Military Expenditure and Employment in a Simple Search Model

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Abstract

This paper builds a dynamic search model to study the effects of military expenditure on employment. It focuses on the determinants of the employment effects of military expenditure and the employment dynamics following military expenditure shocks. It shows that the long-run employment effect depends on labor-market fundamentals like search cost and wage bargaining. It also shows that if military shocks are anticipated by the private sector, the short-run adjustment of employment may move in a direction opposite to the long-run adjustment.

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1 Introduction

The economic effects of military expenditure have been discussed at length. While a large body of the literature focuses on how military spending affects output growth, the effect on employment also receives considerable attention. The literature suggests several channels through which military expenditure may influence employment. The first may be "productivity-improving effects." Military expansion may improve labor productivity in various ways, e.g., through technology spillover from the military sector to the private sector, the security of civilians and property from foreign military threats, and the construction of military infrastructure that also benefits civilians. As labor productivity improves, labor demand increases. Contrary to the last channel, there may be "tax distortion effects." Military spending may need to be financed by levying taxes. Thus, tax burdens may reduce labor demand, if they are on employers, or labor supply if on workers. Moreover, such tax burdens may be quite heavy if the government is inefficient or corrupt. Taking positive and negative effects into account, there is no clear-cut prediction about how employment should respond to changes in military spending.[1]

The empirical literature on military spending and employment provides mixed findings. For instance, Dunne and Smith (1990) show that military spending does not Granger cause unemployment, and vice versa, and that military spending has no significant effect on unemployment in 9 out of 11 OECD countries. In contrast, Abell (1990) shows that defense spending Granger causes unemployment in the US. Barker et al. (1991) examine the economic implications of cuts in military spending for the UK and suggest that such reductions may lead to a significant reduction in unemployment and an increase in output. Using the panel data on the fifty U.S. states, Hooker and Knetter (1997) suggest that military procurement spending affects employment in that it does explain a statistically significant degree of the variation in employment growth across states. Yildirim and Sezgin (2003) find that military expenditure negatively affects employment in Turkey, both in the short and long runs. Using Taiwanese data, Huang and Kao (2005) find that defense spending negatively affects employment growth in the short run, but positively in the long run. These findings indicate that employment effects of military expenditure vary widely across countries. Meanwhile, comparisons

[1] Other channels may include the "reallocation effects." The contraction of the military sector, for instance, may induce workers to move from the military sector into the private sector. Such reallocation may not be smooth and thus may result in frictional unemployment.

between Yildirim and Sezgin (2003) and Huang and Kao (2005) suggest that the response of employment to military spending shocks exhibits intriguing dynamic patterns in that the short-run response may or may not be in accordance with the long-run response.

Motivated by the above findings, we attempt to build a model to explore the determinants of the employment effects of military spending, and to study the employment dynamics following military shocks. Furthermore, as argued by many researchers (e.g., Hall, 1999), labor market frictions and the nature of wage bargaining are central to understanding the adjustments of employment in response to exogenous shocks and government policies. According to this view, the link between military spending and employment may be influenced significantly by the trading frictions inherent in the labor market. We are sympathetic with this view; thus, we also would like to investigate how the labor market structure would influence the employment effects of military expenditure.

To this end, we build a dynamic general equilibrium model along the line of the Mortensen-Pissarides job-search models (see, e.g., Pissarides, 2000). In the Mortensen-Pissarides framework, workers and firms are paired via a search and matching process; certain search costs must be incurred during the search process; and wages are determined by the bilateral bargaining between workers and firms. This structure explicitly characterizes dynamic adjustments of employment and captures many realistic features of labor market frictions.[2] In addition, our model captures potentially positive and negative effects of military spending by assuming that military expenditure is a positive factor in production and is financed by distortionary taxes.

Our main findings can be summarized as follows. First, we show that a permanent increase in military burden may have a positive effect on long-run employment when the productivity-improving effect of this policy change outweighs the accompanying tax distortions, and vice versa if the employment effect is negative. Second, we show that the magnitude of the long-run effect can be affected by labor market fundamentals like search frictions and wage bargaining. This result suggests that cross-country differences in the labor market fundamentals may help explain cross-country differences in the employment effects of military expenditure. Lastly, employment dynamics are found to be dependent on the nature of the military shock. In particular, if the military shock is pre-announced

[2]The Mortensen-Pissarides models have been increasingly popular in studying labor market dynamics and other macroeconomic issues. Yet, to our knowledge, this study is among the first attempts to include military expenditure in a similar model.

and thus anticipated by the private sector, the short-run adjustment of employment may move in a direction opposite to the long-run adjustment.

The remainder of this paper is organized as follows. The next section describes the model economy. The section after examines the employment effect of military spending in the short and long runs. Concluding remarks are presented in the final section.

2 The Model

2.1 Job creation and job destruction

In the labor market, the labor force is fixed and normalized to unity.[3] At time t, if $\overline{n}_t$ denotes aggregate employment, the number of unemployed workers is $1-\overline{n}_t$ and denoted $\overline{u}_t$. The evolution of employment is determined by job creation and destruction processes. Job creation takes place when an unemployed worker is matched with an unfilled vacancy. If $\overline{u}_t$ searching workers and $\overline{v}_t$ vacancies engage in matching, the number of new matches taking place is given by $m(\overline{v}_t,\overline{u}_t)$.[4] As suggested by many empirical studies (e.g., Petrongolo and Pissarides, 2001), the matching technology m is assumed to be constant returns to scale and of the Cobb-Douglas form; i.e., $\psi(\overline{v}_t,\overline{u}_t)=\overline{\psi}\overline{v}_t^{1-\eta}\overline{u}_t^{\eta}$, where $\overline{\psi}>0$ and $0<\eta<1$. The job seekers and unfilled vacancies that are matched are randomly selected from the pools $\overline{u}_t$ and $\overline{v}_t$. Hence, the instantaneous probability for a vacancy to be filled is denoted q_t, given by $q_t=\psi(\overline{v}_t,\overline{u}_t)/\overline{v}_t=\overline{\psi}\theta_t^{-\eta}$, where $\theta_t=\overline{v}_t/\overline{u}_t$ is labor market tightness. Analogously, the probability for an unemployed worker to be matched is given by $\psi(\overline{v}_t,\overline{u}_t)/\overline{u}_t=\overline{\psi}\theta_t^{1-\eta}=q_t\theta_t$. Note that the market tightness θ_t depends on *aggregate* numbers of vacancies and job seekers. Thus, θ_t should be taken as given by individual agents, and so should be q_t.

