Non-productive consumption loans and threshold effects in the inflation-growth relationship

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Abstract

Recent empirical evidence indicates that two inflation thresholds exist in the inflation-growth relationship. Pre-existing theoretical models, however, fail to generate such a pattern. By adding non-productive consumption loans into a standard model of informational imperfection, this paper finds that an increase in the inflation rate may increase, decrease, or have no significant effect on economic growth for inflation rates below a threshold level; however, for inflation rates higher than this threshold level, an increase in the inflation rate significantly reduces economic growth. Moreover, the marginal impact of an increase in the inflation rate in terms of reducing economic growth is increasing along with the rise in the inflation rate, until the inflation rates reach another threshold level, from which such a marginal effect significantly decreases. These results accord well with recent empirical evidence.

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1. Introduction

Ever since the seminal work of Tobin (1965), the effect of inflation on capital investment and economic growth has long been one of the important topics in macroeconomics. Theoretically, depending on how money is introduced into the model, the early literature has established that an increase in the inflation rate may lead to an increase [as in Tobin (1965) where money is a substitute for capital in the portfolio], a decrease [as in Stockman (1981) where capital investment is subject to the cash-in-advance constraint], or have no effect [as in Sidrauski (1967) where money enters into the utility function] on capital investment and economic growth. On the other hand, early empirical studies report a mixed correlation between inflation and economic growth, until recently where numerous studies have found non-linear correlations between inflation and economic growth.

Fischer (1993) first points out a possibility that the effect of an increase in the inflation rate on economic growth may differ at low levels and high levels of inflation. Specifically, by choosing 15% and 40% inflation rates as the break points, Fischer (1993) finds that an increase in the inflation rate leads to an increase in economic growth for inflation rates below 15%; however, such an increase results in a reduction in economic growth for inflation rates above 15%. Moreover, the marginal (and negative) impact of an increase in the inflation rate on economic growth is substantially lower for inflation rates above 40% than for those between 15% and 40%. Consequently, there may exist two inflation thresholds under which the effect of an increase in the inflation rate on economic growth changes.

By adopting the break points proposed by Fischer (1993), Barro (1997) presents a negative correlation between inflation and economic growth for all ranges of inflation; however, the coefficients of the inflation rates in the growth regressions are equal to –0.023 for inflation rates below 15%, –0.055 for inflation rates between 15% and 40%, and –0.029 for inflation rates above 40%. In a sense, Barro's (1997) finding is very consistent with Fischer (1993), as there may be two inflation thresholds in the inflation-growth relationship.[1]

[1] Indeed, according to Barro's (1997) result, the marginal effect of an increase in the inflation rate in reducing economic growth is small for inflation rates below the first threshold level as well as for inflation rates

The only difference between Barro (1997) and Fischer (1993) is the inflation-growth correlation for inflation rates below 15%.

Ghosh and Phillips (1998) also exhibit a pattern of the inflation-growth relationship that is very close to Fischer (1993). In contrast to Fischer (1993), whose break points are picked based on judgment, Ghosh and Phillips (1998) employ a panel regression to allow for a non-linear specification and find a positive correlation between inflation and economic growth for very low levels of inflation rates (around 2-3% a year). For other levels of inflation rates, however, there is a negative correlation between inflation and economic growth, corroborating the existence of the first inflation threshold. Moreover, the negative correlation is convex - namely, that the marginal impact of an increase in the inflation rate in reducing economic growth is higher for inflation rates between 10% and 20% than for those between 40% and 50%, implying that a second threshold level does exist. Khan and Senhadji (2001) and Burdekin, Denzau, Keil, Sitthiyot, and Willett (2004) also confirm Ghosh and Phillips's (1998) findings.[2]

While the aforementioned studies reach an agreement on the existence of two inflation thresholds, they disagree on the inflation-growth correlation for inflation rates below the first threshold. Such a disagreement, in fact, can also be found from other empirical studies. Bullard and Keating (1995) find that an increase in the inflation rate leads to an increase (a decrease) in economic growth for low (high) levels of inflation. Along the same lines, Sarel (1996) uncovers an 8% threshold level of inflation, so that an increase in the inflation rates leads to a significant reduction in economic growth for inflation rates above the threshold level. For inflation rates below this threshold level, inflation has an insignificant effect on economic growth, or it may display a slightly positive effect. Khan and Senhadji (2001), using a newly-developed econometric technique to re-examine this issue, find that a

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above the second threshold level. If inflation rates are located in-between the first and second thresholds, then inflation has a relatively large impact on economic growth.

[2] In particular, Figure 2 in Khan and Senhadji (2001) replicates the inflation-growth relationship that we just outlined. Burdekin, Denzau, Keil, Sitthiyot, and Willett (2004) examine the inflation-growth correlation for developing countries. They find that the first threshold level of inflation is 3% while the second one is 15%.

significant threshold relationship exists between inflation and economic growth in a way similar to that found by Sarel (1996) and Ghosh and Phillips (1998).[3] In Bruno and Easterly (1998) the negative correlation between inflation and economic growth is only observed with high levels of inflation; for low levels of inflation, there is no cross-country correlation between inflation and economic growth.

To sum up, recent empirical studies have found two inflation thresholds in the inflation-growth relationship. Specifically, an increase in the inflation rate may increase, decrease, or have no significant effect on economic growth for the inflation rates below the first threshold level. Once the inflation rates are greater than the first threshold level, inflation displays a significant negative effect on economic growth, with the marginal (negative) impact of an increase in the inflation rate on economic growth substantially decreasing when the inflation rates are greater than the second threshold level. While recent empirical studies have confirmed the existence of two inflation thresholds, pre-existing theoretical models fail to generate such a pattern for the inflation-growth relationship.[4] The purpose of this paper is to develop a model that is able to yield such a pattern for the inflation-growth relationship and thereby provide a possible theoretical explanation.

Ideally, a model that is able to yield such a pattern in the inflation-growth relationship should contain two opposite effects of an increase in the inflation rate on economic growth, for example in the following way. The positive effect may dominate (be dominated by) the negative one at low (high) levels of inflation (the first inflation threshold) and, for high levels of inflation, the difference between these two effects is significant until inflation rates are greater than another threshold level, from which this difference diminishes along with an increase in the inflation rate. To yield a positive effect of inflation on economic growth, however, is not so obvious, considering that there is a consensus that inflation has an adverse effect on resource allocation and is thereby detrimental to capital investment and economic growth.

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[3] Khan and Senhadji (2001) find that this threshold is lower in developed countries than in developing countries.

[4] As the inflation rate does not have any effect on how money is introduced into the theoretical models, early theoretical models are not able to explain this pattern.

By adding non-productive consumption loans into a standard model of informational imperfection, this paper is able to generate a positive effect of inflation on economic growth. As pointed out by Modigliani (1986), informational problems in the financial markets may result in credit rationing of consumption loans, which prevents consumers from borrowing as much as they would like to reach their optimal consumption profile. It turns out, as modeled by Jappelli and Pagano (1994), that credit rationing of non-productive consumption loans may force the economy to save more resources for capital investment and thereby facilitate economic growth. We then follow Bose (2002) by assuming that money is needed for loan transactions between lenders and borrowers. Under this framework, we find that inflation impedes bank lending activity for the purpose of consumption (household credit) so that an inflation rate increases the incidence of credit rationing of consumption loans, which, as modelled by Jappelli and Pagano (1994), is beneficial to capital investment and economic growth. Consequently, the presence of non-productive consumption loans gives rise to a positive effect of inflation on economic growth.

The coexistence of positive and negative effects of inflation on economic growth allows us to capture two inflation thresholds in the inflation-growth relationship. Specifically, an increase in the inflation rate leads to a rise in the incidence of credit rationing of both investment and consumption loans. While an increase in the incidence of credit rationing of investment loans is detrimental to capital investment and economic growth, such an increase in relation to non-productive consumption loans facilitates it. The presence of non-productive consumption gives rise to another channel through which inflation may affect capital investment and economic growth. This can be observed by the fact that an increase in the inflation rate leads to an increase in the loan rate. Since the amount borrowed by consumers (the quantity of consumption loans) is related to the present value of their old-age income, an increase in the loan rate decreases the quantity of each consumption loan, which increases the amount of resources available for capital investment and is thereby beneficial to capital investment and economic growth.

The net effect of an increase in the inflation rate on economic growth depends on the two opposite effects mentioned above. We find that the effect arising from consumption loans may dominate (be dominated by) that from investment loans for low (high) levels of inflation, resulting in the first inflation threshold.[5] Furthermore, for inflation rates above the first threshold level, the magnitude of the effect from investment loans is increasing in the inflation rate while the magnitude of the effect from consumption loans is first decreasing and then increasing in the inflation rate. As the marginal impact of inflation on economic growth is the difference between these two, this implies that the marginal impact of inflation on economic growth is first increasing in the inflation rate and then decreasing, leading to the second inflation threshold.

This paper proceeds as follows. Section 2 presents the basic model and Section 3 describes the equilibrium loan contracts for the purpose of investment and consumption. In Section 4 we first obtain the equilibrium growth rate and then examine how a change in the inflation rate affects the equilibrium rate of economic growth. We also compare our results with recent theoretical studies. Section 5 concludes.

2. Description of the Model

The economy consists of an infinite sequence of three-period-lived overlapping generations (OG).[6] Each generation is of identical size and composition, and contains two kinds of risk neutral agents: lenders and borrowers. Borrowers are further classified into two groups of equal size: entrepreneurs and consumers. For simplicity, the populations of lenders, consumers, and entrepreneurs are normalized to n, m, and one, respectively.

