents an international mixed oligopoly model with asymmetric costs and provides its solutions under three types of the order of moves. Section 3 analyzes the decision on whether to privatize the domestic public firm, and in Section 4 we explore the relationship among foreign competition, orders of moves and privatization. Conclusions are drawn in section 5.

## 2. The Model and Its Solutions

We consider a single market that made up of one domestic public firm and  $m$  foreign private firms ( $m \geq 1$ ) producing a homogeneous good. Let  $q_0$  be the output of the domestic public firm,  $q_i$  be the output of the  $i$ 'th foreign private firm ( $i = 1, \dots, m$ ). The inverse demand function for the homogeneous good is given by  $P = a - Q$ , where  $Q = q_0 + \sum_i q_i$  is the total output of the market. As a consequence, total consumer surplus from the market is  $CS = (q_0 + \sum_i q_i)^2 / 2$ . The cost function of

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<sup>5</sup> Regarding the competitiveness of the market, another reasonable approach is endogenizing the number of private firms as adopted by Anderson *et al* (1997), Matsumura and Kanda (2005), and Fujiwara (2007). They analyzed the welfare-improving privatization policy in the long-run equilibrium in which free entry and exit prevail and all private firm's profit is driven to zero, and within such context the number of private firms is then endogenously determined.

any private firms is  $C(q) = F + q^2/2$ . To simplify the exposition of the results but without the loss of generality, we assume that  $F=0$ .

As well known, the public firm is usually less efficient than private firms. In this paper, we deal with the case in which public and private firms differ in efficiency. Following Pal and White (1998), we assume that before privatization, the public firm has the cost function  $C(q_0) = gq_0^2/2$ . The assumption  $g \geq 1$  indicates that there is an efficiency gap between the public firm and foreign competitors, and the foreign private firms are more cost efficient than the public firm. Furthermore, we allow that privatization improves the public firm's efficiency to the level of private firms.

The government imposes a specific tariff,  $t$ , per unit of foreign firm's output. Its goal is to maximize domestic social welfare, which comprises the consumer surplus,  $CS$ , the profit of domestic public firm,  $\pi_0$ , and tariff revenues. Thus, the domestic social welfare can be expressed as:

$$W = CS + \pi_0 + t \sum_i q_i . \quad (1)$$

The profit functions of domestic public firm and each foreign private firm are:

$$\pi_0 = [a - (q_0 + \sum_i q_i)]q_0 - gq_0^2/2 \quad (2)$$

and

$$\pi_i = [a - (q_0 + \sum_i q_i)]q_i - tq_i - q_i^2/2 . \quad (3)$$

Public and private firms have different objectives: the former maximizes domestic social welfare while the latter maximizes its own profit. However, if the public firm is privatized, there are  $1+m$  private firms competing in the market.

For sake of comparison, we posit three cases. In case I, the domestic firm is a

leader; i.e. it decides its output in advance of foreign private firms. In case II, foreign private firms are leaders. In case III, we consider the classical Cournot competition case; i.e. all firms decide their outputs simultaneously. We restrict our attention to symmetric equilibrium in which each private firm has the same output level.

### **Case I: Domestic Leadership**

We propose a three-stage game with the following timing. In the 1<sup>st</sup> stage, the government selects the tariff rate to maximize domestic social welfare. In the 2<sup>nd</sup> stage, the domestic firm decides its output. In the 3<sup>rd</sup> stage, each foreign private firm decides its output to maximize its own profit.

### **Mixed Oligopoly**

Preprivatization, the domestic public firm seeks for social welfare maximization. We obtain the following results, where the superscript *MD* denotes mixed oligopoly under domestic leadership.