As in the basic matching models (e.g., Pissarides, 2000, ch. 1), the job destruction process is formulated by assuming that existing jobs dissolve at a constant rate $\delta>0$ after engaging in production. Thus, the evolution of employment $\overline{n}_t$ is given by the difference between job creation and job de-

[3] In this paper, military personnel is not counted in the labor force. Also, this paper does not consider the military sector; thus, it abstracts from the mobility of workers between military and private sectors.

[4] As mentioned in Pissarides (2000, pp.3–4), "trade in the labor market is a nontrivial economic activity because of the existence of heterogeneities, frictions, and information imperfections." This matching function m is a modeling device that "captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit."

struction:

$$\dot{\overline{n}}_t = \psi(\overline{v}_t, \overline{u}_t) - \delta \overline{n}_t = q_t \theta_t (1 - \overline{n}_t) - \delta \overline{n}_t. \tag{1}$$

2.2 The household

The economy is populated by many identical households, and the measure of households is normalized to one. The representative household consists of many infinitely-lived members (workers), and the size of the household is normalized to unity. The workers could be employed or unemployed. If a worker is employed, he is assumed to supply one unit of labor. If unemployed, the worker searches for a job. If n_t denotes the fraction of household members in work at time t, $1 - n_t$ is the fraction of household members in search. The workers are assumed to insure one another against labor-income variations by pooling their income at any time.[5] The household's preferences are

$$W_0 = \int_0^\infty U(c_t, n_t) e^{-\rho t} \mathrm{d}t, \; 0 < \rho, \tag{2}$$

where c_t is the household's consumption, and ρ denotes time preference rate. The momentary utility function takes the following form:

$$U(c_t, n_t) = \frac{(c_t - b n_t)^{1-\sigma} - 1}{1 - \sigma}, \; 0 < b, \sigma,$$

where $1/\sigma$ denotes the elasticity of intertemporal substitution; $b n_t$ represents the disutility of working.[6] This momentary utility function implies that the disutility of working is nonseparable from the utility of consumption. As emphasized by Rogerson and Wright (1988), this kind of specification is particularly relevant for the analysis of models with unemployment. Utility functions of the nonseparable form were also adopted in the search models in Hairault (2002) and Heijdra and Ligthart (2002).

Among the $1 - n_t$ unemployed workers, $(q_t \theta_t)(1 - n_t)$ of them can find a job and become employed, where $q_t \theta_t$ is the job-finding probability defined in the last section. As employed workers

[5] In this model, an individual worker faces idiosyncratic risks to earnings. These risks induce distributions of consumption and wealth across workers. To avoid analytical complexity, we assume that workers fully insure one another within a large household. This assumption has been widely adopted by the literature (e.g., Merz, 1995; Andolfatto, 1996).

[6] In accordance with the "indivisible labor" literature (Hansen, 1985; Rogerson, 1988), the disutility of working is assumed to be linear in labor supply.

separate from jobs at the rate δ, employment evolves according to

$$\dot{n}_t = q_t \theta_t (1 - n_t) - \delta n_t. \tag{3}$$

Households are free to borrow or lend at interest rate r_t, and own the firms in the economy. The budget constraint facing the representative household is thus

$$c_t + \dot{a}_t = r_t a_t + w_t n_t + \pi_t, \tag{4}$$

where w_t is wage rate; a_t is the household's lending; and π_t is firm profits.

The household seeks to maximize (2) subject to (3) and (4). The household optimization problem, as well as the firm optimization problem defined later, can be solved with the Pontryagin maximum principle. The optimality conditions for the present problem imply

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma}\left(r_t - \rho\right), \tag{5}$$

where $z_t \equiv c_t - b n_t$.

2.3 The firm

There are a fixed large number of identical firms in the economy, and the measure of firms is normalized to one. The representative firm produces y_t units of final goods by the technology

$$y_t = A m_t^{\alpha} (n_t^d)^{1-\alpha} \equiv f(n_t^d, m_t), \; 0 < A, \; 0 < \alpha < 1,$$

where n_t^d is the number of employed workers at time t, and m_t is military spending. This specification captures the productivity-improving effects of military spending, in accordance with the notion that the defense sector may imply positive externalities for the civilian sector (Sandler and Hartley, 1995, ch. 8).

To hire new workers, the firm must post vacancies in the labor market. Posting a vacancy takes the cost of χ units of final goods. A vacancy posted at time t can be filled with the probability q_t, as mentioned before, and it will be destroyed with zero scrap value if it is unfilled.[7] If v_t vacancies are posted, the firm's labor employment evolves according to

$$\dot{n}_t^d = q_t v_t - \delta n_t^d, \tag{6}$$

where δ is the job destruction rate defined earlier.