2.1. Behavior of Agents

A young lender, endowed with one unit of labor, cares only about his old-period consumption; hence, he will sell his labor to firms to generate wage income and save this income for old-age consumption. Each young lender is endowed with a constant-returns-to-scale (CRTS) technology that can convert one unit of time t output into $Q\varepsilon$ ($\varepsilon < 1$) units of time

[5] More specifically, for inflation rates below the first inflation threshold, an increase in the inflation rate increases (decreases) economic growth if the effect from consumption loans is greater (less) than that from investment loans. If the difference between both effects is not large, then there is no significant correlation between inflation and economic growth for inflation rates below the first threshold.

[6] The structure of this model follows Bose (2002).

t+2 units of capital with certainty.[7] By denoting ρ_{t+2} as the rental rate of capital at time t, it is clear that the rate of return of the CRTS technology between time t and t+2 is $Q\varepsilon\rho_{t+2}$.

A time-t young lender can simply save his wage income by means of this CRTS technology for his old-age consumption. Alternatively, young lenders can extend loans to borrowers in return for time t+2 output. As is the case in Azariadis and Smith (1996), it is assumed that there are financial intermediaries that attract deposits from lenders and offer loans to borrowers.

Entrepreneurs care only about old-age consumption. An entrepreneur is endowed with one unit of labor as well as an investment project in his second period of life. The investment project is risky, and according to its probability of success can be classified as either high-risk (type-*H*) or low-risk (type-*L*). Note that entrepreneurs are not endowed with any output; hence, external funding is necessary for an entrepreneur to implement his project. Following Bose (2002), a young entrepreneur must apply for a loan (from an intermediary) during his first period of life, even though he needs the external funding during middle age.

A middle-aged entrepreneur who obtains a loan from the intermediary can operate his investment project using his own labor to convert one unit of time-t+1 output into Q units of time-t+2 capital, with probability p_i, $i=H, L$. With probability 1-p_i, the operation of the project fails and nothing is produced. By assumption, $0 < p_H < p_L \leq 1$ and the types of entrepreneurs' projects are private information. Moreover, a fraction λ of entrepreneurs is assumed to have type-*H* projects.

If no funds are forthcoming, then the entrepreneur can utilize his labor in the home production of goods during his second period of life.[8] Following Bose (2002), output produced by a type-i entrepreneur at time t in the home production yields $\beta_i^e w_{t-1}$ units of time t consumption goods, where w_{t-1} is the market wage rate at time t-1. It is assumed that $\beta_L > \beta_H = 0$, implying that the opportunity cost of being rejected in regard to a loan is lower

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[7] As will be seen, an entrepreneur's capital project, if successful, can convert one unit of time-t output into Q units of time-t+2 capital. The assumption that $\varepsilon < 1$ implies that the CRTS technology is inferior to the entrepreneur's project in terms of producing capital.

[8] The low- (high-) risk entrepreneurs are those entrepreneurs whose projects belong to type-*L* (-*H*). Entrepreneurs' projects are not tradable.

for a low-risk entrepreneur than for a high-risk one. As will be seen, this assumption makes the separating equilibrium emerge.[9] We also follow Bose (2002) by assuming that a middle-aged entrepreneur has access to a storage technology that can convert one unit of time-t+1 output into one unit of time-t+2 output.[10] Thus, if a low-risk entrepreneur is rejected in regard to a loan, he can engage in the home production and store the proceeds for his old-age consumption.

Each consumer borrower cares about consumption in his second and third periods of life. Consumers have no endowment in the first period; however, with a non-negative probability, each consumer will be endowed with one unit of labor in his final period of life. We assume that the structure of consumers is similar to that of entrepreneurs, namely, that there are two types of consumers and that consumers' types (private information) refer to the probability of getting one unit of labor when old. Specifically, with probability p_i, i=H, L, a type-i consumer will receive one unit of labor in the old period and, with probability $1-p_i$, the consumer will be endowed with nothing. Similar to entrepreneurs, $0 < p_H < p_L \leq 1$ and a fraction λ of consumers is of type-H.

The utility function of a representative (generation-t) consumer is given as

$$U^c(c_t,\ c_{t+1}, c_{t+2}) = c_{t+1} + \beta^c c_{t+2}, \tag{1}$$

where c_t is the consumption in time t and β^c is the discount factor. To induce borrowing, we assume that β^c is sufficiently small; hence, if possible, all consumers intend to borrow from the intermediary and consume all expected old-age income in their middle age. In a way similar to entrepreneur borrowers, consumer borrowers must apply for a loan at a young age, even though they are concerned about middle-age consumption. It should be noted that if no funds are forthcoming, then the expected lifetime utility of a generation-t consumer is $p_L \beta^c w_{t+2}$ for type-L consumers and $p_H \beta^c w_{t+2}$ for type-H ones. Given that $p_L > p_H$,

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[9] As indicated by Bose (2002), the assumption that different types of borrowers with different opportunity costs are denied credit ensures the "single crossing properties" of the indifference curve in the contract plane.

[10] The low-risk entrepreneur can engage in home production in his second period of life if his loan application is rejected. The entrepreneur, however, cares only about old age consumption. Hence, Bose (2002) implicitly assumes that the low-risk entrepreneur in middle age has access to a storage technology. It should be noted that this storage technology is not accessible to young borrowers, so that young borrowers must hold money. See below.

it is clear that type-L consumers have a lower opportunity cost of being denied credit than do type-H ones.

Without a loss of generality, it is assumed that each entrepreneur can operate a firm.[11] The old borrower can utilize capital that he produced himself and/or rent capital from other old entrepreneurs and lenders, plus hire young lenders and old consumers (who obtain labor endowment) as labor input to produce output. The output production in time t is given as

$$y_t = A\psi_t^{\sigma} k_t^{\alpha} L_t^{1-\alpha}, \ A>0, \tag{2}$$

where ψ_t denotes the average capital stock per firm and k_t and L_t are the capital stock as well as labor employed by the firm, respectively. Capital depreciates fully after production. In the capital market equilibrium, each firm employs the same amount of capital; hence, $\psi_t = k_t$. Moreover, following Bose and Cothren (1996) and Bose (2002), it is assumed that $\sigma = 1-\alpha$, implying that the output production technology is linear as in the Ak model. Labor and capital are competitive so that the wage rate (w_t) and the rental rate of capital (ρ_t) at time t are given as

$$w_t = A(1-\alpha)k_t^{\sigma+\alpha} L_t^{-\alpha} = A(1-\alpha)k_t L_t^{-\alpha} \tag{3}$$

and

$$\rho_t = A\alpha\psi^{\sigma} k_t^{\alpha-1} L_t^{1-\alpha} = A\alpha L_t^{1-\alpha}. \tag{4}$$

As will become clear, the per-firm labor employment is constant over time under separating equilibrium in the loans market. Hence, the rental rate of capital is constant over time (which is denoted as ρ) as in the Ak model.[12]

2.2. Financial Intermediation, Money, and Loan Transactions

We assume that lending/borrowing is intermediated.[13] It is also assumed that each young lender can establish an intermediary without incurring any cost. The assumption of free entry into the intermediary activity ensures competitive behavior among intermediaries,

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[11] In other words, the number of firms is normalized to 1 – an assumption also made by Bose (2002).

[12] Labor employment includes young lenders and old-consumers who obtain one unit of labor endowment. Hence, $L_t = L = n + m[\lambda p_H + (1-\lambda)p_L]$.

[13] Implicitly, we assume that direct lending/borrowing between lenders and borrowers is more costly than indirect lending/borrowing (i.e. via financial intermediation). We also consider the limiting case where the cost of intermediation is normalized to zero.

which will drive the intermediary's profits to zero.

It should be recalled that borrowers need external funding during their second period of life while lenders wish to save while still young. As a result, borrowers must contract with financial intermediaries when they are young. Once a young borrower at time t obtains a loan from an intermediary, he must exchange it for money in the same period and then use the money to buy output in the next period for capital investment or consumption. Moreover, the operation of each financial intermediary is subject to a reserve requirement policy that requires that each young intermediary hold a μ $(1 > \mu > 0)$ fraction of total deposits in the form of money between time t and t+1.[14]

At the beginning of time t, each young borrower applies for loans from a young intermediary. Once a young intermediary reaches an agreement with a borrower, he must offer a deposit contract to young lenders. Suppose that a young intermediary at time t agrees to offer q_t units of time-t output to a borrower at time t. Then, in order to fulfill the borrower's need, the intermediary must also offer a deposit contract at time t to young lenders that attracts $q_t/(1-\mu)$ units of time-t output in the form of deposits during the same period. Once the intermediary obtains $q_t/(1-\mu)$ in deposits, he will hand over q_t $(\equiv (1-\mu)q_t/(1-\mu))$ units of deposits to the borrower and hold $q_t\mu/(1-\mu)$ units of deposits in the form of money between time t and t+1 to satisfy the reserve requirement.

The demand for money originates from the young borrowers (who want to store their loans in the form of money) as well as the young intermediary (who needs to hold money to satisfy the reserve requirement).[15] Following Bose (2002), the government accomplishes any monetary injection by a lump-sum transfer to old lenders and borrowers; hence, the suppliers of money at any point of time include these agents (old lenders and borrowers) plus the middle-aged intermediaries (who hold money as a reserve requirement during their young period in life). On the other hand, young borrowers (who obtain loans) and young

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[14] For simplicity, the young intermediary is an intermediary operated by a young lender.

[15] In Bose (2002), young lenders who intend to lend to borrowers need to hold money. We have modified our model to be consistent with Bose (2002) in such a way that young lenders, instead of young borrowers, need to hold money. The conclusion derived below, however, does not change. The results are available upon request.

intermediaries (who are required to hold money) must hold money.