**Lemma 1.** *In an international mixed oligopoly with asymmetric costs and domestic leadership, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*<sup>6</sup>

$$t^{MD} = \frac{2ag}{4 + g(4 + m)}, \quad q_0^{MD} = \frac{4a}{4 + g(4 + m)}, \quad q_i^{MD} = \frac{ag}{4 + g(4 + m)},$$

$$Q^{MD} = \frac{a(4 + gm)}{4 + g(4 + m)}, \quad CS^{MD} = \frac{a^2(4 + gm)^2}{2[4 + g(4 + m)]^2}, \quad \pi_0^{MD} = \frac{8a^2g}{[4 + g(4 + m)]^2},$$

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<sup>6</sup> Derivation for lemma 1 is provided in appendix. Lemmas 2-6 can then be obtained by using the similar method. Details for the derivation of the following lemmas are available upon request.



$$\pi_i^{MD} = \frac{3a^2 g^2}{2[4 + g(4 + m)]^2}, \quad W^{MD} = \frac{a^2(4 + gm)}{2(4 + 4g + gm)}.$$

### Private Oligopoly

Postprivatization, the domestic firm seeks for profit maximization. We obtain the following results, where the superscript *PD* denotes private oligopoly under domestic leadership.

**Lemma 2.** *In an international private oligopoly with domestic leadership, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*

$$t^{PD} = \frac{2a(14 + 8m + m^2)}{(4 + m)(18 + 10m + m^2)}, \quad q_0^{PD} = \frac{4a(2 + m)(3 + m)}{(4 + m)(18 + 10m + m^2)}$$

$$q_i^{PD} = \frac{a(10 + 6m + m^2)}{(4 + m)(18 + 10m + m^2)}, \quad Q^{PD} = \frac{a(6 + 6m + m^2)}{18 + 10m + m^2},$$

$$CS^{PD} = \frac{a^2(6 + 6m + m^2)^2}{2(18 + 10m + m^2)^2}, \quad \pi_0^{PD} = \frac{8a^2(2 + m)(3 + m)^2(6 + m)}{(4 + m)^2(18 + 10m + m^2)^2}$$

$$\pi_i^{PD} = \frac{3a^2(10 + 6m + m^2)^2}{2(4 + m)^2(18 + 10m + m^2)^2}, \quad W^{PD} = \frac{a^2(32 + 34m + 10m^2 + m^3)}{2(4 + m)(18 + 10m + m^2)}.$$

### Case II: Foreign Leadership

We propose a three-stage game with the following timing. In the 1<sup>st</sup> stage, the government selects the tariff rate to maximize domestic social welfare. In the 2<sup>nd</sup> stage, each foreign private firm decides its output to maximize its own profit. In the 3<sup>rd</sup> stage, the domestic firm decides its output. The superscripts *MF* and *PF* denote mixed oligopoly and private oligopoly under foreign leadership, respectively.

### Mixed Oligopoly

**Lemma 3.** *In an international mixed oligopoly with asymmetric costs and foreign leadership, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*

$$t^{MF} = \frac{2ag}{(1+g)(4+m)}, \quad q_0^{MF} = \frac{a}{1+g}, \quad q_i^{MF} = \frac{ag}{(1+g)(4+m)}, \quad Q^{MF} = \frac{a(4+gm+m)}{(1+g)(4+m)},$$
$$CS^{MF} = \frac{a^2(4+gm+m)^2}{2(1+g)^2(4+m)^2}, \quad \pi_0^{MF} = \frac{a^2g(4-m)}{2(1+g)^2(4+m)}, \quad \pi_i^{MF} = \frac{3a^2g^2}{2(1+g)^2(4+m)^2},$$
$$W^{MF} = \frac{a^2[4+m+g(4+gm+m)]}{2(1+g)^2(4+m)}.$$

### Private Oligopoly

**Lemma 4.** *In an international private oligopoly with foreign leadership, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*

$$t^{PF} = \frac{7a(3+2m)}{54+51m}, \quad q_0^{PF} = \frac{2a(3+2m)}{18+17m}, \quad q_i^{PF} = \frac{5a}{18+17m}, \quad Q^{PF} = \frac{3a(2+3m)}{18+17m},$$
$$CS^{PF} = \frac{9a^2(2+3m)^2}{2(18+17m)^2}, \quad \pi_0^{PF} = \frac{6a^2(3+2m)^2}{(18+17m)^2}, \quad \pi_i^{PF} = \frac{25a^2(3+4m)}{6(18+17m)^2},$$
$$W^{PF} = \frac{a^2(24+31m)}{6(18+17m)}.$$

### Case III: Cournot Competition

We propose a two-stage game with the following timing. In the 1<sup>st</sup> stage, the government selects the tariff rate to maximize domestic social welfare. In the 2<sup>nd</sup> stage,

each foreign private firm and domestic firm decide their outputs simultaneously. The superscripts  $MC$  and  $PC$  denote mixed oligopoly and private oligopoly under Cournot competition, respectively.