The firm seeks to maximize its present value of profit flows by solving the problem,

$$\max \int_0^\infty e^{-\int_0^t r_s \mathrm{d}s} \pi_t \mathrm{d}t$$

subject to (6), where π_t is the profit flow given by

$$\pi_t = (1-\tau_t) y_t - w_t n_t^d - \chi v_t,$$

where τ_t is a tax on sales.[8] The optimal firm behavior can be characterized by

$$\chi = q_t \tilde{\lambda}_t, \tag{7}$$

$$\dot{\tilde{\lambda}}_t = (r_t + \delta)\tilde{\lambda}_t - \left[(1-\tau_t) f_n(t) - w_t\right], \tag{8}$$

$$\lim_{t\to\infty} e^{-\int_0^t r_s \mathrm{d}s} \tilde{\lambda}_t n_t^d = 0, \tag{9}$$

where $\tilde{\lambda}_t$ is the multiplier for (6), and $f_n(t) \equiv (1-\alpha)A(m_t/n_t^d)^\alpha$ is the marginal product of labor. The multiplier $\tilde{\lambda}_t$ is the shadow price of an additional worker to the firm; thus, $q_t\tilde{\lambda}_t$ is the expected value of a posted vacancy. Equation (7) characterizes the optimal decision on vacancy, as it equalizes the marginal cost (χ) with the expected return ($q_t\tilde{\lambda}_t$) of vacancy posting. Thus, (7) may be referred to as the *job creation condition*. Equation (8) is the Euler equation governing the evolution of $\tilde{\lambda}$, while (9) is the transversality condition.

[7] Thus, the vacancy cost is irreversible, as is commonly assumed in the literature.

[8] The tax rate τ is used to capture the tax distortions accompanying military spending. We could have considered other taxes, but that would only complicate analysis without gaining much economic insight.

2.4 Wage bargaining

When a producer and a worker form a match, they determine the wage rate according to the Nash criterion. Specifically, at any given point of time they negotiate a wage rate in order to maximize the weighted product of the household and the producer surpluses. By surplus we mean the difference between what a party can get from the employment relationship and his outside option, i.e., what he can get outside that relationship. Following Shi and Wen (1997, 1999) and Mortensen (2005), we assume that it is impossible to search while negotiating. Accordingly, the producer and the worker have no immediate outside options. Hence, the household surplus is the utility gains from working, $(w_t - b)z_t^{-\sigma}$, while the producer surplus is the profit gains from hiring, $(1-\tau_t)f_n(t) - w_t$. With normalization, the Nash bargaining solution solves

$$\max_{w_t} (w_t - b)^{\beta} \left[(1-\tau_t)f_n(t) - w_t\right]^{1-\beta},$$

where β and $1-\beta$ are the bargaining weights of the worker and the firm, respectively. The parameter β can also be interpreted as the worker's bargaining power. The first-order condition with respect to w_t yields

$$w_t = (1-\beta)b + \beta(1-\tau_t)f_n(t). \tag{10}$$

Thus, w_t is a weighted sum of unemployment income and the marginal product of labor.

2.5 The government and the equilibrium

At any given point of time the government chooses the tax rate τ_t and keeps the budget balanced. The government has no expenditures other than military spending. Thus, the government behavior is characterized by

$$m_t = \tau_t y_t = \tau_t A m_t^{\alpha} n_t^{1-\alpha}. \tag{11}$$

Here, τ_t also represents the *military burden*, i.e., the share of military spending in GDP.

To close the model, market-clearing conditions must be stated. When the labor market clears, labor supply, labor demand, and aggregate employment are equal; therefore, $n_t = n_t^d = \overline{n}_t$, and $u_t = \overline{u}_t$. Meanwhile, since all firms are identical, $v_t = \overline{v}_t = \theta_t(1-n_t)$ in equilibrium. When the final-goods market clears, the quantity supplied equals the quantity purchased for private consumption, government consumption, and vacancy posting. Thus,

$$f(n_t, m_t) = c_t + m_t + \chi\theta_t(1 - n_t). \tag{12}$$

Given the foregoing analysis, the equilibrium of the model economy can be found by solving the system of (1), (5), and (7)—(12). However, the system would be easier to solve if appropriately transformed. Specifically, the shadow price $\tilde{\lambda}_t$ is multiplied by the marginal utility of consumption and transformed into a utility-based shadow price $\lambda_t = \tilde{\lambda}_t z_t^{-\sigma}$. Thus, the job creation condition (7) becomes

$$\chi = q_t \lambda_t z_t^{\sigma}. \tag{13}$$

Using (5) and the fact that $\dot{\lambda}_t / \lambda_t = \dot{\tilde{\lambda}}_t / \tilde{\lambda}_t - \sigma \dot{z}_t / z_t$, (8) can be rewritten as

$$\dot{\lambda}_t = \left(\delta + \rho\right) \lambda_t + z_t^{-\sigma} \left[w_t - (1 - \tau_t) f_n(t) \right]. \tag{14}$$

The market-clearing condition (12) becomes

$$f(n_t, m_t) = z_t + b n_t + m_t + \chi\theta_t(1 - n_t). \tag{15}$$

Equation (5) implies $r_t = \rho + \sigma \dot{z}_t / z_t$. Integrating this over time yields $\int_0^t r_s \, \mathrm{d}s = \rho t + \sigma \log(z_t / z_0)$. Using this expression, the transversality condition (9) can be rewritten as

$$\lim_{t \to \infty} e^{-\rho t} \lambda_t n_t = 0. \tag{16}$$

In summary, an equilibrium of the model is a time path of $\{n_t, \lambda_t, \theta_t, z_t, w_t, m_t\}_{t=0}^{\infty}$ that solves the system of (1), (10), (11), and (13)—(16), given the value of n_0 and the time path of $\{\tau_t\}_{t=0}^{\infty}$.

3 Results

3.1 The steady state

In the steady state, all variables are stationary. Setting $\dot{n}_t = 0$ and manipulating (1) yield

$$n^* = \frac{q^*\theta^*}{q^*\theta^* + \delta} = \frac{\overline{\psi}(\theta^*)^{1-\eta}}{\overline{\psi}(\theta^*)^{1-\eta} + \delta}, \tag{17}$$

where asterisks denote steady-state values. This equation describes a positive relationship between n^* and θ^*. Setting $\dot{\lambda}_t = 0$ and combining (14) with (10) yield

$$\lambda^*(z^*)^\sigma = \frac{(1-\beta)\left[(1-\tau)f_n^* - b\right]}{\delta + \rho}. \tag{18}$$

This indicates that the shadow price of employment, the left-hand side, equals the capitalized value of the firm surplus, the right-hand side, where $\delta + \rho$ is used as a discounting factor.