Denote δ_t as the fraction of total wage incomes of time-*t* young lenders that are lent to young borrowers (via intermediaries). The market-clearing price (P_t) is then determined by an equation similar to Bose (2002), which can be written as $\delta_t n w_t P_t /(1-\mu) = \delta_{t-1} n P_{t-1} w_{t-1} /(1-\mu) + M_t$.[16] As stated by Bose (2002), w_t is proportional to k_t (eq. (2)), which is predetermined in time *t*-1, and P_{t-1}, δ_{t-1}, w_{t-1}, and w_{t-2} are also predetermined. Therefore, the inflation rate is determined by the change in monetary injections (or withdrawals) as well as by the ratio of the reserve requirement μ. As both are policy variables determined by the monetary authority, we follow Bose (2002) by treating the inflation rate, denoted by $1+\tau$, as a policy variable.

3. Loan Markets and the Equilibrium Contracts

We shall now determine the equilibrium contracts for investment and consumption loans. At the beginning of each period, each time-*t* young intermediary must decide whether or not to finance borrowers after receiving deposits. To finance the borrower, the young intermediary announces one set of contracts intended for entrepreneur borrowers and the other set for consumer borrowers. If a young intermediary's offer is not dominated by others, then he is approached by potential young borrowers. Since money is needed in loan transactions, the contract terms offered by the intermediary must take the inflation rate into account. After the completion of loan transactions, the intermediary offers a deposit contract to young lenders and attracts deposits to fulfill the needs of borrowers.

As in Bencivenga and Smith (1993), the equilibrium loan contracts at time *t* are defined such that there is no incentive for any intermediary to offer an alternative contract, taking w_t, w_{t+2}, ρ_{t+2}, $1+\tau$ (the inflation rate), and other intermediaries' offers as given. We also

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[16] Recall that the population of lenders is equal to *n* and each lender is endowed with one unit of labor when young. Hence, young lenders' total wage income amounts to nw_t. If young borrowers intend to borrow $\delta_t n w_t$, financial intermediaries must attract $\delta_t n w_t /(1-\mu)$ units of deposits, of which $(1-\mu)\delta_t n w_t /(1-\mu)$ ($=\delta_t n w_t$) is handed over to borrowers and $\mu\delta_t n w_t /(1-\mu)$ is held in the form of money. Note that borrowers will sell $(1-\mu)\delta_t n w_t /(1-\mu)$ for money and will use the money in exchange for output in the next period. Hence, the total demand for money at time *t* is equal to $P_t\{[\mu\delta_t n w_t + (1-\mu)\delta_t n w_t]/(1-\mu)\}$, which is equal to $\delta_t n w_t P_t /(1-\mu)$.

follow Bencivenga and Smith (1993) and Bose (2002) by focusing on the separating equilibrium such that an intermediary offers contracts that separate borrowers according to their type. Before determining the equilibrium contracts for entrepreneurs and consumers, we first specify the equilibrium loan rate for both types of loans and the deposit contract extended to young lenders.

3.1. The Deposit Contract and the Equilibrium Loan Rates to Borrowers

Competition ensures that each intermediary earns zero profit from lending/borrowing. Recall that the rate of return from lenders' CRTS technology between time *t* and *t*+2 is $Q\varepsilon\rho$. The intermediaries' zero-profit condition implies that the rate of return in relation to the deposits between time *t* and *t*+2 must be equal to $Q\varepsilon\rho$, the rate of return on the lenders' CRTS technology.

Let $q_{t,i}^{j}$ and $R_{t,i}^{j}$, $i = H, L$ and $j = c, e$, be respectively the loan quantity and loan rate offered by an intermediary at time *t* to a type-*i* consumer or entrepreneur (the superscript *c* refers to the consumer while *e* refers to entrepreneurs). Since deposits are subject to the reserve requirement policy, each intermediary at time *t* must attract $q_{t,i}^{j}/(1-\mu)$ units of deposits, of which $q_{t,i}^{j}$ is handed over to the borrower and $q_{t,i}^{j}\mu/(1-\mu)$ is exchanged for money to satisfy the reserve requirement. Note that the money obtained at time *t* can be exchanged for time-*t*+1 output in the next period that is equal to $q_{t,i}^{j}\mu/(1-\mu)(1+\tau)$, where $(1+\tau)$ is the inflation rate between time *t* and *t*+1. We assume that, similar to entrepreneurs, each middle-aged intermediary at time *t*+1 has access to a storage technology that can convert one unit of time-*t*+1 output into one unit of time-*t*+2 output.[17] Thus, the intermediary can simply store $q_{t,i}^{j}\mu/(1-\mu)(1+\tau)$ units of time-*t*+1 output and repay them to his depositors at time *t*+2. We assume that $Q\varepsilon\rho > 1/(1+\tau)$; hence, the intermediary will only attract the amount of deposits that is exactly equal to $q_{t,i}^{j}/(1-\mu)$.

The zero-profit condition for each intermediary in terms of lending/borrowing activity is

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[17] We have assumed that the young intermediary is asked to hold the money between time *t* and *t*+1. If the intermediary is required to hold the money for two periods, then the rate of return from money is equal to $1/(1+\tau)^2$ and the loan rate (i.e. eq. (6) below) is equal to $R_{t,i}^{j} = \{Q\varepsilon\rho - [\mu/(1+\tau)^2]\}/p_i(1-\mu)$. We have verified that our results as obtained below do not change. The results are available upon request

given by[18]

$$\frac{q_{t,i}^j}{1-\mu} Q\varepsilon\rho = p_i q_{t,i}^j R_{t,i}^j + \frac{\mu}{(1-\mu)(1+\tau)} q_{t,i}^j, \tag{5}$$

which further implies that

$$R_{t,i}^j = \frac{Q\varepsilon\rho - \dfrac{\mu}{(1+\tau)}}{p_i(1-\mu)}, \quad i = H,\ L, \quad j = c,\ e. \tag{6}$$

An increase in the inflation rate leads to an increase in the loan rate. This result is quite intuitive, because money is needed for loan transactions, but is inessential for lenders' CRTS technology. Note that the expected return to an intermediary from lending to entrepreneurs should be equal to that from lending to consumers; otherwise, one type of loan will disappear. In other words, eq. (6) should be offered to both entrepreneurs and consumers.

3.2. Equilibrium Contracts to Entrepreneurs

As in Bencivenga and Smith (1993) and Bose (2002), the contract offered by an intermediary to a type-*i* entrepreneur (denoted as $C_{t,i}^e$) at time *t* comprises a 3-tuple $\{\pi_{t,i}^e,\ q_{t,i}^e,\ R_{t,i}^e\}$, where $\pi_{t,i}^e \in [0,1]$ is the probability with which an intermediary offers the loan. Given this, a generation-*t* entrepreneur's expected utility at time *t*+1 is

$$p_L \pi_{t,L}^e [\frac{q_{t,L}^e}{(1+\tau)} Q\rho - R_{t,L}^e q_{t,L}^e] + (1-\pi_{t,L}^e)\beta_L^e w_t \tag{7}$$

for a type-*L* entrepreneur and is

$$p_H \pi_{t,H}^e [\frac{q_{t,H}^e}{(1+\tau)} Q\rho - R_{t,H}^e q_{t,H}^e]. \tag{8}$$

for a type-*H* entrepreneur.

Note that eq. (6) implies that $R_{t,H}^e > R_{t,L}^e$. Since a borrower's type is private

[18] The LHS of eq. (5) is the expected return for the lenders who invest $q_{t,i}^j/(1-\mu)$ units at time *t* in the CRTS technology while the RHS is the expected return to the intermediary at time *t* who lends $q_{t,i}^j$ to the borrower and holds $q_{t,i}^j\mu/(1-\mu)$ in the form of real money between time *t* and *t*+1. The zero-profit condition implies that both are equal.

information, this will induce type-*H* entrepreneurs to pretend to be type-*L* ones and to enjoy a lower loan rate. To prevent this from occurring, the contract terms must satisfy the following self-selection constraints for type-*H* entrepreneurs:

$$p_H \pi^e_{t,H} [\frac{q^e_{t,H}}{(1+\tau)} Q\rho - R^e_{t,H} q^e_{t,H}] \geq p_H \pi^e_{t,L} [\frac{q^e_{t,L}}{(1+\tau)} Q\rho - R^e_{t,L} q^e_{t,L}]. \tag{9}$$

The LHS of eq. (9) shows the expected payoffs of a type-*H* entrepreneur when he reveals his true type in applying for a loan that is intended for a type-*H* entrepreneur (i.e. $C^e_{t,H}$), while the RHS is that of a type-*H* entrepreneur when he pretends to be a type-*L* entrepreneur in applying for $C^e_{t,L}$. Under eq. (9), type-*H* entrepreneurs prefer revealing their true type to misrepresenting themselves. Similarly, the separating equilibrium requires that type-*L* entrepreneurs have no incentive to pretend to be type-*H* entrepreneurs; hence,

$$p_L \pi^e_{t,L} [\frac{q^e_{t,L}}{(1+\tau)} Q\rho - R^e_{t,L} q^e_{t,L}] + (1-\pi^e_{t,L}) \beta^e_L w_t$$

$$\geq p_L \pi^e_{t,H} [\frac{q^e_{t,H}}{(1+\tau)} Q\rho - R^e_{t,H} q^e_{t,H}] + (1-\pi^e_{t,H}) \beta^e_L w_t. \tag{10}$$

Under the separating equilibrium, at least one of equation (9) or equation (10) must hold with the strict inequality.