### Mixed Oligopoly

**Lemma 5.** *In an international mixed oligopoly with asymmetric costs and Cournot competition, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*

$$t^{MC} = \frac{2ag}{(1+g)(4+m)}, \quad q_0^{MC} = \frac{a}{1+g}, \quad q_i^{MC} = \frac{ag}{(1+g)(4+m)}, \quad Q^{MC} = \frac{a(4+gm+m)}{(1+g)(4+m)},$$

$$CS^{MC} = \frac{a^2(4+gm+m)^2}{2(1+g)^2(4+m)^2}, \quad \pi_0^{MC} = \frac{a^2g(4-m)}{2(1+g)^2(4+m)}, \quad \pi_i^{MC} = \frac{3a^2g^2}{2(1+g)^2(4+m)^2},$$

$$W^{MC} = \frac{a^2[4+m+g(4+gm+m)]}{2(1+g)^2(4+m)}.$$

### Private Oligopoly

**Lemma 6.** *In an international private oligopoly with Cournot competition, in equilibrium, the optimal tariff, the output levels of the firms, the total output of the market, the consumer surplus, the profits of the firms and domestic social welfare are, respectively:*

$$t^{PC} = \frac{14a}{36+5m}, \quad q_0^{PC} = \frac{12a}{36+5m}, \quad q_i^{PC} = \frac{5a}{36+5m}, \quad Q^{PC} = \frac{a(12+5m)}{36+5m},$$

$$CS^{PC} = \frac{a^2(12+5m)^2}{2(36+5m)^2}, \quad \pi_0^{PC} = \frac{216a^2}{(36+5m)^2}, \quad \pi_i^{PC} = \frac{75a^2}{2(36+5m)^2},$$

$$W^{PC} = \frac{a^2(16+5m)}{2(36+5m)}.$$

Comparing Lemma 3 and 5,<sup>7</sup> we obtain the following result.

**Corollary 1.** *In an international mixed oligopoly with asymmetric costs, all variables have exactly the same equilibrium values between foreign leadership case and Cournot competition case.*

The optimal tariffs preprivatization and postprivatization under three types of the order of moves are summarized in Table 1.

**(Insert Table 1 here)**

**Proposition 1.** (1) *Without an efficiency gap, the privatization of a public firm unambiguously increases the optimal tariff regardless of the order of moves.*

(2) *In an international mixed oligopoly with tariff, the optimal tariff is the highest under domestic leadership regardless of the value of  $g$ .*

(3) *In an international private oligopoly with tariff, the optimal tariff under foreign leadership is higher (lower) than under Cournot competition if  $m \geq 2$  ( $m=1$ ); it is always the lowest under domestic leadership.*

**Proof.** See appendix.

### **3. The Decision on Whether to Privatize**

It remains for us to study whether the government should privatize the domestic public firm or not. First of all, we relegate the equilibrium domestic social welfare to Table 2.

**(Insert Table 2 here)**

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<sup>7</sup> Although the public firm earns a negative profit before privatization in case II and III when  $m > 4$ , the government, seeking for social welfare maximization, may provide some financial support or transfer the public funds or other acts of protection to ensure its survival.

So far we have assumed that asymmetric cost functions for the public and the private firms. Actually, there are a large number of existing papers concerned with identical cost functions.<sup>8</sup> Therefore, it is worthwhile for us to briefly analyze the decision on whether to privatize the domestic public firm when all  $1+m$  firms share identical cost functions. Let  $g=1$  and compare the differences in domestic social welfare in Table 2. Therefore, we obtain the following proposition.