Note that the government budget constraint (11) implies $m_t / n_t = (\tau_t A)^{\frac{1}{1-\alpha}}$. Thus, the marginal product of labor can be written as $f_n(t) = (1-\alpha)A^{\frac{1}{1-\alpha}}\tau_t^{\frac{\alpha}{1-\alpha}}$. Using this relation, we can obtain from (13) and (18) that

$$\frac{(\delta+\rho)\chi}{(1-\beta)q^*} = \frac{(\delta+\rho)\chi\theta^{*\eta}}{(1-\beta)\overline{\psi}} = (1-\tau)f_n^* - b = \phi(\tau) - b, \tag{19}$$

where

$$\phi(\tau) \equiv (1-\alpha)A^{\frac{1}{1-\alpha}}(1-\tau)\tau^{\frac{\alpha}{1-\alpha}}.$$

Substituting for θ^* with (17) and rearranging the terms, we can obtain from (19) that

$$B\left(\frac{n^*}{1-n^*}\right)^{\eta/(1-\eta)} = \phi(\tau) - b,$$

where B is a composite labor-market parameter:

$$B \equiv \frac{(\delta+\rho)\chi}{(1-\beta)\overline{\psi}}\left(\frac{\delta}{\overline{\psi}}\right)^{\eta/(1-\eta)}. \tag{20}$$

The last equation implies a unique steady state:

$$n^* = \frac{\left[\phi(\tau) - b\right]^{\frac{1-\eta}{\eta}}}{B^{\frac{1-\eta}{\eta}} + \left[\phi(\tau) - b\right]^{\frac{1-\eta}{\eta}}}. \tag{21}$$

To ensure the existence of n^*, it is assumed that $\phi(\tau) > b$. Once n^* is determined, the steady-state values of other variables can easily be deduced from equilibrium conditions.

We next examine the stability properties in the neighborhood of the steady state. The equilibrium conditions are combined and linearized around the steady state to yield the system:

$$\begin{pmatrix} \dot{\hat{n}}_t \\ \dot{\hat{\lambda}}_t \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} \hat{n}_t \\ \hat{\lambda}_t \end{pmatrix} + \begin{pmatrix} M_{11} \\ M_{21} \end{pmatrix} \hat{\tau}_t \equiv J \begin{pmatrix} \hat{n}_t \\ \hat{\lambda}_t \end{pmatrix} + M\hat{\tau}_t, \tag{22}$$

where a caret-bearing variable $\hat{x}_t$ denotes the percentage deviation from the steady state, $(x_t - x^*)/x^*$. As derived in Appendix A.1, the coefficient matrices J and M are given by

$$\begin{aligned}
J_{11} &= \delta(1-\eta)(z^* + \chi\theta^*)/\Phi - (q^*\theta^* + \delta), \\
J_{12} &= \delta(1-\eta)z^*/(\sigma\Phi) > 0, \\
J_{21} &= \eta(\delta+\rho)(z^* + \chi\theta^*)/\Phi > 0, \\
J_{22} &= \eta(\delta+\rho)z^*/(\sigma\Phi) > 0, \\
M_{11} &= \delta(1-\eta)(\alpha-\tau)y^*/((1-\alpha)\Phi), \\
M_{21} &= \eta(\delta+\rho)(\alpha-\tau)y^*/((1-\alpha)\Phi) - (\alpha-\tau)(1-\beta)q^* y^*/(\chi n^*),
\end{aligned}$$

where

$$\Phi \equiv \chi\theta^*(1-n^*) + \eta z^*/\sigma > 0.$$

As the system (22) contains one predetermined variable (n) and one jump variable (λ), stability of the system requires that the matrix J has one positive and one negative eigenvalues. It can be shown that the determinant of J is

$$\det(J) = -(q^*\theta^* + \delta)J_{22} < 0.$$

This implies that J has two eigenvalues with opposite signs, for $\det(J)$ equals the product of the eigenvalues. Thus, the stability of the system is ensured.

3.2 The long-run effect of military expenditure on employment

After establishing the existence and uniqueness of the steady state, we can obtain the comparative static result regarding the employment effect of military burden. This effect consists of two parts. The first part can be obtained from (19):

$$\frac{\mathrm{d}\theta^*/\theta^*}{\mathrm{d}\tau/\tau} = \frac{\phi'(\tau)\tau}{\eta\left[\phi(\tau)-b\right]} = \frac{(\alpha-\tau)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}}{\eta\left[\phi(\tau)-b\right]}. \tag{23}$$

This part indicates how τ would affect market tightness θ. If $\alpha > \tau$, the labor productivity that is marginally raised by the military spending outweighs the accompanying tax distortions; thus, raising military spending has a positive net effect on production. Consequently, the firms' surplus from job matches would increase, and so would the firms' incentives to open new vacancies. This would imply a positive effect of τ on θ. Conversely, if $\alpha < \tau$, raising τ would have a negative effect on θ. For expository convenience, this channel may be labeled as the "surplus effect."

The second part can be obtained from (17):

$$\frac{\mathrm{d}n^*/n^*}{\mathrm{d}\theta^*/\theta^*} = (1-\eta)(1-n^*) = \frac{1-\eta}{1+(\phi(\tau)-b)^{\frac{1-\eta}{\eta}}B^{\frac{\eta-1}{\eta}}} > 0, \tag{24}$$

where the last equality is established using (21). This part represents how market tightness would affect employment. Given a fixed number of unemployed workers, a greater θ implies a greater chance that a worker can find a job; thus, it has a positive effect on employment. For convenience, this channel may be labeled as the "tightness effect."