To determine the equilibrium contract intended for entrepreneurs, it should first be noted that the entrepreneurs' capital technology is linear (in the case of success); hence, each will intend to implement his project at the maximal scale.[19] As pointed out by Bencivenga and Smith (1993), the maximal scale of the project in the presence of financial intermediaries should be related to k_t. As w_t is linear in k_t, we assume that the maximal scale for each generation-*t* entrepreneur's project is equal to w_t. Note that in order to obtain w_t in time *t*+1, each young entrepreneur at time *t* must borrow $(1+\tau)w_t$, where $(1+\tau)$ is the inflation rate between time *t* and *t*+1. Hence, $q^e_{t,L} = q^e_{t,H} = (1+\tau)w_t$. Moreover, there are

[19] It should be noted that the assumption that entrepreneurs' capital projects have a maximal scale is commonly encountered in the literature on asymmetric information. See Bencivenga and Smith (1993) for the case of adverse selection and Ma and Smith (1996) for the case of costly state verification.

participation constraints, which ensure that entrepreneurs are willing to borrow. Note that if a type-L entrepreneur does not apply for a loan, then his expected lifetime utility is equal to $\beta_L^e w_t$. As a result, to induce a type-L entrepreneur to borrow, it must be the case that eq. (7) is greater than (or equal to) $\beta_L^e w_t$; i.e.

$$p_L \pi_{t,L}^e [\frac{q_{t,L}^e}{(1+\tau)} Q\rho - R_{t,L}^e q_{t,L}^e] - \pi_{t,L}^e \beta_L^e w_t \geq 0,$$

or by using $q_{t,L}^e = q_{t,H}^e = (1+\tau)w_t$,

$$\pi_{t,L}^e w_t \{p_L[Q\rho - (1+\tau)R_{t,L}^e] - \beta_L^e\} \geq 0.$$

Obviously, both $\pi_{t,L}^e \geq 0$ and $p_L[Q\rho - (1+\tau)R_{t,L}^e] - \beta_L^e \geq 0$ are necessary for the above condition to hold. Under the condition that $p_L[Q\rho - (1+\tau)R_{t,L}^e] - \beta_L^e \geq 0$, one sees that eq. (7) is increasing in $\pi_{t,L}^e$.[20] Similarly, if a type-H entrepreneur does not apply for a loan, then his lifetime utility is $\beta_H^e w_t$ $(=0)$. Consequently, eq. (8) must be greater than (or equal to) 0, indicating that $[Q\rho - (1+\tau)R_{t,H}^e] \geq 0$. Assuming that this is the case, one sees that eq. (8) is increasing is $\pi_{t,H}^e$.

As in Bencivenga and Smith (1993), the separating equilibrium is obtained by offering type-H borrowers their corresponding first-best contract ($C_{t,H}^e$) while the contract offered to type-L ($C_{t,L}^e$) borrowers is distorted in such a way that type-H entrepreneurs are indifferent between accepting $C_{t,H}^e$ and $C_{t,L}^e$ contracts. Given that the expected payoff of a type-H entrepreneur in eq. (8) is increasing in $\pi_{t,H}^e$, it is clear that $\pi_{t,H}^e = 1$. Moreover, since the expected utility of a type-L entrepreneur in eq. (7) is increasing in $\pi_{t,L}^e$, the optimal contract's value of $\pi_{t,L}^e$ should be as large as possible, making the incentive constraint in eq. (9) binding. Hence,

$$\pi_{t,L}^e = \frac{Q\rho - (1+\tau)R_{t,H}^e}{Q\rho - (1+\tau)R_{t,L}^e} = \frac{p_L\{Q\rho[p_H(1-\mu) - (1+\tau)\varepsilon] + \mu\}}{p_H\{Q\rho[p_L(1-\mu) - (1+\tau)\varepsilon] + \mu\}}. \tag{11}$$

We focus on the case where the parameters satisfy $\pi_{t,L}^e < 1$; hence, some type-L entrepreneurs are denied loans and are thereby credit rationed. By substituting the terms of

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[20] The condition for $p_L[Q\rho - (1+\tau)R_{t,L}^e] - \beta_L^e \geq 0$ will be given below.

the equilibrium contract into eq. (10), we can verify that eq. (10) holds with a strict inequality, which proves that the separating equilibrium indeed exists.

It is instructive to examine how a change in the inflation rate influences the terms of the contracts extended to entrepreneurs. Note that

$$\frac{\partial \pi^e_{t,L}}{\partial(1+\tau)} = \frac{p_L (Q\rho)^2 \varepsilon (p_H - p_L)(1-\mu)}{p_H \{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu\}^2} < 0,$$

indicating that an increase in the inflation rate $(1+\tau)$ reduces the probability of type-*L* entrepreneurs obtaining a loan. This is so because a change in the inflation rate has an asymmetric effect on the loan rates in relation to type-*H* and type-*L* entrepreneurs, which exacerbates the problem of asymmetric information and thereby raises the amount of credit rationing in regard to good (type-L) borrowers. Moreover, it is easy to verify that

$$\frac{\partial^2 \pi^e_{t,L}}{\partial(1+\tau)^2} = \frac{p_L (Q\rho)^2 \varepsilon (p_H - p_L)(1-\mu) 2Q\varepsilon\rho}{p_H \{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu\}^3}$$

$$= \frac{2Q\varepsilon\rho}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu} \frac{\partial \pi^e_{t,L}}{\partial(1+\tau)} < 0; \qquad (11')$$

hence, the marginal impact of this negative effect on $\pi^e_{t,L}$ is reinforced as the inflation rate increases.

Note that $\pi^e_{t,L} \geq 0$ and $Q\rho - (1+\tau)R^e_{t,H} \geq 0$ if

$$(1+\tau) \leq \frac{Q\rho p_H (1-\mu)+\mu}{Q\varepsilon\rho} \equiv (1+\overline{\tau}^e_1).$$

Moreover, $p_L[Q\rho - (1+\tau)R^e_{t,L}] - \beta^e_L \geq 0$ if

$$(1+\tau) \leq \frac{Q\rho p_L (1-\mu)+\mu - \beta^e_L(1-\mu)}{Q\varepsilon\rho} \equiv (1+\overline{\tau}^e_2).$$

We also assume that $(1+\tau) < p_H(1-\mu)/\varepsilon \equiv (1+\overline{\tau})^*$, which implies that the entrepreneurs' capital project is better than the lenders' CRTS technology in terms of producing capital.[21]

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[21] A high-risk entrepreneur, who implements his capital project with w_t, can produce Qw_t of time t+2 capital. A lender, instead of lending $w_t(1+\tau)/(1-\mu)$ units to the entrepreneur, can convert $w_t(1+\tau)/(1-\mu)$ units into $w_t(1+\tau)Q\varepsilon/(1-\mu)$ units of time t+2 capital. Hence, the assumption that

We should consider the case where $(1+\tau)\le \max\{\ (1+\overline{\tau}_2^e),\ (1+\overline{\tau}_1^e),\ (1+\overline{\tau})^*\}$.

3.3. The Equilibrium Contract to Consumers

The equilibrium contract extended to consumers shares a similar feature with that extended to entrepreneurs. Specifically, the contract offered by a lender to a type-*i* consumer (denoted as $C_{t,i}^c$) at time *t* comprises a 3-tuple $\{\pi_{t,i}^c,\ q_{t,i}^c,\ R_{t,i}^c\}$, where $\pi_{i,i}^c \in [0,1]$ is the probability with which a lender offers the loan, $q_{t,i}^c$ is the quantity of loan offered, and $R_{t,i}^c$ is the loan rate that the consumer must pay back in time *t*+1 when he receives the labor endowment. The expected payoffs for a type-*i*, *i*=*H*, *L*, consumer are

$$\pi_{t,i}^c[\frac{q_{t,i}^c}{1+\tau}+\beta^c p_i(w_{t+2}-q_{t,i}^c R_{t,i}^c)]+(1-\pi_{t,i}^c)\beta^c p_i w_{t+2}. \tag{13}$$

The self-selection conditions that prevent type-*H* consumers from pretending to be type-*L* consumers and vice versa are given respectively by

$$\pi_{t,H}^c[\frac{q_{t,H}^c}{(1+\tau)}+\beta^c p_H(w_{t+2}-q_{t,H}^c R_{t,H}^c)]+(1-\pi_{t,H}^c)\beta^c p_H w_{t+2}$$

$$\ge \pi_{t,L}^c[\frac{q_{t,L}^c}{1+\tau}+\beta^c p_H(w_{t+2}-q_{t,L}^c R_{t,L}^c)]+(1-\pi_{t,L}^c)\beta^c p_H w_{t+2} \tag{14}$$

for a type-*H* consumer and

$$\pi_{t,L}^c[\frac{q_{t,L}^c}{(1+\tau)}+\beta^c p_L(w_{t+2}-q_{t,L}^c R_{t,L}^c)]+(1-\pi_{t,L}^c)\beta^c p_L w_{t+2}$$

$$\ge \pi_{t,H}^c[\frac{q_{t,H}^c}{(1+\tau)}+\beta^c p_L(w_{t+2}-q_{t,H}^c R_{t,H}^c)]+(1-\pi_{t,H}^c)\beta^c p_L w_{t+2} \tag{15}$$

for a type-*L* one.

To obtain the equilibrium contracts, we first assume that β^c is sufficiently small so that each type of consumer will likely borrow all of his expected old-age income for young-age consumption.[22] This implies that $q_{t,i}^c = w_{t+2}/R_{t,i}^c$.[23] By using this result, the

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$(1+\tau)<(1+\overline{\tau})^*$ implies that the entrepreneurs' capital project is better than the lenders' CRTS technology in terms of producing capital.