**Proposition 2.** *In an international mixed oligopoly with symmetric costs, the government never privatizes the public firm in a sequential play game; however, the government should privatize the public firm in a Cournot competition game if and only if  $m \geq 3$ .*

**Proof.** See appendix.

Another reason for the above analysis is that it is the starting point when we focus on the asymmetric cost case. We will study this issue in what follows to explore the role of asymmetric cost structure under each case.

### **Case I: Domestic Leadership**

We define a new variable:

$$\alpha(g, m) \equiv W^{MD} - W^{PD}. \quad (4)$$

When  $\alpha < 0$ , it is welfare-improving for the domestic government to privatize the public firm; if  $\alpha > 0$ , it is welfare-decreasing. First note that when  $g=1$ ,  $\alpha > 0$ ; that is what proposition 2 shows. Second, we evaluate  $\partial\alpha/\partial g$ :

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<sup>8</sup> See e.g. De Fraja and Delbono (1989) and Bárcena-Ruiz and Garzón (2003 and 2006).

$$\frac{\partial \alpha}{\partial g} = -\frac{8a^2}{(4 + 4g + gm)^2}. \quad (5)$$

We find that  $\partial \alpha / \partial g$  is definitely negative regardless the value of  $m$ . Taken together, these observations indicate that: at a low level of  $g$ ,  $\alpha$  is positive and the government should not privatize the public firm. Since  $\alpha$  is always decreasing with  $g$ , and then  $\alpha$  must fall below zero at some critical value of  $g$ , which strictly exceeds unity. We can derive the critical value of  $g$ ,  $g^* = 1 + \frac{1}{2+m} - \frac{1}{4+m} > 1$ . The importance of the critical value is that it corresponds to the boundary between welfare-improving privatization and welfare-decreasing privatization. Therefore, we obtain the following proposition.

**Proposition 3.** *Under domestic leadership, provided there exists an efficiency gain by privatization, the government should privatize the public firm if and only if  $g \geq g^*$ .*

### Case II: Foreign Leadership

By using the similar analytical method as in case I, we define a new variable:

$$\beta(g, m) \equiv W^{MF} - W^{PF}. \quad (6)$$

$\beta$  has the same power with  $\alpha$  to indicate the government decision on whether to privatize the public firm. Proposition 2 argues that when  $g=1$ ,  $\beta > 0$ . Now we evaluate  $\partial \beta / \partial g$ :

$$\frac{\partial \beta}{\partial g} = \frac{a^2(-4 - 4g + gm - m)}{2(1+g)^3(4+m)}. \quad (7)$$

The sign of equation (7) appears to be ambiguous. For  $g$  close to one,  $\partial \beta / \partial g$  approaches  $-\frac{a^2}{2(4+m)}$ , which is negative. When  $m > 4$ ,  $\partial \beta / \partial g$  alters to positive

with the increase of  $g$ . It is  $g_1 = \frac{m+4}{m-4}$  that let  $\partial\beta/\partial g$  be zero, which yields the minimum of  $\beta$  for each  $m$ .<sup>9</sup> Define that:

$$\beta_{\min}(m) \equiv \beta\left(\frac{m+4}{m-4}, m\right). \quad (8)$$

It is straightforward to check that when  $m \geq 6$ ,<sup>10</sup>  $\beta_{\min}(m) \geq 0$ . Considering privatization decreases the market competition, we can interpret the case in which  $m \geq 6$ . This is a case in which market competition is less intensive so that the government never privatizes the public firm no matter how large the efficiency gain is. When  $m < 6$ , a sufficient efficiency gain ensures that privatization is welfare-improving.<sup>11</sup> The efficiency gain is represented by a critical value of  $g$ ,

$$g^{**} = \frac{24 - 38m - 11m^2 + \sqrt{3}\sqrt{(6-m)(4+m)(18+17m)(36+29m)}}{4(48 + 47m - 10m^2)} \quad \text{and} \quad g^{**} > 1 \quad \text{for}$$

$m < 6$ . Therefore, we obtain the following proposition.