The employment effect of military burden, denoted by ζ_τ, is the product of the surplus and tightness effects:

$$\zeta_\tau \equiv \frac{\mathrm{d}n^*/n^*}{\mathrm{d}\tau/\tau} = \left(\frac{\mathrm{d}n^*/n^*}{\mathrm{d}\theta^*/\theta^*}\right)\left(\frac{\mathrm{d}\theta^*/\theta^*}{\mathrm{d}\tau/\tau}\right) = \frac{(\alpha-\tau)(1-\eta)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}}{\eta\left[\phi(\tau)-b\right]\left[1+(\phi(\tau)-b)^{\frac{1-\eta}{\eta}}B^{\frac{\eta-1}{\eta}}\right]}. \tag{25}$$

Since the tightness effect is always positive, the sign of ζ_τ depends on the surplus effect. Clearly, (25) reveals that ζ_τ is positive if $\alpha > \tau$, and negative if $\alpha < \tau$.

3.3 The role of labor market fundamentals

Further conclusions can be drawn from the comparative static analysis. It can be easily obtained from (21) that employment is decreasing in B; that is, $\mathrm{d}n^*/\mathrm{d}B < 0$. As defined in (20), the composite labor-market parameter B depends on "deep" labor-market parameters. Particularly, B is decreasing in matching efficiency $\overline{\psi}$ but increasing in the worker bargaining power β, job destruction rate δ, and vacancy-posting cost χ. An increase in B may represent deterioration in $\overline{\psi}$ or increment in β, δ, and χ, which would be unfavorable for the firms to create new jobs. Thus, it is not surprising that a greater B leads to a lower employment or a higher unemployment.

Labor market fundamentals, as captured by the parameter B, are also likely to affect the employment effects of military expenditure. By inspecting (23) and (24), one can see that B has no influence on the surplus effect, but has a positive impact on the tightness effect. Consequently, as ζ_τ is the product of the surplus and tightness effects, the magnitude of ζ_τ would be amplified by B; i.e., $\mathrm{d}|\zeta_\tau|/\mathrm{d}B > 0$. This result can also be obtained by directly differentiating (25) with respect to B. The intuition underlying this result is clear. The tightness effect, as revealed by (24), is more pronounced when unemployment $(1-n^*)$ is high; meanwhile, as shown earlier, unemployment is increasing in B. Hence, a greater B leads to a stronger tightness effect. As the structure of the labor market varies widely across countries (Blau and Kahn, 1999), this result suggests that cross-country differences in labor market fundamentals may help explain the cross-country differences in the employment effect of military expenditure.

3.4 Dynamic effects of military expenditure

In the remainder of this section, we study employment dynamics following a permanent change in the military burden. To the end, graphical analysis based on phase diagrams will be undertaken. Before proceeding, note that the sign of M_{11} depends on $\alpha - \tau$, but the signs of J_{11} and M_{21} are ambiguous. To simplify the graphical analysis, we assume that $J_{11} < 0$ and $M_{21}/M_{11} < 0$. Roughly speaking, this assumption would require that σ do not deviate far away from zero. Note that when $\sigma \to 0$, $\Phi \to \infty$; thus, $J_{11} \to -(q^*\theta^* + \delta) < 0$, and $M_{21}/M_{11} \to -\infty$. When σ is positive but sufficiently small, continuity in σ implies that both J_{11} and M_{21}/M_{11} would remain negative. Appendix A.2 provides a numerical example.

Figure 1 displays the phase diagram. As $J_{11} < 0$ and $J_{12} > 0$, the schedule $\dot{\hat{n}} = 0$ is upward sloping, and as $J_{21}, J_{22} > 0$, the schedule $\dot{\hat{\lambda}} = 0$ is downward sloping. The two schedules intersect at the steady state and divide the $(\hat{n}, \hat{\lambda})$-space into four regions, where the arrows show the directions of motion in each region. The stable manifold is the downward-sloping line along which the economy converges to the steady state.

Figure 2 illustrates the employment adjustments following a policy change in τ. Without loss of generality, we assume $\alpha > \tau$, implying that $\zeta_\tau > 0$, $M_{11} > 0$, and $M_{21} < 0$. Suppose at $t = 0$ the economy is at the steady state E_1, where the $\dot{\hat{n}} = 0$ line intersects with the $\dot{\hat{\lambda}} = 0$ line. The government announces that the military burden will increase by ε percent from date T onward. That is, $\hat{\tau}_t = 0$ for $t < T$, and $\hat{\tau}_t = \varepsilon$ for $t \geq T$. As $M_{11} > 0$ and $M_{21} < 0$, both the schedules $\dot{\hat{n}} = 0$ and $\dot{\hat{\lambda}} = 0$ move right once if the policy change takes place. The point E_2, where the two new schedules intersect, represents the new steady state. As the policy change is permanent, (25) reveals that the employment level associated with E_2 should be $\varepsilon\zeta_\tau$ percent greater than the employment associated with E_1.

While it shows a long-run accumulation of employment, Figure 2 also reveals that employment may exhibit interesting short-run dynamics, depending on the nature of the policy change. Suppose the stable manifold around E_2 passes below E_1. For example, the new stable manifold may be represented by the thin, solid line going through E_2. Upon the announcement, employment stays at its original level, because it is predetermined. If the policy change takes place immediately after the announcement (i.e., $T = 0$), stability implies that the shadow price λ would jump down and hit a point on the stable manifold associated with E_2. Suppose that point is $\tilde{O}$. From $\tilde{O}$, the economy would converge to E_2 along $\tilde{O}E_2$. This trajectory indicates that if the policy change is unanticipated, n_t would accumulate monotonically until reaching the new steady state.