[22] Note that each old consumer is endowed with a unit of labor with probability p_i. If he consumes all of his labor income when old, then his expected lifetime utility is $p_i\beta^c w_{t+2}$. Instead, if he borrows all of his

participating constraint for a type-*L* consumer is[24]

$$\pi_{t,L}^{c} w_{t+2}[\frac{1}{(1+\tau)R_{t,L}^{c}} - \beta^{c} p_{L}] \geq 0 .$$

For this condition to hold, both $1/(1+\tau)R_{t,L}^{c} - \beta^{c} p_{L}$ and $\pi_{t,L}^{c}$ must be non-negative. Assuming that $1/(1+\tau)R_{t,L}^{c} - \beta^{c} p_{L}$ is non-negative, one sees that the expected utility of a type-*L* consumer (when he applies for a loan) is increasing in $\pi_{t,L}^{c}$. Similarly, the participating constraint for a type-*H* consumer is

$$\pi_{t,H}^{c} w_{t+2}[\frac{1}{(1+\tau)R_{t,H}^{c}} - \beta^{c} p_{H}] \geq 0 .$$

To satisfy this constraint, both $\pi_{t,H}^{e}$ and $1/(1+\tau)R_{t,H}^{c} - \beta^{c} p_{H}$ are non-negative. Assuming that $1/(1+\tau)R_{t,H}^{c} - \beta^{c} p_{H} \geq 0$, the expected utility of a type-*H* consumer (when he applies for a loan) is increasing in $\pi_{t,H}^{c}$.

Following similar logic, the separating equilibrium is obtained by setting $\pi_{t,H}^{c}=1$ and the value of $\pi_{t,L}^{c}$ as large as possible. The latter implies that the incentive constraint of eq. (14) is binding; hence,

$$\pi_{t,L}^{c} = \frac{[q_{t,H}^{c}/(1+\tau)] - \beta^{c} p_{H} w_{t+2}}{[q_{t,L}^{c}/(1+\tau)] - \beta^{c} p_{H} w_{t+1}} = \frac{p_{H}(1-\mu) - \beta^{c} p_{H}[(1+\tau)Q\varepsilon\rho - \mu]}{p_{L}(1-\mu) - \beta^{c} p_{H}[(1+\tau)Q\varepsilon\rho - \mu]} . \tag{16}$$

Obviously, $\pi_{t,L}^{c} < 1$. An increase in inflation raises the loan rate in an asymmetric way such that the magnitude of the reduction in the quantity of loans to type-*H* consumers is greater than that in relation to type-*L* ones. This increases the incentive of type-*H* consumers to pretend to be type-*L* ones. Thus, $\pi_{t,L}^{c}$ must decrease to prevent this behavior from arising. Mathematically, we have

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expected old-age labor income when he is young and consumes it in the second period of life, then his expected lifetime utility is $w_{t+2}/(1+\tau)R_{t,i}^{c}$. Obviously, if $p_i \beta^{c} < 1/(1+\tau)R_{t,i}^{c}$, then it is likely that each consumer will borrow all of his old-age income for consumption during the second-period.

[23] Note that $w_{1+2}/R_{t,i}^{c}$ is the maximal amount a consumer can borrow. Hence, in a way that is similar to investment loans, consumers will also borrow the maximal amount.

[24] Note that if a type-*L* consumer does not apply for a loan, then his expected lifetime utility is equal to $\beta^{c} p_{L} w_{t+2}$. Therefore, a type-*L* consumer will have the incentive to borrow if

$$\pi_{t,L}^{c}[q_{t,L}^{c}/(1+\tau) + \beta^{c} p_{L}(w_{t+2} - q_{t,L}^{c} R_{t,L}^{c})] + (1-\pi_{t,L}^{c})\beta^{c} p_{L} w_{t+2} \geq \beta^{c} p_{L} w_{t+2} .$$

$$\frac{\partial \pi_{t,L}^c}{\partial(1+\tau)} = \frac{(p_H - p_L)\beta^c p_H Q\varepsilon\rho}{\{p_L(1-\mu) - \beta^c p_H[(1+\tau)Q\varepsilon\rho - \mu]\}^2} < 0 . \tag{17}$$

One can easily verify that $\partial^2 \pi_{t,L}^c / \partial(1+\tau)^2 < 0$. Hence, the negative effect of an increase in the inflation rate on $\pi_{t,L}^c$ is intensified as the inflation rate increases. Note that an increase in the inflation rate also reduces the quantity of consumption loans to each type of consumer, which is equal to the present value of consumers' income in their old age.

Note that $1/(1+\tau)R_{t,L}^c - \beta^c p_L \geq 0$, if

$$(1+\tau) \leq \frac{(1-\mu)+\beta^c \mu}{\beta^c Q\varepsilon\rho} \equiv (1+\overline{\tau})^c ,$$

which is also the condition for $\pi_{t,L}^c \geq 0$ and $1/R_{t,H}^c - \beta^c p_H \geq 0$.

4. Inflation, Capital Formation, and Economic Growth

After we obtain the equilibrium contracts for consumers and entrepreneurs, we can examine the correlation between inflation and economic growth. Once we obtain this correlation, we will compare our result with recent theoretical studies.

4.1. The Non-linear Correlation between Inflation and Economic Growth

Recall that borrowers may not apply for loans if the initial levels of inflation are too high. Under such circumstances, capital is converted by means of lenders' CRTS technology. On the other hand, for relatively low levels of inflation, capital investment is affected by the equilibrium contracts extended to entrepreneurs and consumers. To examine how the joint consideration of investment and consumption loans affects the inflation-growth relationship, we should focus on the inflation rate $1+\tau$ that is less than $(1+\overline{\tau})^c$, $(1+\overline{\tau})_1^e$, $(1+\overline{\tau})_2^e$, and $(1+\tau)^*$. In other words, both entrepreneurs and consumers are willing to borrow.[25]

From the equilibrium contracts, we can see that the total amount used in consumption loans at time *t* is equal to $m[\lambda q_{t,H}^c + (1-\lambda)q_{t,L}^c \pi_{t,L}^c]/(1-\mu)$, while the total amount needed

[25] If the inflation rate is too high, then the loans market is disrupted. In this case, inflation has no effect on bank lending activity. Boyd, Levine, and Smith (2001) find evidence of this.

to finance entrepreneurs at time t is $(1+\tau)[\lambda+(1-\lambda)\pi^e_{t,L}]w_t/(1-\mu)$, which produces an amount of time-t+2 capital equal to $Q[\lambda p_H+(1-\lambda)p_L\pi^e_{t,L}]w_t$. Denoting k_{t+1} as the per firm capital at time t+1, we see that

$$k_{t+1}=\{nw_t-\frac{(1+\tau)}{(1-\mu)}[\lambda+(1-\lambda)\pi^e_{t,L}]w_t-\frac{m}{(1-\mu)}[\frac{\lambda}{R^c_{t,H}}+\frac{(1-\lambda)}{R^c_{t,L}}\pi^c_{t,L}]w_{t+2}\}Q\varepsilon$$

$$+Q[\lambda p_H+(1-\lambda)p_L\pi^e_{t,L}]w_t. \tag{18}$$

The first part of the RHS of eq. (18) is the amount of capital produced by lenders' CRTS technology while the second part is that produced by entrepreneurs' investment projects. By substituting eq. (2) into eq. (18) and after performing some algebraic manipulations, we obtain the rate of economic growth between time t and t+1 (denoted as g) which is given as[26]

$$\frac{k_{t+1}}{k_t}=g=\frac{\{\varepsilon n+\lambda[p_H-\frac{(1+\tau)\varepsilon}{(1-\mu)}]+(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{(1-\mu)}]\pi^e_{t,L}\}}{1+\frac{mQ\varepsilon}{(1-\mu)}[\lambda p_H+(1-\lambda)p_L\pi^c_{t,L}]\frac{A(1-\alpha)L^{-\alpha}}{Q\varepsilon\rho-[\mu/(1+\tau)]}}QA(1-\alpha)L^{-\alpha}. \tag{19}$$

To see the effect of a change in the inflation rate on economic growth in this case, we take logs of both sides of eq. (19) and differentiate them with respect to $1+\tau$, which gives rise to

$$\frac{\partial\ln g}{\partial(1+\tau)}=\frac{\partial}{\partial(1+\tau)}\ln E^e-\frac{\partial}{\partial(1+\tau)}\ln E^c, \tag{20}$$

where

$$E^e=\varepsilon n+\lambda[p_H-\frac{(1+\tau)\varepsilon}{(1-\mu)}]+(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{(1-\mu)}]\pi^e_{t,L} \tag{21}$$

and

$$E^c=1+\frac{mQ\varepsilon}{(1-\mu)}[\lambda p_H+(1-\lambda)p_L\pi^c_{t,L}]\frac{A(1-\alpha)L^{-\alpha}}{Q\varepsilon\rho-[\mu/(1+\tau)]}. \tag{22}$$

It is obvious that $\partial\ln E^e/\partial(1+\tau)$ represents the effects from investment loans while $\partial\ln E^c/\partial(1+\tau)$ denotes the effects from consumption loans. Note that the effects of a change in the inflation rate on the value of E^e contain two parts. The first one is the

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[26] It is assumed that $p_H(1-\mu)>\varepsilon(1+\tau)$. Hence, capital produced by entrepreneurs is positive.

quantity effect, such that an increase in $1+\tau$ induces entrepreneurs to borrow more resources (in order to obtain w_t at time *t*) and this thereby reduces the amount of resources available for capital produced by lenders' CRTS technology. This is captured by the terms $-(1+\tau)\varepsilon/(1-\mu)$ in the numerator of eq. (21) (with the presence of a reserve requirement). The second one is related to the quality of investment loans such that an increase in $1+\tau$ reduces the probability of more efficient capital borrowers (i.e. type-*L* entrepreneurs) getting a loan ($\pi_{t,L}^e$). As both are detrimental to economic growth, the value of $\partial \ln E^e / \partial(1+\tau)$ is negative.