**Proposition 4.** *Under foreign leadership, provided there exists an efficiency gain by privatization, the government never privatizes the public firm when  $m \geq 6$ . When  $m < 6$ , the government should privatize the public firm if and only if  $g \geq g^{**}$ .*

### Case III: Cournot Competition

Define:

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<sup>9</sup>  $\left. \frac{\partial^2 \beta}{\partial g^2} \right|_{g=g_1} > 0$ . Therefore, the 2<sup>nd</sup>-order condition is satisfied.

<sup>10</sup>  $\{m \geq 6\}$  is the subset of  $\{m > 4\}$ .

<sup>11</sup> When  $m < 5$ ,  $\partial\beta/\partial g$  is always negative. When  $m=5$ ,  $\partial\beta/\partial g$  may be negative (initially) or positive (subsequently); furthermore, for  $g$  close to the positive infinity,  $\beta(g,5)$  is negative and  $\partial\beta/\partial g$  approaches zero, which accordingly indicate that a critical value of  $g$  does exist in this case. Therefore, we demonstrate that privatization is welfare-improving at some critical value of  $g$  when  $m < 6$ .

$$\gamma(g, m) \equiv W^{MC} - W^{PC}. \quad (9)$$

We evaluate  $\partial\gamma/\partial g$ :

$$\frac{\partial\gamma}{\partial g} = \frac{\partial\beta}{\partial g} = \frac{a^2(-4-4g+gm-m)}{2(1+g)^3(4+m)}. \quad (10)$$

Proposition 2 states that when  $g=1$ ,  $\gamma$  is negative provided  $m \geq 3$ . Considering  $\partial\gamma/\partial g$  is always negative ( $m=3$  and 4) or approaches zero for  $g$  close to the positive infinity ( $m > 4$ ), hence we demonstrate that  $\gamma$  is always negative as long as  $m \geq 3$ . When  $m < 3$ , although  $\gamma$  begins with a positive level, a sufficient efficiency gain ensures that privatization is welfare-improving. We can derive the critical value

of  $g$ ,  $g^{***} = \frac{16-16m-5m^2 + \sqrt{(4+m)(36+5m)(144-24m+5m^2)}}{128}$ , and  $g^{***} > 1$  for

$m=1$  and 2. In sum, we can re-write the expression of  $g^{***}$  as:

$$g^{***} = \begin{cases} \frac{16-16m-5m^2 + \sqrt{(4+m)(36+5m)(144-24m+5m^2)}}{128} & \text{when } m < 3; \\ 1 & \text{when } m \geq 3. \end{cases}$$

Therefore, we obtain the following proposition.

**Proposition 5.** *Under Cournot competition, provided there exists an efficiency gain by privatization, the government should privatize the public firm if and only if  $g \geq g^{***}$ .*

#### 4. Foreign Competition, Order of Moves and Privatization

In Matsumura (2003) and Chang (2005), the results they obtained came from the model with single foreign private firm. Although the results in the previous section may be not so striking, the above analysis does provide us a framework to explore the relationship among foreign competition, orders of moves and privatization.

It is easy to see the likelihood that  $W^P > W^M$ , so that the domestic government



should privatize the public firm, depends on a sufficiently large number of foreign competitors. It is straightforward to check that under domestic leadership a higher value of  $m$  raises the range of values of  $g$  at which privatization improves domestic social welfare ( $\partial g^* / \partial m < 0$ ).

Under foreign leadership, however, as  $m$  gets large, the critical value of  $g$  deviates away from unity ( $\partial g^{**} / \partial m > 0$  for  $m < 6$ ); when the number of foreign competitors is large enough, i.e.  $m \geq 6$ , privatization never improves domestic social welfare.

Under Cournot competition, when the number of foreign competitors is not so large, i.e.  $m < 3$ , the range of values of  $g$  at which privatization improves domestic social welfare also increases with the increase of  $m$  ( $\partial g^{***} / \partial m < 0$  for  $m < 3$ ). This effect is similar to the case of domestic leadership. When the number of foreign competitors is large enough, i.e.  $m \geq 3$ , privatization unambiguously improves domestic social welfare at any value of  $g$ .