In contrast, suppose that the policy shock is pre-announced (i.e., $T > 0$) and thus anticipated by the households. At $t = 0$, when the policy shock is announced, the shadow price λ would jump down to somewhere above $\tilde{O}$, say O. Continuity in T implies that O must be close to $\tilde{O}$ if T is small, and close to E_1 if T is great. As O lies in the southwest sector of the phase diagram centered by E_1, the economy moves southwesterly from O. At $t = T$, when the policy change takes place, stability implies that the economy would hit a point on the new stable manifold, say O'. From O', the economy would converge to E_2 along $O'E_2$. Along this trajectory, the path of $\hat{\lambda}_t$ must remain continuous for all $t > 0$, as implied by (22). Most importantly, this trajectory indicates that employment stays below the pre-shock level for a certain period. Figure 3 illustrates the time path of employment following the policy change. Particularly, the figure shows that employment declines since $t = 0$ and until $t = T$, although it eventually accumulates and exceeds the pre-shock level. Such a pattern suggests that military spending may negatively affect employment in the short run, but positively in the long run.

Intuitively, higher future employment results in higher future consumption, but it also may reduce present consumption because employment accumulation takes greater hiring costs. Thus, increasing long-run employment involves a trade-off between present consumption and future consumption. If their preference for intertemporal substitution is weak (i.e., $1/\sigma$ is low), employers would be reluctant to trade present consumption for future consumption. Thus, before the policy change takes place, employers even may reduce hiring costs to prevent their consumption from falling drastically. Consequently, employment may decrease in the short run.

Nevertheless, we cannot rule out the case in which the stable manifold around E_2 is relatively steep and passes above E_1. For example, the new stable manifold may be represented by the thin, broken line through E_2. Suppose the policy change is pre-announced. At $t = 0$, n stays at its original level, but λ jumps up to somewhere below the new stable manifold, say P. Then, the economy moves northeasterly and hits a point on the new stable manifold, say P', at $t = T$. From P', the economy would converge to E_2 along $P'E_2$. This trajectory indicates that employment would increase even before the policy change takes place. This case is likely to occur when the households have strong preferences for intertemporal substitution (i.e., $1/\sigma$ is high) and are willing to trade present consumption for future consumption.

4 Concluding Remarks

This paper studies the influences of military expenditure on employment in a search-theoretic model. It shows that a permanent increase in military burden may have a positive long-run effect on employment when the productivity-improving effect induced by this policy change outweighs the accompanying tax distortions, and vice versa if the employment effect is negative. Moreover, the magnitude of the long-run effect is found to be dependent on labor market fundamentals, suggesting that cross-country differences in labor market fundamentals may help explain cross-country differences in the employment effects of military expenditure. This paper also shows that the dynamic response of employment to a military shock depend on the nature of the shock and the households' preferences for intertemporal substitution. This result suggests that the search model can provide plausible explanations for the employment dynamics that have been empirically identified.

This paper is one step toward understanding the employment effects of military expenditure. For tractability, the model has abstracted from physical capital and has been restricted to one sector. It is thus important to incorporate physical capital and a two-sector setup into the present model. The extended models then can be used to address such important issues as economic growth and the reallocation of productive factors between civilian and military sectors. These interesting avenues are left for future research.

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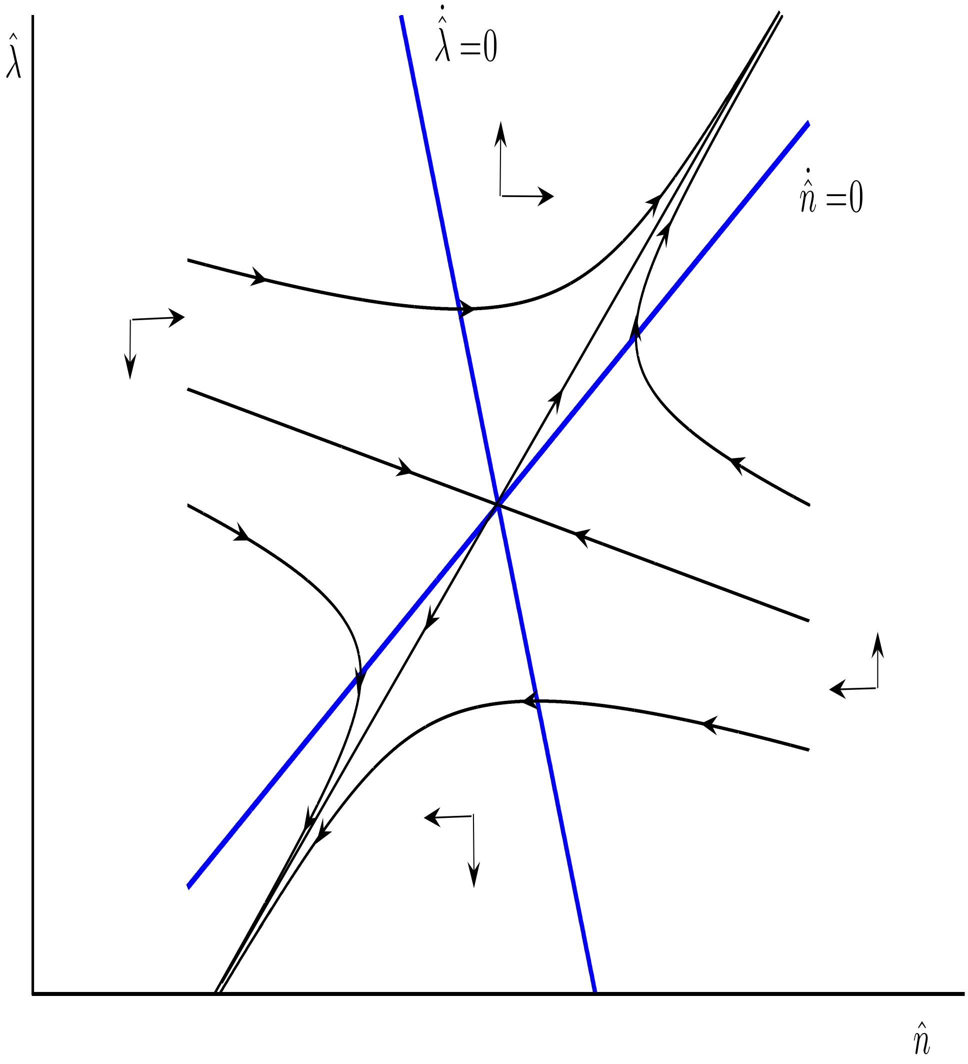