A change in $1+\tau$ similarly gives rise to quantity and quality effects on the value of E^c. An increase in $1+\tau$ increases the loan rate, which reduces the quantity of each consumption loan. This quantity effect, however, increases the resources available for producing capital using the CRTS technology. The quality effect is observed by the fact that an increase in $1+\tau$ reduces $\pi_{t,L}^c$, the probability of more efficient consumers (type-*L* consumers) getting a loan. Note that the quantity effect is represented by the term $A(1-\alpha)L^{-\alpha}/[Q\varepsilon\rho-\mu/(1+\tau)]$ in the denominator of eq. (22), which for future reference is denoted as *TC*. An increase in $1+\tau$ reduces both the values of *TC* and $\pi_{t,L}^c$; hence, the value of $\partial \ln E^c / \partial(1+\tau)$ is also negative.[27] As a consequence, the sign of $\partial \ln g / \partial(1+\tau)$ is determined by the relative magnitudes of $\partial \ln E^e / \partial(1+\tau)$ and $\partial \ln E^c / \partial(1+\tau)$ in the absolute values. Specifically, an increase in the inflation rate will lead to a decrease (an increase) in the growth rate if the absolute value of $\partial \ln E^e / \partial(1+\tau)$ is greater (less) than that of $\partial \ln E^c / \partial(1+\tau)$.

Note that the inflation rate plays a key role in determining the relative magnitudes of $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$. Nevertheless, to determine which is bigger is quite complicated. We then try to determine the relative magnitudes of $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$ by depicting their loci as functions of $1+\tau$ (i.e. whether or not $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$ are increasing or decreasing in $1+\tau$). To do so, note first that $\left|\partial \ln E^e / \partial(1+\tau)\right| = -\partial \ln E^e / \partial(1+\tau)$ and $\left|\partial \ln E^c / \partial(1+\tau)\right| = -\partial \ln E^c / \partial(1+\tau)$. Moreover, $-\partial \ln E^e / \partial(1+\tau) =$ $-[\partial E^e / \partial(1+\tau)]/E^e$ and $-\partial \ln E^c / \partial(1+\tau) =$

[27] The effect arising from consumption on economic growth is represented by $-\partial \ln E^c / \partial(1+\tau)$ and is therefore positive.

$-[\partial E^c / \partial(1+\tau)] / E^c$.

Note that

$$\frac{\partial}{\partial(1+\tau)}[\frac{\partial E^e}{\partial(1+\tau)}] = (1-\lambda)[p_L - \frac{\varepsilon(1+\tau)}{1-\mu}]\frac{\partial^2 \pi^e_{t,L}}{\partial(1+\tau)^2} - 2\frac{(1-\lambda)\varepsilon}{1-\mu}\frac{\partial \pi^e_{t,L}}{\partial(1+\tau)}$$

$$= -\frac{2(1-\lambda)\varepsilon}{(1-\mu)}\frac{\partial \pi^e_{t,L}}{\partial(1+\tau)}\{1 - \frac{[p_L(1-\mu)-(1+\tau)\varepsilon]Q\rho}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu}\} > 0,$$

where the last equality is obtained by using eq. (11'). Note also that

$$\frac{\partial}{\partial(1+\tau)}[-\frac{\partial \ln E^e}{\partial(1+\tau)}] = \frac{\partial}{\partial(1+\tau)}[-\frac{\partial E^e / \partial(1+\tau)}{E^e}] = -\frac{[\partial^2 E^e / \partial(1+\tau)^2]E^e - [\partial E^e / \partial(1+\tau)]^2}{(E^e)^2},$$

implying that the sign of $\partial[-\partial \ln E^e / \partial(1+\tau)] / \partial(1+\tau)$ is ambiguous. However, we have verified that if the value of ε is not too large, then the sign of $\partial[-\partial \ln E^e / \partial(1+\tau)] / \partial(1+\tau)$ is positive and independent of $1+\tau$.[28]

To determine the effect of an increase in $1+\tau$ on $-\partial \ln E^c / \partial(1+\tau)$ is more difficult. Note that

$$\frac{\partial}{\partial(1+\tau)}[-\frac{\partial \ln E^c}{\partial(1+\tau)}] = \frac{\partial}{\partial(1+\tau)}[-\frac{\partial E^c / \partial(1+\tau)}{E^c}] = -\frac{[\partial^2 E^c / \partial(1+\tau)^2]E^c - [\partial E^c / \partial(1+\tau)]^2}{(E^c)^2}, \quad (23)$$

where

$$\frac{\partial E^c}{\partial(1+\tau)} = \frac{(1+\tau)mQ\varepsilon A(1-\alpha)L^{-\alpha}}{(1+\tau)Q\varepsilon\rho-\mu}\frac{(1-\lambda)}{1-\mu}p_L\frac{\partial \pi^c_{t,L}}{\partial(1+\tau)}$$

$$-\frac{[\lambda p_H + (1-\lambda)p_L \pi^c_{t,L}]\mu mQ\varepsilon A(1-\alpha)L^{-\alpha}}{(1-\mu)[(1+\tau)Q\varepsilon\rho-\mu]^2}$$

$$= TC(1-\lambda)\frac{p_L mQ\varepsilon}{(1-\mu)}\frac{\partial \pi^c_{t,L}}{\partial(1+\tau)} + [\lambda p_H + (1-\lambda)p_L \pi^c_{t,L}]\frac{mQ\varepsilon}{1-\mu}\frac{\partial TC}{\partial(1+\tau)}, \quad (24)$$

and

$$\frac{\partial^2 E^c}{\partial\theta^2} = TC(1-\lambda)\frac{p_L mQ\varepsilon}{1-\mu}\frac{\partial^2 \pi^c_{t,L}}{\partial(1+\tau)^2} + [\lambda p_H + (1-\lambda)p_L \pi^c_{t,L}]\frac{mQ\varepsilon}{1-\mu}\frac{\partial^2 TC}{\partial(1+\tau)^2}$$

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[28] This is a sufficient condition. See the Appendix for the derivation.

$$+2\frac{\partial TC}{\partial(1+\tau)}(1-\lambda)\frac{p_L mQ\varepsilon}{1-\mu}\frac{\partial \pi^c_{t,L}}{\partial(1+\tau)}, \tag{25}$$

where

$$\frac{\partial TC}{\partial(1+\tau)}=-\frac{\mu A(1-\alpha)L^{-\alpha}}{[(1+\tau)Q\varepsilon\rho-\mu]^2}<0$$

and

$$\frac{\partial^2 TC}{\partial(1+\tau)^2}=\frac{2\mu Q\varepsilon\rho A(1-\alpha)L^{-\alpha}}{[(1+\tau)Q\varepsilon\rho-\mu]^3}>0.$$

As is shown in eq. (23). the sign of $\partial\left|\partial \ln E^c/\partial(1+\tau)\right|/\partial(1+\tau)$ crucially depends on the sign of $\partial^2 E^c/\partial(1+\tau)^2$. If $\partial^2 E^c/\partial(1+\tau)^2<0$, then $\left|\partial \ln E^c/\partial(1+\tau)\right|>0$. If $\partial^2 E^c/\partial(1+\tau)^2>0$, then the sign of $\left|\partial \ln E^c/\partial(1+\tau)\right|$ is ambiguous. Nevertheless, a comparison between eqs. (24) and (25) reveals that $\partial^2 E^c/\partial(1+\tau)^2>[\partial E^c/\partial(1+\tau)]^2$, because $[\partial TC/\partial(1+\tau)][\partial \pi^c_{t,L}/\partial(1+\tau)]>0$. Moreover, eq. (22) indicates that $E^c>1$. These results imply that $[\partial^2 E^c/\partial(1+\tau)^2]E^c-[\partial E^c/\partial(1+\tau)]^2>0$ and thereby the sign of $\left|\partial \ln E^c/\partial(1+\tau)\right|$ is negative if $\partial^2 E^c/\partial(1+\tau)^2>0$.

From eq. (25), the sign of $\partial^2 E^c/\partial(1+\tau)^2$ depends on three terms. An increase in $1+\tau$ reduces the value of $\pi^c_{t,L}$ and the marginal impact of such a reduction is reinforced as the inflation rate increases (i.e. $\partial^2 \pi^c_{t,L}/\partial(1+\tau)^2<0$). This first effect implies that the sign of $\partial^2 E^c/\partial(1+\tau)^2$ is negative and thereby the sign of $\partial\left|\partial \ln E^c/\partial(1+\tau)\right|/\partial(1+\tau)$ is positive. On the other hand, an increase in $1+\tau$ reduces *TC*, but the marginal impact of this reduction diminishes as the inflation rate increases (i.e. $\partial^2 TC/\partial(1+\tau)^2>0$). This second effect implies that the sign of $\partial^2 E^c/\partial(1+\tau)^2$ is positive and thereby the sign of $\partial\left|\partial \ln E^c/\partial(1+\tau)\right|/\partial(1+\tau)$ is negative. Moreover, the cross (third) effect between the first and second effects also implies that the sign of $\partial^2 E^c/\partial(1+\tau)^2$ is positive (i.e. the third term on the RHS of eq. (25)), so that the sign of $\partial\left|\partial \ln E^c/\partial(1+\tau)\right|/\partial(1+\tau)$ is negative.[29] We find that the second and third effects dominate the first effect for low levels of the inflation rate; hence, the sign of $\partial^2 E^c/\partial(1+\tau)^2$ is positive and the sign of $\partial\left|\partial \ln E^c/\partial(1+\tau)\right|/\partial(1+\tau)$ is negative for low levels of inflation. On the other hand, for

[29] The cross effect is represented by $2[\partial TC/\partial(1+\tau)](1-\lambda)p_L mQ\varepsilon[\partial \pi^c_{t,L}/\partial(1+\tau)]/(1-\mu)$ in eq. (25).

high levels of inflation the first effect dominates the second and third effects and thereby the sign of $\partial^2 E^c / \partial(1+\tau)^2$ is negative and the sign of $\partial \left| \partial \ln E^c / \partial(1+\tau) \right| / \partial(1+\tau)$ is positive.