We can explicitly depict the relationship among  $g^*$ ,  $g^{**}$ ,  $g^{***}$  and  $m$  via a geometric exposition.

**(Insert Figure 1 here)**

Figure 1 suggests how the ranking of the critical value of  $g$  interacts with  $m$ . When  $m = 1$ , the ranking of the critical value of  $g$  is  $g^* < g^{**} < g^{***}$ . When  $m = 2$ , the ranking is  $g^* < g^{***} < g^{**}$ . When  $m \geq 3$ , note that  $g^{***}$  has been revised since  $g \geq 1$ , and Cournot competition has the largest likelihood. When  $3 \leq m < 6$ ,  $g^{***} < g^* < g^{**}$ . When  $m \geq 6$ , there does not exist such a  $g^{**}$ , and the ranking of  $g^*$  and  $g^{***}$  remains at  $g^{***} < g^*$ .

From Figure 1, we can also see the tendency of the critical value of  $g$  under each case more clearly and obtain the following proposition.

**Proposition 6.** *Under domestic leadership and Cournot competition, higher foreign competition tends to raise the likelihood that privatization improves domestic welfare. But, under foreign leadership, privatization more likely improves domestic welfare when the market is less competitive.*

This proposition has policy implications. Nowadays, many countries alter their economic policies towards free trade by opening markets that allows entry of foreign private firms. This proposition provides valuable insight into the issue of open market policy. According to the proposition, making market more competitive may provoke an undesirable increase in the privatization barrier, causing a detrimental effect on welfare. The government must take both the order of moves and the existing foreign competitors into account.

## 5. Conclusions

In this paper, we examined privatization policy in a domestic market consisting of one domestic public firm and one or more foreign private firms with asymmetric cost structure. To obtain sharper results, we presented the international mixed oligopoly model under domestic leadership, foreign leadership and Cournot competition, respectively.

For each case, the government's decisions on privatization are associated with the magnitude of the efficiency gain by privatization. The main results obtained in the three cases are as follows. Under domestic leadership, the government will choose to

privatize the public firm if the efficiency gain is sufficient. Under foreign leadership, if foreign competition is intensive, the public firm should not be privatized regardless of the magnitude of the efficiency gain. If foreign competition is not-so-intensive, the public firm should be privatized as long as the efficiency gain is sufficient. Under Cournot competition, if foreign competition is intensive, the public firm should be privatized regardless of the magnitude of the efficiency gain.

These results reveal that the impact of foreign competition upon the likelihood that privatization improves domestic social welfare differs with respect to the order of moves. Namely, under domestic leadership and Cournot competition (foreign leadership), higher foreign competition tends to raise (reduce) the likelihood that privatization improves domestic social welfare.

In conclusion, this paper has shown that, first, the efficiency gain is crucial in determining the government's decision on whether to privatize the public firm and that, second, the privatization decision differs when we are taking into account the order of moves.

## APPENDIX

### A. Derivation of lemma 1.

As usual in such models, a backward induction method is applied in order to obtain the sub-game perfect equilibrium. In the case of mixed oligopoly under domestic leadership, we begin with the maximization of the profit of  $i$ 'th foreign private firm.

Its reaction function is:

$$q_i = (a - q_0 - t - \sum_{j \neq i} q_j) / 3,$$

and thus

$$q_i = (a - q_0 - t) / (2 + m), \quad (\text{A-1})$$

by taking symmetric assumption into account. It follows that

$$Q = (am - mt + 2q_0) / (2 + m). \quad (\text{A-2})$$

Substituting (A-1) and (A-2) into (1), the domestic social welfare becomes:

$$W = \frac{(am - mt + 2q_0)^2}{2(2 + m)^2} + \frac{(2a + mt - 2q_0)q_0}{2 + m} - \frac{gq_0^2}{2} + \frac{tm(a - q_0 - t)}{2 + m}. \quad (\text{A-3})$$

In the second stage, maximization of (A-3) yields the domestic public firm's reaction function:

$$q_0 = \frac{4a(1 + m) - 2mt}{4(1 + m) + g(2 + m)^2}. \quad (\text{A-4})$$

In the first stage, making use of (A-4), differentiation of (A-3) with respect to  $t$  gives:

$$t^{MD} = \frac{2ag}{4 + g(4 + m)}.$$

Having derived the optimal tariff rate, we can obtain other equilibrium variables in lemma 1 by making simple substitutions.