Figure 1: Local structure of the high-employment steady state

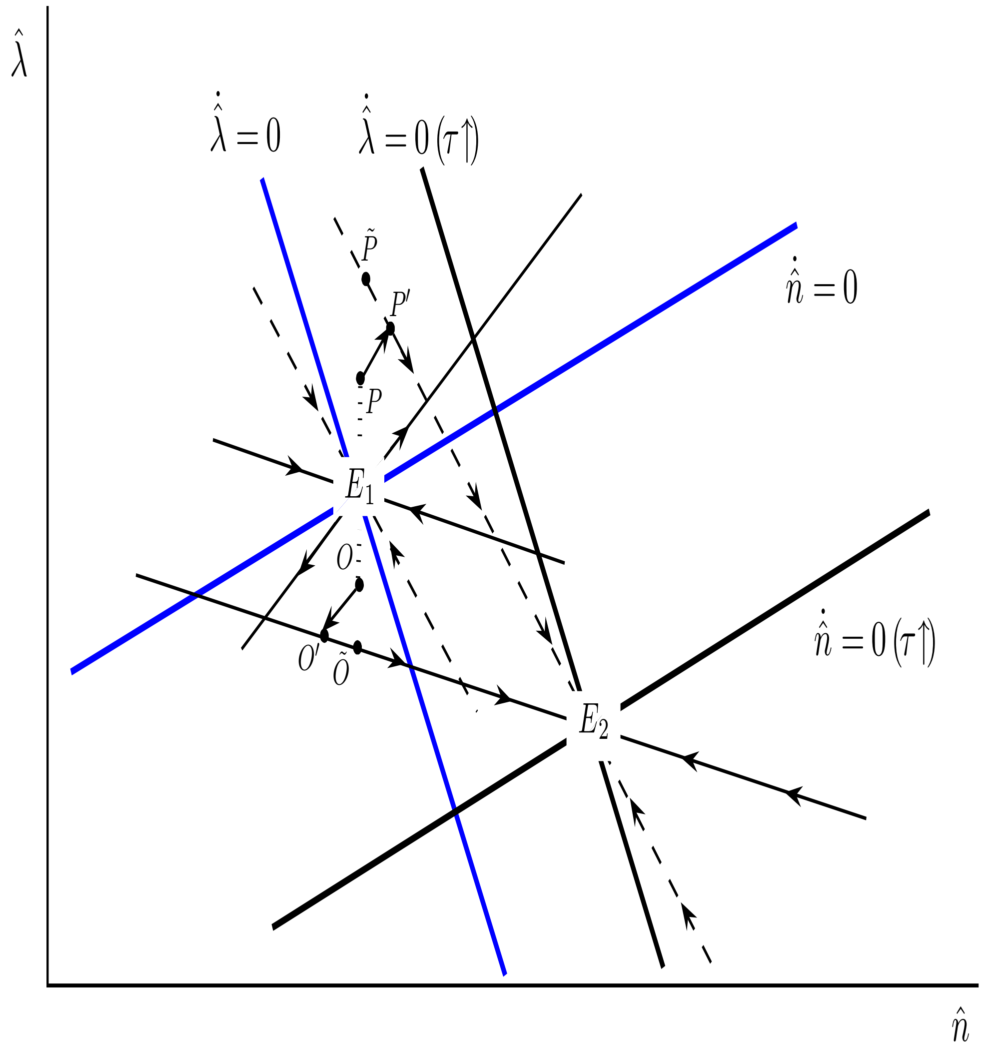

Figure 2: Transitional dynamics induced by an anticipated increase in military burden