To better illustrate our above analyses, we resort to numerical simulations. To do so, first note that capital's income share (α) is roughly 0.33. The reserve ratio varies quite a lot across countries, ranging from 0.05% to 25%.[30] There is no empirical evidence for choosing other parameters, however. Since our purpose is to illustrate the existence of two inflation thresholds, we intend to choose other parameters that can produce a case consistent with recent empirical evidence. Specifically, we choose other parameters to reproduce the empirical evidence by Ghosh and Phillips (1998) such that the first threshold level of inflation is located in between 10% and 20% while the second one is located in between 40% and 50%. Moreover, the chosen parameters should yield reasonable rates of economic growth. We then consider the following example:[31]

Consider an economy with $\lambda = 0.45$, n=4, m=2.5, $p_L = 0.7$, $\mu = 0.23$, $p_H = 0.53$, $\varepsilon = 0.21$, $\alpha = 0.33$, Q=1.5, A=2, $\beta^c = 0.67$, and $\beta_L^e = 1$. In this economy, $(1+\overline{\tau})_1^e = 2.29$, $(1+\overline{\tau})_2^e = 1.74$, $(1+\overline{\tau})^* = 1.94$, and $(1+\overline{\tau})^c = 2.10$; hence, we consider the inflation rate τ with $0 < \tau < 0.74$.[32] The loci of $\left| \partial \ln E^e / \partial(1+\tau) \right|$ and $\left| \partial \ln E^c / \partial(1+\tau) \right|$ are depicted in Figure 1. As is shown, $\left| \partial \ln E^e / \partial(1+\tau) \right|$ is increasing in τ while $\left| \partial \ln E^c / \partial(1+\tau) \right|$ is decreasing (increasing) in τ for $\tau \leq (>)\ 0.43$.[33]

[Insert Figure 1 about here]

Recall that whether or not an increase in $1+\tau$ leads to an increase or a decrease in economic growth depends on the relative magnitudes of $\left| \partial \ln E^e / \partial(1+\tau) \right|$ and $\left| \partial \ln E^c / \partial(1+\tau) \right|$. As is shown, $\left| \partial \ln E^c / \partial(1+\tau) \right| > \left| \partial \ln E^e / \partial(1+\tau) \right|$ for low levels of inflation, such that an increase in the inflation rate leads to an increase in the rate of economic growth. If τ is greater than 11.5%, then the effects from investment loans dominate those from consumption loans (i.e. $\left| \partial \ln E^e / \partial(1+\tau) \right| > \left| \partial \ln E^c / \partial(1+\tau) \right|$), leading to a negative

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30 These figures are reserve coefficients reported by Giorgio (1999).

31 Note that changing the parameter values within a reasonable range of the chosen values does not alter the shapes of $\left| \partial \ln E^e / \partial(1+\tau) \right|$ and $\left| \partial \ln E^c / \partial(1+\tau) \right|$.

32 As our purpose is to examine the inflation thresholds, we do not consider the case of disinflation, which arises for $\tau < 0$.

33 That is, the sign of $\left| \partial \ln E^c / \partial(1+\tau) \right|$ is positive if the inflation rate is greater than 43%.

correlation between inflation and economic growth. Consequently, there is a threshold level of inflation (about 11.5%) under which the correlation between inflation and economic growth changes. Figure 2 depicts the correlation between inflation and economic growth.

[Insert Figure 2 about here]

The marginal impact of an increase in $1+\tau$ on economic growth is determined by the difference between $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$. As shown in Figure 1, after the first threshold level (i.e. $\tau > 0.115$) the difference between $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$ becomes large as the inflation rate increases. This implies that the marginal impact of an increase in $1+\tau$ on economic growth becomes more significant along with the increase in $1+\tau$. Moreover, after $\tau > 0.43$, the difference between $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$ is decreasing in terms of the inflation rate, implying that the marginal impact of an increase in $1+\tau$ on economic growth decreases substantially along with an increase in $1+\tau$ after $\tau > 0.43$. In other words, there exists a second threshold level. As a result, this numerical example accords with the empirical evidence obtained by Ghosh and Phillips (1998).

It is also possible that the value of $\left|\partial \ln E^e / \partial(1+\tau)\right|$ is greater than that of $\left|\partial \ln E^c / \partial(1+\tau)\right|$ starting from $\tau = 0$. In this case, we observe a negative correlation between inflation and economic growth for all ranges of inflation. However, one can still find two inflation thresholds in this case. Specifically, the difference between $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$ is small for low levels of inflation rates and hence the coefficient (negative value) of inflation is relatively low and may be insignificant. Once a threshold level is reached, this difference becomes large and increases along with an increase in the inflation rate, yielding the first inflation threshold. The second inflation threshold is obtained when the value of $\left|\partial \ln E^c / \partial(1+\tau)\right|$ is increasing in $1+\tau$, implying that the marginal impact of inflation on economic growth decreases substantially. This case is consistent with Barro (1997).

4.2 Discussion

We have shown that adding non-productive consumption into a standard model of asymmetric information can yield two inflation thresholds in the inflation-growth relationship that is consistent with recent empirical studies. It is worth noting that recent theoretical studies by Azariadis and Smith (1996), Huybens and Smith (1999), and Bose (2002) successfully capture some aspects (but not all) of this pattern of non-linearity in the inflation-growth relationship. In a neoclassical growth model, Azariadis and Smith (1996) consider an overlapping generations model incorporated with private information. It is shown that credit rationing arises in investment loans for high levels of inflation rates, and an increase in the inflation rate in this case exacerbates informational problems.[34] As a result, inflation tends to increase the total amount of credit rationing on investment loans for high levels of inflation and is thereby detrimental to capital investment.[35] For low levels of inflation, however, credit is not rationed and in this case an increase in the inflation rate increases the total amount of investment loans, which facilitates capital investment. Consequently, their model is able to explain the existence of the first threshold level of inflation in the relationship between inflation and the steady-state capital stock. Nevertheless, the possibility that inflation has a negative or insignificant effect on economic growth below the first threshold level does not appear in their model. Similarly, the second threshold level does not exist.[36]

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[34] Azariadis and Smith (1996) consider the possibility that the *lenders* may apply for loans if the rate of return on capital investment is too low. In such a case, the lenders must employ storage technology (with a constant rate of return) in order to save. In a model where money is a close substitute for capital loans, Azariadis and Smith (1996) show that an increase in the inflation rate, which decreases the deposit rate, will raise the lender's incentive to pretend to be a borrower. To prevent this, an incentive constraint arises such that the size of the investment loans must decrease. It is shown that this incentive constraint is not binding for low levels of inflation so that an increase in the inflation rate simply increases the size of the investment loans and thereby facilitates capital investment. However, for high levels of inflation, this incentive constraint is binding and therefore an increase in the inflation rate reduces the size of each investment loan, which is detrimental to capital investment.

[35] Espinosa-Vega and Yip (1999) also develop a theoretical model whereby the inflation-growth correlation depends on the agents' degree of risk aversion. Since the degree of risk aversion is not correlated with the inflation rate in the model, their model may not capture the empirical evidence as does Bullard and Keating (1995).

[36] In fact, Azariadis and Smith (1996) do find a second threshold level of inflation such that if inflation is greater than this threshold level, the dynamics of the economy becomes indeterminate. As the two inflation thresholds are obtained by recent empirical studies that estimate the long-run relationship between inflation and

Boyd and Smith (1998) develop a neoclassical growth model in an OG model with the presence of asymmetric information. They find two steady states in a monetary economy: one with a low capital stock and output while the other has a high capital stock and output. An increase in the money growth rate increases the steady state capital stock under the low-capital-stock steady state. However, such an increase in the high-capital-stock steady state reduces the steady state capital stock. Consequently, their model implies that the relationship between the money growth rate and the steady state capital stock depends on the initial capital stock. This is obviously not consistent with recent empirical work, which reports that the correlation between inflation and economic growth depends on the initial inflation rate.[37]

Huybens and Smith (1999) examine a model with a costly-state-verification problem. They show that an increase in the inflation rate always leads to a reduction in real activity and, in particular, this negative correlation appears more pronounced at higher rates of inflation. Similarly, Bose (2002) extends the framework of Bose and Cothren (1996) by showing that an increase in the inflation rate increases the amount of credit rationing on investment loans and is thereby detrimental to economic growth. Moreover, he finds an inflation threshold such that the borrowing regime switches from rationing to screening. Given that inflation exacerbates the problem of asymmetric information to a greater extent in the screening regime than in the rationing one, his work is able to capture the empirical fact that the negative effect of inflation on growth is less (more) pronounced in low (high) levels of inflation. While both studies are able to capture the fact that the marginal impact of an increase in the inflation rate on economic growth increases along with an increase in the inflation rate, their models are not able to yield the second threshold level under which the marginal effect decreases substantially. Moreover, the possibility that an increase in the inflation rate may lead to an increase in economic growth for low levels of inflation rates also

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growth, the second threshold level found by Azariadis and Smith (1996) may not be related to recent empirical evidence.