## B. Proof of proposition 1.

**Proof of part 1.** When there is not an efficiency gap,  $g=1$ .

$$t^{PD} - t^{MD} = \frac{4a(20 + 10m + m^2)}{(4 + m)(8 + m)(18 + 10m + m^2)} > 0.$$

$$t^{PF} - t^{MF} = \frac{2a(15 + 13m + 7m^2)}{(54 + 51m)(4 + m)} > 0.$$

$$t^{PC} - t^{MC} = \frac{a(20 + 9m)}{(36 + 5m)(4 + m)} > 0.$$

**Proof of part 2.**

$$t^{MD} - t^{MF} = \frac{2agm}{[4 + g(4 + m)](1 + g)(4 + m)} > 0.$$

$$t^{MD} - t^{MC} = \frac{2agm}{[4 + g(4 + m)](1 + g)(4 + m)} > 0.$$

**Proof of part 3.**

$$t^{PD} - t^{PF} = \frac{am(66 - 182m - 115m^2 - 14m^3)}{(4 + m)(54 + 51m)(18 + 10m + m^2)} < 0.$$

$$t^{PD} - t^{PC} = -\frac{4am(24 + 11m + m^2)}{(4 + m)(36 + 5m)(18 + 10m + m^2)} < 0.$$

$$t^{PF} - t^{PC} = \frac{35am(-3 + 2m)}{(54 + 51m)(36 + 5m)} > (<) 0, \text{ if } m \geq (<) 2.$$

## C. Proof of proposition 2.

When  $g=1$ :  $W^{MD} - W^{PD} = \frac{16a^2}{(4 + m)(8 + m)(18 + 10m + m^2)} > 0.$

$$W^{MF} - W^{PF} = \frac{a^2(48 - 22m + 29m^2)}{24(4 + m)(18 + 17m)} > 0$$

$$W^{MC} - W^{PC} = \frac{a^2(32 + 4m - 5m^2)}{8(4 + m)(36 + 5m)} > (<) 0, \text{ if } m < (>) 3.$$

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## TABLES

TABLE 1: Optimal Tariff: Preprivatization and Postprivatization

	Preprivatization	Postprivatization
Domestic Leadership	$t^{MD} = \frac{2ag}{4 + g(4 + m)}$	$t^{PD} = \frac{2a(14 + 8m + m^2)}{(4 + m)(18 + 10m + m^2)}$
Foreign Leadership	$t^{MF} = \frac{2ag}{(1 + g)(4 + m)}$	$t^{PF} = \frac{7a(3 + 2m)}{54 + 51m}$
Cournot Competition	$t^{MC} = \frac{2ag}{(1 + g)(4 + m)}$	$t^{PC} = \frac{14a}{36 + 5m}$

TABLE 2: Equilibrium Domestic Social Welfare: Preprivatization and Postprivatization

	Preprivatization	Postprivatization
Domestic Leadership	$W^{MD} = \frac{a^2(4 + gm)}{2(4 + 4g + gm)}$	$W^{PD} = \frac{a^2(32 + 34m + 10m^2 + m^3)}{2(4 + m)(18 + 10m + m^2)}$
Foreign Leadership	$W^{MF} = \frac{a^2[4 + m + g(4 + gm + m)]}{2(1 + g)^2(4 + m)}$	$W^{PF} = \frac{a^2(24 + 31m)}{6(18 + 17m)}$
Cournot Competition	$W^{MC} = \frac{a^2[4 + m + g(4 + gm + m)]}{2(1 + g)^2(4 + m)}$	$W^{PC} = \frac{a^2(16 + 5m)}{2(36 + 5m)}$



# FIGURE

FIGURE 1. The Relationship among  $g^*$ ,  $g^{**}$ ,  $g^{***}$  and  $m$ .