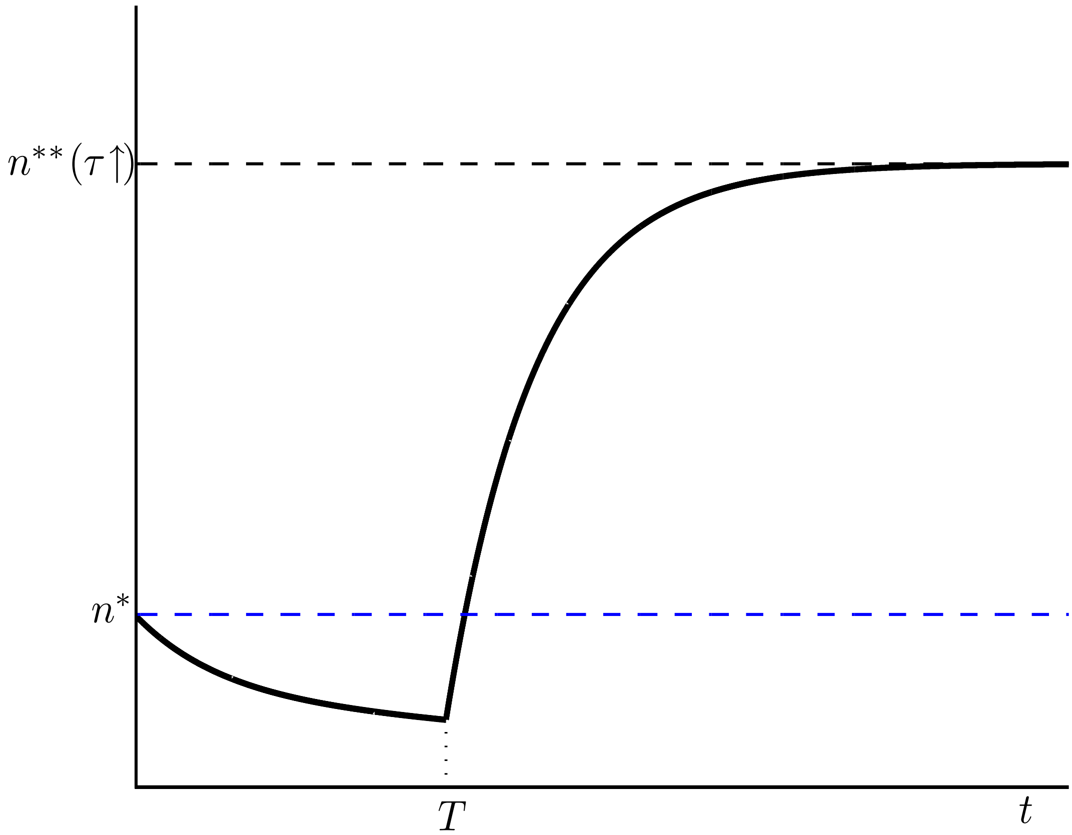

Figure 3: Time path of employment following an anticipated increase in military burden

A Mathematical Appendix

A.1 Derivation of (22) in the main text

Here we derive (22) in details. Equations (11), (13), and (15) are log-linearized to obtain

$$\hat{m}_t = \frac{\hat{\tau}_t}{1-\alpha} + \hat{n}_t, \tag{26}$$

$$0 = -\eta\hat{\theta}_t + \sigma\hat{z}_t + \hat{\lambda}_t, \tag{27}$$

$$0 = \left(1-\alpha+\frac{\chi\theta^* n^*}{y^*} - \frac{bn^*}{y^*}\right)\hat{n}_t - \frac{\chi\theta^*(1-n^*)}{y^*}\hat{\theta}_t - \frac{z^*}{y^*}\hat{z}_t + (\alpha-\tau)\hat{m}_t. \tag{28}$$

Equations (1) can be log-linearized to yield

$$\dot{\hat{n}}_t = \delta(1-\eta)\hat{\theta}_t - (q^*\theta^* + \delta)\hat{n}_t. \tag{29}$$

Equations (14) and (10) are log-linearized and combined to yield

$$\dot{\hat{\lambda}}_t = (\delta+\rho)\eta\hat{\theta}_t - \frac{(\alpha-\tau)(1-\beta)q^* y^*}{\chi n^*}\hat{\tau}_t. \tag{30}$$

The system (22) in the main text can be obtained by replacing (26)—(27) into (29) and (30).

A.2 The signs of J_{11} and M_{21}/M_{11}

To ensure that $J_{11} < 0$ and $M_{21}/M_{11} < 0$, we assume

$$\max\left\{\frac{\kappa_1\kappa_3 - \kappa_2}{\eta z^*}, \frac{\kappa_4 - \kappa_2}{\eta z^*}\right\} < \frac{1}{\sigma}, \tag{31}$$

where

$$\kappa_1 \equiv z^* + \chi\theta^*,\ \ \kappa_2 \equiv \chi\theta^*(1-n^*),\ \ \kappa_3 \equiv \frac{\delta(1-\eta)}{q^*\theta^* + \delta},\ \ \kappa_4 \equiv \frac{\eta(\delta+\rho)\chi n^*}{(1-\alpha)(1-\beta)q^*}.$$

To see the implications of (31), note that by its definition,

$$J_{11} = \delta(1-\eta)\left(\frac{\kappa_1}{\kappa_2 + \eta z^*/\sigma} - \frac{1}{\kappa_3}\right).$$

Thus, if $(\kappa_1\kappa_3-\kappa_2)/(\eta z^*) < 1/\sigma$, $J_{11} < 0$. If $(\kappa_4-\kappa_2)/(\eta z^*) < 1/\sigma$, it can be obtained from the definition of M_{11} and M_{21} that

$$\frac{M_{21}}{M_{11}} = \frac{(\delta+\rho)\eta}{\delta(1-\eta)} - \frac{(1-\beta)(1-\alpha)q^*\left[\chi\theta^*(1-n^*)+\eta z^*/\sigma\right]}{\delta(1-\eta)\chi n^*} = \frac{(1-\beta)q^*}{\delta(1-\eta)\chi n^*}\left(\kappa_4-\kappa_2-\eta z^*/\sigma\right) < 0.$$

As shown above, J_{11} is negative unless $\kappa_1\kappa_3-\kappa_2$ is positive and $1/\sigma$ is small. Meanwhile, M_{11} and M_{21} have opposite signs unless $\kappa_4-\kappa_2$ is positive and $1/\sigma$ is sufficiently small.

For illustrative purposes, we provide a numerical example. In line with the literature (e.g., Shi and Wen, 1999), we set $A = \overline{\psi} = 1$, $\eta = \beta = 0.5$, and $\delta = 0.1$. The values of τ and α are 0.05 and 0.1, respectively. The value of τ is a reasonable estimate for the military burden. The value of α is admittedly arbitrary, but the results of this example is quite robust under alternative values of α. The value of χ is 0.23, which is selected such that the vacancy cost, $\chi\theta^*(1-n^*)$, is approximately 5% of GDP. As the GDP share of vacancy cost ranges from 1% to 10% in the literature, our calibration seems to be reasonable. The value of b is calibrated at 0.53, implying that $n^* = 0.94$. Under this parameterization, the equilibrium conditions imply $n^* = 0.94$, $q^* = 0.64$, $\theta^* = 2.45$, and $z^* = 0.1$. Therefore, $(\kappa_1\kappa_3-\kappa_2)/(\eta z^*) = -0.26$, and $(\kappa_4-\kappa_2)/(\eta z^*) = 0.15$. Thus, $J_{11} < 0$ for all values of σ; $M_{21}/M_{11} < 0$ if $1/\sigma > 0.15$ (or $\sigma < 6.67$).