[37] Boyd and Smith (1998) also find that if the inflation rate is too high, the dynamics of the economy could display limiting cycles, implying that there is no equilibrium path approaching the high-capital-stock steady state.

disappears.

5. Conclusion

This paper extends the work of Bencivenga and Smith (1993) and Bose (2002) by adding non-productive consumption loans into a standard model of informational imperfection in order to examine the threshold effects in the inflation-growth relationship. Without considering consumption loans, an increase in the inflation rate always leads to a reduction in the rate of economic growth, as obtained by Huybens and Smith (1999) and Bose (2002). However, the inclusion of consumption loans gives rise to an opposite effect of inflation on economic growth.

We find that the effect arising from consumption loans may dominate (be dominated by) that arising from investment loans for inflation rates below the first threshold level of inflation. For inflation rates above the first threshold level of inflation, we find that the difference between these two effects is increasing in the inflation rate for inflation rates below a second threshold level of inflation and decreasing in the inflation rate for inflation rates above this second threshold, ensuring the existence of the second threshold level in the inflation-growth relationship. These observations accord well with recent empirical evidence on the inflation-growth relationship.

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Figures

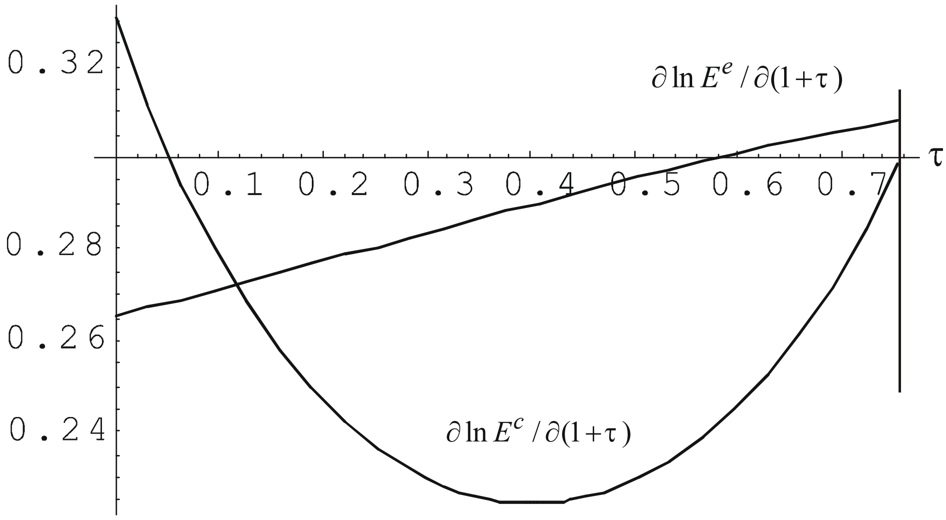

Figure 1. The loci of $\left|\partial \ln E^e / \partial(1+\tau)\right|$ and $\left|\partial \ln E^c / \partial(1+\tau)\right|$

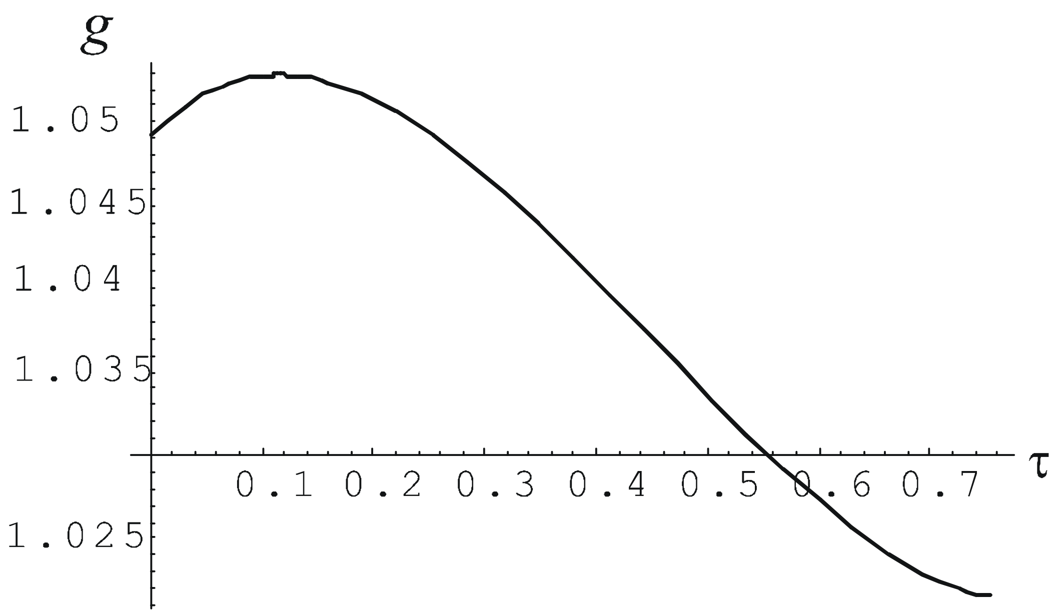

Figure 2. Inflation and Economic Growth

Appendix (Not Intended for Publication)

This appendix shows that the value of $\left|\partial \ln E^e / \partial(1+\tau)\right|$ is increasing in $1+\tau$ if the value of ε is not too large.

Note that

$$\frac{\partial}{\partial(1+\tau)}[-\frac{\partial \ln E^e}{\partial(1+\tau)}]=\frac{\partial}{\partial(1+\tau)}[-\frac{\partial E^e / \partial(1+\tau)}{E^e}]=-\frac{(\partial^2 E^e / \partial\theta^2)E^e-(\partial E^e / \partial\theta)^2}{(E^e)^2},$$

where

$$\frac{\partial E^e}{\partial(1+\tau)}=-\lambda\frac{\varepsilon}{1-\mu}-(1-\lambda)\frac{\varepsilon}{1-\mu}\pi^e_{t,L}+(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}<0$$

and

$$\frac{\partial^2 E^e}{\partial(1+\tau)^2}=-2(1-\lambda)\frac{\varepsilon}{1-\mu}\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}+(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]\frac{\partial^2\pi^e_{t,L}}{\partial(1+\tau)^2}$$

$$=\frac{2(1-\lambda)\varepsilon}{(1-\mu)}\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}\{\frac{[p_L(1-\mu)-(1+\tau)\varepsilon]Q\rho}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu}-1\}>0\,.$$

It is then clear that

$$(\partial^2 E^e / \partial\theta^2)E^e-(\partial E^e / \partial\theta)^2=\frac{2(1-\lambda)\varepsilon}{(1-\mu)}\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}\frac{\mu}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu}E^e$$

$$-\{(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}\}^2-\{\frac{\varepsilon}{1-\mu}[\lambda+(1-\lambda)\pi^e_{t,L}]\}^2$$

$$+2\frac{(1-\lambda)\varepsilon}{1-\mu}[p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}[\lambda+(1-\lambda)\pi^e_{t,L}]$$

$$=\frac{2(1-\lambda)\varepsilon}{(1-\mu)}\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}\{[\lambda+(1-\lambda)\pi^e_{t,L}][p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]-\frac{\mu E^e}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu}\}$$

$$-\{(1-\lambda)[p_L-\frac{(1+\tau)\varepsilon}{1-\mu}]\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}\}^2-\{\frac{\varepsilon}{1-\mu}[\lambda+(1-\lambda)\pi^e_{t,L}]\}^2\,. \qquad \text{(A1)}$$

The first term in eq. (A1) may be positive or negative while the last two terms are negative.

Define

$$S \equiv [\lambda + (1-\lambda)\pi_{t,L}^{e}][p_L - \frac{(1+\tau)\varepsilon}{1-\mu}]$$

and

$$D \equiv \frac{\mu E^e}{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu}.$$

For given parameters, the value of *S* is decreasing in ε while the value of *D* is increasing in ε; hence, if the value of ε is small enough, then $S - D > 0$ and hence the first term in eq. (A1) is negative, implying that $(\partial^2 E^e / \partial\theta^2)E^e - (\partial E^e / \partial\theta)^2 < 0$ is independent of $(1+\tau)$. This further implies that[38]

$$\frac{\partial}{\partial(1+\tau)}[-\frac{\partial \ln E^e}{\partial(1+\tau)}] = \frac{\partial}{\partial(1+\tau)}[-\frac{\partial E^e / \partial(1+\tau)}{E^e}] = -\frac{(\partial^2 E^e / \partial\theta^2)E^e - (\partial E^e / \partial\theta)^2}{(E^e)^2} > 0.$$

Note that if the value of ε is relatively large, then the first term in (A1) may be positive for high levels of the inflation rates $1+\tau$. This is so because the value of *S* is decreasing in $(1+\tau)$ while the value of *D* is increasing in $1+\tau$. In other words, in the case where ε is relatively large, the sign of $\partial[-\partial \ln E^e / \partial(1+\tau)] / \partial(1+\tau)$ is positive (negative) for low (high) levels of inflation. As can be inferred from Figures 1 and 2 in the paper, this case strengthens our conclusion regarding the existence of the second threshold level.

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[38] We have performed many numerical simulations to verify whether or not $\left|\partial \ln E^e / \partial(1+\tau)\right|$ is increasing in $1+\tau$. It is verified that $\left|\partial \ln E^e / \partial(1+\tau)\right|$ is increasing in $1+\tau$ if the value of ε is small.